

Meta-Heuristic Multi-Objective as an Affordable Method for Improving the Grating Lobe in a Wide Scan Phased Array Antenna

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Abstract—In electronic beam scanning, the number of phase shifters is an obvious challenge. So, there are several methods to reduce the number of phase shifters. The aim of this paper is to investigate the use of the meta-heuristic algorithm to lower the grating lobe level in the subarray antenna. Improve the result obtained by group subarray optimization techniques to determine topology and space between elements, and complex optimization of weight, simultaneously. Uniform subarray and random subarray are analyzed in Matlab to determine the coefficient of excitation by the evolutionary algorithm, as well as swarm and hybrid. The results of the simulation are shown; this method leads to radiation pattern without grating lobe in wide scanning angle. It indicates that there is a possibility of obtaining wide electronic scanning with minimum number of phase shifters and improving result.

1. INTRODUCTION

Phased-array antennas have been used in a wide range of applications, from military systems to commercial cellular communications networks such as wireless communication systems, wireless power transmission systems, and radar systems. Electronic scanning, a key role of phased array antenna, is defined as a method of positioning a beam with the antenna remaining fixed. The basic electronic scanning techniques are phase shifting, true time delay, frequency scanning, and feed switching [1].

If the phase shifter technique is used, the design complexity would be higher. Therefore, the key feature of the phased array antenna is to use the lowest number of phase shifters to achieve, as much as possible, a wide range of high-gain radiation patterns with lower cost [2, 3]. Typically, in an optimal phased array with minimal number of phase shifters the distance between elements is greater than half wavelength. With respect to array antennas when the distance between elements is smaller than or equal to half-wavelength, the grating lobe usually appears by increasing the scan angle up to 60 degrees. Therefore, in antennas in which distance between elements is greater than half wavelength, the grating lobes are likely to occur at a lower angle of scan [4, 5]. Consequently, what will be critical is to provide a phased array antenna with the minimum number of phase shifters without the existence of grating lobes within the scan range.

One way to compensate the side effect of using a minimal number of phase shifter is using tapering distance (optimizing inter element spacing) [4, 6–9]. Controlling the distance between the elements in arrays will further reduce the level of the GL. Extensive research has been done on the subarray antenna [4, 10–12]. Such subarrays were initially selected uniformly. The random subarray (RSA) was later used to eliminate the second array periodicity in uniform subarray [2]. Also, an overlay subarray (OSA) technique was suggested to provide an effective phased array by reducing the distance [13]. The most current research is a combination of random and overlap subarrays (OSA) [13], or randomly overlap subarray (ROSA) [14].

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Another approach that some researchers have proposed is to use complex coefficient tapering [11, 15] based on two categories of algorithms. The first category is based on Stochastic gradient descent (an iterative method of optimizing), which involves the least mean square (LMS), constant modulus algorithm (CMA), and recursive least-squares (RLS) algorithm. These algorithms are characterized by different complexity levels and periods of convergence. Second, the synthesis of the desired pattern is based on the optimization algorithm. Determining excitation coefficients with the optimization algorithm depends on the cost functions. So, the starting point or range of varying complex amplitudes affects the algorithm's accuracy and speeds.

In this paper, the combination of the two approaches mentioned is used to achieve a more efficient phased array than previous work. Distance tapering is managed by ROS configuration, and the optimum coefficients of excitation are achieved using hybrid optimization algorithm. Compared to earlier studies, the presented paper has different variations which eventually produce better results. First, using a hybrid optimization algorithm with a suitable cost function does not result in a local solution getting stuck. Secondly, the focus is on estimating the optimal coefficients of the second array by consciously selecting the starting point which makes the optimization algorithm more accurate and quicker.

The paper is organized as follows. Section 2 explains the detail of the optimization algorithm, and the proposed method to control GL with minimum SLL is illustrated. The simulation results are provided in Section 3. Finally, Section 4 presents the conclusions of this work.

2. OPTIMIZATION METHOD TO DESIGN RANDOM SUBARRAY ANTENNA

2.1. Hybrid Optimization Algorithm

Over the last two decades, meta-heuristic optimization techniques have become very popular [16]. They have been mostly inspired by very simple concepts, in which the inspirations are typically related to physical phenomena, animals' behaviours, or evolutionary concepts. These algorithms may be classified into three main classes: evolutionary algorithm (EA) [17], physics-based, and swarm intelligence (SI) algorithms [18]. EAs are usually influenced by the evolutionary ideas of nature.

The most popular algorithm in this branch is particle swarm (PSO) [19] and genetic algorithm (GA) [20]. GA [21], which Holland proposed in 1992, is a search and optimization approach inspired by genetic science and Darwinian evolution and focuses on the survival of the best or nature's choice. In general, the algorithm procedure is as follows.

