Progress In Electromagnetics Research C, Vol. 104, 171–186, 2020

Friction-Free Permanent Magnet Bearings for Rotating Shafts: A Comprehensive Review

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Abstract—This article presents a comprehensive review of modeling, analysis, and development of permanent magnet bearings (PMB) for rotating shafts. The different configurations of PMB are highlighted with relevant approaches to estimate their features. The progress in mathematical approaches adopted and optimization of the static and dynamic bearing characteristics in terms of accuracy are discussed in depth. Further, key developments on instability issues and their realization in combination with other bearings for rotors stability in low and high-speed applications are reviewed. Finally, concluding remarks on key aspects to be followed in the design and development of PMB are presented.

1. INTRODUCTION

Magnetic bearings, in contrast to conventional bearings, are null-friction bearings wherein the moving part is supported by magnetic forces. The attractive friction-free characteristic of these bearings offers many interesting features such as operation without lubrication, less-maintenance, and long life. Magnetic bearings are categorized as active and passive ones. Active magnetic bearings (AMB) use electrical and electronic devices to levitate the rotors [1, 2] whereas rotor levitation is achieved passively in the absence of external energy sources, electrical and electronic devices in passive magnetic bearings. These bearings are realized with permanent magnets, superconductors, and use of eddy current effect. Superconducting magnetic bearing exploits the reluctance force generated between the superconducting material and the permanent magnets [3, 4]. Generated Lorentz force due to eddy currents in a moving conductor within the constant magnetic field of the permanent magnets is the main cause of rotor levitation in electrodynamics bearings [5–7]. In PMB, the reluctance force generated between two different permeability materials is the source for the bearing characteristics, and generated attractive or repulsive forces are utilized between the magnets for the realization of its property. Baermann [8] introduced the concept of PMB in 1954, and detailed experimentation and analysis was performed for the first time by Backers [9] in 1961. Absence of electrical and electronic components and friction-free features of PMB attracted researchers to use them in both low- and high-speed applications such as fan [10], conveyor [11], wind turbine [12], energy storage flywheel [13, 14], and compressors [15, 16]. In this article, Section 2 details about PMB configurations and modeling methods. In Section 3, we review the progress in modeling and analysis of PMB for force, stiffness, damping and dynamic characteristics using mathematical approaches and finite element analysis (FEA). Finally, the key developments to overcome instability issues and their usage in different applications are detailed in Section 4.

Received 4 June 2020, Accepted 22 July 2020, Scheduled 11 August 2020
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2. PERMANENT MAGNET BEARING CONFIGURATIONS AND MODELING METHODS

A bearing can be easily realized with two permanent magnet (PM) rings, one fitted to the moving component (rotor) and the other to stationary (stator) as shown in Fig. 1. The magnetization of rings can be in the axial or radial directions. Axially and/or radially magnetized rings can be used in combination or separately to form a simple single ring pair PMB as shown in Figs. 1(a)–(c). The classification and synthesis of PM thrust and radial bearing configurations were described in detail in Refs. [17–19]. The magnetic interactions between the rings of a bearing are modeled by using two different approaches namely dipole and surface density methods. In the dipole method, the ring radius is assumed to be infinite by neglecting its curvature, and magnetic interactions between long parallel bar-shaped magnets (magnetic dipoles) are considered as shown in Fig. 2. The interaction forces between the surfaces of the magnets are determined using either Coloumbian or Amperian model in the surface density method [20]. Radially polarized PM ring is represented as two equivalent surface current densities on the top and bottom plane faces in Amperian model (Fig. 3(a)), whereas the inner and outer cylindrical faces of the magnet are as two surface charge densities in Coloumbian approach (Fig. 3(b)). Diploe method is applicable only when the mean air gap radius is considerably larger than the air gap between the rings, and surface density method is best suitable for PM rings having real dimensions.

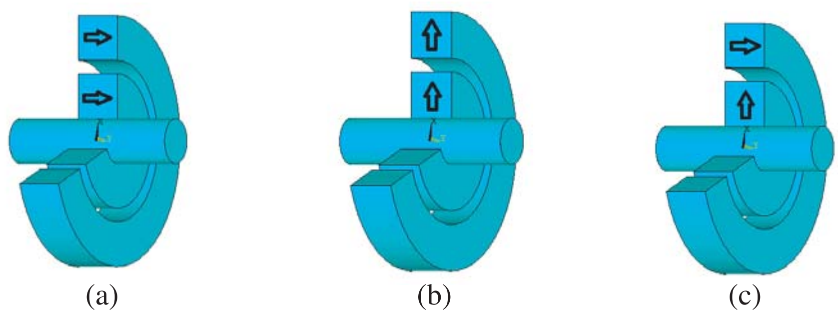


Figure 1. PMB with rings magnetized. (a) Axially. (b) Radially. (c) Perpendicularly.

