Phase-Only Nulling with Limited Number of Controllable Side Elements

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Abstract—In this paper, the required array patterns with controlled nulls are obtained by optimizing only the excitation phases of a small number of elements on both sides of the array. A genetic algorithm is used to appropriately find which elements of the array to be optimized and also to find the required number of the excitation phases. The performance of the proposed phase-only method is compared with some other exciting methods, and it is found to be competitive, fulfil all the desired radiation characteristics, and represent a good solution for interference mitigation. Moreover, the proposed phaseonly array is designed and validated under realistic electromagnetic effects using CST full wave modeling. Experimental results are found in a good agreement with the theoretical ones and show realistic array patterns with accurate nulls.

1. INTRODUCTION

Currently there are increasingly demands for new wireless communication systems especially when 5G applications are emerging. These demands impose more constraints on the available spectrum and sharing the frequencies which will not only make electromagnetic spectrum crowded but also be more prone to interference signals. Thus, adequate reception of the target signals in such an environment with the presence of many undesired signals becomes impossible. Null steering in the radiation patterns of the antenna arrays can effectively overcome this problem by placing some selected nulls toward those undesired signals directions. Generally, null steering can be achieved by controlling the amplitude-only weighting [1–4], phase-only weighting [5–12], and complex (both amplitude and phase) weighting [13–18] of array elements excitations. It can also be achieved by controlling the separation distances between array elements [19, 20].

Among all of these controlling methods, phase-only methods have shown advantages, thus used widely in practice. One of the major advantages is its need for only a single power divider network to realize the uniform amplitude distribution which enjoys a very simple implementation in practice. In addition, some researchers for example see [14] suggested that the use of the discrete phase shifters instead of their continuous counterpart may get better results.

Controlling phase-only excitations of all elements in the array does not guarantee a simplified feeding network implementation especially for large phased arrays. The use of the partially controllable elements instead of its fully counterparts may greatly reduce the cost, the complexity of the feeding network, and the required number of the RF components. Also, the strategies of the partially controllable elements generate the required nulls with faster response time which is an important and essential feature for the array pattern formulation in the phased arrays. One way to get faster response is by using fewer numbers of the optimized variables during the optimization process and limiting the searching space.

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Toward these desired goals, in [6], a simple null steering method was presented by controlling only the excitation phases of a part of the elements. The authors employed the required phase excitations in the objective function according to the minimum mean square error sense. Then, the minimization of the array pattern was performed by using the quasi-Newton method. In [15, 16], a new null steering method has been suggested by controlling only the amplitude and phase excitations of two or more side elements. It is found that the amplitude and phase excitations of the elements located near the array edges have the greatest effect on forming the required nulls, on while the central elements have less effect. Moreover, the amplitude and phase excitations of the selected side elements were appropriately optimized by means of the genetic algorithm to obtain wide and deep nulls.

In this paper, the method presented in [16] is further extended and simplified by controlling only the phase excitations of a number of the selected side elements while preserving the following desired characteristics: generating multiple wide and deep nulls in the interference directions; reducing the main beam perturbations; and the desired nulls are performed very quickly. The amplitude excitations of these selected side elements as well as the other remaining elements of the array are chosen to be constant.

In addition, the directivity of the proposed array is further improved compared to the method presented in [16].

2. THE PHASE-ONLY NULLING METHOD

Consider a linear array with even number of elements $2N$ (i.e., N elements on each side of the array) and uniform current excitation (i.e., the amplitudes of all elements excitations are chosen to be unity, $A_n = 1$ for $n = 1, 2, ..., 2N$). The separation distance between the array elements is also considered constant and symmetrically located about the origin. To place a number of wide nulls equal to Q, a subset of only M controllable elements out of N number of array elements on each side of the array antenna is considered. To accurately achieve the required nulls, the number of controllable elements on each side of the array, M, must be larger than the number of required nulls, $M > Q$. On the other hand, to maintain the complexity of the feeding network as simple and cheap as possible, the controllable number of elements on each side of the array should be $M \ll N$. It is also worth to mention that by perturbing only a small number of array elements, the main beam distortion can be kept within the acceptable range. Then, each side of the array will have a number of uncontrollable elements equal to $N - M$ and M controllable elements. The overall far field pattern due to the $2(N - M)$ uniformly excited array elements and the $2M$ controllable elements can be written as [16]:

