Adaptive Antijamming Based on Space-time 2-D Sparse Array for GNSS Receivers

Ruiyan Du[1], Fulai Liu[1, *], Kai Tang[2], and Hui Song[2]

Abstract—Space-time adaptive antijamming problem has received significant attention recently for global navigation satellite system (GNSS). It can jointly utilize spatial filters and temporal filters to suppress interference signals. However, most of the works on space-time antijamming problem presented in the literature require a space-time two-dimension (2-D) array with multiple antennas and delay taps. In this paper, an effective adaptive antijamming method based on a space-time 2-D sparse array is proposed. The maximum array gain is utilized to construct a space-time 2-D sparse array. The space-time antijamming weight vector is given by minimizing the 2-D sparse array output power. Compared with the previous works, the presented method can have better antijamming performance than a space-time 2-D uniform array. Simulation results verify the effectiveness and feasibility of the proposed method.

1. INTRODUCTION

With the development of the global navigation satellite system (GNSS), location and navigation are widely applied in military and civilian fields, such as agriculture, aviation, land-vehicle navigation, and marine-navigation. However, the GNSS receiver is easily disrupted by intentional or unintentional jamming since the navigation signal is extremely weak when it reaches the ground. Thus, the antijamming technology has become a key to maintaining the integrity and functionality for GNSS receivers.

Spatial filter is recognized as a fundamental antijamming method for receiving the satellite signal of a specific direction while suppressing interference signals of other directions through an antenna array. The minimum variance distortionless response (MVDR) algorithm is one of the popular approaches for spatial antijamming. It can minimize the array output power and maintain a distortionless mainlobe response toward the desired signal. Unfortunately, MVDR beamformer may have unacceptably high sidelobes, which may lead to significant performance degradation in the event of unexpected interference signals [1]. A modified MVDR method is proposed, which utilizes a series of constraints to enforce the direction of arrival (DOA) of the desired signal to be far away from the DOAs of the interfering signals [2]. Based on projection processing, an effective beamforming technique is given for uniform linear arrays [3]. The covariance matrix is reconstructed against both the covariance matrix uncertainty and the signal steering vector mismatch, which can improve the performance of blanket jamming. A simple and effective spatial filter is proposed for uniform linear arrays [3]. In the presented method, the covariance matrix is constructed and the desired-signal steering vector is estimated by projection processing. The antenna structure and geometrical parameters are optimized in [4], and it has excellent radiating performance and small size. The beam pattern of proposed switched-beam antenna array has low side-lobe levels and is not sensitive to noise level. It is well known that the spatial degree of freedom

(DOF) is equal to $M - 1$ in the case of using an M-element antenna array. The application of above approaches will be limited when the number of interference signals exceeds the DOF of the antenna array. Therefore, the space-time adaptive antijamming method is developed to overcome the aforementioned problem, and several delay taps are placed behind each antenna element, namely, a temporal filtering is added on the basis of the spatial filtering. Thus, the space-time adaptive antijamming method can suppress more interference signals owing to the addition of temporal DOF [5–10]. A distortionless space-time adaptive antijamming method is presented [5], which can not only obtain a higher DOF but also enhance the gain of GNSS signals. A proportion differentiation algorithm is proposed for space-time adaptive antijamming [6]. It can achieve excellent performance for interference suppression without increasing the computational load. In [7], a two-stage antijamming approach is developed, in which the first stage estimates an interference-free subspace before the acquisition and tracking phases and projects all received signals into this subspace, and the second stage estimates the array attitude parameters based on detecting and employing GNSS signals. Through the aforementioned processing, the two-stage antijamming approach can significantly decrease distortion while maximizing signal-to-noise ratio (SNR). To achieve higher antijamming performance, a distortionless space-time adaptive processor based on minimum variance distortionless response (MVDR) beamformer is proposed [8]. It can maintain the linearity of space-time filter response, when odd number of delay taps is exploited. A virtual space-time adaptive beamforming is proposed for space-time antijamming [9]. Via temporal smoothing technique, a virtual space-time two-dimension (2-D) antijamming structure is constructed. It has better antijamming performance such as lower computational complexity than the antijamming structure without temporal smoothing. A new robust LCMV beamforming approach is presented and extended to space-time 2-D adaptive processing for sidelobe and mainlobe optimization problem [10]. In the proposed method, the robust beamformer is formulated as a nonconvex quadratically constrained quadratic programming problem. Furthermore, it is transformed into a semidefinite programming problem so that the weight vector can be effectively solved.

