

Fast Method of Calculating the Air-Cored Coil Impedance Using the Filamentary Coil Model

Grzegorz Tytko*

Abstract—This paper presents a method for calculating the air-cored coil impedance with the employment of a mathematical model of an ideal filamentary coil. The proposed algorithm enables assigning, in a very quick way, each cylindrical air-cored coil to the corresponding filamentary coil using only two equivalent parameters. The first of them is the radius of the coil, whereas the second one is the distance between the coil and the surface of the tested material. The changes both in the parameters of the system under consideration and in the tested material bring about the same change in the impedance of the air-cored coil and the corresponding filamentary coil. This property brings a lot of advantages, since it allows using simpler final formulas for the filamentary coil and performing the calculations in a much shorter time, while obtaining the same results as in the case of the air-cored coil. At the same time, the creation of the scale of the measuring instrument and its calibration becomes far simpler since it is based on only two equivalent parameters.

1. INTRODUCTION

Surface probes have a practical application in non-destructive testing carried out using the eddy current method. What constitutes such a probe in the basic case is a coil fed with alternating current. When it is brought close to a conductive material, it induces eddy currents in the material. Consequently, the magnetic field created by the eddy currents causes changes in the coil impedance, whose value depends on the properties of the conductive workpiece. This feature constitutes the basis for eddy-current testing since it enables the determination of some material parameters and of geometrical dimensions of the tested workpiece.

In eddy-current testing, the air-cored coil (Fig. 1) is commonly used for the measurement of: material thickness [1–3], magnetic permeability [4–6], electrical conductivity [7, 8], calculations of magnetic field [9, 10], or in a fuzzy logic approach [11–14]. In mathematical models, the geometric dimensions of the air-cored coil are usually determined by the means of four parameters: inner radius r_1 , outer radius r_2 , lower height h_1 , and upper height h_2 (Fig. 2). However, this approach causes some difficulties during the designing, calibration, or scale construction of the measuring instrument. Unfortunately, these processes usually turn out to be quite complicated, because each coil should be assigned as many as 4 parameters determining its position and geometric dimensions. The need for a simpler mathematical model also arises while determining optimal parameters of the probe and creating mechanisms of compensation for adverse effects (e.g., liftoff) [15–17]. In the case of applications that require multiple iterations, such as multi-frequency eddy current testing [18, 19], what prove to be particularly useful are final formulas that allow doing calculations in the shortest possible time.

What may be the solution that meets these expectations is the employment of the model of the ideal filamentary coil whose all N turns are concentrated in a circle with a radius r_0 , in the distance of h_0 from the surface of the tested material (Fig. 3). The method that enables substituting the corresponding

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* Corresponding author: Grzegorz Tytko (grzegorz.tytko@wp.pl).

The author is with the Institute of Electronics, The Silesian University of Technology, Akademicka 16, Gliwice, Poland.

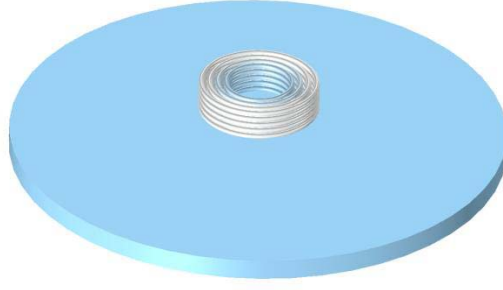


Figure 1. Air-cored coil located above a conductive material.

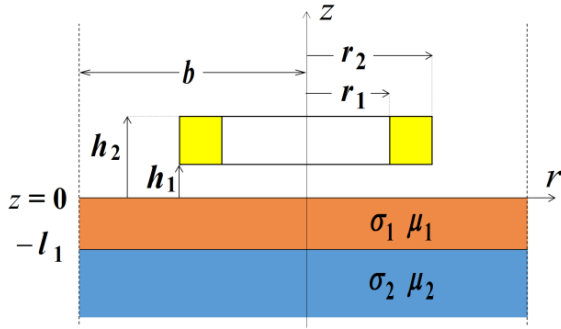


Figure 2. Cross section of the air-cored coil in a cylindrical coordinate system.

