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GPR Data Regression and Clustering by the Fuzzy Support Vector Machine and Regression

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Abstract—In this paper, the problem of determining the depth and radius of a circular pipe along with the soil characteristics is studied, using electromagnetic waves with a fuzzy support vector machine as well as a fuzzy support vector machine. To this end, three neural network based fuzzy support vectors are used to determine the soil, depth, and dimensions. Also, using the 2D time domain numerical simulations of electromagnetic field scattering, along with MATLAB software, 1030 data are generated for training as well as neural network verification. Given the fact that for each of the three parameters the nature of the problem is different, separate neural networks are considered with different parameters, thus the number of different data for the network training is considered. In all three cases, the neural network parameters are optimized using genetic algorithm to reduce the error and also reduce the number of support vectors. It should be noted that the objective function of the genetic algorithm consists of two components of the error, as well as the number of membership functions, which can be determined by determining a control parameter. For soil permittivity, the algorithm can accurately predict 93% of permittivities, and it decreases to 89.8 for the pipe depth determination. For diameter it is seen that for 69.3 of the cases the algorithm can correctly classify the pipes.

1. INTRODUCTION

Ground penetrating radar (GPR) has been considered as a non-destructive evaluation tool. In this type of radar, electromagnetic waves are sent from the transmitter antenna to the ground, and by comparing the wave in the receiver with the transmitted signal, it attempts to find the sub-layers. The details of the GPR structure and signal processing can be found in [1–3].

Due to the obscure effect of various factors in the signal received from the GPR (such as soil characteristics, depth, and physical dimensions of buried objects, noise), automatic treatment of the signal is a complex task. The problem becomes more complicated for low depth objects, in which the return signal is close to the strong direct signal. In this case, the ordinary signal processing algorithms fail in obstacle determination. Currently, this work is carried out by experts operators. The training of such people is an expensive task.

Proper clustering of these signals can reduce the complexity and increase the accuracy of the processing. By clustering the processing accuracy increases, and operator training costs are also reduced. A straightforward method is to assume high-order auto regressive time series model as:

$$r(t) = \sum_j \alpha_j s(t - \tau_j) + n(t) \tag{1}$$

for the received signal, to find the delays and coefficients, where $r(t)$, α_j, $s(t-\tau_j)$, and $n(t)$ are received signal, loss, delayed transmitted signal, and noise, respectively. Because delays (i.e., τ_js) are in the

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range of ns, sampling and digital analysis of these types of signals require nano-second scale high-speed A/D converters. Local maximum points of the cross-correlation function of the transmitted and received signals indicate the delays. However, because of the stationary nature of the received signals, the signal can be reconstructed and analyzed using slow A/Ds in several periods of time. Yule-Walker algorithm may be classified in this category [4, 5].

In practice, this method requires high frequency bandwidth for data analysis. However, when the resolution of the layers is negligible or for the thin layers, the time series method loses its effectiveness. So other methods of signal analysis are taken into account. Also, in practice, due to the low speed ADCs, the number of received signal samples is low, which mathematically reduces the accuracy of the time series method. Therefore, it is necessary to look for methods that are able to extract the information from the signal with a limited number of samples.

The first candidate in this regard is the array processing methods. Array processing techniques employ multiple antennas at the receiver. The receiver receives highly correlated signals, allowing the signal to be processed quickly and accurately with a low number of received samples. For example, using the MUSIC algorithm, Thomas and Roy have attempted to calculate the thickness of the coal layer [6].

Batard et al. have also attempted to determine the thickness of asphalt using ground penetrating radar data and also compared the array processing algorithms with Yule-Walker algorithm [7]. It is found that array processing algorithms generally perform better than Yule-Walker algorithm. Among the three array methods presented, the ESPRIT method exactly determines the delays, and the noise effect is completely eliminated.

Shrestha and Arai have used the MUSIC array processing algorithm with Fourier transform to process the data. It should be noted that the algorithm has been applied on the FMCW radar. The algorithm has demonstrated the overwhelming advantage of array processing methods over direct methods. It seems that for the direct methods for near targets, the waveforms overlap, which is less likely for array processing methods [8].

However, array processing methods increase the complexity of the receiver. Also, the processing time in the receiver increases, which makes the system slower and in real-time practical applications interrupts the device users. In addition, array processing methods need to calculate and process (compute eigenvalue and eigenvector) the correlation matrix between the sensors, which increases the processing load and reduces the program speed.