$$
AF(u) = 2\sum_{n=1}^{(N-M)} \cos\left(\frac{(2n-1)}{2}kdu\right) + \sum_{m=N-M+1}^{N} \left\{ A_{mr} e^{j\left(\frac{2m-1}{2}kdu + P_{mr}\right)} + A_{ml} e^{-j\left(\frac{2m-1}{2}kdu + P_{ml}\right)} \right\} (1)
$$

where $k=2\pi/\lambda$, $u=\sin(\theta)$, θ is the observation angle around the array axis, d the element spacing, and λ the wavelength in free space. The amplitudes and phases on the left and right sides of the controllable elements are represented by A_{ml} , A_{mr} , P_{ml} , and P_{mr} , respectively.

Clearly, asymmetric amplitude and phase excitations of each controllable element on each side of the array result in a total number of degrees of freedom equal to $4M$ which is quite enough to generate the multiple wide nulls as shown in [16]. However, this comes at the cost of increasing the number of attenuators equal to 2M and another set of phase shifters equal to $2M$. Thus, the complexity is on the order of (4M).

This complexity can be reduced by using phase-only weighting instead of both amplitude and phase weighting of the controllable elements. Many other advantages can be observed by using the phase-only weighting as shown below. Applying the phase-only weighting, Equation (1) can be rewritten in the following form

$$
AF(u) = 2\sum_{n=1}^{(N-M)} \cos\left(\frac{(2n-1)}{2}kdu\right) + \sum_{m=N-M+1}^{N} \left\{e^{j\left(\frac{2m-1}{2}kdu + P_{mr}\right)} + e^{-j\left(\frac{2m-1}{2}kdu + P_{ml}\right)}\right\}
$$
(2)

Here, A_{mr} and A_{ml} are both assumed to be 1. Note that the only P_{mr} and P_{ml} for $m = 1, 2, ..., M$ are the controllable parameters that used for generating the required Q nulls as shown in Fig. 1. Equation (2)

Figure 1. Configuration of the proposed phase-only method.

can be further simplified to

$$
AF(u) = 2\sum_{n=1}^{(N-M)} \cos\left(\frac{(2n-1)}{2}kdu\right) + 2\sum_{m=N-M+1}^{N} \cos\left(\frac{(2m-1)}{2}kdu + P_m\right) \tag{3}
$$

Ideally, the array pattern has a response equal to zero at the required nulls directions, $AF(u)|_{u=u_q}=0$ for $q = 1, 2, \ldots, Q$. This can be realized only when the magnitudes of the first and second terms of Eq. (3) are equal and out of phase. Then Eq. (3) can be rewritten in the following form

$$
\sum_{n=1}^{(N-M)} \cos\left[\frac{(2n-1)}{2}kdu\right] = -\sum_{\substack{m=N-M+1\\ \text{array with } 2N \text{ controllabel elements}}}^{N} \cos\left(\frac{2m-1}{2}kdu + P_m\right) \tag{4}
$$

Clearly, the main beam magnitude of the controllable elements array (right hand side of Eq. (4)) is lower than that of the fixed elements array (left hand side of Eq. (4)) by a factor equal to 2M. This factor may play a major role in perturbing the main beam of the proposed array and reduces its gain as will be shown later.