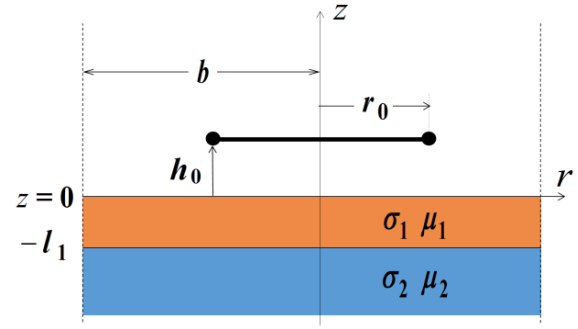


Figure 3. Cross section of the filamentary coil in a cylindrical coordinate system.

filamentary coil for any air-cored coil has been described in [20, 21]. The equivalent parameters r_0 , h_0 are determined directly from integral expressions, so that the change in the impedance resulting from bringing the coils close to the conductive material is the same for both the coils. In this way, each air-cored coil may be unambiguously characterized by only two parameters (r_0 , h_0) instead of four ones. This means that the changes in the values of the system parameters (such as the frequency of the current feeding the coil, magnetic permeability of the material, or liftoff) bring about the same changes in the impedance of both the air-cored coil and the corresponding filamentary coil.

In order to use the model of the filamentary coil, it is necessary to work out a solution that enables quick and precise calculation of the equivalent radius r_0 and equivalent height h_0 . The method proposed in this paper constitutes such a solution — it employs the ideal filamentary coil model developed by using the truncated region eigenfunction expansion (TREE) method, in which the time-consuming integral expressions have been replaced by the sums of the series [22]. The developed algorithm enables effective determination of the values of the parameters r_0 , h_0 for any cylindrical air-cored coil whose geometrical dimensions are known. These parameters are to be determined only once and they characterize unambiguously a particular coil. The search for equivalent parameters is based on the values of the changes in the components of the air-cored coil impedance, which are caused by bringing the coil to the surface of the tested material. They can be obtained in two ways:

- directly from the measuring device during carrying out the tests, or in the process of scaling the coil in order to assign the measured values to the instrument indications. The relationship between the measured values and the indications is determined using the model of the filamentary coil.

- on the basis of the calculations carried out with any numerical or analytical method using an appropriate mathematical model. The calculations are made at the stage of designing the coil or configuring the measuring stand. The proposed algorithm for calculating the parameters r_0 , h_0 will allow optimal selection of the test conditions and the geometrical dimensions of the real coil, while maintaining the correct number of the turns.

Thus, i.e., by using simpler formulas for the filamentary coil, it is possible to determine the changes in the impedance of the air-cored coil in a much shorter time. Owing to the adoption of such a solution,

the process of creating the scale of the measuring instrument becomes simpler since it can be carried out using only two parameters r_0, h_0 . This method may also be used for performing sensitivity analysis, creating mechanisms of compensating for adverse effects, or measuring the porosity of surfaces.

2. SOLUTION

The determination of the equivalent parameters r_0, h_0 of the air-cored coil can be carried out for any mathematical model. In this paper, models of the ideal filamentary coil [22] and air-cored coil [23, 24], derived with the employment of the TREE method [25], are used. In both the cases, formulas for the change in the coil impedance resulting from bringing the coil close to the tested material were used.

The method of determining the impedance of the air-cored coil with N turns, while the coil is located over the two-layer material with electrical conductivity σ_1, σ_2 and magnetic permeability μ_1, μ_2 (Fig. 2) was described by Dodd and Deeds in [26, 27]. The axisymmetric problem was considered in a cylindrical coordinate system with an infinite solution domain. The integral expressions obtained in this way were then reformulated, by the means of TREE method, into the form that facilitates numerical implementation [28]:

$$\Delta Z = \frac{j\omega 2\pi\mu_0 N^2}{(r_2 - r_1)^2 (h_2 - h_1)^2} \sum_{i=1}^{Ns} \frac{\chi^2(q_i r_1, q_i r_2)}{b^2 q_i^7 J_0^2(q_i b)} \left(e^{-q_i h_1} - e^{-q_i h_2} \right) \frac{C_{3i}}{B_{3i}}, \quad i = 1, 2, 3, \dots, Ns \quad (1)$$

$$\frac{C_{3i}}{B_{3i}} = \left(1 \pm \mu_1 \frac{q_i}{s_{1i}} \right) C_{4i} + \left(1 \mp \mu_1 \frac{q_i}{s_{1i}} \right) B_{4i}, \quad (2)$$

$$\frac{C_{4i}}{B_{4i}} = e^{\mp s_{1i} l_1} \left(\mu_1 \mp \mu_2 \frac{s_{1i}}{s_{2i}} \right), \quad (3)$$