The array feature is critical to improving the the adaptive processing performance. Recently, several sparse arrays with higher DOFs are proposed to improve the adaptive processing performance with a reduced number of antennas. For example, nested and coprime arrays have attracted considerable interest due to their capability of providing increased array aperture, enhanced DOFs, and reduced mutual coupling effect compared to uniform linear arrays (ULAs) [11]. A compressed sparse array (CSA) scheme is presented for DOA estimation [12]. By combining the compressive measurement method and the sparse array together, the CSA has not only a higher number of DOFs but also a lower system complexity. Via a sparse receiving array, a semi-definite programming-total least squares (SDP-TLS) method is presented for eliminating the influence of the the gain/phase uncertainties and the basis mismatch problem [13]. A sparse wideband array design method is addressed for wideband beamforming problem [14]. A subset of array antennas is selected from a full array by discarding useless information [15], and it guarantees jamming nulling while declining the cost of hardware and computational load. A design of an electronically reconfigurable planar antenna array is proposed, in which discontinuities can be selected by RF switches [16]. It can achieve low cost, limited hardware complexity, and excellent reconfigurable capabilities. The output maximum signal to interference-plus-noise ratio (MSINR) criterion is the most essential criterion for adaptive beamforming. Therefore, optimum array configuration for MVDR adaptive procession is proposed in [17, 18], which designs a sparse array via giving a number of antennas and offering new insights into adaptive beamforming.

In this paper, an effective space-time adaptive antijamming method based on space-time 2-D sparse array is developed for GNSS receivers. The rest of the paper is organized as follows. The data model is introduced in Section 2. Section 3 presents the details of the proposed algorithm. Simulations are given to demonstrate the performance of the proposed method in Section 4. Section 5 concludes the paper.

2. DATA MODE

Consider that GNSS receiver with a ULA which consists of M antennas. Each antenna is equally spaced with P taps, and the space-time filter structure is given in Figure 1.

Assume that there are an interesting satellite signal $s(t)$ and q narrow-band interference signals $j_k(t) \, (k = 1, \cdots, q)$ impacting on the ULA. Then, the $MP \times 1$ received signals $\mathbf{x}(t)$ at time instant t
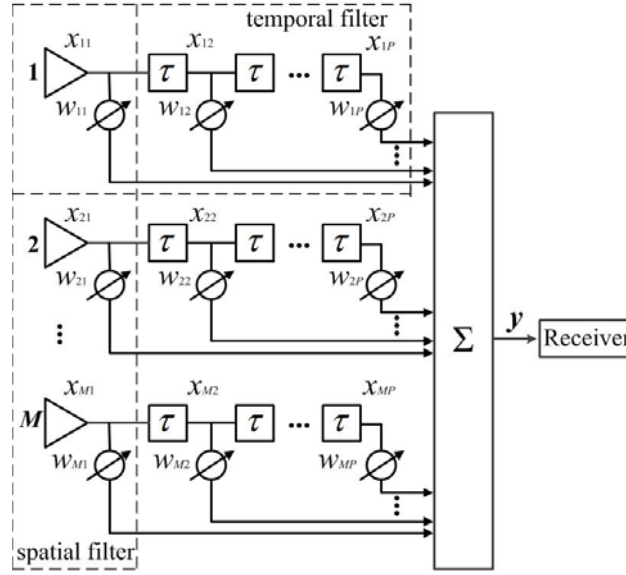
Figure 1. Space-time filter structure.

can be expressed as [8]

$$\mathbf{x}(t) = \mathbf{a}_0 s(t) + \sum_{k=1}^{q} \mathbf{a}_k j_k(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_{11}, x_{12}, \cdots, x_{1P}, \cdots, x_{M1}, \cdots, x_{MP}]^T$, $s(t)$ and $j_k(t)\{k = 1, 2, \cdots, q\}$ represent satellite signal and the kth interference signal, respectively. $\mathbf{n}(t)$ denotes the white Gaussian noise of the space-time filter structure. $\mathbf{a}_0$ and $\mathbf{a}_k\{k = 1, 2, \cdots, q\}$ represent steering vectors of satellite and interference signals, respectively, which have the following forms [8]

$$\mathbf{a}_l = \mathbf{a}_s(\theta_l) \otimes \mathbf{a}_t(f_l) \text{ for } l = 0, 1, 2, \cdots, q$$

$$\mathbf{a}_s(\theta_l) = [1, \exp\{-j2\pi f_l d \sin(\theta_l)/c\}, \cdots, \exp\{-j2\pi f_l(M-1)d\sin(\theta_l)/c\}]^T \tag{2}$$

$$\mathbf{a}_t(f_l) = [1, \exp\{-j2\pi f_l \tau\}, \cdots, \exp\{-j2\pi f_l(P-1)\tau\}]^T$$

where θ_l and f_l denote the DOA and frequency for the lth signal, respectively. τ stands for the delay time. The superscript $(\cdot)^T$ denotes the transpose operation.