In recent years, artificial intelligence based methods have been considered as an alternative for time series methods. These methods are either independent or in combination with the time series method [9]. Pan et al. have used a combination of time series forecasting methods (forward-backward linear prediction method (FBLP)) and support vector regression to solve the problem for continuous wave radars. The support vector regression (SVR) is robust to small number of samples. Radar signal is considered as time series of finite order, and series coefficients are obtained by SVR. The SVR function is formulated for time series coefficients. The comparison of the simulation results for a sample problem reveals that there are advantages of FBLP-SVR over the alternative method (FBLP) in the resolution and required samples for the signal processing [9].

Xie et al. have used a support vector machine approach to estimate the locations of the cavities in concrete for experimental and simulated data. They have used simulation to generate training data and compared simulated results with experimental data. For the support vector machine, four linear, Gaussian, sigmoid, and polynomial functions have been utilized. The average accuracy of the method is 78.35, 88.90, 21.23, and 65.32 percent for the experimental data, respectively [10].

Williams et al. have used a support vector machine with a Markov chain to detect gaps in the ice layers. The Markov chain is first used to process data and identify suspicious locations in images for subsequent processing. The Markov chain output is then applied to a support vector machine to determine the location of the layers (horizontal distance and depth). The paper reports an error probability of 0.0007 for 129 experiments [11].

Zou and Yang have attempted to find disturbances within the asphalt on the airport road using a support vector machine. Results for the three sets of data report the accuracy of the methods 97.9, 95.9, and 93.9 and the mean accuracy of 95.9, and compared to the conventional neural network with 87% accuracy, the results showed that the support vector machine method was more accurate. The

Gaussian kernel has been used [12].

El-Mahallawy and Hashim have used a support vector machine method with discrete cosine transform to cluster materials. First, by applying the cosine transform on the signal, the signal characteristics are extracted and applied as input to the support vector machine. It has been found that the noise-free signal is able to achieve a 100% true detection rate, and the detection probability in the presence of noise decreases. It should be noted that the number of features used by the cosine Fourier transform must be optimally selected. Selecting more or less attributes results in an error in the method [13].

Bastard et al. have used support vector regression to determine the delay and material characteristics. The paper argues that using the super-resolution capabilities of the support vector regression method can prevent the distractive effects of the correlated echo signals. The Gaussian kernel is used for the support vector regression. The problem is solved for two cases with interfering and non-interfering echoes. It has been found that for non-interfering echoes the results are slightly better than interfering echoes, although they are acceptable in both cases [14].

Shao et al. have proposed an automated classification method for GPR signals in the ground, with the aim of examining the effect of gravel and sand around railway tracks. Signal analysis is performed in the frequency domain. It is observed that for the gravel bed with 50% clay, the maximum signal frequency spectrum is lower than that for the clean gravel bed and 50% coal bed. Also in the frequency range 800 MHz to 1200 MHz, the coal bed signal loss rate is higher for most other platforms. The analysis is based on three steps of preprocessing, feature extraction and class based on the support vector machine. In the preprocessing the DC signal is removed, and harmful interference signals are separated from the main signal. In the feature extraction, distinct frequencies are first separated from the signal after Fourier transform and signal normalization. At the end of the extraction phase, the local maximum features are extracted at the specified frequencies. It is claimed that for the number of frequencies equal to 17, the support vector method is able to classify correctly at 100% rate [15].

In addition to the above references, the application of neural networks to analyze GPR data has been recently considered. The use of SF-GPRs, neural networks for non-metallic pipe detection [16], pipe crack detection algorithm [17], identification of concealed targets inside the book [18], and object location classification [19] are some examples in this regard. Dumin et al. provided a brief review in this regard [20].

The purpose of this paper is to present a suitable procedure for designing neural networks based on a fuzzy support vector machine and fuzzy support vector regression. The article is arranged in 5 sections. The second section will provide a formulation appropriate to the problem. The appropriate membership function as well as the method of extracting the appropriate attributes from the GPR signal will be presented. The third section will be devoted to the algorithm implementation procedure. Section 4 also presents the results of the algorithm implementation. Finally, the conclusion of the paper will be provided.

