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Advanced EMC Assessment of Composites Material: Monte Carlo Statistical Description with Spherical Inclusions and Improvement with SROM

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Abstract—This article proposes an advanced methodology to deal with the complexity of composite materials modeling up to 60 GHz. For radiofrequency (RF) requirements, it has been demonstrated that the distribution of conductive inclusions plays a major role. Since their locations are intrinsically subject to uncertain assumptions, the Monte Carlo (MC) technique is considered as a golden standard. Unfortunately, the computational costs involved by coupling full-wave electromagnetic (EM) simulations and MC remains prohibitive. The aim of this proposal is to demonstrate the interest of stochastic reduced order method (SROM) to tackle computational constraints, jointly with the statistical precision needed for a realistic description of RF composites.

1. INTRODUCTION

Nowadays, various industrial areas (automotive, aerospace, communication) are expressing an increasing interest for composite materials. In this framework, a constant demand exists for reliable and efficient numerical tools to characterize such media over large frequency bandwidth (from direct current to tens of GHz), including but not restricted to applications such as electromagnetic interference/compatibility (EMI/EMC), 5G networks, more electrical aircrafts/automobiles. The state-of-the-art regarding full-wave electromagnetic (EM) modeling shows that various methods were successfully used to simulate composite fiber-reinforced media including models in frequency domain (finite element method [1], integral equation [2]), and time domain (finite difference [3], discontinuous Galerkin [4], finite integral technique (FIT) [5]). However, due to the intrinsic variability of composite materials (e.g., conductivity, locations or shapes of inclusions, ...), full-wave modeling is often needed to assess EM shielding properties (shielding effectiveness for instance) as accurately as possible. Obviously, the computing resources available have to be taken carefully into account in this process (involving the achievement of a realistic number of simulations, here 10 maximum considering computing costs in Table 1). Mostly, the variability of inputs is taken into account through MC procedures [6]. Nevertheless, the computing costs required by full-wave simulations through MC is often a crucial constraint, leading to a limited number of simulations [1, 2]. In this framework, the next sections 2 and 3 will put the focus on the interest of alternative technique, through stochastic reduced order method (SROM).

2. PROBLEM STATEMENT

The aim of this work is to provide accurate and efficient homogenisation procedures to characterise RF properties of composite materials. This relies on results previously obtained regarding Maxwell-Garnett

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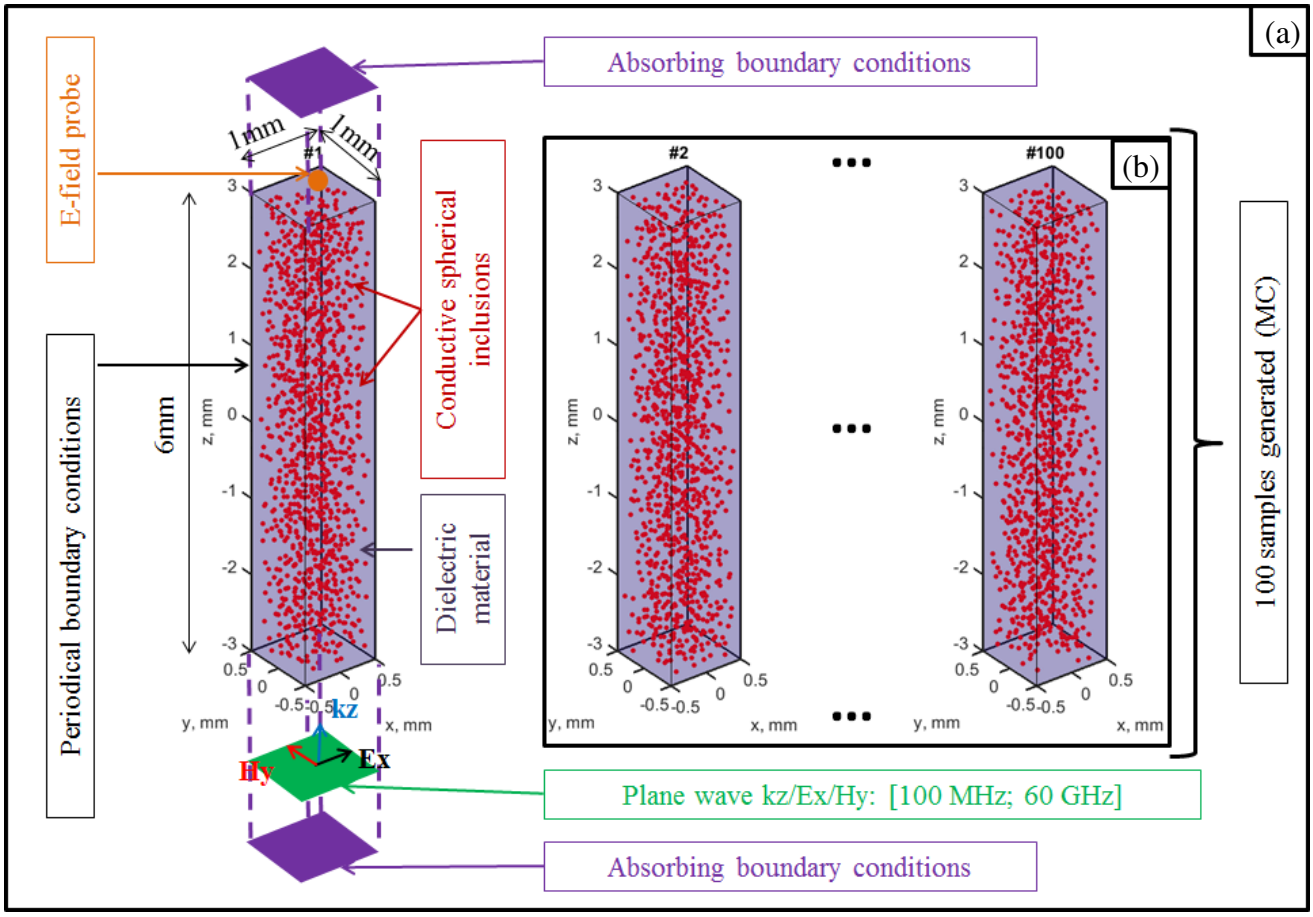