$$\chi^2(q_i r_1, q_i r_2) = q_i^2 \int_{r_1}^{r_2} r J_1(q_i r) dr, \quad (4)$$

where $J_1(x)$ is the Bessel function of the first kind, ω the angular frequency, and $s_n = (q_i^2 + j\omega\mu_n\mu_0\sigma_n)^{1/2}$. The limitation of the solution domain in the radial direction leads to the occurrence of discrete eigenvalues q_i . However, the determination of them is relatively simple as it boils down to finding the positive roots of function $J_1(q_i b) = 0$. The integral in Eq. (4), which is troublesome in numerical calculations, was expanded using the Bessel and Struve functions in the following way [29]:

$$q_i^2 \int_{r_1}^{r_2} r J_1(q_i r) dr = \frac{\pi}{2} q_i \left\{ r_2 [J_0(q_i r_2) H_1(q_i r_2) - J_1(q_i r_2) H_0(q_i r_2)] \right. \\ \left. - r_1 [J_0(q_i r_1) H_1(q_i r_1) - J_1(q_i r_1) H_0(q_i r_1)] \right\}. \quad (5)$$

By placing a filamentary coil with N turns (Fig. 3) above the same conductive material, the changes in its impedance can be determined from the formula [22]:

$$\Delta Z = j\omega 2\pi\mu_0 r_0 N^2 \sum_{i=1}^{Ns} q_i^{-1} \left[\frac{J_1(q_i r_0) e^{-q_i h_0}}{J_0(q_i b) b} \right]^2 \frac{e^{-2s_{1i} l_1} k_{1i} k_{3i} + k_{2i} k_{4i}}{e^{-2s_{1i} l_1} k_{1i} k_{4i} + k_{2i} k_{3i}}, \quad (6)$$

$$k_{1i} = s_{1i} \mu_2 - s_{2i} \mu_1, \quad (7)$$

$$k_{2i} = s_{1i} \mu_2 + s_{2i} \mu_1, \quad (8)$$

$$k_{3i} = q_i \mu_1 + s_{1i}, \quad (9)$$

$$k_{4i} = q_i \mu_1 - s_{1i}. \quad (10)$$

The substitution of the corresponding filamentary coil for the air-cored coil (whose geometrical dimensions r_1, r_2, h_1, h_2 are known) requires determining the appropriate values of the equivalent parameters r_0, h_0 , so that the changes in the impedance of both the coils are equal. What was obtained

by comparing the right sides of the formulas (1) and (6) was a complex equation whose solution allows determining the searched pair r_0, h_0 . This, in consequence, enables determination of the filamentary coil that unequivocally characterises the given air coil.

3. ALGORITHM

The task of working out the analytical formulas for the searched parameters r_0, h_0 is relatively troublesome because of the form of the formula (6). For this reason, a different approach is proposed in this paper. The operating principle of the developed algorithm is based on the iterative selection of the parameters r_0, h_0 , with an adaptively determined step. This step is determined through dividing the search ranges into smaller and smaller parts until the assumed accuracy of determining the changes in the coil impedance has been achieved. The source code of the algorithm implemented in Matlab is presented in Appendix.

Before the algorithm starts to operate, the input data are entered, i.e., the search ranges ($r0_min, r0_max$) and ($h0_min, h0_max$), the initial steps of the incrementation $r0_step, h0_step$, and the value of the change in the air-cored coil impedance dZa obtained from Eq. (1). After entering the input data, Equation (6) is solved for the iteratively selected values r_0, h_0 , which are incremented with the step $r0_step$ and respectively $h0_step$. The pair r_0, h_0 , for which the smallest difference in the impedance change was obtained in comparison to the reference value dZa , is assigned as $r0_best, h0_best$. On this basis new search ranges ($r0_best \pm k * r0_step$) and ($h0_best \pm k * h0_step$), and consequently also new values of the incrementation steps: $r0_step, h0_step$ are created. The parameter $k > 1$ makes the search ranges wider, and therefore the final solution may be found even in the situation when it is outside the ranges ($r0_min, r0_max$), ($h0_min, h0_max$). Thus, the greater the value of the parameter k , the greater the chance of working out the correct values of r_0, h_0 , despite the fact that the provided input data may be incorrect. However, this requires carrying out additional iterations, which consequently results in a longer time of the calculations.