Nevertheless, the sidelobe regions of these two arrays are very close and can be matched in magnitude if the phases of the controllable elements are adjusted properly. To further highlight this important feature which is the key idea of this work, let us examine some results. Fig. 2 shows the matching range between these two sub-arrays according to Eq. (4) for $M = 3$ controllable elements, $N = 1$ elements, and $N - M = 7$ uncontrollable elements on each side of the array. Here, two wide nulls, each with width equal to 0.05, are considered. To fully confirm the effectiveness of the proposed method compared to the method in [16], let us reproduce the results of Fig. 2 with the method of [16] and then compare between them. Fig. 3 shows the results of method [16]. It can be seen that the sidelobe structures of the two sub-arrays for phase-only weighting method according to Eq. (4) have more matching tendency than that of the complex weighting method. This, of course, will help the genetic algorithm to faster find the optimized values of the required phases and reach the required nulls. More importantly, the magnitude of the main beam at $\theta = 0^{\circ}$ of the phase-only method will be reduced by only 1.18 compared with 1.959 for the complex weighting method. Again, the phase-only method shows its superiority in maintaining the lowest main beam perturbations.

Figure 2. Matching between two patterns in the case of phase only weighting method.

Figure 3. Matching between two patterns in the case of complex weighting method.

3. SIMULATION RESULTS

To verify the performance of the proposed phase-only method, various examples under different numbers of controllable and uncontrollable array elements are presented. In all examples, the parameters of the genetic algorithm are chosen as: population size of 20; selection is roulette; crossover is single point; mutation rate is 0.15; mating pool is chosen to be 4. The upper and lower bounds of the phases are chosen between $-\pi/2$ and $\pi/2$, while for amplitudes, in the case of complex weighting method, they are chosen between 0 and 1. Also, the inter-element spacing in all considered arrays is chosen to be half-wavelength, and the amplitude excitations of the arrays elements are set to one.

In the first example, an array with the total number of elements $N = 10$ on each side of the array

Figure 4. Array patterns of three methods to obtain single wide null.

and $M = 3$ are chosen to place a single wide null centered at 20[°], ranged from 18[°] to 23[°], and the number of controllable elements which is represented by the variable M is investigated. Note that each wide null in this work is generated by forming two adjacent narrow nulls, thus, M should be at least 2. Fig. 4 shows the results of the proposed array with phase-only control. The results of the complex weighting (i.e., the method of [16]) under the same number of controllable elements, $M = 3$, is also shown in Fig. 4 for comparison purpose. Furthermore, the result of the fully controllable array elements where all of its 20 phases are chosen to be controllable is shown in Fig. 4. The corresponding elements excitations of these three methods are also shown in Fig. 4 and Table 1. From this figure, it can be seen that all the methods are accurately placing the required nulls with specified width and depth. Of course, there is a clear superiority with the fully controllable method as the number of degrees of freedom is much higher than the required ones. However, the complexities of these three methods (proposed phase-only, complex weighting, and fully phase-only) are: 30%, 60%, 100%, respectively, and the convergence rates of the optimizer to reach the desired nulls are found to be 6.27 sec, 7.07 sec, and 7.31 sec, respectively. Clearly, the proposed phase-only method enjoys the lowest complexity and faster convergence than the other two methods.

Furthermore, to show the nulling capability of the three tested methods, the instantaneous depths of the generated nulls as a function of the iteration number by the proposed phase-only method, complex weighting method that was presented in [16] and the fully phase-only method that was presented in [12] are depicted in Fig. 5. It can be seen that the proposed phase-only method reaches a satisfactory depth, −60 dB, just after 50 iterations while the other two methods are slower, and they need more iterations to reach the required depth.