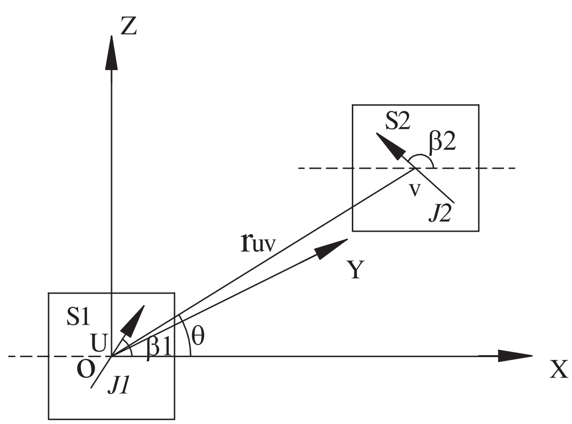
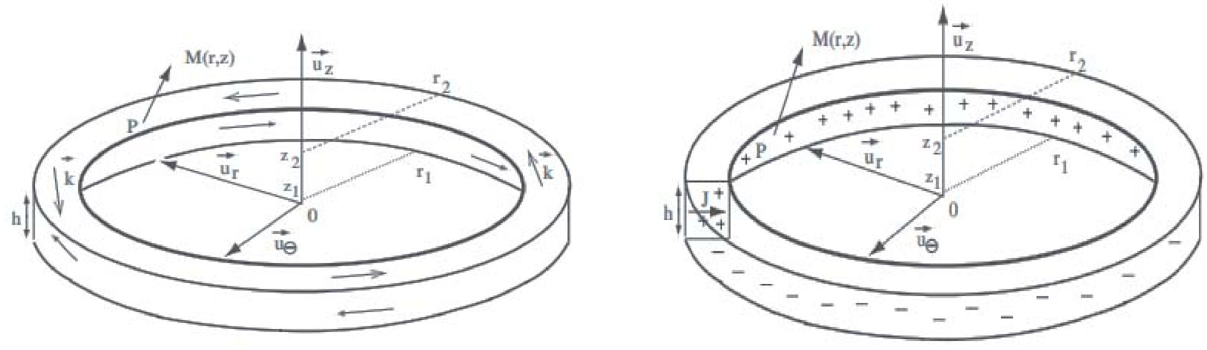


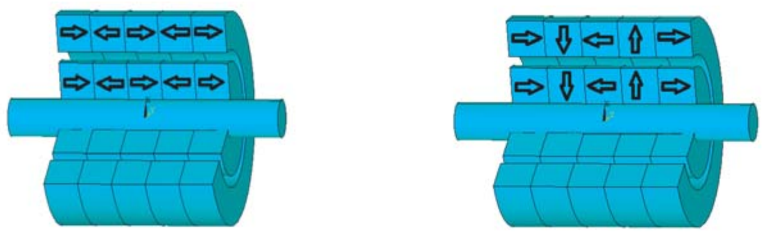
Figure 2. Interactions between cross sections of two long parallel magnets in dipole method [17].

Force and stiffness generated in PMB with only two rings are low, and the enhancement of these limited values was proposed by introducing stacked structures in Ref. [21]. Axially or radially magnetized rings were arranged in opposition to form a conventional structure, and a rotation magnetized direction (RMD) stacked structure was obtained by arranging axially and radially magnetized rings in opposition as shown in Fig. 4.



(a) (b)

Figure 3. Representation of radially polarized ring magnets in (a) Amperian and (b) Coulombian models [20].



(a) (b)

Figure 4. Stacking of rings in (a) conventional and (b) rotating magnetization direction (RMD) structures.

3. EXPLORATIONS ON BEARING CHARACTERISTICS

Permanent magnet bearing is characterized by the interaction forces generated between the surfaces of magnets and the stiffness that is generated due to the movement of one magnet with respect to the other. This section discusses the calculation of force, stiffness, and damping properties of single as well as multi-ring PMB along with mathematical approaches dealing with these characteristics in terms of accuracy and simplicity.

3.1. Estimation of Force and Stiffness Parameters

Force and stiffnesses developed in all basic configurations of axial and radial PMB with two rings can be easily determined with simple two-dimensional (2D) analytical Eqs. (1) & (2) presented in Refs. [17–19] using the dipole method as shown in Fig. 2. A new improved method [22] was proposed for calculating radial stiffness in PMB by considering the effect of small displacement of the rotor using the surface current model (refer Fig. 3(a)). Lang [23] introduced the force and stiffness calculation method with less computational time. A simplified method [24] of determining the axial force between axially polarized two rings PMB using magnetic conductance principle was proposed, and results were validated with experimental results.

$$F_z \cong -p\frac{J_1 J_2}{2\pi\mu_0}\frac{2}{R^3}S_1 S_2 \sin\theta \tag{1}$$

$$K_z \cong -p\frac{J_1 J_2}{2\pi\mu_0}\frac{6}{R^4}\, S_1 S_2 \tag{2}$$

where F_z = axial force, K_z = axial stiffness of the bearing, p = average perimeter of the bearing, K_r = radial stiffness of the bearing, S_1 and S_2 = average cross-sections of two ring magnets, R = average distance between the cross-sections, and J = magnetic polarization of the permanent magnet.