First population production: the initial population in the algorithm is generated randomly according to the size of the population of interest which is determined by the fitness function. Parental selection: selection of parents as the next generation is important. There are various methods for parent selection, such as roulette, competitive, and random selection.

Parents are chosen on the basis of fitness in all those processes. So, the parent roulette form is much more likely to be chosen, which has a higher fitness value. Crossover: at this stage by combining the two parents, two children are produced. Intersection methods in the genetic algorithm can be referred to single point intersection, the intersection of two points, the intersection of multiple points, or uniform intersection. All the intersection methods, except the uniform intersection method, are used for discrete problems, and only the uniform intersection method is used for continuous problems. Mutation: a mutation is one of the phenomena of genetic science that rarely occurs in some chromosomes, in which children find features that do not belong to any parent. The role of mutation in GA is to restore lost genetic material in the population. Generating a new population: at this stage of the algorithm, the fitness value of the population generated during the crossover and mutation is calculated, then the current population is merged with the former population and the population sorted based on the fitness level. After sorting the population, the size of population required by half of the population is selected as the next generation (new population), and the first one has the best value that is considered as the best answer to the problem.

A chromosome is a set of parameters in GAs which defines a proposed solution to the problem that the GA is trying to solve. The mutation operator and transverse operator are used to take into account the configuration of the chromosome.

The PSO algorithm [19] was proposed by Kennedy and Eberhard [22] and inspired from the social behaviour of birds flocking. In the PSO algorithm, there are a number of organisms, which are called

a particle and distributed in the search space. Each particle calculates the objective function value in a location of the space.

All particles choose the direction to move, and then a step from the algorithm ends. These steps are repeated several times in order to obtain the desired response. In fact, the swarm of particles that search the minimum value of a function acts like a flock of birds looking for food. In the first stage of the algorithm, the initial location and velocity of each particle are initialized randomly, and then using a fitness function or error, these experiences are considered as the best practice of each particle, then the particle having the best value is selected as the best particle in all particles, and the algorithm is inserted into its original loop. Then per iteration of the process of optimizing the velocity of each particle is updated. After updating the particle location and speed of each particle, in the second stage of the optimization process, the location of each particle must be updated. At this point, the particle velocity is calculated after updating its position. If the current fitness value is better than the previous best value, this value will be saved as the best position of the particle. The best amount of fitness of each particle in the current position is compared with the best value of all previous particle sizes, and if its value is better than the value of the best particle in previous iterations, it is considered the best particle. The termination condition of this algorithm is similar to the termination conditions of the genetic algorithm [23].

Although both of these approaches are highly capable of solving the problem, each has its own advantages and disadvantages [24, 25]. For example, one of the disadvantages of PSO algorithm is that the algorithm can be stuck at the local optimum point because of its structure, but the advantage of this is that the algorithm converges rapidly into the optimal solution. In addition, a larger query space than the PSO algorithm is available in GA. The crossover and mutation functions used in this algorithm offer the advantage. Despite GA's drawbacks, it should be noted that this algorithm is more suited for discrete problems. Some researchers have suggested a hybrid algorithm [26, 27] to minimize these drawbacks. So far, in the context of a hybrid algorithm, various combinations of these two optimization approaches have been provided. In the first model, the obtained results from GA are introduced as the initial values to the PSO algorithm, and the optimization process is performed by the algorithm. The second type of hybrid algorithms is GA crossover, and mutation functions are used in the PSO algorithm in order to escape the local optimum. Moreover, the third model of combination uses GA and PSO in parallel to optimize the problems.

In this paper, the third model hybrid algorithm is used to get better results and to achieve the advantages of GA and PSO simultaneously. Thus, the first half of the population is optimized with GA, and the remaining half will join the PSO algorithm, then the optimization process will continue until the end conditions are met, and the results are compared with either of the two approaches.

2.2. Problem Specification and Optimization Model

The optimal phased array design is to find optimum configuration and array factor. For linear antenna arrays with uniform and nonuniform elements' spacing, design is considered throughout the whole range of scan angle and random value of distance between elements with minimum number of phase shifters. The array factor varies as a function of the distance between element, excitation coefficients, and antenna scan angle. So, the minimax criterion is:

$$\max_{\theta} \min_{N_{psh}} [|AF(d_n \cdot w_n \cdot \theta_{scan})|] \quad n = 1 \dots N_{psh} \quad (1)$$

Suppose that there are m sampling angles belonging to scan range, and minimized SLL is required according to the dined mask. The optimization model can be described as follows

$$\begin{aligned} \min_{n_1 \dots n_{N_{psh}}} E(n_1 \dots n_{N_{psh}}) &= \max_{j=1 \dots m} [|AF(d_n \cdot w_n \cdot \theta_{scan})|] \\ s.t. \quad n_{\min} &\leq n_i \leq n_{\max} \quad i = 1 \dots N_{psh} \\ d_i &\geq \lambda/2 \end{aligned} \quad (2)$$

where n_{\min} and n_{\max} denote the lower and upper number bounds of number of phase shifters.