Figure 1. Description of problem statement: EM propagation through composite (two-phase) material with conductive spherical inclusions. (a) Numerical setup (CST simulations). (b) Randomizing the location of inclusions (100 different samples).

(MG) effective medium principles [7], dynamic homogeneisation method (DHM) [1], and use of full-wave 3D time domain simulations to accurately match DHM equivalent material [5]. Thus, Fig. 1(a) and Table 1 briefly summarize the numerical characteristics of FIT simulations provided with CST.

As an extension to [5], one hundred full-wave 3D simulations were achieved in order to provide a reference set of results for the EM assessment of the shielding properties of RF composite. Indeed, in order to properly initiate the two-phase (complex permittivity ϵ_1 and ϵ_2, Table 1) composite model with

Table 1. Numerical characteristics of CST simulations, referring to Fig. 1.

EM source	Plane wave excitation $[0.1; 60]$ GHz
Output	Outer (composite) E-field probe, see Fig. 1(a)
Matrix characteristics	$\epsilon_1 = 5\epsilon_0$, $\sigma_1 = 0$ S/m, 6 mm^3-parallelepiped
Conductive inclusions	$\epsilon_2 = \epsilon_0$, $\sigma_2 = 1,000$ S/m, spheres 0.1 mm-diameter
Composite volumetric rate	$\nu = 10\%$/1,146 inclusions in total
EM solver	Transient, total duration=400 ps/meshing $\approx \lambda/40$
Computing hardware	PC, Quad-core Intel Xeon processor, RAM 12 GB
Computing time costs	≈ 6 hours per composite sample, see Fig. 1(b)

mixing DHM rule, it is necessary to define ϵ_∞ standing for the complex permittivity of the so-called infinite medium [5] as follows:

$$\epsilon_\infty = \epsilon_1 + \epsilon_2 \left(d/\lambda\right)^\gamma , \tag{1}$$

where d represents the equivalent size of inclusions, λ the assumed wavelength, and γ the equivalent coefficient optimized from mean value given by full-wave simulations [1]. This allows replacing expensive 3D simulations (see Table 1) with realistic homogenised equivalent model.

As depicted in Fig. 1 and Table 1, the whole procedure requires important computational resources for 100 time domain simulations. The reference statistical result (averaged EM attenuation due to the presence of composite material) will be considered in the following from the latter dataset.

3. THEORETICAL PRINCIPLES AND METHODOLOGY

It is noticed that the proposed work covers the whole methodology depicted in Fig. 2.