The covariance matrix of $\mathbf{x}(t)$ can be expressed as

$$\mathbf{R}_x = \mathrm{E}\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \sigma_s^2 \mathbf{a}_0 \mathbf{a}_0^H + \mathbf{A}_j \mathbf{R}_b \mathbf{A}_j^H + \sigma_n^2 \mathbf{I} \tag{3}$$

where $\mathrm{E}\{\cdot\}$ represents the statistical average operation, and the superscript $(\cdot)^H$ denotes the conjugate transpose. $\sigma_s^2 = \mathrm{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the power of the interesting satellite signal. $\mathbf{R}_b = \mathrm{E}\{\mathbf{j}(t)\mathbf{j}^H(t)\}$, where $\mathbf{j}(t) = [j_1(t), j_2(t), \cdots, j_k(t), \cdots, j_q(t)]$ indicates q interference signals. $\mathbf{R}_b$ is a diagonal matrix whose kth diagonal element is the power of the kth interference signal for $k = 1, 2, \cdots, q$. The $MP \times q$ steering vector matrix $\mathbf{A}_j = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_k, \cdots, \mathbf{a}_q]$. σ_n^2 and $\mathbf{I}$ indicate Gaussian noise power and identity matrix, respectively.

The beamformer output $y(t)$ of the space-time filter can be written as

$$y(t) = \mathbf{w}^H \mathbf{x}(t) \tag{4}$$

where $\mathbf{w} \in \mathbf{C}^{MP \times 1}$ is the space-time antijamming weight vector.

The mean square power output P_0 of the beamformer can be expressed as

$$P_o = \mathrm{E}\left\{y(t)y^H(t)\right\} = \mathbf{w}^H \mathbf{R}_s \mathbf{w} + \mathbf{w}^H \mathbf{R}_j \mathbf{w} + \mathbf{w}^H \mathbf{R}_n \mathbf{w} = \mathbf{w}^H \mathbf{R}_s \mathbf{w} + \mathbf{w}^H \mathbf{R}_{j+n} \mathbf{w} \tag{5}$$

where $\mathbf{R}_s = \sigma_s^2 \mathbf{a}_0 \mathbf{a}_0^H$ denotes the interesting satellite signal covariance matrix. $\mathbf{R}_j = \mathbf{A}_j \mathbf{R}_b \mathbf{A}_j^H$ stands for the covariance matrix of interference signals. $\mathbf{R}_n = \sigma_n^2 \mathbf{I}$ represents the noise covariance matrix. $\mathbf{R}_{j+n} = \mathbf{R}_j + \mathbf{R}_n = \mathbf{A}_j \mathbf{R}_b \mathbf{A}_j^H + \sigma_n^2 \mathbf{I}$ denotes the interference-plus-noise covariance matrix.

3. ALGORITHM FORMULATION

In this section, an adaptive antijamming algorithm based on space-time 2-D sparse array is described in detail. Firstly, via the output maximum signal to interference-plus-noise ratio (MSINR) criterion, a target function is constructed based on the output SINR and the gain of space-time 2-D array. Secondly, space-time 2-D sparse array configuration is determined by solving selection vectors. Finally, the closed-form solution of space-time antijamming weight vector is given.

3.1. Space-Time 2-D Array Output SINR and Gain

3.1.1. Array Output SINR

The output SINR of a beamformer can be written as

$$\text{SINR}_o(\mathbf{w}) = \frac{\mathbf{w}^H \mathbf{R}_s \mathbf{w}}{\mathbf{w}^H \mathbf{R}_{j+n} \mathbf{w}} = \frac{\sigma_s^2 |\mathbf{w}^H \mathbf{a}_0|^2}{\mathbf{w}^H \mathbf{R}_{j+n} \mathbf{w}} \tag{6}$$

where $\mathbf{w}$ is the the space-time 2-D weight vector.

Invoking MVDR beamformer [8], the optimal solution $\mathbf{w}_{\text{MVDR}}$ of space-time 2-D weight vector can be expressed as

$$\mathbf{w}_{\text{MVDR}} = \mu \mathbf{R}_{j+n}^{-1} \mathbf{a}_0 = \frac{\mathbf{R}_{j+n}^{-1} \mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}_{j+n}^{-1} \mathbf{a}_0} \tag{7}$$

where the superscript $(\cdot)^{-1}$ denotes matrix inverse and $\mu = \frac{1}{\mathbf{a}_0^H \mathbf{R}_{j+n}^{-1} \mathbf{a}_0}$.

Substituting Eq. (7) into Eq. (6), the output SINR with the weight vector $\mathbf{w}_{\text{MVDR}}$ is given by

$$\text{SINR}_o(\mathbf{w}_{\text{MVDR}}) = \text{SNR}_i \mathbf{a}_0^H \left(\mathbf{I} + \frac{1}{\sigma_n^2} \mathbf{A}_j \mathbf{R}_b \mathbf{A}_j^H \right)^{-1} \mathbf{a}_0 \tag{8}$$

where the signal-to-noise ratio $\text{SNR}_i = \sigma_s^2/\sigma_n^2$, in which σ_s^2 and σ_n^2 denote the power of interesting satellite signal and the power of noise, respectively. $\mathbf{a}_0$ stands for the steering vector of interesting satellite signal. $\mathbf{A}_j$ and $\mathbf{R}_b$ represent the steering matrix and the covariance matrix for the multiple interference signals, respectively.