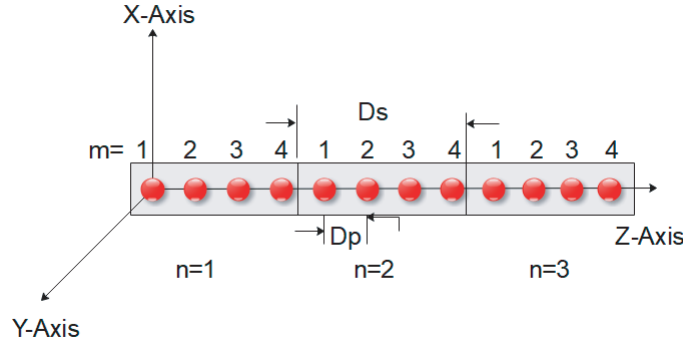


Figure 1. Example of array uniform subarrays, $M = 4$, $N = 3$.

Assume, as shown in Fig. 1, a linear antenna array of isotropic elements located along the z -axis, and the array factor is specified by Eq. (4)

$$AF(\theta) = \sum_{n=1}^N \sum_{m=1}^M a_{nm} e^{j \frac{2\pi}{\lambda} d_m \sin \theta} \quad (3)$$

where $AF(\theta)$ is the synthesized array factor. The general element's excitation coefficient is a_{nm} , and the wavenumber of free space is considered.

As mentioned, the variations in the pattern of the linear array result from the variations in the magnitude and phase of the coefficients, the distance between the elements, and the number of phase shifters. Therefore, these three parameters must be calculated at the end of the optimization, and the efficiency of the algorithm is implemented by the given upper and lower mask with minimum mean square error (LMSE) between the desired and synthesized pattern.

In addition, the efficiency is calculated by a numerical value that characterizes how well the trial solution performs by defining a cost function (CF).

By repeatedly evaluating the CF with different trial solutions, the optimizer adjusts the trial solutions and attempts to converge to the best solution, defined by the solution returning the lowest cost. The CF used here is given by Eq. (4) in which the user needs to determine the limits of the desired pattern as upper and lower masks. So, it should be minimized.

$$score = \min \left[\frac{1}{P} \sum_{i=1}^P (AF(i) - Mask(i)) \right] \quad (4)$$

where P is the number of pattern points that should be sufficiently large enough to cover the variations in the desired pattern. For a given desired radiation pattern, each pattern point that lies outside the specified limits contributes a value to the cost function. In addition, to see finer improvement in score, it is better to use a logarithmic scale. The interaction of the optimizer with cost function to find a fitness solution is shown in Fig. 2.

Based on the Eq. (4), two strategies are available for generating low side lobes to perform amplitude tapering. The first involves conducting an exact taper of amplitude at each of the elements in the array, such as uniform array. The second is available with a uniform primary array for amplitude tapering only at subarray output.

Amplitude weighting at the subarray's ports simplifies the antenna architecture, but the resulting quantized amplitude taper in subarray degrades the side lobe performance. Defining a suitable starting point for optimization is expected to be efficient at convergence speed with lower local response probability.

We choose a close approximation to the optimal amplitude distribution of the uniformly weighted elements of the related antenna elements in each subarray to improve the estimation of the exact element amplitude tapering. The average of the coefficients forms our starting point as shown in Fig. 3. Furthermore, critical approximation of amplitude could change the slope of data to quantize.

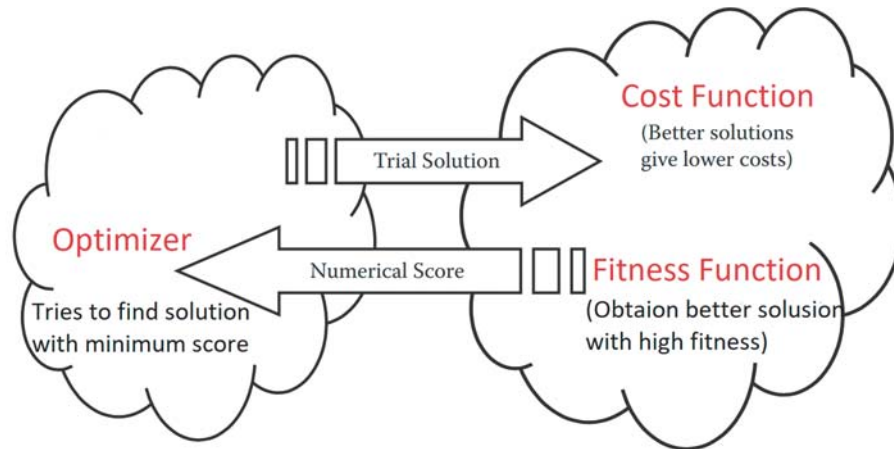


Figure 2. Interaction of optimizer with cost function to find fitness solution.