2. PROBLEM FORMULATION

2.1. Support Vector Machine and Regression

The support vector machine is a neural network with supervised learning and is capable of performing clustering (linear and nonlinear using kernel tricks) and regression. A brief history of the evolution of the method is given in [21]. Since 1936, many researchers have been involved in the evolution of the method. However, in most sources, Vapnik and his co-workers provide most of the role that in 1963 presented the standard formulation for the method. Of course, the relevant references were originally written in Russian, with the 1976 English translation of the method presented. The aforementioned researchers in 1992 proposed the use of kernel tricks for classification as well as regression of nonlinear functions. The formulation proposed by Cortes was improved in 1993.

The purpose of binary linear clustering is to group the points in the dimensional P dimensional space using a dimensional $P-1$ hyperplane. Obviously, infinite number of hyperplanes can be found to do this. The purpose of the support vector machine is to find the optimal hyperplane for performing the clustering operation. The optimal hyperplane is the one with the maximum Euclidean distance from all members of each cluster. In this regard forbidden area is assumed. No data should be in the restricted

area. By properly formulating the problem of finding the optimal hyperplane, it becomes a convex bounded optimization problem which can be solved by the Lagrangian multipliers method. The points that lie on the border of the forbidden area are considered as support vectors, and we only need these support vectors to find the characteristics of the hyperplane. None of the data can be located in the forbidden area. In practice due to the noise and measurement errors, the hard support vector machine method is not able to perform the clustering properly. The first step in solving the above problem is to provide the possibility of a limited violation in the formulation of the support vector machine, which is commonly called soft support vector machine. Also with the kernel trick and application of nonlinear maps, it is possible to cluster the space with the curved hyperplane.

The support vector machine for clustering can be extended to fit the curve or approximation of functions. This topic is known as the support vector regression method. The purpose of the problem is to find a function that maps the input vector to the scalar output. The problem was initially formulated for linear regression. Using kernel tricks, the problem can be generalized to approximate nonlinear functions.

2.2. Fuzzy Support Vector Machine and Regression

The problem of support vector machine as well as support vector regression is the error due to noisy training data. This problem can be solved by fuzzy logic by defining membership for each training datum. Membership is a measure of confidence that a datum belongs to a class. With slight modifications to the support vector machine formulation and support vector regression, membership functions can be applied to the formulation. It should be noted that various formulations for the support vector machine are mentioned [22–26]. Due to its simplicity of the formulation and similarity to the standard formulation, the formulation introduced by Wu etand Yap is used in this paper [26]. The probability of determination of data with low membership as a support vector is very low. By this approach and proper membership function, the noise effect can be mitigated. A similar idea can be utilized to the support vector regression.

The success of fuzzy methods to capture outliers depends on the selection of appropriate membership functions for the training data. Several functions are recommended in this regard. The membership functions can be divided into two categories of general membership functions and application-oriented membership functions [27]. In general membership functions, the membership of a datum is determined based on its distance from the center of the cluster or hyperplane. To the points close to the center of the cluster (or hyperplane), the larger memberships are assigned, and the other data play less role in the support vector training process.

An application-oriented approach to membership function is considered for application. Initially, non-fuzzy support vector is applied to the data, and operational error is extracted. Then the corresponding error is applied to the following membership function:

$$\mu(x_i) = \alpha + (1-\alpha)\tanh(\gamma|e_i|) \tag{2}$$

where α and γ are constants to be heuristically determined. e_is are misclassification or mis-regression penalty. Three parameters appear in membership function, which will be properly assigned. Fig. 1 depicts the membership versus error for various parameters. The proposed membership function has the following properties. When the function argument tends to zero, the membership function tends to α. It should be determined in such a way that the role of the data with negligible regression error can be ignored. In this case, with the steep slope of the membership function, it responds more to small errors. For large error, this function tends to one. Therefore, the role of data with large error becomes more pronounced.