The gradual narrowing of the search ranges and the decreasing of the incrementing step lead to the results that contain lesser and lesser error. The algorithm stops operating after determining the values of r_0, h_0 , for which the relative error of the change of resistance and the change of reactance err meet the assumed criteria.

4. RESULTS AND DISCUSSION

Formulas (1) and (6) that describe the change in the impedance of the air-cored coil and the filamentary coil respectively, as well as the algorithm for determining the parameters r_0, h_0 were implemented in Matlab (Appendix). The calculations were done on a computer equipped with an Intel Core 2 Duo T7200 processor and 4 GB of RAM, assuming the number of summation terms $Ns = 50$ and the domain radius $b = 12r_2$. The equivalent parameters r_0, h_0 were determined for 3 coils with different geometrical dimensions, containing 500 turns and fed with alternating current of frequency 100 kHz (Table 1). The coils were placed over a non-ferromagnetic ($\mu_1 = \mu_2 = 1$) two-layer material with electrical conductivity $\sigma_1 = 8 \text{ MSm}^{-1}$, $\sigma_2 = 32 \text{ MSm}^{-1}$ and thickness $l_1 = 4 \text{ mm}$. In the algorithm, $r0_min = r_1, r0_max = r_2, h0_min = h_1, h0_max = h_2$ were assumed as the initial search ranges. The resulting intervals were divided into 5 parts ($r0_parts = h0_parts = 5$), and in the next stages of the algorithm new search ranges were created for the value $k = 1.25$. The accuracy of the coil impedance components is 0.1% ($err > 0.1 + 0.1 * i$). The roots of the function $J_1(x)$, which were used to determine the discrete eigenvalues of q_i , were calculated using the Newton-Raphson method.

The obtained results show that the parameters r_0, h_0 were determined for all the coils with a similar number of iterations. Thus, it was found that the geometric dimensions of the coil did not significantly affect the time of determining the searched equivalent parameters. This time turned out to be relatively short, and for each of the coils it was about 0.2 s. It was noted that in the case of solenoid coils ($h_2 - h_1 \gg r_2 - r_1$) and pancake coils ($h_2 - h_1 \ll r_2 - r_1$), the pair r_0, h_0 may take values outside the ranges (r_1, r_2) and (h_1, h_2). In this situation, it is recommended to broaden the initial search ranges or increase the value of the parameter k .

Table 1. Geometric dimensions of the coils and values of equivalent parameters r_0 , h_0 .

		Rectangular coil	Solenoid coil	Pancake coil
Inner coil radius	r_1 [mm]	2	2	2
Outer coil radius	r_2 [mm]	6	6	16
Lower height	h_1 [mm]	0.1	0.1	0.1
Upper height	h_2 [mm]	5.1	15.1	5.1
Equivalent radius	r_0 [mm]	4.13	3.56	10.63
Equivalent height	h_0 [mm]	2.44	4.32	4.41
Iterations		380	381	338

Table 2. Input data sets used in the calculations.

Tests	f [kHz]	μ_1	μ_2	σ_1 [MSm ⁻¹]	σ_2 [MSm ⁻¹]	liftoff [mm]
1	1	1	1	8	32	0.1
2	10	1	1	58	58	2
3	50	1	1	16	8	1
4	100	1	1	8	32	0.1
5	100	1	1	16	58	2
6	1	25	50	8	32	0.1
7	1	100	5	58	16	1
8	10	10	5	16	32	2
9	50	25	25	8	8	0.1
10	100	10	100	8	58	0.1

The calculations of the changes in the impedance ΔZ were carried out for 10 sets of input data (Table 2). For this purpose, a rectangular air-cored coil and a corresponding filamentary coil were used (Table 1). All the values of the changes in the filamentary coil impedance were determined for the same pair r_0 , h_0 . Subsequently, the values were compared with the results obtained from Eq. (1) and by using the finite element method in the Comsol Multiphysics package. The achieved results for tests 1–10 are presented in Table 3, whereas the values of ΔZ normalized in relation to the reactance in the air space X_0 and obtained for tests 1 and 6 are shown in Figs. 4–5. In the case of ferromagnetic materials, the difference in the changes of the components of the filamentary coil impedance (TREE) and the air-cored coil impedance (FEM) did not exceed 1.5%, whereas for the non-ferromagnetic materials it was lower than 0.5%. The results obtained in the TREE method both for the filamentary coil and the air-cored coil were almost identical in the entire studied frequency range.