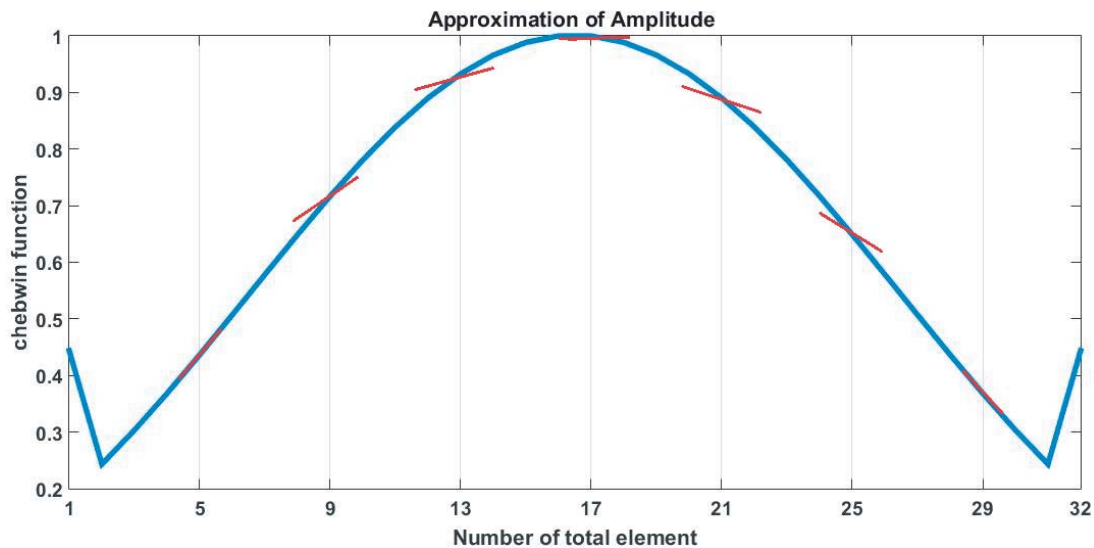


Figure 3. Tapering of amplitude for uniform subarray, $M = 32$, $N = 8$.

3. SIMULATION RESULT

3.1. Hybrid Optimization Algorithm

In our simulation, the linear uniform subarray (USA) with 32 total elements and 8 subarrays (To equate the findings according to [2]) is considered as a case study, which is shown in Fig. 1. Related results are shown in Fig. 4, using the Chebyshev window to evaluate the desired array element. The broadside array factor is a suitable result, as it is obtained by the null primary array product to the secondary array grating lobes. Thus, the grating lobe appears by scanning beam.

One factor to compensate is excitement. Therefore, we use GA, PSO, and GAPSO algorithm to determine this complex coefficient. The cost function considers the same for all of optimization algorithms. The results of the GA and PSO algorithm are shown in Fig. 5. More grating lobes usually rise, but there is the grating lobe on one side compared to the main lobe.

GAPSO is used in order to have hybrid advantages of each algorithm, and the result and the defined upper and lower masks are shown in Fig. 6. The GAPSO algorithm, however, is effective, and the result could be better. The large distance between the elements is the principal reason for the