		Proposed phase-only		Complex weighting		Fully phase-only	
		method		method [16]		method $[12]$	
Single null	$\mathbf{N}=\mathbf{10}$ $\mathbf{M}=\mathbf{3}$			Amplitude Phase/rad Amplitude Phase/rad		Amplitude Phase/rad	
		1	-1.0025	0.6427	-0.1525		0.6815
		$\mathbf{1}$	0.9569	0.4469	0.3789	$\mathbf 1$	0.0444
		$\mathbf{1}$	-1.3506	0.5198	-0.4661	$\mathbf{1}$	-0.4384
		1	Ω	1	Ω	$\mathbf 1$	0.5016
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	0.0111
		$\mathbf 1$	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	0.5542
		$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	0.7764
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	0.4624
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	1.0187
		$\mathbf 1$	$\overline{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\overline{1}$	1.0114
		$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	-0.0908
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	1.1112
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	1.1753
		$\mathbf 1$	$\boldsymbol{0}$	1	$\boldsymbol{0}$	$\mathbf{1}$	0.7527
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\boldsymbol{0}$	$\mathbf{1}$	0.5121
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\overline{1}$	0.6042
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	1.3360
		$\mathbf{1}$	-1.3063	0.7780	-1.0031	$\overline{1}$	1.3180
		$\mathbf 1$	0.7031	0.2143	-0.2241	1	-0.8151
		$\mathbf 1$	-1.4590	0.1843	-0.7281	$\mathbf{1}$	1.5708

Table 1. Amplitude and phase excitations of the tested three methods for the case of a single wide null.

Figure 5. Instantaneous depths of the generated nulls for the three tested methods.

Figure 6. Array patterns of three methods to obtain two wide nulls.

In the second example, two wide nulls centered at $-20°$ and $20°$ are investigated. The number of controllable phases is chosen to be $M = 3$ out of $N = 10$ on each side of the array. The results of the phase-only method compared to that of the complex weighting method are shown in Fig. 6. The corresponding amplitude and phase excitations of the tested methods are shown in Fig. 6 and Table 2.

In the next example, the effect of the controllable elements on the depth of the generated nulls, response time of the used optimizer, complexity in terms of the number of RF components including the phase shifters and attenuators, main beam perturbation, and the directivity of both the proposed phase-only method and the method of [16] are investigated. Figs. 7, 8, 9, 10 ,and 11 show the results. In all of these five figures, the total number of array elements on both sides of the array is chosen to be 20 elements, and a single wide null centered at $20°$ is considered. The number of controllable elements, M, on each side of the array is varied from 1 to 20 (when M reaches 20 this is the fully controllable array elements). From Fig. 7 and as expected, it can be seen that the complex weighting method [16] is able to produce slightly deeper nulls than that of the proposed phase-only method under the use of the same number of controllable elements. The deep nulls are come at the cost of higher complexity, longer response time, and more main beam reduction as shown in Figs. 8, 9, and 10.

From Figs. 10 and 11, it can be seen that as the number of controllable elements increases, higher perturbation will occur (more main beam reduction), and lower directivity will be obtained. Nevertheless, the proposed phase-only method with limited number of controllable elements provides less perturbation and higher directivity than the complex weighting method [16]. This is mainly because the phases have less effect than the amplitudes on the main beam pattern. Moreover, Figs. 10 and 11

Figure 7. Depth of the required nulls.

Table 2. Amplitude and phase excitations of the tested methods for the case of two wide nulls.