Let $\mathbf{R}_i = \mathbf{I} + \sigma_n^{-2} \mathbf{A}_j \mathbf{R}_b \mathbf{A}_j^H$. According to matrix inversion lemma [19], the inverse of the matrix $\mathbf{R}_i$ can be written as $\mathbf{R}_i^{-1} = \mathbf{I} + \mathbf{A}_j (\mathbf{A}_j^H \mathbf{A}_j + \sigma_n^2 \mathbf{R}_b^{-1})^{-1} \mathbf{A}_j^H$. Substituting $\mathbf{R}_i^{-1}$ into Eq. (8), the output SINR can be rewritten as

$$\text{SINR}_o(\mathbf{w}_{\text{MVDR}}) = \text{SNR}_i G = \text{SNR}_i \left[D - \mathbf{a}_0^H \mathbf{A}_j (\mathbf{R}_o + \mathbf{A}_j^H \mathbf{A}_j)^{-1} \mathbf{A}_j^H \mathbf{a}_0 \right] \tag{9}$$

where $\mathbf{R}_o = \sigma_n^2 \mathbf{R}_b^{-1}$. The array gain $G = [D - \mathbf{a}_0^H \mathbf{A}_j (\mathbf{R}_o + \mathbf{A}_j^H \mathbf{A}_j)^{-1} \mathbf{A}_j^H \mathbf{a}_0]$, in which $D = \mathbf{a}_0^H \mathbf{a}_0$ with D being the sum of the numbers of antennas and taps actually used to receive the signals.

3.1.2. Space-Time 2-D Array Gain

Define a selection vector $\mathbf{z} = (z_1, \cdots, z_{MP})^T$ with $z_k \in \{0, 1\}$ $k = 1, 2, \cdots, MP$, where "1" indicates that the unit is retained, and "0" means elimination. For each selection vector $\mathbf{z}$, there is a corresponding selection matrix $\mathbf{Z} = \text{diag}(\mathbf{z})$, and it is easy to know that $\mathbf{Z}^T \mathbf{Z} = \text{diag}(\mathbf{z})$. Suppose that the steering matrix of the input signals can be written as $\overline{\mathbf{A}} = [\overline{\mathbf{A}}_j, \overline{\mathbf{a}}_0]$, where $\overline{\mathbf{A}}_j = [\overline{\mathbf{a}}_1, \overline{\mathbf{a}}_2, ..., \overline{\mathbf{a}}_q]$, $\overline{\mathbf{a}}_0$ and $\overline{\mathbf{a}}_k$, $\{k = 1, 2, ..., q\}$ are the steering vectors of the satellite and the interference signals, respectively. Via the selection matrix $\mathbf{Z}$, the steering matrix $\mathbf{A}$ of a sparse array can be expressed as $\mathbf{A} = \mathbf{Z}\overline{\mathbf{A}} = \mathbf{Z}[\overline{\mathbf{A}}_j, \overline{\mathbf{a}}_0]$ and $\mathbf{a}_0 = \mathbf{Z}\overline{\mathbf{a}}_0$, $\mathbf{A}_j = \mathbf{Z}\overline{\mathbf{A}}_j$.

Let $\mathbf{C}_j = \mathbf{A}_j^H \mathbf{A}_j + \mathbf{R}_o$, and $\mathbf{C}_j$ can be further expressed as

$$\mathbf{C}_j = \overline{\mathbf{A}}_j^H \text{diag}(\mathbf{z}) \overline{\mathbf{A}}_j + \mathbf{R}_o \tag{10}$$

where $\mathbf{R}_o = \sigma_n^2 \mathbf{R}_b^{-1}$.

Similarly, the matrix $\mathbf{C}$ is written as follows

$$\mathbf{C} = \overline{\mathbf{A}}^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}} + \mathbf{R} = \begin{bmatrix} \mathbf{C}_j & \overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{a}_0} \\ \overline{\mathbf{a}}_0^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}}_j & \overline{\mathbf{a}}_0^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{a}_0} \end{bmatrix} \tag{11}$$

where $\mathbf{C}_j$ and $\overline{\mathbf{a}}_0^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{a}}_0$ are invertible.

According to partitioned matrix determinant [19], the determinant of $\mathbf{C}$ is calculated as follows

$$|\mathbf{C}| = \begin{vmatrix} \mathbf{C}_j & \overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{a}_0} \\ \overline{\mathbf{a}}_0^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}}_j & \overline{\mathbf{a}}_0^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{a}_0} \end{vmatrix} = |\mathbf{C}_j||D - \mathbf{a}_0^H \mathbf{A}_j (\mathbf{R}_o + \mathbf{A}_j^H \mathbf{A}_j)^{-1} \mathbf{A}_j^H \mathbf{a}_0| \tag{12}$$

where the operation $|\cdot|$ represents the determinant of matrix.