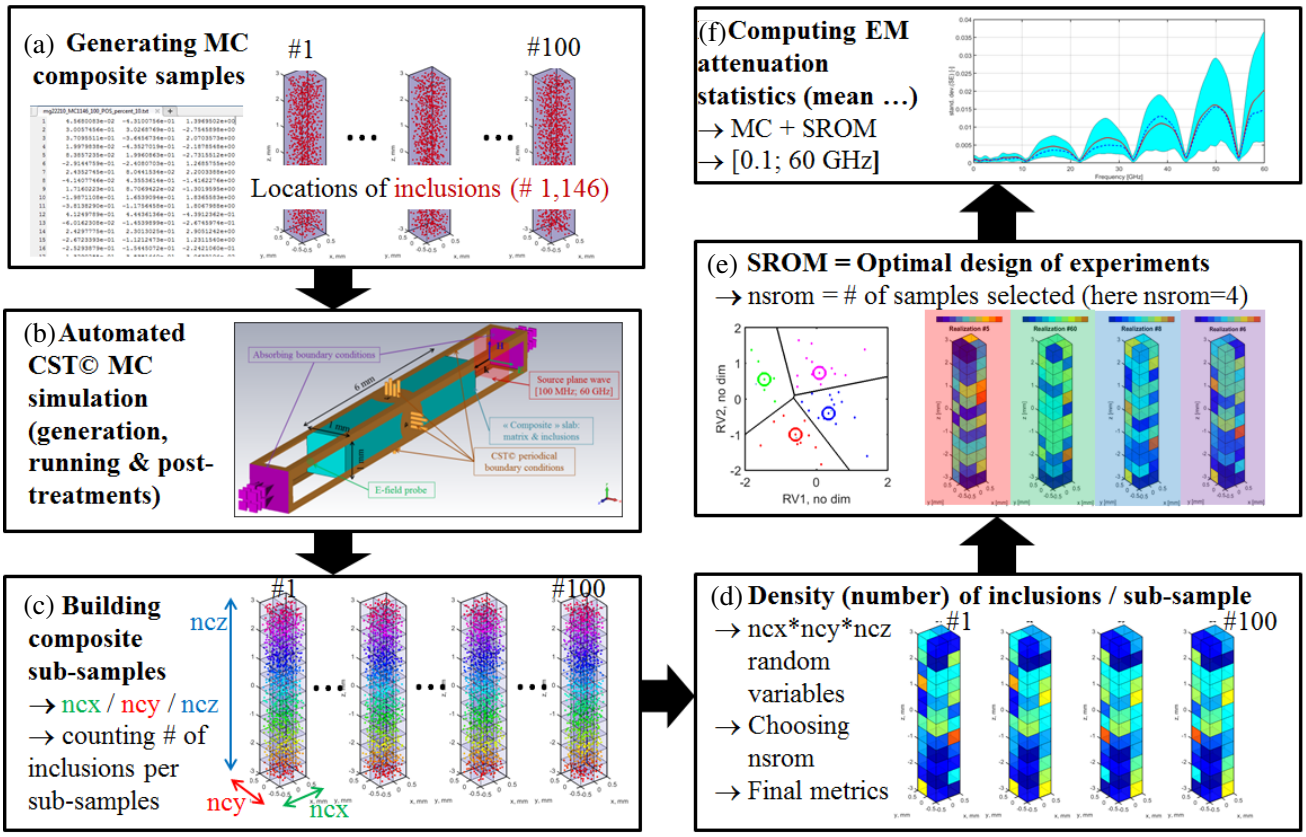

Figure 2. Flow chart summarizing modeling steps [(a)–(f)]. [(a); (c)–(f)]: using Matlab. [b]: from automated CST simulations.

The process described in Fig. 2 is monitored (with Matlab) through:

- Steps A–B: the respective levels A and B are dedicated to the generation of composite samples and CST monitoring through COM-interface communication with Matlab. The details about the automated pre-/post-treatments and launching of deterministic time domain simulations are given in [5].
- Steps C–D: considering the huge number of random variables (RV) needed ($3 \times 1,146 = 3,438$ considering each Cartesian component of the inclusions), the C-level is devoted to the construction of a restricted number of sub-samples depending on an uniform meshing of the composite sample (here through $ncx \times ncy \times ncz = 2 \times 2 \times 12 = 48$ 0.5 mm-cubes). It is independent of the statistical

assumption considering inclusions inside dielectric matrix, and the main goal is to construct metrics well-fitted to achieve levels C-D in Fig. 2. Depending on the physical nature of composite, this sub-sampling might be freely adapted by defining alternative meshing (see for instance the deposit of fibers evoked in [8]). Step D is devoted to the filtering of initial $3,438$ random parameters to a restricted number of RVs (here 48), still allowing a physical interpretation of the random distributions of conductive inclusions. The density (in terms of number of inclusions) of each sub-sample given in step C is finally computed. The impact of the subsampling steps has been studied: the results with increasing number of subcells (i.e., $ncx, ncy, ncz/6 = 3$, $ncx, ncy, ncz/6 = 4$, ...) validate the convergence of the proposed methodology (data not shown here).