The work of Backers was extended by developing the closed-form 2D analytical expressions for radial force and stiffness in stack structured PMB in terms of curves useful for machine designers [25]. A new analytical solution for thrust characters of a novel stack structured PMB using the current sheet model was reported, and results were validated with FEA and experimentation [26]. While developing 2D equations, researchers approximated rings as two long parallel magnets by neglecting curvature effect [27–29] necessitating the development of three dimensional (3D) equations to increase accuracy in bearing characteristics. Tan et al. [30] proposed a permanent magnetic-hydrodynamic hybrid journal bearing with bearings mounted side by side and presented a 3D equation for the magnetic force developed between two axially magnetized rings. The total force is due to the combined effect of hydrodynamic film and magnetic field. The authors validated theoretical results with experimental values. An attempt was made to increase the radial load and stiffness of radial PMB with two rings by developing 3D equations using surface charge density method [31]. They stressed the concept of stiffness variation with the axial displacement of the rotor magnet and demonstrated the performance of different bearing configurations with different magnet sectors by validating theoretical predictions with experimental results. Further, they have shown the improvement in dynamic performance of the magnetic bearing using hydrodynamic lubrication. Three-dimensional complex semi-analytical equations (Eqs. (3) and (4)) consisting of elliptical integrals [32–34] requiring low computational time for bearing features were presented in PMB having axially, radially, and perpendicularly magnetized rings. They extended the use of equations for analysing stacked structures with three and five ring pairs for one degree of freedom of the rotor magnets (rotor ring magnets are assumed to be concentric with stator ring magnets). Jiang et al. [35, 36] presented the calculation of force, stiffness, and moment parameters in axially magnetized PMB and presented a 5×5 stiffness matrix for axially polarized radial bearings. The 3D mathematical approaches [37–40] for characteristics in PMB made of axially, radially, and perpendicularly magnetized rings by subjecting the rotor movement in five directions (three translational and two angular) are reported to overcome the difficulty associated with solving force and stiffness equations involving elliptical integrals. They used a simple vector approach (using Coloumbian model) (refer Fig. 5) and presented a 5×5 stiffness matrix for stacked configurations with three-ring pairs in addition to one ring pair. FEA results were used to validate the mathematical model results.

$$F_z = \frac{J^2}{4\pi\mu_0} \sum_{i,k=1}^{2} \sum_{j,l=3}^{4} (-1)^{i+j+k+l} \left(A_{i,j,k,l}\right) \quad + \quad \frac{J^2}{4\pi\mu_0} \sum_{i,k=1}^{2} \sum_{j,l=3}^{4} (-1)^{i+j+k+l} \left(S_{i,j,k,l}\right) \tag{3}$$

with

$$A_{i,j,k,l} \;=\; -8\pi\, r_i \,\in\, E\left[-\frac{4\, r_i r_j}{\in}\right]$$

$$S_{i,j,k,l} \;=\; -2\pi r_j^2 \int_0^{2\pi} \cos\left(\theta\right) \ln\left[\beta + \alpha\right] d\theta$$

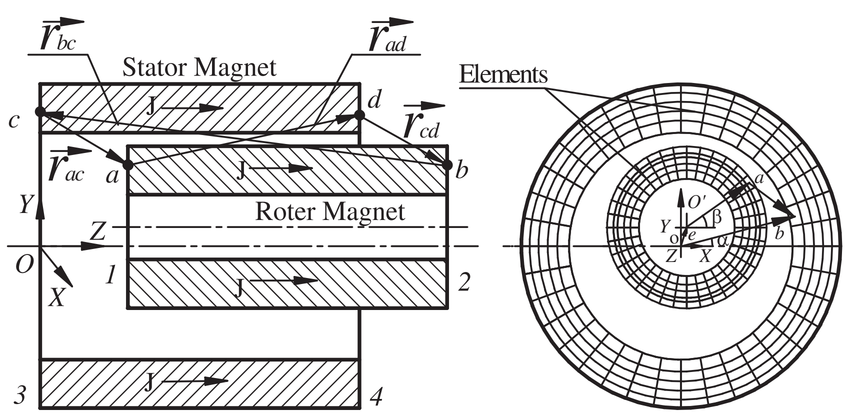


Figure 5. Force calculation model using simple vector approach (Coloumbian model) [37].

where $E[m]$ gives the complete elliptical which is expressed as follows:

$$E\left[m\right]=\int\limits_{0}^{\pi/2}\sqrt{1-m\,\sin\left(\theta\right)}^{2}d\theta$$

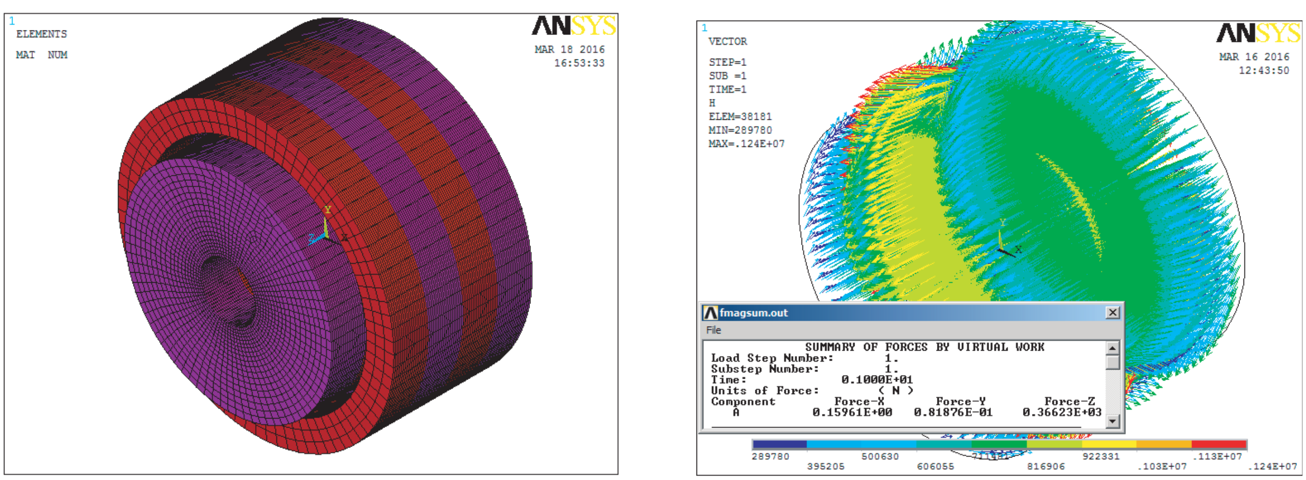
The parameters ϵ, α, and β depend on the ring permanent magnet dimensions and are defined as follows:

$$\begin{aligned}
&\in=\left(r_i-r_j\right)^2+\left(z_k-z_l\right)^2\\
&\alpha=\sqrt{r_i^2+r_j^2-2r_ir_j\cos\left(\theta\right)+\left(z_k-z_l\right)}^2\\
&\beta=r_i-r_j\cos\left(\theta\right)\\
&K_z=\frac{J^2}{4\pi\mu_0}\sum_{i,k=1}^{2}\sum_{j,l=3}^{4}\left(-1\right)^{i+j+k+l}\left(k_{i,j,k,l}\right)
\end{aligned}\tag{4}$$

with

$$k_{i,j,k,l}=-\int\limits_{0}^{2\pi}\frac{r_j\left(z_k-z_l\right)\left(\alpha+r_i\right)}{\alpha\left(\alpha+\beta\right)}d\theta$$

The dependence of bearing characteristics on the number of rings in stacked structures motivated researchers towards the development of generalized force and stiffness equations for PMB made of 'n' number of rings. For the first time, a generalized 2D analytical planar model of force and stiffness in multi-ring PMB was proposed by Tian et al. [41] and Marth et al. [42] based on virtual work principle. They used FEA results to check the accuracy of the analytical model. The proposed 2D equations for multi-rings PMB are modified and utilized by the researchers for the optimization in Ref. [43]. The generalized 3D mathematical model for force and stiffness in multi-ring PMB was presented in Ref. [44] and demonstrated the application of the same model to three types of multi-ring PMB with axially, radially, and perpendicularly magnetized ring pairs. The configurations are modeled (Fig. 6) in the mechanical APDL of ANSYS, and results of axial force were used to validate mathematical model results of all the three structures. Equally distributed sequences based Monte Carlo method [45] was used to solve multiple integrals in force and stiffness equations of multi-ring PMB based on equivalent surface charge method. The results of proposed method were validated with results of FEA and experiments for PMB with four ring pairs.



(a) (b)

Figure 6. Modelling of multi-ring PMB in ANSYS (a) arrangement of ring magnets with different magnetic polarizations, (b) generated axial force in PMB [44].

3.1.1. Optimization of Geometrical Dimensions

Attractive friction-free feature of PMB and higher force and stiffness requirements in both low- and high-speed applications drive the researchers to explore the optimization of geometrical variables for maximum bearing characteristics. The optimization in PMB with single and multi-ring pairs could be carried out in two ways: first, maximizing the bearing properties in the given volume of the magnet; secondly, minimizing the volume of the magnet against force and stiffness requirements of applications. In both types, geometrical dimensions that could be optimized are axial offset, number of rings, inner and outer radii of the stator, and rotor rings. Initially, the optimization of PMB with only two rings was carried out in Refs. [32, 33] in terms of only limited dimensions like axial offset, height, and width of rings by using 3D semi-analytical equations. Results show that separate optimized dimensions exist either for a force or stiffness, and maximum force and stiffness values corresponding to optimized dimensions were highlighted. Analytical equations presented in Ref. [21] for force and stiffness of axially polarized single ring pair radial bearing configuration were modified in terms of several variables affecting the design [46]. They demonstrated an optimization technique with objective function and constraints using the interior trust region method in MATLAB for the given load by minimizing the volume of the magnet. Several configurations presented in the literature were selected as case studies and pointed out the percentage reduction in the volume of the magnet after optimization. A complete and effective procedure of designing and optimizing single ring pair radial PMB was presented in Ref. [47]. The authors proposed generalized equations for different important parameters of the design in terms of the inner radius of the rotor and outer radius of the stator. For the first time, repulsive multi-ring PMB was optimized for maximum radial stiffness in a given control volume by Moser et al. [48] using the finite element simulation method (FEMM package). Results show that optimal bearing does not depend upon its aspect ratio but on the air gap between stator and rotor. The authors provided design curves representing optimized variables for different air gaps in terms of an outer diameter of the bearing for designing and selecting an optimum configuration of bearing in a given control volume. The optimization of axial and Halbach array of PM thrust bearing using 2D equations (derived using equivalent current sheet model) for attaining the required load-carrying capacity in the least volume of magnet was presented in Ref. [49]. They highlighted that the importance of a Halbach array in meeting force requirement with less magnet volume than the axial array and highlighted an existence of an optimal axial air gap between rings in the axial array with the results of analytical, FEA, and experiments. Optimization for all possible topologies of the PM thrust bearing was carried out and reported in Ref. [50] using 2D analytical equations for the maximum force. Optimization of axially and perpendicularly polarized multi-ring PM thrust bearings was presented by the authors in Ref. [51, 52] for maximum bearing characteristics in a cylindrical volume. The generalized plots are provided for the optimum thickness of each ring, inner diameters of inner and outer rings concerning the outer diameter of the bearing. The guidelines were also suggested for optimizing multi-ring PMB configurations in a control volume using presented generalized plots with application examples. The effectiveness of the optimization could be seen by referring Fig. 7 indicating the enhanced values of force and stiffness in the optimized RMD configuration as compared to the only two ring pairs in the control volume. Designers can use these plots for optimization in a control volume instead of complicated 3D equations. A complete optimization technique for axially stacked radial PMB was reported in Ref. [53] to overcome less accurate optimization techniques proposed in the literature. Equations to calculate parameters such as mean radius and clearance were presented for the given axial length and outer radius of the stator for maximum bearing capacity.