Consequently, the relationship between $|\mathbf{C}|$ and $|\mathbf{C}_j|$ can be obtained as follows

$$\frac{|\mathbf{C}|}{|\mathbf{C}_j|} = \left| D - \mathbf{a}_0^H \mathbf{A}_j \left(\mathbf{R}_o + \mathbf{A}_j^H \mathbf{A}_j \right)^{-1} \mathbf{A}_j^H \mathbf{a}_0 \right| = G \tag{13}$$

where G denotes the array gain given by Eq. (9).

From Eqs. (10), (11), and (13), we can get

$$\log G = \log \left| \overline{\mathbf{A}}^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}} + \mathbf{R} \right| - \log \left| \overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}}_j + \mathbf{R}_o \right| \tag{14}$$

where the array gain G is converted into its logarithmic.

3.2. Space-Time 2-D Sparse Array Configuration

In this section, via the array gain given by Eq. (14), an effective space-time 2-D sparse array configuration is designed.

3.2.1. Antenna Sparse Array Design

Invoking the array gain in Eq. (14), the sparse array design problem can be formulated in terms of maximizing the logarithm of G,

$$\max_{\mathbf{z}} \log \left| \overline{\mathbf{A}}^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}} + \mathbf{R} \right| - \log \left| \overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}}_j + \mathbf{R}_o \right|$$
$$\text{s.t.} \begin{cases} \mathbf{1}^T \mathbf{z} = K, \\ 0 \leq \mathbf{z} \leq 1 \end{cases} \tag{15}$$

where $0 \leq \mathbf{z} \leq 1$ means componentwise, i.e., $0 \leq z_k \leq 1$ for all k.

However, the above objective function belongs to two concave functions [20]. Via the sequential convex programming (SCP) based on iteratively linearizing the concave function [21], the optimization problem (15) can be rewritten as

$$\max_{\mathbf{z}} \log \left| \overline{\mathbf{A}}^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}} + \mathbf{R} \right| - \Delta \mathbf{g}_1^T \left(\mathbf{z}^{(k)} \right) \left(\mathbf{z} - \mathbf{z}^{(k)} \right)$$
$$\text{s.t.} \begin{cases} \mathbf{1}^T \mathbf{z} = K, \\ 0 \leq \mathbf{z} \leq 1 \end{cases} \tag{16}$$

where $\Delta \mathbf{g}_1^T(\mathbf{z}^{(k)}) = [\overline{\mathbf{a}}_{r,i}^H (\overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z}^{(k)})\overline{\mathbf{A}}_j + \mathbf{R}_o)^{-1} \overline{\mathbf{a}}_{r,i}, i = 1, 2, ..., M]^T$ is the gradient of concave function $\log |\overline{\mathbf{A}}_j^H \operatorname{diag}(\mathbf{z})\overline{\mathbf{A}}_j + \mathbf{R}_o|$. $\overline{\mathbf{a}}_{r,i}$ represents the ith column of the matrix $\overline{\mathbf{A}}_j^H$.

Note that the SCP depends on different initial values $\mathbf{z}^{(0)}$. It is necessary to select different initial values, run multiple times, and ultimately select the maximum operation result of the objective function. The optimization problem in Eq. (16) can be solved by using the software package CVX embedded in Matlab [22].

3.2.2. Tap Sparse Array Design

Based on the antenna sparse array design in Eq. (16), a space-time 2-D array with spatial sparse configuration can be obtained, if P delay taps are uniformly added to each antenna element. Thus, our design goal is to choose an appropriate $D(D < P)$ sparse tap position so that the designed space-time 2-D sparse array can have better performance than other array configurations.

As the antenna position has been determined, the space-time steering vector is redefined as $\overline{\mathbf{a}}_{s,s}$. The steering vector of the satellite and interference signals can be redefined as $\overline{\mathbf{a}}_{0,s}$ and $\overline{\mathbf{a}}_{k,s}\{k = 1, 2, \cdots, q\}$, respectively. Similarly, the steering matrix of the interference signals and input signals can be written as $\overline{\mathbf{A}}_{j,s} = [\overline{\mathbf{a}}_{1,s}, \overline{\mathbf{a}}_{2,s}, ..., \overline{\mathbf{a}}_{q,s}]$, $\overline{\mathbf{A}}_s = [\overline{\mathbf{A}}_{j,s}, \overline{\mathbf{a}}_{0,s}]$. Define a tap selection vector $\mathbf{z}_t = [z_i, i = 1, 2, ..., KP] \in \{0, 1\}^{KP}$. In the sparse process, the antenna positions are all set to "1" in the selection vector, thereby ensuring that only the taps are sparse. The selection vector diagonal expansion also gets the selection matrix, that is, $\mathbf{Z}_t = \text{diag}(\mathbf{z}_t)$. The selection matrix is used to obtain sparse steering vectors and matrix such as $\mathbf{a}_{0,s} = \mathbf{Z}_t\overline{\mathbf{a}}_{0,s}$ and $\mathbf{A}_{j,s} = \mathbf{Z}_t\overline{\mathbf{A}}_{j,s}$. Similarly, the selection vector is associated with the array gain, which translates into an optimization problem similar to Eq. (15) as follows

$$\max_{\mathbf{z}_t} \log \left| \overline{\mathbf{A}}_s^H \text{diag}(\mathbf{z}_t)\overline{\mathbf{A}}_s + \mathbf{R} \right| - \log \left| \overline{\mathbf{A}}_{j,s}^H \text{diag}(\mathbf{z}_t)\overline{\mathbf{A}}_{j,s} + \mathbf{R}_o \right|$$

$$\text{s.t.} \begin{cases} \mathbf{1}^T\mathbf{z}_t = D, \\ 0 \leq \mathbf{z}_t \leq 1 \end{cases} \tag{17}$$

The solution of the above optimization can refer to Eq. (16). According to the obtained tap selection vector $\mathbf{z}_t$, the final sparse array configuration can be obtained.