2.3. Feature Extraction

The sensor output in a period consists of a large number of samples, of which the samples due to the soil type, pipe location, and pipe diameter occupy a small fraction of time interval relative to the entire time interval. Proper selection has a great impact on the accuracy of measurements. Studying sample figures, it is observed that the return signal presents finite time windows. The time delay of the first

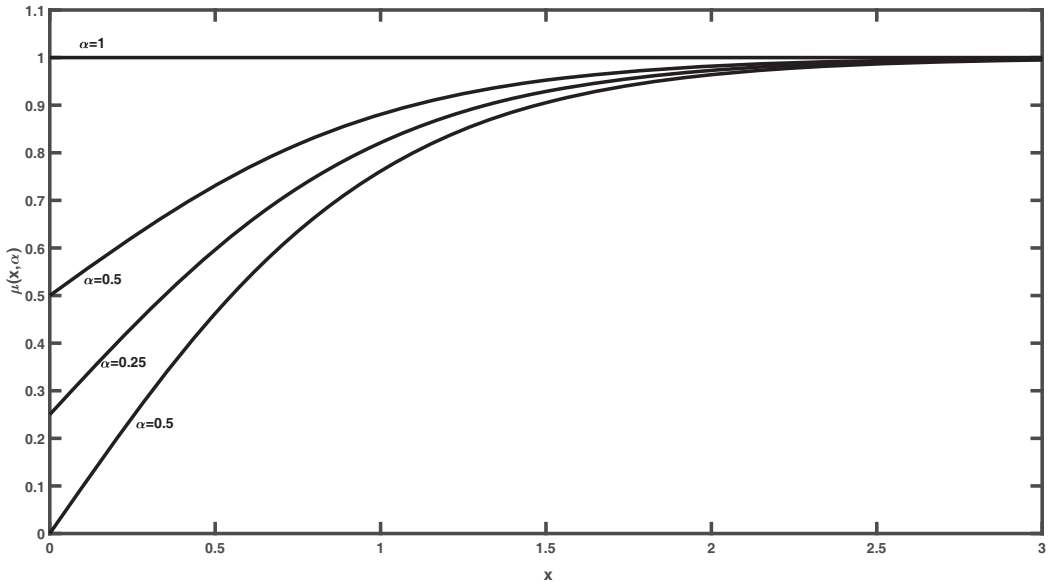

Figure 1. The proposed membership function.

window can represent the soil type. The second window represents the return signal of the pipe, the location of the pipe beneath the soil. The width of the second signal depends on the target diameter.

Therefore, it seems that the location of the average signals in the window as well as the signal width in the window can be suitable parameters for determining the characteristics of the targets. Given the Gaussian shape of the transmitted signals, the Gaussian mean (μ_{w_i}) and standard deviation (σ_{w_i}) of the measured signal (i.e., $E(t)$) is suggested as a suitable characteristic for fuzzy vector machine input.

$$\mu_{w_i} = \frac{\int_{w_i} t\,|E(t)|}{\max(|E(t)|)} \tag{3}$$

$$\sigma_{w_i} = \sqrt{\frac{\int_{w_i} (t-\mu_{w_i})^2\,|E(t)|^2}{\max\left(|E(t)|^2\right)}} \tag{4}$$

Therefore, the signal reception interval is subdivided into a number of sub-intervals. When the signal is only inside a window, the non-zero mean represents the target location, and the non-zero standard deviation can represent the target diameter in the window. In the absence of a signal in the window, the mean and standard deviation are obviously zero. If the target is located in adjacent windows, the mean and standard deviation of both windows are non-zero, which may indicate the target between the two windows. Obviously, the smaller the windows are, the greater the resolution of the measurements are, and consequently, the accuracy of the measurements is. This can lead to an increase in the number of attributes. Therefore, the neural network structure becomes more complex, and large amounts of training data are required to train the network, which is not desirable. Increasing the window size also reduces the resolution, which reduces the accuracy of the measurement. Therefore, to determine the size of the window, it is necessary to make a trade-off between the number of training samples and the accuracy of the measurement.

3. ALGORITHM IMPLEMENTATION

In order to produce training data, circular pipes with diameters of 5 to 20 cm at depths of 0.5 to 2 m in clay, sand, and mixed soils are considered. Pipe diameter, pipe depth, and soil type are considered

randomly with a uniform statistical distribution. Ground penetrating radar signals are simulated using time domain finite element method in MATLAB. MATLAB simulations are carried out on a computer with a 3.4 GHz processor and 16 GB memory. 1100 training and validation data are extracted. The whole process of getting data on the above computer took about a week (152 hours 34 minutes 18 seconds). The transmitted pulse is a Gaussian pulse-modulated centroid with a frequency of 250 MHz and a period of 8 nanoseconds with which the pulse is expected to detect targets at a depth of 2 m and a diameter of 20 cm [1]. The pulse repetition period is set at 80 ns. Each simulation period generates 1000 samples of the signal.