- Steps E–F: the E-level stands for the SROM choice of a weighted (optimised) design of experiments (DoE) considering the inital 100 samples generated (see level A). Step F enables computing final statistical results (mean, variance, ...) from the set of simulations given by E-level and SROM (see details in equation 2). The assessment of the averaged EM attenuation is crucial with respect to (wrt) EMT-like procedures, the DHM final step relies on the mean characteristics of EM shielding [5].

The core of the proposed methodology relies on filtering the huge number of RVs (i.e., $3,438$ due to Cartesian locations of inclusions); in this framework, classical stochastic techniques (e.g., stochastic collocation, unscented transform, ...) are facing *curse of dimensionality* [9]. Although alternative methods exist to cope with this problem (see for instance works in [10]), the high number of RVs and the computing costs expected from full-wave simulations required to drastically decrease the number of simulations, still preserving accuracy (steps [C-F] in Fig. 2). The main principles of SROM [11] are derived from pattern classification techniques [12] as follows:

- Level 1: providing an initial set composed of N samples (here 100 full-wave simulations, Fig. 1) of composites as described in step D (Fig. 2).
- Level 2: generating n_{SROM} Voronoi subregions (see Fig. 2(e) for two RVs) from random choice of n_{SROM} samples from N initial set.
- Level 3: computing the Euclidian distance between n_{SROM}-size samples (Level 2) to N initial dataset (Level 1).
- Level 4: optimisation of n_{SROM}-size samples with a figure of merit highlighting the compliance between well-chosen statistics from the original N dataset (e.g., density of inclusions in sub-samples, see Fig. 2(d)). This step provides a weighted set of samples (see equation 2).

Here, an enhanced algorithm has been developed to ensure the convergence of the SROM DoE (latter levels 1–4). The E-step in Fig. 2 is reproduced ten times checking the uniqueness of the SROM weighted set of samples (data not shown). In the following, we consider $ncx \times ncy \times ncz = 2 \times 2 \times 12 = 48$ RVs (Fig. 2(c)). The random inputs are initially generated 100 times (those *reference* results are simulated in time domain with CST), and the random parameters are given through random matrix $\mathbf{X}_i^j$ ($i = 1, \ldots, N$, and $j = 1, \ldots, 48$), standing for the number of inclusions per sub-sample j for full-wave simulation number i. The SROM procedure enables choosing the optimised dataset $(\hat{x}_i, w_i)$ for $i = 1, \ldots, n_{SROM}$, and $\hat{x}_i$ is the sample number from the initial N samples. Finally, the mean value of the chosen output (e.g., EM attenuation S) is defined as follows:

$$\langle S(f) \rangle = \sum_{i=1}^{n_{SROM}} w_i S_{\hat{x}_i}(f) \tag{2}$$

where $\langle S(f) \rangle$ is the averaged EM attenuation needed for DHM extraction (frequency f), and $S_{\hat{x}_i}(f)$ stands for the EM shielding extracted from simulation with composite sample number $\hat{x}_i$. The next section will lay emphasis on the accuracy of results obtained with $n_{SROM} = 10$ and numerical characteristics listed in Table 1.

4. NUMERICAL RESULTS FROM MC AND SROM

Figure 3 shows the precision of SROM (black dashed line) wrt the *reference* MC data (i.e., initial set of 100 simulations, red solid line).