3.3. Solution for Space-Time Antijamming Weight Vector

As discussed above, an effective space-time 2-D sparse array can be given, and the antenna selection vector $\mathbf{z}_s$ and tap selection vector $\mathbf{z}_t$ are calculated by Eqs. (15) and (17), respectively. Determine the weight vector $\mathbf{w}$ as the solution to the following optimization problem

$$\min_{\mathbf{w}} \mathbf{w}\mathbf{R}_{s,t}\mathbf{w}^H \text{ s.t. } \mathbf{w}^H\mathbf{a}_{0,s,t} = 1 \tag{18}$$

where $\mathbf{R}_{s,t}$ and $\mathbf{a}_{0,s,t}$ denote the covariance matrix and the space-time steering vector of the interesting signal for the space-time 2-D sparse array given by Eqs. (15) and (17), respectively.

Thus, the optimal weight vector of the problem in Eq. (18) can be expressed as

$$\mathbf{w} = \frac{\mathbf{R}_{s,t}^{-1}\mathbf{a}_{0,s,t}}{\mathbf{a}_{0,s,t}^H\mathbf{R}_{s,t}^{-1}\mathbf{a}_{0,s,t}} \tag{19}$$

Summary of the Proposed Method

Step 1 By Eqs. (15) and (17), design the space-time 2-D sparse array.

Step 2 Collect the sample data, and estimate the covariance matrix $\hat{\mathbf{R}}$ according to $\hat{\mathbf{R}} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k)$, where K denotes the snapshot number.

Step 3 Calculate the space-time 2-D antijamming weight vector $\mathbf{w}$, according to Eq. (19).

4. SIMULATION RESULTS

In this section, several simulation results are given to verify the performance of the proposed method. Consider a ULA with $M = 8$ antenna elements whose separation distances are half-wavelength, and each element is equally spaced with $P = 6$ taps. Assume that there are four far field signals impinging on the array. Among them, the DOA of the interesting satellite signals is set to $\theta_1 = 0°$, and its normalized center frequency is $f_1 = 1.0\,\text{GHz}$. The directions of arrival of three interference signals are $\theta_2 = -10°$, $\theta_3 = 5°$, $\theta_4 = 0°$, respectively. Their corresponding center frequencies are $f_2 = 0.9\,\text{GHz}$, $f_3 = 1.1\,\text{GHz}$,

$f_4 = 1.2\,\text{GHz}$. The SNR of satellite signal is equal to $-20\,\text{dB}$, and the interference-to-noise ratio (INR) of the three interfering signals is equal to $40\,\text{dB}$.

Figure 2 shows the sparse diagram which is given by solving Eqs. (15) and (16) for the uniform array with eight antennas in which each antenna is equipped with six equally spaced taps. In the case of ensuring the maximum array gain, the first 5 antennas are selected, the 6th and 8th antennas discarded, and then the taps are sparse on the basis of the selected 6 antennas, so that a space-time 2-D sparse array is obtained with 6 antennas and 18 taps.

Figure 3(a) shows the space-time 2-D response diagram of the designed sparse array. As can be seen in the pattern, it has better mainlobe for the interesting satellite signal at $\theta_1 = 0°$ and $f_1 = 1.0\,\text{GHz}$, and the three interference signals are perfectly suppressed. Figure 3(b) is a contour plot of the response shown in Figure 3(a). It is easy to know that the coordinates of the three interference signals are located at $(\theta_2, f_2) = (-10°, 0.9\,\text{GHz}), (\theta_3, f_3) = (5°, 1.1\,\text{GHz}), (\theta_4, f_4) = (0°, 1.2\,\text{GHz})$, respectively. As shown in Figure 3, the presented method based on space-time 2-D sparse array not only effectively suppresses
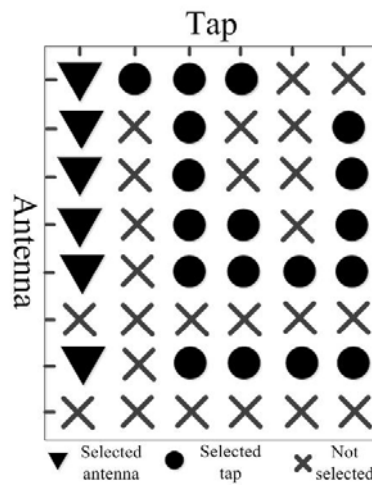

Figure 2. Optimum space-time sparse array for $M = 8$, $P = 6$.