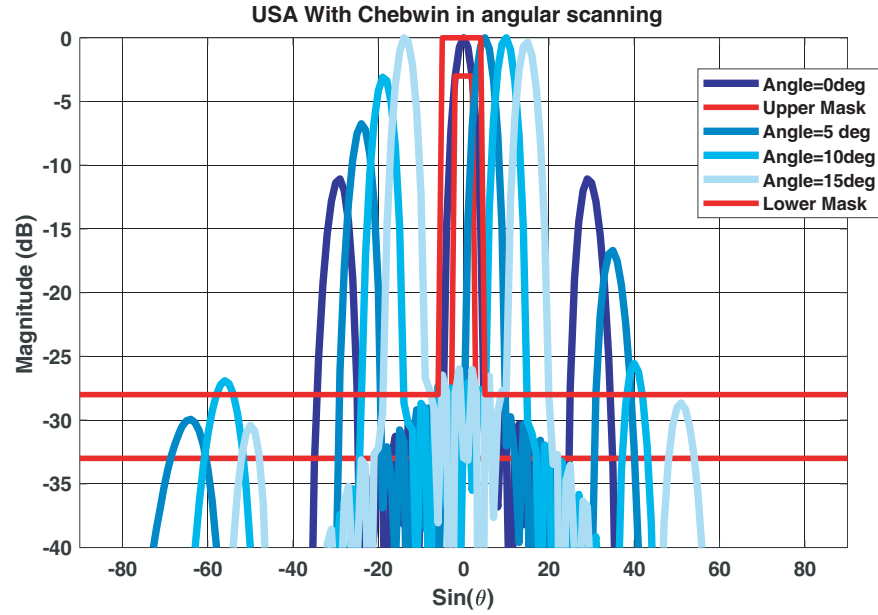


Figure 4. Radiation pattern of uniform subarray ($N_t = 32$, $N_p = 4$, $N_{psh} = 8$) with applying Chebyshev window for a 12. angular scan with a 4° resolution.

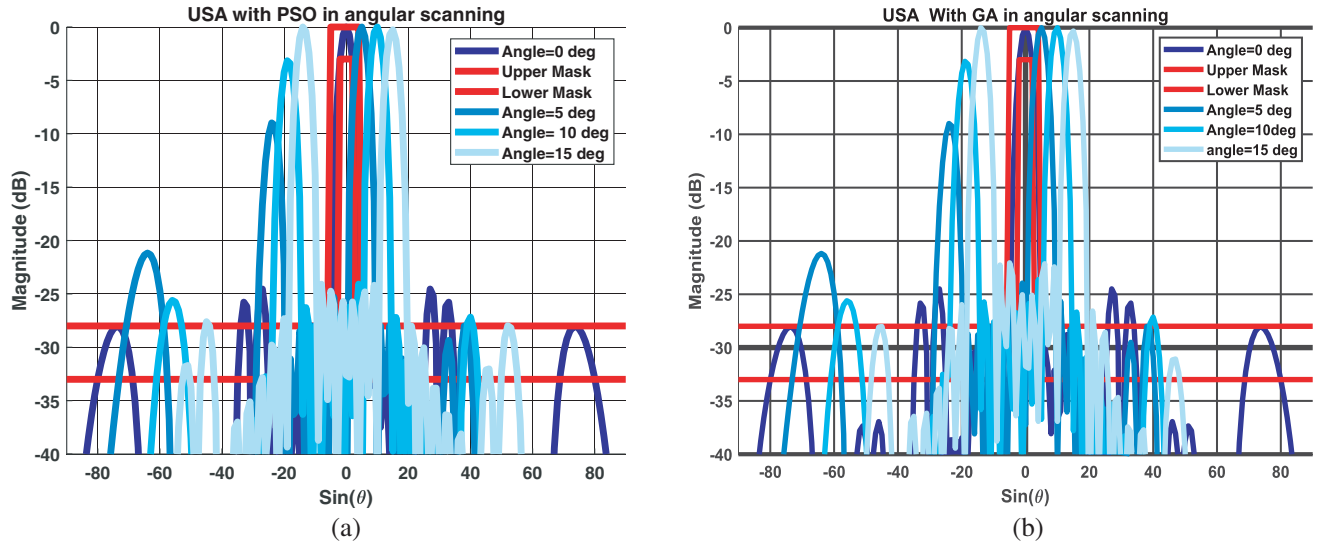


Figure 5. Radiation pattern of uniform subarray ($N_t = 32$, $N_p = 4$, $N_{psh} = 8$) for a 12. angular scan with a 4° resolution. (a) with applying GA and (b) with applying PSO algorithm to amplitude weighting.

occurring of grating lobe. In the mentioned example, the distance between the elements is 2 times of the wavelength. So, although the GAPSO has been applied, the grating lobes have not been eliminated. So, it is necessary to determine the best configuration with complex excitation coefficients.

In the end, the convergence of optimization algorithm during iteration should be shown by describing the cost function's score of the trial solution. For USA the convergence was investigated, and it is shown in Fig. 7, because of allowing the user to see finer improvements as the optimizer converges choosing logarithm scale. The findings indicate that the following is true:

- The GA has fewer iterations in comparison.