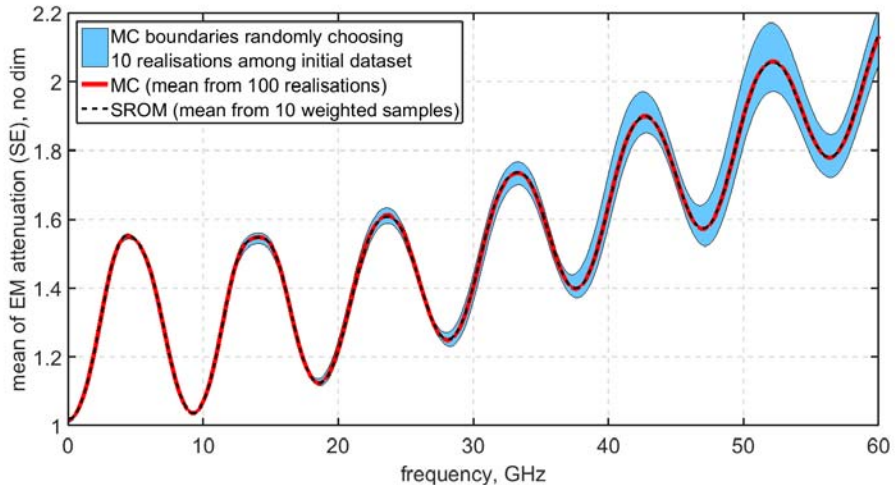

Figure 3. Averaged EM attenuation from: the whole initial simulated dataset (100 samples) or part of it (10 realisations randomly chosen or given by SROM). Reference results are given by MC mean (red solid line) extracted from the initial set of 100 composite samples (see Fig. 1(b)). Maximum/minimum caliber wrt $\langle S \rangle$ computed from 10 randomly chosen simulations (light blue area, process iterated 10,000 times). Mean EM shielding as given by SROM procedure (black dashed line).

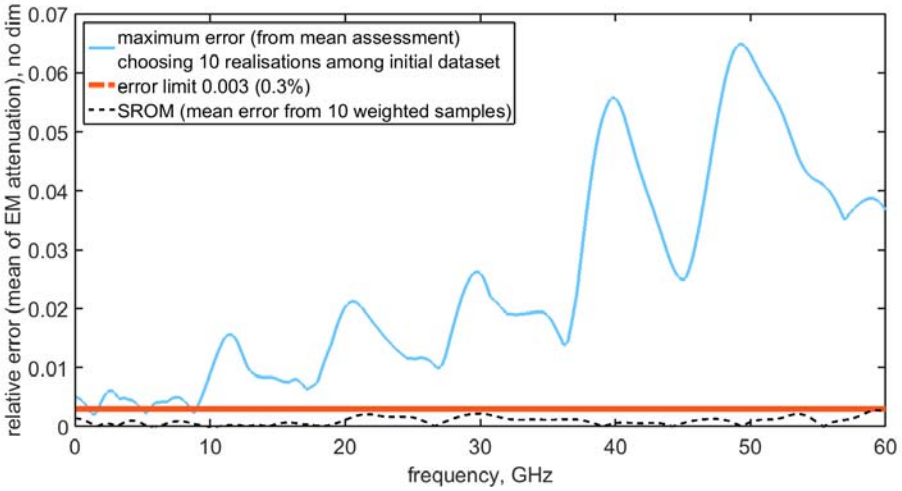

Figure 4. Computing $RE^{\beta}(f)$ from $\langle S \rangle$ due to composite samples: from 10-realisation MC (light blue solid line), from the 10-simulation SROM optimised DoE (black dashed line), and 0.3%-error isocurve (orange line)

In order to emphasis the accordance between 100-simulation MC results and SROM ones, 10 simulations are randomly chosen from the initial dataset (process iterated 10,000 times, light blue area): the results show discrepancy (increasing with frequency) between previous results and MC. Furthermore, an error criterion is proposed to quantify the quality of the 10-realisation MC data and SROM:

$$RE^{\beta}(f) = \left| S^{\beta}(f) - S^{MC}(f) \right| / \left| S^{MC}(f) \right| \tag{3}$$

with $RE^{\beta}(f)$ the relative error of mean EM attenuation at frequency f due to the use of method β (β standing for 10-realisation MC or SROM); reference is still given by full MC simulations (S^{MC}). The results are shown in Fig. 4: SROM averaged EM shielding is below 0.3% error isocurve over the

whole frequency bandwidth (orange line), whereas the error gap from 10-realisation MC increases wrt frequency (up to 6.5%). This demonstrates the interest of weighted set of points from SROM in terms of precision and computing costs (less than 3 days needed since pre-treatment with SROM only requires few seconds).

5. CONCLUDING REMARKS

This article demonstrated the efficiency and precision of SROM in the framework of RF composite modeling: the method offers a 90%-gain regarding the overall full-wave modeling of the random material, without spoiling the precision of results (maximum 0.3% with reference data). The whole methodology allows dealing with high number of RVs (hundreds). Regarding the type of inclusions given in this work (sub-wavelength spheres), the SROM methodology is useful to decrease the number of simulations needed, comparatively to MC. By keeping the level of accuracy offered by MC, SROM is an efficient technique to assess electromagnetic field attenuation, and its statistical average may offer attractive prospects when coupled with DHM procedures. Actual developments are in progress to enhance the DHM extraction, including second order statistical data (variance) for sensitivity analysis purposes with different inclusions (varying shapes, sizes, ...).

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