Since the force and stiffness are contradicting parameters in the optimization of PMB, multi-objective optimization with a single objective for achieving both maximum force and stiffness was presented by Lijesh et al. [54]. The authors presented the optimization results using constraints, constants, and bounds of the dimensions available in the literature. Optimization was performed for three (single ring pair, conventional stacked structure, and RMD) radial PMB configurations for maximizing force and stiffness and determine the optimum dimensions. Equations to calculate the optimized values of load, stiffness, and design variables in stacked PMB were presented in terms of a single layer. The methodology of optimization for all three configurations was proposed, and the same was validated by discussing different cases from the existing literature.

It has been shown in preceding literature that a significant increase (as compared with conventional

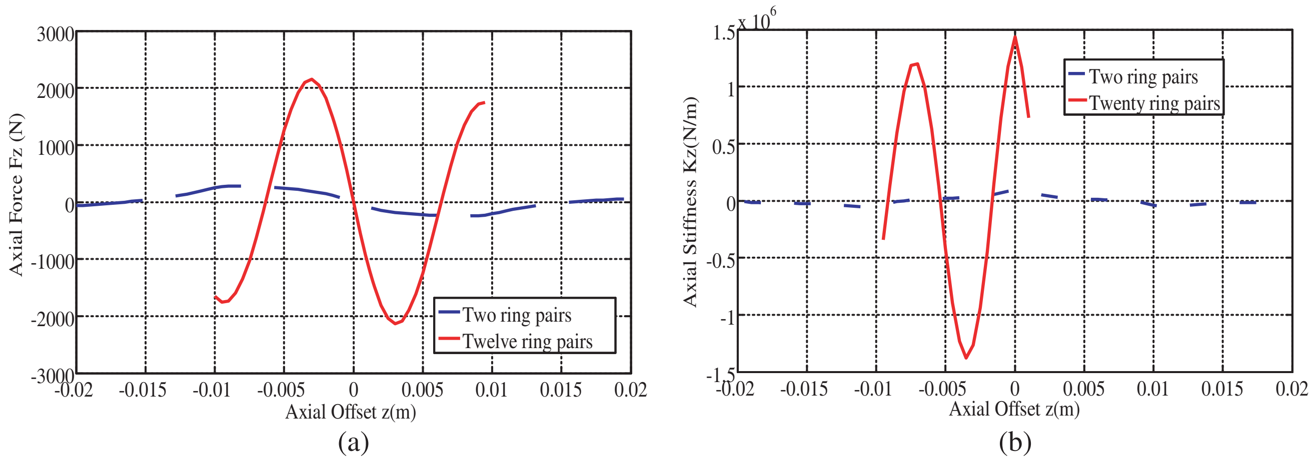


Figure 7. Variation of maximized values of bearing characteristics in the optimized RMD configuration thrust bearing with aspect ratio = 0.75, total axial length = 0.0375 m, air gap = 1 mm, diameter of control volume = 0.05 m. (a) Axial force. (b) Axial stiffness [52].

bearings) in the bearing characteristics was achieved by optimizing design variables in multi-ring radial as well as thrust PMB.

3.2. Estimation of Damping Characteristics

In rotating systems, damping plays an important role in suppressing excessive transverse and lateral vibrations. Despite the advantages of PMB over conventional and AMB, low damping is a major concern, and it is necessary to provide damping to reduce excessive vibrations of the rotor. Damping can be provided by active [55–57] or passive means. Passive damping is simple and can be achieved by including a conductive material in the magnetic space (i.e., eddy current damper) [58–60] or by using viscoelastic materials [61–63]. Eddy current dampers (ECD) are non-contact in nature and damping force is generated due to interaction between the applied magnetic field and the field generated due to the eddy currents formed in the conductive material. This article focuses only on the review of non-contact eddy current dampers used in combination with PMB.

In ECD, the conductive material can be provided either on a stator or rotor. An eddy current damping mechanism in which conductive material provided on the rotor can be modelled by determining magnetic flux density between concentric ring magnets and then damping force between current density and magnetic field. A simple model of an ECD [64] is shown in Fig. 8 in which the radial movement of a spinning conductor (mounted on the rotor) inside a developed axisymmetric magnetic field can be considered. This model is also valid when the conductor is not spinning (mounted on the stator). In [65], the authors presented 3D equations to calculate damping force and coefficients in transverse and

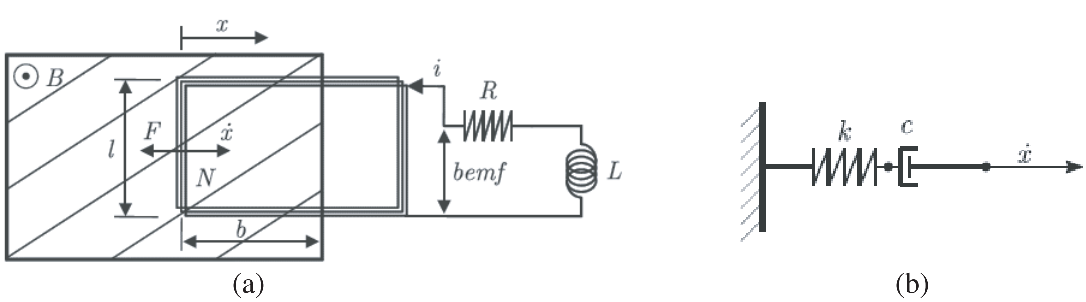


Figure 8. A simple model of ECD (a) a coil of wire in motion in a constant magnetic field (b) equivalent mechanical model of a motional ECD [64].

lateral directions in axially magnetized single ring pair PMB with conductive material on the rotor. The results of the mathematical model are compared with the experimentally measured values and report higher errors between the two.