		Proposed phase-only		Complex weighting		Fully phase-only	
		method		method [16]		method [12]	
		Amplitude Phase/rad		Amplitude Phase/rad		$Amplitude$ Phase/rad	
		1	1.0501	0.3559	-0.7024	1	1.2990
		$\mathbf{1}$	-1.0410	0.7627	0.3165	$\mathbf{1}$	-0.8028
		$\mathbf{1}$	1.2747	0.2315	-1.4698	$\mathbf{1}$	1.5662
		1	$\overline{0}$	1	Ω	$\mathbf{1}$	0.2857
		$\mathbf 1$	$\boldsymbol{0}$	$\mathbf 1$	θ	$\mathbf{1}$	0.3621
		$\mathbf 1$	$\boldsymbol{0}$	$\mathbf 1$	$\boldsymbol{0}$	$\mathbf{1}$	0.3910
		1	$\boldsymbol{0}$	1	$\boldsymbol{0}$	$\mathbf{1}$	0.5106
Two	$N = 10$	$\mathbf 1$	$\overline{0}$	1	$\overline{0}$	$\mathbf{1}$	0.3918
wide null	$M=3$		$\overline{0}$	1	$\overline{0}$	$\mathbf{1}$	0.5090
		$\mathbf 1$	$\overline{0}$	1	$\overline{0}$	$\mathbf{1}$	0.5180
		$\mathbf 1$	$\overline{0}$	$\mathbf 1$	θ	$\mathbf{1}$	0.5180
		1	$\boldsymbol{0}$	$\mathbf 1$	$\boldsymbol{0}$	$\mathbf{1}$	0.5090
		$\mathbf 1$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	0.3910
		1	$\boldsymbol{0}$	1	θ	$\mathbf{1}$	0.5106
		1	$\overline{0}$	1	θ	1	0.3910
		$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	$\overline{0}$	$\mathbf{1}$	0.3621
		1	$\overline{0}$	1	$\overline{0}$	$\mathbf{1}$	0.2857
		1	1.2747	0.2315	-1.4698	$\mathbf{1}$	1.5662
		1	-1.0410	0.7627	0.3165	$\mathbf{1}$	-0.8028
		1	1.0501	0.3559	-0.7024	$\mathbf{1}$	1.2990

also show that a small number of controllable elements are better in terms of directivity and main beam perturbation than the fully controllable array elements. This is an additional advantage feature to the proposed method.

Figure 8. Convergence time of the optimizer.

Figure 9. Complexity of the feeding network.

Table 3. Parameters of the antenna design.

	Length (mm)	Width (mm)	Other specification
Patch	$Lp = 31.5$	$Wp = 38.03$	Thickness $= 0.035$ mm
Ground	$Lg = 37.9$	$Wg = 47.6$	Thickness $= 0.035$ mm
Substrate	$Ls = 37.9$	$Ws = 47.6$	$\varepsilon_r = 3.4$
			$Thickness = 1.6 \text{ mm}$
Inset fed	$Lins = (Ls/2 - Lp/2) + Lp/3$	$Wf=3$	$Thickness = 0.035 \,\mathrm{mm}$

Figure 10. The main beam reduction.

Figure 11. The directivity versus the number of controllable elements.

4. VERIFICATION OF THE PROPOSED METHOD

To validate the performance of the proposed phase-only method, an array with 16 rectangular patch elements operating at frequency 2.4 GHz is designed and tested under realistic electromagnetic environment. Fig. 12 and Table 3 show the specification of the designed single patch.

In this design, the phases of 3 elements on each side of the array are optimized to impose a wide null centered at $\theta = 20^\circ$. Fig. 13 shows the actual pattern of the designed array with the generated null. For comparison, the theoretical pattern for the same array with the Matlab software is also plotted in Fig. 13. It can be seen that the theoretical result of the proposed array is in a good agreement with that of the actual one. Moreover, the required wide nulls have been achieved.

Figure 12. (a) Single antenna element design, (b) S_{11} parameter of the designed array for $N = 16$ patches.

Figure 13. The theoretical and the actual patterns of the designed array for a single wide null.

5. CONCLUSIONS

A method for phase-only nulling with limited number of controllable elements has been presented. Previous results obtained by the complex weighting method [16] and fully phase-only weighting method [12] have been improved, and wide nulls with width more than 5◦ and depth more than −60 dB are achieved. The complexity of the feeding network in terms of the required number of RF components, convergence time of the optimizer in terms of the required number of iterations, main beam reduction, and directivity are computed and compared for the three methods using a core-i3 laptop with 2.4 GHz processors. The proposed method can be easily extended to the planar arrays where the controllable phase-only elements can be selected either by the use of genetic algorithm or a more advanced optimization approach called compressed sensing.

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