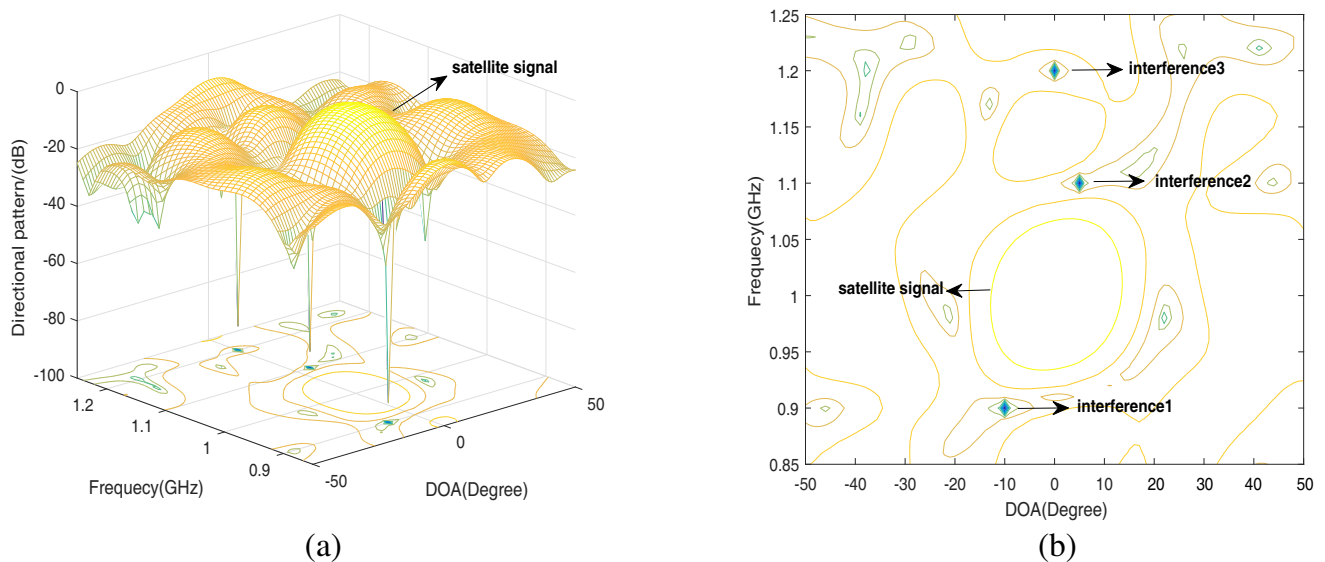(a)   (b)

Figure 3. Three-dimensional beam pattern and contour plot of optimum sparse arrays. (a) Beam pattern. (b) Contour plot.

interference signals, but also maintains a distortionless response to satellite signal.

For convenience, the uniform linear array with four antennas in which each antenna is equipped with six equally spaced taps is named as A1, and the uniform linear array with six antennas in which each antenna is equipped with four equally spaced taps is named as A2. Figure 4 shows the array gain curves of the proposed method, A1, and A2, with input SNRs ranging from $-35\,$dB to $-5\,$dB. Compared with A1 and A2, the proposed method had better array gain.
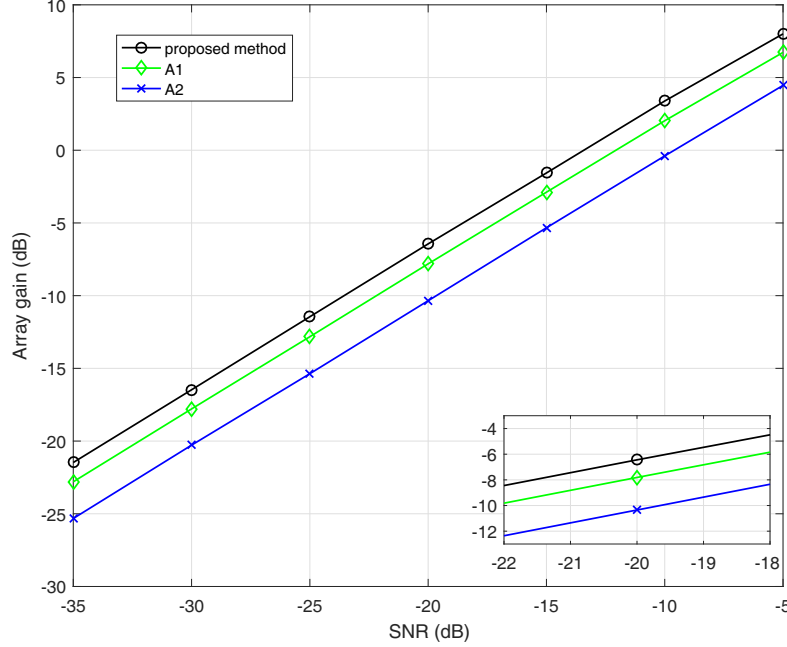

Figure 4. Array gain versus input SNR for related array configuration.