A passive magnetic bearing with a damping system was proposed and developed for a cup-shaped rotor of a high-speed compressor [15]. A Halbach magnet array and axial passive bearing that is provided inside and outside the rotor cup provides sufficient stiffness as well as damping. An analytical model for calculating stiffness and damping was provided. An optimal design procedure for maximizing axial stiffness and improving the damping coefficient was also suggested. They measured stiffness and damping coefficient experimentally and validated the analytical results. A novel method of modelling and identification of ECD characteristics for rotors supported by PMB was presented by Detoni et al. [64]. The method proposed involves an analytical model and curve fitting with the results of electromagnetic FEA models for determining parameters characterizing for ECD mechanical impedance. The accuracy of the proposed model was verified with the experimental results. The design of compact PMB with enough stiffness and damping was proposed in Ref. [66] using Halbach arrays and eddy current damper. A two-dimensional analytical model for calculating stiffness, axial, radial, and rotating damping coefficients was presented, and 3D finite element simulations results were used to check the accuracy of analytical model results. All design parameters were normalized and optimized for maximum stiffness and damping. The general procedure of designing and optimizing a compact PMB with eddy current damper for high-speed applications was discussed. The inclusion of conductive materials between inner and outer rings of PMB to have a damping effect creates a larger air gap and thereby decreases force and stiffness. In Ref. [67], the authors presented the optimization of standard and Halbach structures with air intervals for including conductive materials for achieving larger stiffness and required amount of damping using 2D equations. Also, they have shown that the stiffness of the Halbach structure does not improve by providing air intervals.

Two dynamic damping systems (one-mass and two-masses) were investigated by Marth et al. [68] for calculating their optimal behaviour theoretically and their realization based on actual specifications and limitations. Correlations between physical parameters and an optimal choice of the same were discussed using the results of dimensionless formulations. Proposed two systems were compared in terms of efficiency and feasibility by considering eddy current and viscoelastic damping examples and suggested that a two-mass system is preferable over one-mass based on a broader range of possible damping.

3.3. Rotor Dynamic Analysis

Permanent magnet bearings and passive dampers are characterized by nonlinear behaviour, and therefore passive magnetic suspension is also nonlinear. Research contributions to gain insight into the nonlinear dynamics of passive magnetic suspensions are discussed in this section.

The passive magnetic suspension was modeled using springs and rare-earth permanent magnets in [69], and it is shown that natural frequency strongly depends on nonlinear magnetic force. Nonlinear frequency-response curves as a soft and hard spring were discussed. The semi-analytical approach was used to determine asymptotic oscillation amplitudes and stability properties. The contact vibrations of a rotating shaft supported by PMB [70] during the passage of critical speed were studied. It was mentioned that the rotational speed of the escapement of the shaft from the contact vibration decreases with an increase in axial displacement. Dynamic characteristics such as amplitudes of vibration at different frequencies and orbit plots of a rotor supported by RMD configuration were reported in Ref. [71]. In RMD configuration, radially polarized rings were replaced by square-shaped magnets and aluminum ring, and the static and dynamic characteristics of the proposed structure are compared with those of conventional stacked structure. They demonstrated the superiority of the proposed structure over conventional concerning dynamic performance (Fig. 9). Dynamic modelling of a vertical rotor supported by two stack-structured radial PMB and one axial AMB with viscoelastic damper was carried out in Ref. [72]. Rotor dynamic analysis was performed by taking into account the nonlinear behavior of viscoelastic damper. Optimization of the entire rotor system was carried out, and analytical model results were verified by conducting the experiments on the manufacturer's test system.

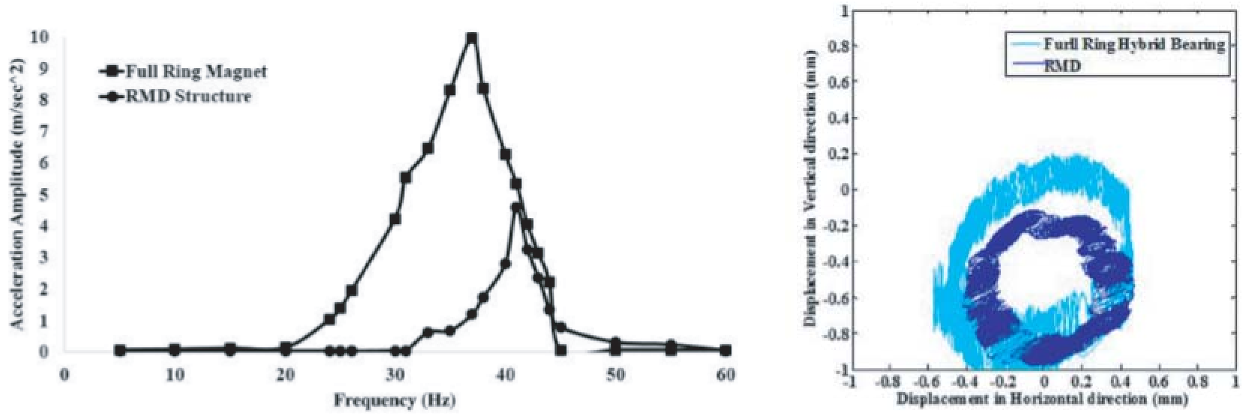


Figure 9. Dynamic characteristics of proposed RMD and conventional full ring structure. (a) Acceleration at different frequencies. (b) Orbit plot [71].