What is one of the major advantages of performing the calculations with the employment of the filamentary coil is relatively simple final formulas that allow obtaining results in a much shorter time than it is in the case of the air-cored coil. Table 4 presents the times of determining the changes in the

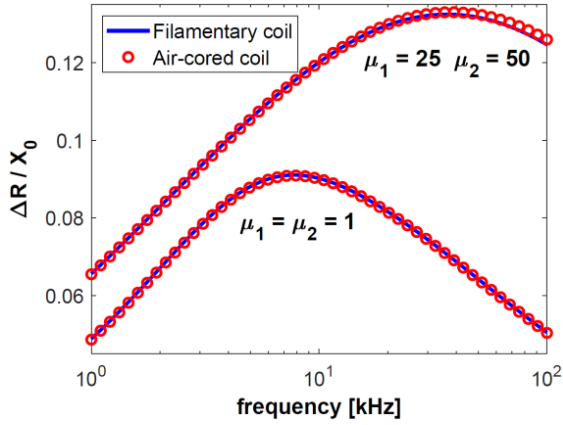


Figure 4. Normalized changes in the resistance of the filamentary coil (TREE) and the air-cored coil (FEM).

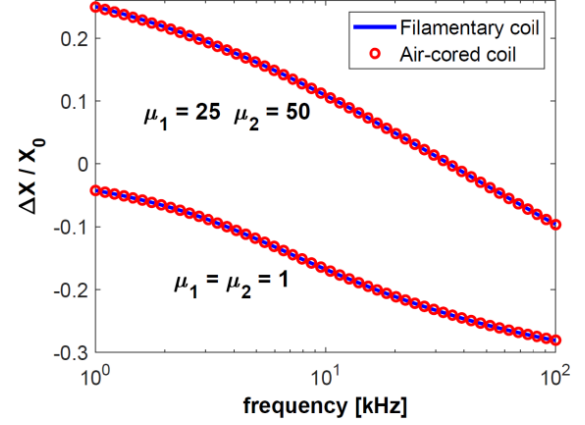


Figure 5. Normalized changes in the reactance of the filamentary coil (TREE) and the air-cored coil (FEM).

Table 3. Changes of the impedance of the rectangular air-cored coil and the filamentary coil.

Tests	ΔZ [Ω]		
	FEM	TREE	TREE
	Air-cored coil	Air-cored coil	Filamentary coil
1	$0.35 - j0.31$	$0.35 - j0.31$	$0.35 - j0.31$
2	$1.12 - j6.88$	$1.12 - j6.89$	$1.12 - j6.90$
3	$9.45 - j59.59$	$9.43 - j59.69$	$9.42 - j59.71$
4	$36.45 - j202.99$	$36.44 - j203.17$	$36.44 - j203.17$
5	$7.41 - j74.15$	$7.34 - j74.34$	$7.35 - j74.38$
6	$0.47 + j1.81$	$0.47 + j1.81$	$0.47 + j1.81$
7	$0.38 + j0.88$	$0.37 + j0.86$	$0.37 + j0.86$
8	$3.02 - j1.96$	$3.01 - j2.00$	$3.01 - j2.01$
9	$47.84 - j12.37$	$47.58 - j12.46$	$47.58 - j12.46$
10	$77.05 - j121.41$	$76.87 - j121.60$	$76.86 - j121.60$

Table 4. Comparison of the calculation time of the coil impedance ΔZ for different mathematical models.

	Computation time [s]		
	FEM	TREE	TREE
Iterations	Air-cored	Air-cored	Filamentary
1	6	0.03	0.01
100	443	1.52	0.21
1000	3982	13.66	0.86
10000	— — —	141.75	7.32

coil impedance ΔZ using expressions (1), (6) and the FEM model. The calculations were carried out for independent iterations by changing the value of the inner radius r_1 and the outer radius r_2 of the coil. In the TREE method, the calculation time depends mainly on the number of summation terms

Ns . The value of this parameter ($Ns = 50$) was selected in such a way so that the best accuracy of the results for both the coils was achieved. The same assumption was made in the FEM method, where in each iteration in the Comsol Multiphysics package a dense mesh consisting of 41342 triangular elements, 674 edge elements, and 12 vertex elements was created. In the case of an acceptable lower accuracy of the results, the time of the calculation with the employment of this method can be significantly reduced by using a mesh with fewer elements or by reducing the radius of the solution domain b .