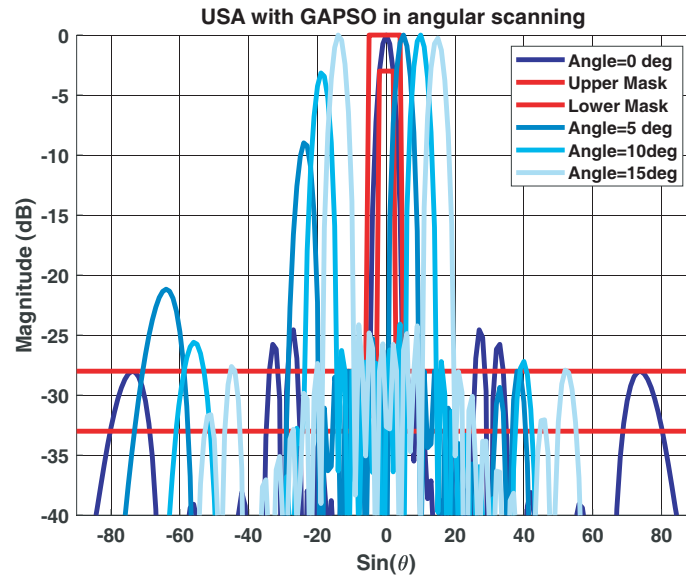


Figure 6. Radiation pattern of uniform subarray ($N_t = 32$, $N_p = 4$, $N_{psh} = 8$) for a 12. angular scan with a 4° resolution with applying GAPSO algorithm to amplitude weighting.

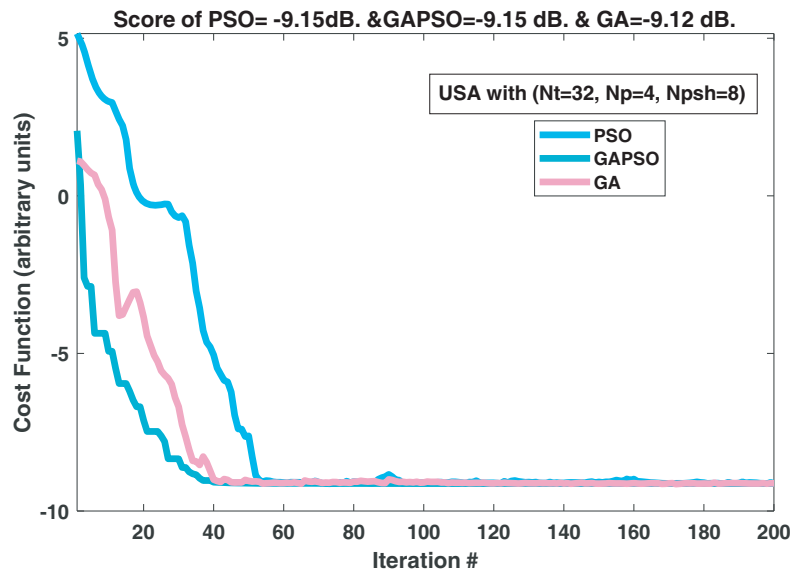


Figure 7. The convergence of algorithms (GA, PSO, GAPSO) of USA with 200 iterations and the same cost function.

- The PSO rapidly converges into the optimal solution.
- Greater query space is needed in the GA.
- The GA has expensive computational cost.
- The PSO has better reliability and accuracy.

3.2. Improvement the Specification of Linear Array Antenna

From Subsection 3.1, we understand that electromagnetic antenna parameters could not be achieved by only using the proper coefficient of excitation. Distance between elements is another key parameter.

Because of the periodicity in distance in USA with minimum number of phase shifters, GL will be accrued in any way.

Firstly, we should choose the desirable configuration of OSA in order to decrease the distance between the elements, or the desirable configuration of RSA this is an aperiodic structure with less distance between elements. In the USA, only one topology exists by specifying the total number of elements (N_t), the number of primary arrays (N_p), and the number of secondary arrays (N_{psh}). Unlike in the USA, when selecting the RSA to boost the array parameters, the optimal solution should be found. There are various combinations of RSA for the particular values of N_t , N_p , and N_{psh} . A sample of OSA topology and RSA (with the total number as USA in the prior section) is shown in Fig. 8.

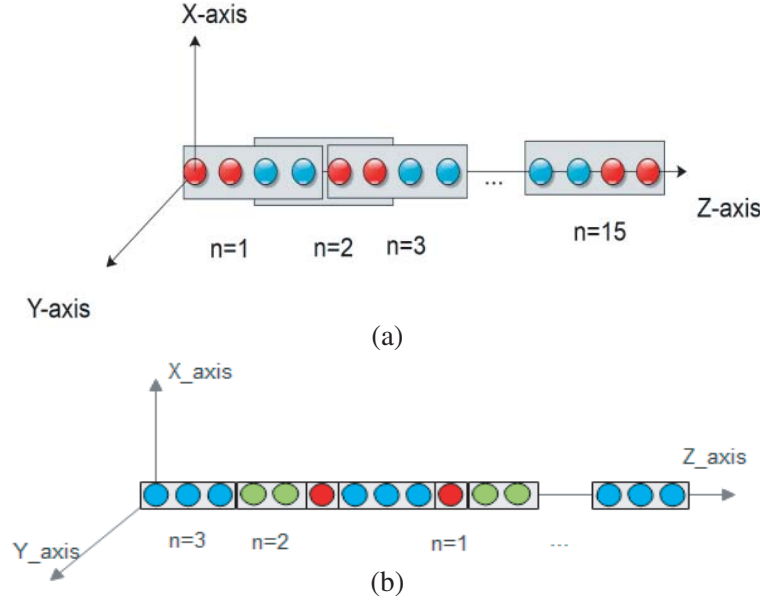


Figure 8. Configuration array with $N_p = 4$ and $M = 32$ (a) overlap subarray with 15 number of phase shifters and (b) random subarray $N_p = [1, 2, 3]$ with 12 phase shifter.