4. DEVELOPMENT OF PERMANENT MAGNET BEARINGS

Rotor's stability cannot be achieved using PMB alone as per Earnshaw's theorem [73]. At least one of the five degrees of freedom has to be constrained by an active or superconducting or mechanical bearing. Instability is the key point to be addressed while designing and developing PMB. The net stiffness of PMB in the Cartesian coordinate system is zero.

$$K_x + K_y + K_z = 0 \tag{5}$$

In the case of circular PM rings,

$$K_x = K_y = K_r, \quad K_z = -2K_r \tag{6}$$

The axial PMB designed for positive axial stiffness will be unstable in the radial direction. Thus, the levitated ring of the radial bearing is unstable in an axial direction and vice versa. It was shown that any instability of the two rings PMB might be removed by subjecting one of the two to a parametric excitation using Mathieu Functions [74]. In Refs. [75, 76], it was reported that small stability spaces can be identified by setting one of the two rings to an axial excitation using the parametric equation of Mathieu. Radial instability [77] of axial PMB was addressed by developing a nonlinear equation of motion of a levitated ring and solving it analytically and numerically without the aid of Mathieu equations. The axial motion of radial PMB was stabilized using a piezoelectric actuator with different strokes experimentally in Ref. [78]. Further, Bassani [79] presented the possibility of addressing radial instability of axial PMB having one ring pair by the exploitation of magnetoelastic properties of the PM ring, and stable vibrations of the ring with small amplitude in both axial and radial directions were reported. The instability of PMB can also be nullified by combining this with another type (conventional or active or superconducting) of bearing along the rotor shaft. The different combinations of bearings along with PMB for stabilizing the rotor in axial as well as radial directions are discussed in the following section.

A magnetic suspension in which rotor was supported by radial PMB for radial and angular stability and unidirectional AMB for axial stability was reported in 1994 [80]. Radial PMB were used in combination with jewel bearings to support the flywheel as shown in Fig. 10, and the rotor was operated much beyond the critical speed at 5500 rpm [81]. In Ref. [82], the authors demonstrated the importance of higher speed and inertia of rotating component in achieving stability of permanent magnetic levitator and developed uni- and bi-ventricular assist pumps (Fig. 11) in which the first was driven axially and second one radially. The impeller of axially driven was supported by two radial PMB and point contact with the stator axially. Then the rotor floats without any contact point at a higher speed due to hydrodynamic force acting on the impeller. In bi-ventricular pump, the rotor with two end impellers

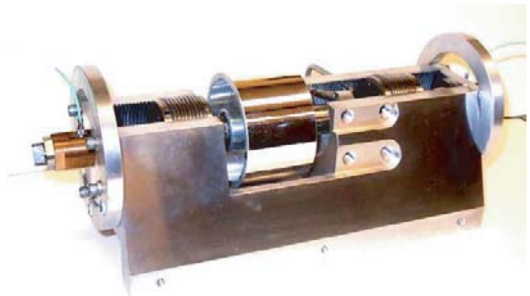


Figure 10. Passive magnetic bearing flywheel [81].

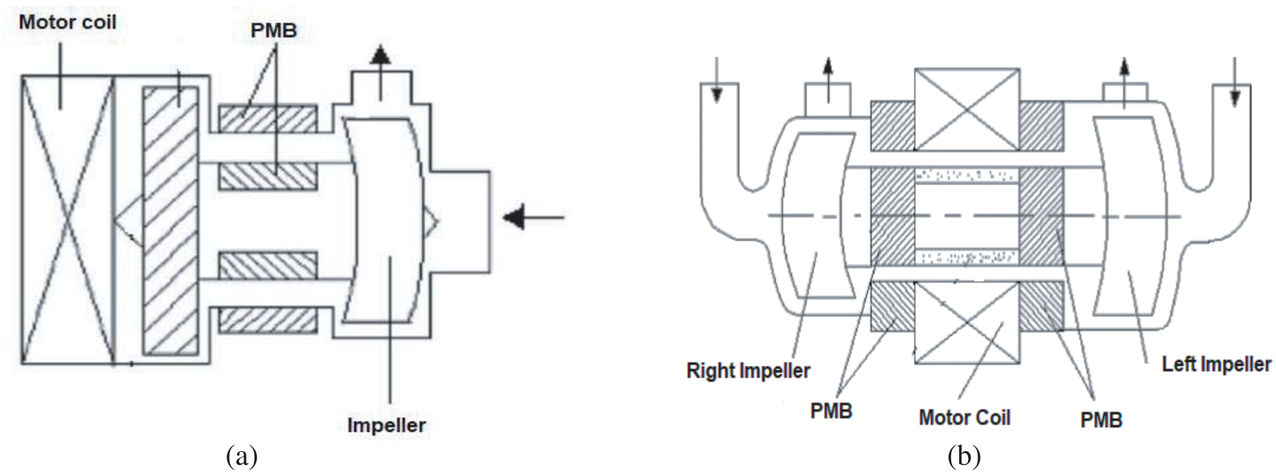


Figure 11. Schematic representation of PMB supported. (a) Axially driven uni-ventricular assist pump. (b) Radially driven bi-ventricular assist pump [82].

was supported by two PMB as shown in Fig. 11(b). In Refs. [83, 84], for a horizontal-shaft, researchers reported a new hybrid magnetic bearing system in which axial control was with a servo controller in addition to radial PMB. Analysis of PMB was done using FEA, and forces and stiffnesses were calculated in three directions. A prototype of rotor bearing configuration was designed and fabricated, and experimental results were also presented. Mukhopadhaya et al. [85] used a typical arrangement of PM for supporting the vertical shaft radially and AMB in the axial direction to address the instability of radial PMB in a prototype model for the application in the dairy industry.