5. CONCLUSION

The method proposed in this paper allows calculating changes in the impedance of the air-cored coil using the corresponding ideal filamentary coil. The parameters r_1 , r_2 and h_1 , h_2 that describe the geometrical dimensions of the air-cored coil have been replaced by the parameters r_0 , h_0 . The calculation of equivalent parameters is done with little computational effort using an algorithm developed for this purpose. The pair r_0 , h_0 adopts such values that for the same input data, the changes in the impedance of the air-cored coil and the corresponding filamentary coil are equal. The values r_0 , h_0 need be determined only once, because changes in the input parameters of the system (such as: electrical conductivity, liftoff, magnetic permeability, and frequency) bring about the same changes in the impedance of both the air-cored coil and the corresponding filamentary coil.

Doing the calculations with the employment of the ideal filamentary coil has a lot of advantages. First of all, the final formulas are simpler than those in the case of the air-cored coil, which greatly facilitates the numerical implementation in both mathematical packages and the microprocessor system located in the measuring device. Less complex formulas also result in a shorter time of the calculations. What is more, the use of only two parameters (r_0 , h_0) defining the geometrical dimensions of the coil makes it possible to significantly simplify the process of calculating the optimal parameters of the probe and creating the scale of the measuring instrument.

The proposed approach for determining the equivalent parameters r_0 , h_0 may be successfully used in the research into magnetic and non-magnetic materials, when making use of both analytical and numerical methods. In the calculations of the changes in the coil impedance in the range from 1 kHz to 100 kHz, a very good agreement was obtained, while the time of the calculations was by far the shortest. It shows how great the possibilities offered by the employment of the filamentary coil are. For this reason, in the subsequent research it is planned to include coils with cores, probes with more complex shapes, and systems with several coils.

APPENDIX A.

Equivalent parameters r_0 , h_0 and changes in the filamentary coil impedance ΔZ in Eq. (6) can be calculated using this algorithm implemented in Matlab.

% Input data

```
mm = 0.001; N = 500; m0 = 4e - 07 * pi; r1 = 2 * mm; r2 = 6 * mm; h1 = 0.1 * mm;
h2 = 5.1 * mm; l1 = 4 * mm; m1 = 1; m2 = 1; omega = 2 * pi * 10^5; Ns = 50;
b = 12 * r2; sig1 = 8 * 10^6; sig2 = 32 * 10^6; k = 1.25; r0_min = r1;
r0_max = r2; r0_parts = 5; h0_min = h1; h0_max = h2; h0_parts = 5;
```

% Change of the air-cored coil impedance

```
dZa = 36.44019 - 1i * 203.17402;
```

% Algorithm

```
r0 = r0_min; h0 = h0_min; err = 1; q = zerobess_J1(Ns)'/b; err_best = 200;
s1 = sqrt(q.^2 + 1i * omega * m0 * m1 * sig1);
s2 = sqrt(q.^2 + 1i * omega * m0 * m2 * sig2);
```

```
while err > 0.1 + 1i * 0.1
while r0 <= r0_max
while h0 <= h0_max
```

```

y = besselj(1, q * r0).^2 ./ (q * besselj(0, q * b).^2);
k1 = s1 * m2 - s2 * m1; k2 = s1 * m2 + s2 * m1; k3 = q * m1 + s1; k4 = q * m1 - s1;
f = (exp(-2 * s1 * l1) * k1 * k3 + k2 * k4) ./ (exp(-2 * s1 * l1) * k1 * k4 + k2 * k3);
fil = 1i * omega * m0 * 2 * pi * N^2 * b^-2 * r0^2 * sum(y * exp(-2 * q * h0) * f);
e1 = 100 * abs(real(dZa) - real(fil)); e2 = 100 * abs(imag(dZa) - imag(fil));
err = e1/real(dZa) + e2/abs(imag(dZa));

if err < err_best
r0_best = r0; h0_best = h0; err_best = err; fil_b = fil;
end

h0 = h0 + (h0_max - h0_min)/h0_parts;
end
r0 = r0 + (r0_max - r0_min)/r0_parts; h0 = h0_min;
end

r0_step = (r0_max - r0_min)/r0_parts; h0_step = (h0_max - h0_min)/h0_parts;
r0_min = r0_best - k * r0_step; r0_max = r0_best + k * r0_step; r0 = r0_min;
h0_min = h0_best - k * h0_step; h0_max = h0_best + k * h0_step; h0 = h0_min;
end

% Results
disp(r0_best); disp(h0_best); disp(fil_b);

```

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