In our simulation, the minimum of two and the maximum of four separate groups can be considered as the primary arrays. If we consider a vector to describe the number of primary arrays ($[a_1, a_2, a_3, a_4]$), each of them can be changed from 1 to 8, and the array configuration can be a combination of the following modes:

1. Combination of four groups: $[1, 2, 3, 4]$, $[1, 3, 5, 7]$, $[2, 4, 6, 8]$
2. Combination of three groups: $[1, 2, 3]$, $[1, 3, 5]$, $[2, 3, 4]$
3. Combination of two groups: $[1, 2]$, $[2, 3]$, $[3, 4]$

To simulate our algorithm, we choose that the primary array groups are composed of $[1, 2, 3]$ elements between which the spacing is half of the wavelength. The desired SLL is considered -15 dB, scan up to $\pm 27^\circ$ with 32 elements and 12 phase shifters. If we want lower SLL (20 dB), it can be available with less scanning angle ($\pm 16^\circ$). Then, according to the result of USA, GAPSO is considered to determine the excitation coefficient to obtain the desired array factor while optimizing the distance between elements by different combinations to form RSA. Only amplitude control of each element in primary array and amplitude and phase control of elements in secondary array are assumed. The maximum and minimum levels of the amplitude and phase are defined before running the algorithm. The total array factor for RSA as the product of the primary array and secondary array and the results of beam scanning are shown in Fig. 9 for both the GAPSO algorithms. In general, a maximum scanning with 4° and 2° resolutions is considered to be 27° . GAPSO algorithm's analysis has a similar response to the Chebyshev window in broadside, but for angular scanning this does not occur. As shown that the Chebyshev window uses additional grating lobe also occurs. Unlike this, the result is appropriate in

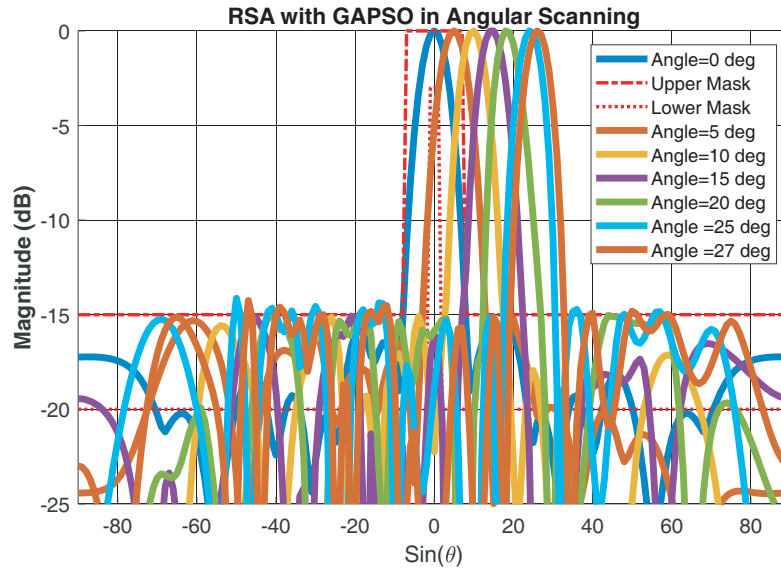


Figure 9. Radiation pattern of RSA ($N_t = 32$, $N_p = [1, 2, 3]$, $N_{psh} = 12$) for a 27° angular scan with a 4° and 2° resolution with applying GAPSO algorithm to amplitude weighting.