It is focused on determining the three parameters of soil permittivity, target depth, and radius of curvature of the target. For this purpose, three support vector machines have been trained. Fuzzy support vector regression is utilized to determine soil permittivity and depth. The support vector regression is unable to determine the radius of curvature, and in this case the support vector machine is used to classify the pipe diameter (diameter greater or less than 10 cm). In each case, the results of the fuzzy neural network are compared with the results of the conventional network. Linear kernel is used to determine the soil permittivity and also to classify the targets according to the radius of curvature. To calculate the target depth, the Gaussian kernel is utilized.

The genetic algorithm is used to determine the network parameters, i.e., C fine, ε, window size, and membership function parameters (α and γ). The objective function of the genetic algorithm is set to minimize computational error for training data and try to minimize the number of support vectors to maintain the generalizability of the network. For complex network, large amount of training and test data is required. It is considered as

$$\left(\sum_{i=1}^{N}|t_i - y_i|\right) \times \left(0.5\left(\frac{S}{S_d} + \frac{S_d}{S}\right)\right)^{\zeta} \tag{5}$$

where y_i and t_i are the network outputs and the desired values, respectively; N is the number of training data; S and S_d are the number of the actual and desired support vectors. ζ is a control parameter that can be used to trade-off between the error rate and the number of support vectors. The expression inside the bracket is minimized when this value is equal to one. If $\zeta = 0$ is considered, the expression in brackets has no effect on the minimization process of the genetic algorithm, and if it is greater than zero, the support vectors approach the desired value. It should be noted that for the implementation of the genetic algorithm we have used the GUI of OPTIMTOOL from MATLAB software.

4. RESULTS AND DISCUSSIONS

4.1. Soil Permittivity

As mentioned before, the soil is determined by the amount of relative permittivity. It is 7 for dry sandy soil and 30 for wet clay soil, and depending on the grain it varies between two above mentioned values for other types of soil. Two neural networks based on support vector regression have been trained in this regard. In the neural network, the usual training method with linear kernel functions is used. Based on the computational error of the first non-fuzzy network for training the second network data, the network data are assigned membership values. In this case, 200 training data would be enough to train network data. In this case, the value of the support vectors, S_d, is set to 50, and the control parameter is set to 6. After optimization for the window, a value of 15.68 ns (196 times the sampling period) is obtained. An error of $\varepsilon = 0.59$ and a penalty parameter of $C = 163$ are obtained.

Due to the small number of samples used for network training, many data are left unused for verification purposes. After training the network for 930 validation data, the performance of the network is evaluated. Fig. 2 shows the error for 930 samples. From this value, it is observed that for 756 samples the measurement error is below 5%, for 131 samples a computational error between 5 and 10%, and for 43 samples a computational error greater than 10%. For the five samples (half percent of the cases), the computational error is greater than 20 percent, which seems unacceptable.