A micro mass measurement system [86] supported by radial PMB and a controlled electromagnet to stabilize it in the axial direction was proposed and developed. The use of a hybrid bearing set for a prototype flywheel system was reported in Ref. [14]. The bearing set was made of radial PMB using four Nd-Fe-B magnet rings and thrust SMB. The authors used increased stiffness of PMB to compensate the load of the rotor and flywheel. Halbach magnetized radial PMB was designed and developed using N35 magnets [87]. The load-carrying capacity (90 N) and stiffness (129297.075 N/m) were measured experimentally. Rotor supported by developed radial PMB along with AMB was rotated at 2000 rpm. Due to the difficulty associated with realizing radially magnetized rings, Lijesh and Hirani [88] used cubical shaped magnets and aluminum rings to replace radially polarized magnets in RMD configuration. The theoretical model for calculating load carrying capacity in the proposed RMD configuration was introduced, and calculated results were validated by conducting the experiments on the developed RMD structure (Fig. 12). They demonstrated that the radial magnetized rings could be easily replaced with cuboidal magnets for achieving the same bearing features.

In Ref. [89], the authors used a hybrid-bearing set (PMB and foil bearings) to levitate the rotor completely as shown in Fig. 13. The RMD structure was used to support the rotor in the axial direction and discrete bump foil bearings in the radial direction, and rotor speed of 40,000 rpm was achieved


(a)


(b)

Figure 12. Cubical shaped magnets on a (a) Stator. (b) Rotor of a RMD configuration [88].

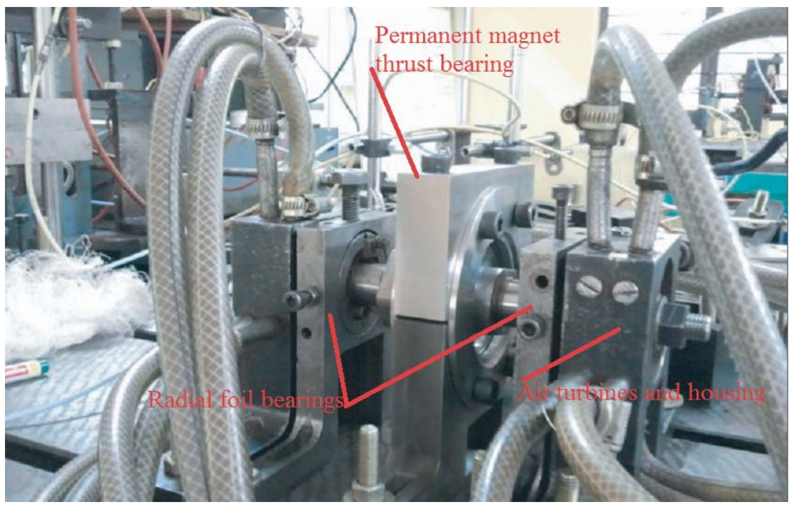


Figure 13. Test rig representing complete passive levitation of rotor supported by PMB and Foil bearings [89].

using turbines driven by compressed air. Wear in hydrodynamic journal bearing operating in boundary lubrication was reduced by designing and developing a hybrid (hydrodynamic and RMD configuration PMB) bearing [90]. Experiments were performed on the proposed bearing by selecting the best out of four RMD configurations suggested based on the results of load-carrying capacity. Based on the measurement of wear of the stator, it was proved that the initial wear of the journal bearing could be reduced to a great extent by using the hybrid bearing concept.

5. CONCLUSIONS

With the support of enough contributions towards maximizing the bearing characteristics, instability, and damping solutions by many researchers in the recent past, PMB could be used to replace conventional bearings for supporting low- and high-speed rotating shafts in different applications. The following is the conclusions based on the review in connection with modeling, analysis, and utilization of PMB for supporting rotating shafts.

- As surface charge density method is best suited for PMB with real dimensions, 3D mathematical equations of force and stiffness derived based on Coloumbian or Amperian approach could be used for their accurate design so that undermining effect of the curvature of PM rings is taken into account.

- The stacking of PM rings has to be carried out either in conventional or RMD patterns to form multi-ring PMB structures with enhanced force and stiffness characteristics. RMD structure is much superior to the conventional one in terms of force and stiffness values.
- Conventional bearings in high-speed applications where friction and maintenance are the major concerns could be replaced with PMB effectively if their designs have to be supported with optimization of design variables for maximum force and stiffness. Optimization in PMB can be carried out either by maximizing the bearing characteristics in a given control volume or minimizing the magnet volume for the required bearing characteristics. General guidelines of optimization presented in the literature based on generalized plots or design equations of optimized variables might be useful for designers in designing and optimizing multi-ring axial and radial PMB for industrial applications.
- Rotor supported on PMB can be stabilized by constraining its unstable axis by either active or passive means. It was also reported that instability can be addressed by subjecting a levitated ring with parametric excitation or by exploiting the magnetoelastic property of the PM ring.
- Low damping property of PMB could be enhanced by using simple eddy current or viscoelastic passive damper. The design and optimization procedure suggested in the literature will help designers in developing an eddy-current damping system efficiently for rotor supported by PMB stabilized in all directions.

In general, while designing PMB for a rotor efficiently, introducing sufficient damping is necessary, in addition to its stability in all directions.

ACKNOWLEDGMENT

Authors acknowledge the support provided by Manipal Institute of Technology, Manipal Academy of Higher Education, Manipal and ME Department of National Institute of Technology Karnataka, Surathkal for carrying out the research work.

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