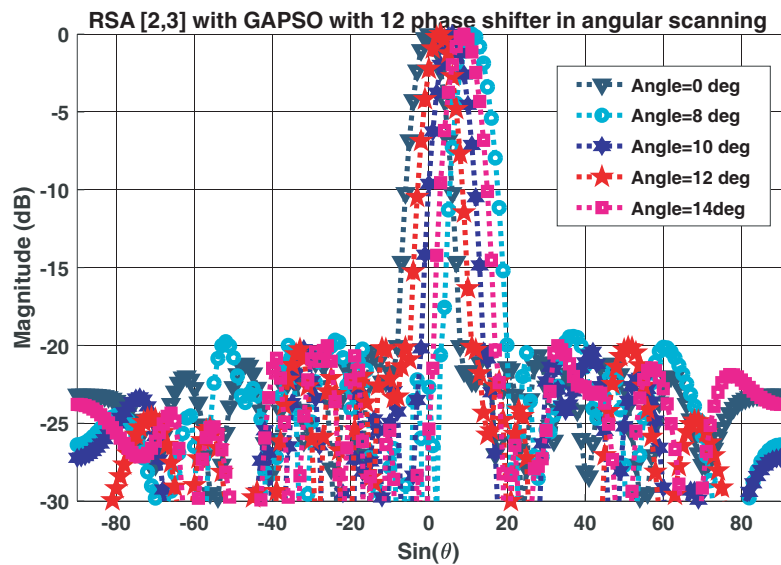


Figure 10. Radiation pattern of RSA ($N_t = 30$, $N_p = [2, 3]$, $N_{psh} = 12$) for a 14° angular scan with a 2° resolution with applying GAPSO algorithm to amplitude weighting.

another angle as well with GAPSO, because of the possibility to provide complex weight with GAPSO algorithm and to optimize the topology array simultaneity. As a result, the array factor by reducing the number of phase shifters is obtained by the GAPSO algorithm for RSA.

Certain combinations of primary array are considered to demonstrate the validity of our algorithm, and the results are clarified in Fig. 10 and Fig. 11.

Also, if we need a lower SLL, Fig. 10 shows that we can obtain a topology with an excitation coefficient and the same number of phase shifters as it has less SLL than previous references. Fig. 11 shows that with our procedure, a wider scanning angle with SLL equal to -15 dB can be accessed based on the references.

In comparison, according to previous papers, our method will boost parameters such as SLL and

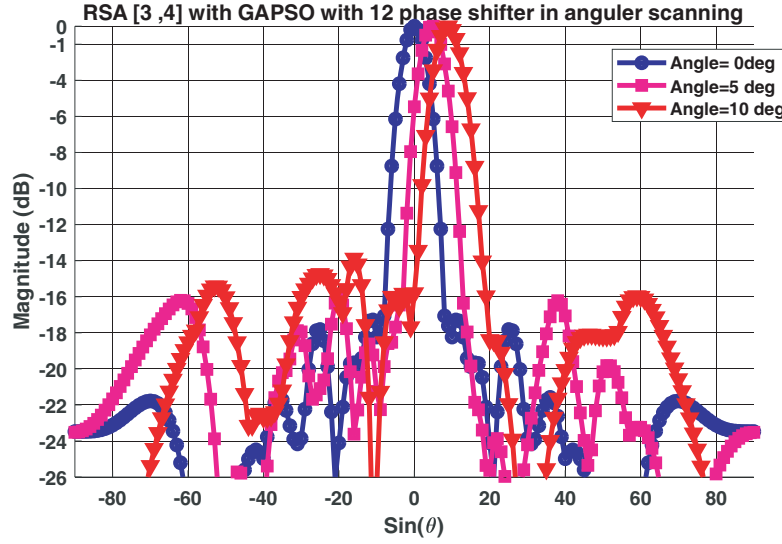


Figure 11. Radiation pattern of RSA ($N_p = [3, 4]$, $N_{psh} = 12$) for a 15° angular scan with a 5° resolution with applying GAPSO algorithm to.

Table 1. Comparing different topology with our algorithm and other references.

	our	Ref. [2]	our	Ref. [2]	our	Ref. [17]
N_t	30	30	30	30	42	42
N_p	[1, 2, 3]	[1, 2, 3]	[2, 3]	[2, 3]	[3, 4]	[3, 4]
N_{psh}	12	12	12	12	12	12
SLL (dB)	-15	-15	-20	-15	-15	-15
Scan angle	$\pm 27^\circ$	$\pm 21^\circ$	$\pm 12^\circ$	$\pm 12^\circ$	$\pm 15^\circ$	$\pm 8^\circ$

scanning angle with the same number of phase shifters. As shown in Table 1, the results obtained from our approach are compared to previous ones, and the proposed method is more effective in determining the topology and excitation coefficients of the aperiodic linear phased array antenna with minimum number of phase shifters.

4. CONCLUSION

An efficient algorithm based on a combination of the GA and PSO algorithm (GAPSO) is presented in this paper. The algorithm is efficiently used to reduce the grating lobe with estimating the excitation coefficients of the antenna array and configuration of the array. The proposed algorithm provides a strong synthesis tool for shaping array factor. The two distributed subarrays antennas, as the USA and RSA, have been considered, and the simulation results are shown. Finally, by applying the GAPSO algorithm to RSA, minimum grating lobe during beam scanning could be acceptable. It is also characterized by fast convergence and high accuracy compared to other weighting functions.

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