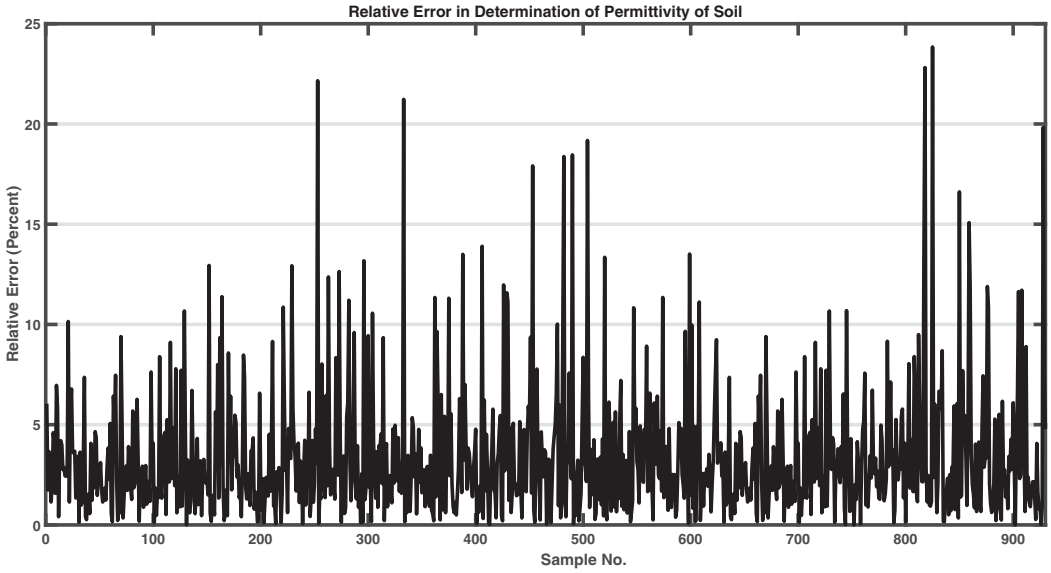

Figure 2. Relative error for soil permittivity determination.

4.2. Pipe Depth

Experience has shown that using the linear kernel function to determine the depth of the pipe results in a high computational error. For this purpose, the Gaussian kernel function with constant width is considered. Large amounts of training are required for the training. So from the 1030 available data, we use 300 measurements for network training and 730 data for network validation. Given that in practical applications for urban installations, the depth of the pipe in cold regions is 90 cm, we consider the depth of pipe from 50 cm to two meters. The simulation data are provided randomly with a uniform distribution in this regard. Genetic algorithm is used to determine the number of support vectors, error value, violation penalty, window size, control parameter, and Gaussian function width. After optimizing the values $S_d = 194$ for the number of support vectors, $\varepsilon = 0.02$ for error, $C = 805$ for violations penalty, 519 samples equivalent to 41.52 ns for the window and 7.3 for the Gaussian function width. The value for the control parameter is $\gamma = 10$.

After training the network, for 730 validation data, the performance of the network is evaluated. Fig. 3 reveals the error for 730 samples. It is found that for 590 samples 80.8% of the sampling error is below 5%, for 66 samples 9%, computational error between 5 and 10%, and for 47 samples, 6.4% is greater than 10%. For 27 samples (3.7% of the cases) the computational error is greater than 20%, which seems unacceptable. Further examination shows that of the 27 tubing depth measurement errors, 17 are for depths greater than 60 cm (between 50 and 60 cm), which is acceptable given the interference of the signals in this error. It should be noted that from the training data, 69 depth samples fall within the above range, with 77% of the depths found with acceptable error. The next 6 samples are for pipe in clay soil 1 m deep and above, which is justified by the high loss of the electromagnetic waves in the clay soil. It should be noted that 288 of the samples are clayey. 4 samples also have depths greater than 60 cm. In this case, it is observed that the crude soil has a dielectric coefficient of less than 5, which interferes with the first signal due to the larger diffusion rate in these types of soils. Second, it causes errors. It should be noted that 33 out of 730 samples are from these soils and 12% of the calculations are incorrect.

4.3. Pipe Radius

It is simply not possible to accurately determine the diameter of the pipe using electric field data. To address this problem, the approach was changed as follows. The purpose of the problem is to cluster the pipes into two large clusters (greater than 10 cm in diameter designated by +1 class) and small clusters (less than 10 cm in diameter designated by class 1). Instead of support vector regression, support vector

machine was used. The criterion of Eq. (5) was adopted to minimize the number of support vectors. 800 data from 1030 available data were used for network training and 230 data for network validation. The linear kernel function was used in the support vector machine.

Genetic algorithm is used to determine the support vectors, violation penalty, window size, and control parameter. After applying the genetic algorithm for the window, a value of 1 ns is obtained, for a delay of 91.3 and for a control parameter of 9.3. The number of support vectors is also 354. The difference between the actual class number value and the network output class number is used to display the network results. Of the 230 validation data, 152 (66.1%) were clustered correctly. For 44 training data (19.2% of cases), the small pipe was clustered as large pipes. This is 34 (14.7%) for large pipes. It appears that the network error for smaller pipes is greater than that for large pipes. Fig. 4 summarizes the results. Further investigation shows that in the validation data, the number of large tubes is 119, of which 75 tubes (63%) are correctly clustered. The number of small tubes is 111, of which 77 (69.3%) are appropriately clustered.

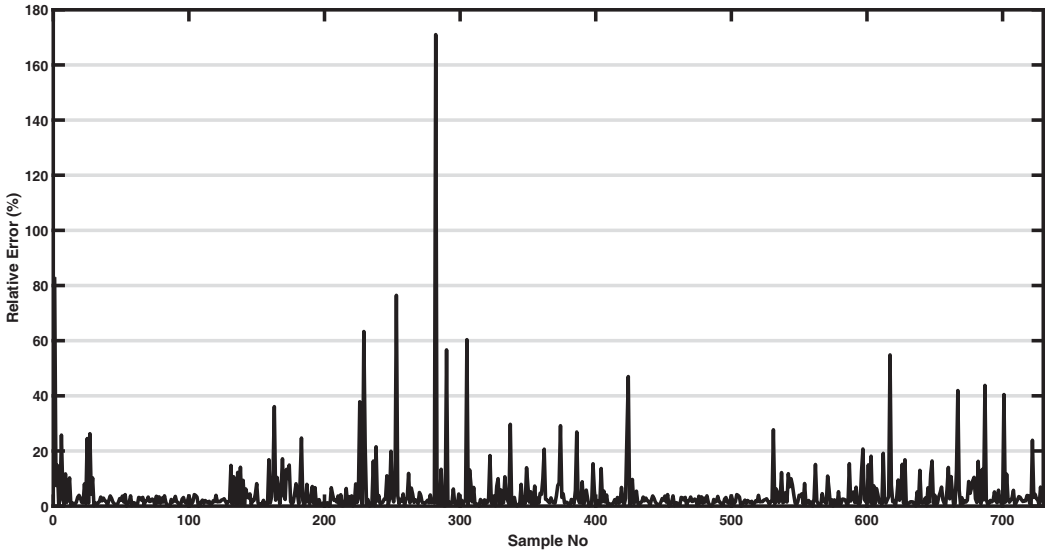

Figure 3. Relative error for pipe depth determination.

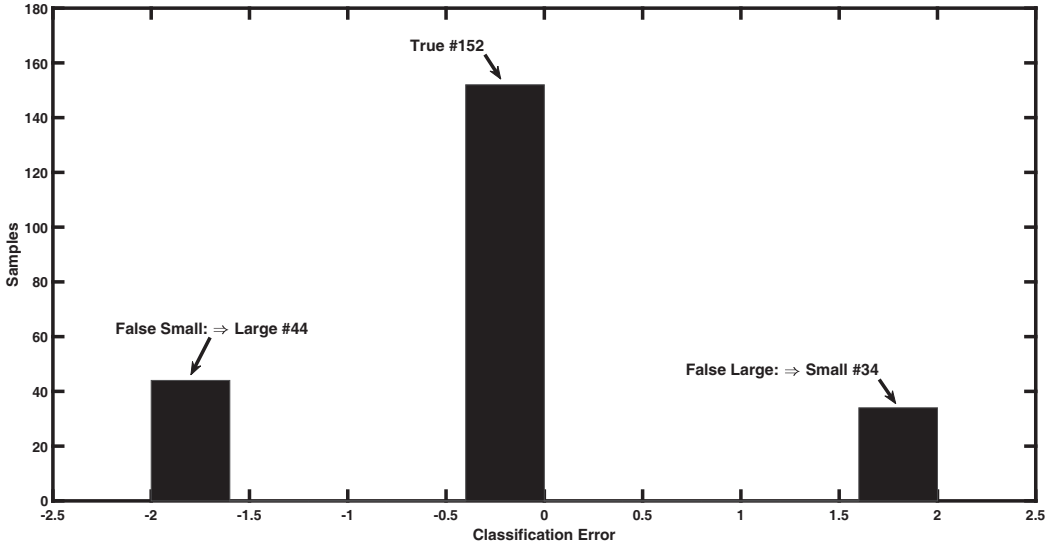

Figure 4. Pipe radius classification results.

5. CONCLUSION

In this paper, the problem of soil characteristic determination and buried obstacle was investigated using optimized fuzzy support vector machine and fuzzy support vector machine. An application oriented fuzzy membership function was proposed in this regard. Genetic algorithm was utilized to optimize the neural network. Fuzzy support regression networks were designed to find the soil permittivity and depth of the underground pipe. Also a fuzzy support vector machine was developed to classify pipe to large (with diameter larger than 10 cm) and small pipes, according to the pattern of the received signal. Simulation experiments reveal that the trained networks usually predict the required parameter with acceptable accuracy.

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