5. CONCLUSION

This paper presents a space-time adaptive antijamming algorithm based on space-time 2-D sparse array for GNSS receivers. Via maximizing the array gain, the selection vectors of antennas and taps are given, respectively, so that an effective space-time 2-D sparse array is designed. Finally, an optimal space-time 2-D antijamming weight vector is calculated by minimizing the designed space-time 2-D sparse array output power. Simulation results show that the proposed method has better antijamming performance.

REFERENCES

1. Souden, M., J. Benesty, and S. Affes, "A study of the LCMV and MVDR noise reduction filters," *IEEE Transactions on Signal Processing*, Vol. 58, No. 9, 4925–4935, 2010.
2. Huang, Y. W., M. K. Zhou, and S. A. Vorobyov, "New designs on MVDR robust adaptive beamforming based on optimal steering vector estimation," *IEEE Transactions on Signal Processing*, Vol. 67, No. 14, 3624–3638, 2019.

3. Saeed, M. and K. Osman, "Adaptive beamforming based on theoretical interference-plus-noise covariance and direction-of-arrival estimation," *IET Signal Processing*, Vol. 12, No. 10, 819–825, 2018.

4. Donelli, M. and P. Febvre, "An inexpensive reconfigurable planar array for Wi-Fi applications," *Progress In Electromagnetics Research C*, Vol. 28, 71–81, 2012.

5. Chen, F. Q., J. W. Nie, B. Y. Li, and F. X. Wang, "Distortionless space-time adaptive processor for global navigation satellite system receiver," *Electronics Letters*, Vol. 51, No. 25, 2138–2139, 2015.

6. Lu, Z. K., J. W. Nie, F. Q. Chen, H. M. Chen, and G. Ou, "Adaptive time taps of STAP under channel mismatch for GNSS antenna arrays," *IEEE Transactions on Instrumentation and Measurement*, Vol. 66, No. 11, 2813–2824, 2017.

7. Saeed, D., J. Ali, B. Ali, and L. Grard, "GNSS space-time interference mitigation and attitude determination in the presence of interference signals," *Sensors*, Vol. 15, No. 6, 12180–12204, 2015.

8. Dai, X. Z., J. W. Nie, F. Q. Chen, and G. Ou, "Distortionless space-time adaptive processor based on MVDR beamformer for GNSS receiver," *IET Radar Sonar & Navigation*, Vol. 11, No. 10, 1488–1494, 2017.

9. Liu, F., R. Du, and X. Bai, "A virtual space-time adaptive beamforming method for space-time antijamming," *Progress In Electromagnetics Research M*, Vol. 58, 183–191, 2017.

10. Xu, J. W., G. S. Liao, S. Q. Zhu, and L. Huang, "Response vector constrained robust LCMV beamforming based on semidefinite programming," *IEEE Transactions on Signal Processing*, Vol. 63, No. 21, 5720–5732, 2015.

11. Zheng, Z., W. Q. Wang, Y. Y. Kong, and Y. D. Zhang, "MISC array: A new sparse array design achieving increased degrees of freedom and reduced mutual coupling effect," *IEEE Transactions on Signal Processing*, Vol. 67, No. 7, 1728–1741, 2019.

12. Muran, G., Y. D. Zhang, and T. Chen, "DOA estimation using compressed sparse array," *IEEE Transactions on Signal Processing*, Vol. 66, No. 15, 4133–4146, 2018.

13. Hu, B., X. C. Wu, X. Zhang, Q. Yang, and W. B. Deng, "Adaptive beamforming for sparse array based on semi-definite programming," *IEEE Access*, Vol. 6, 64525–64532, 2018.

14. Hawes, M. B. and W. Liu, "Sparse array design for wideband beamforming with reduced complexity in tapped delay-lines," *IEEE/ACM Transactions on Audio Speech and Language Processing*, Vol. 22, No. 88, 1236–1247, 2014.

15. Amin, M. G., X. R. Wang, Y. D. Zhang, and F. Ahmad, "Sparse arrays and sampling for interference mitigation and DOA estimation in GNSS," *Proceedings of the IEEE*, Vol. 104, No. 6, 1302–1317, 2016.

16. Donelli, M., T. Moriyama, and M. Manekiya, "A compact switched-beam planar antenna array for wireless sensors operating at Wi-Fi band," *Progress In Electromagnetics Research C*, Vol. 83, 137–145, 2018.

17. Wang, X. R., M. Amin, and X. Cao, "Analysis and design of optimum sparse array configurations for adaptive beamforming," *IEEE Transactions on Signal Processing*, Vol. 66, No. 2, 340–351, 2017.

18. Wang, X. R., M. Amin, and X. H. Wang, "Robust sparse array design for adaptive beamforming against DOA mismatch," *Signal Processing*, Vol. 146, 41–49, 2018.

19. Meyer, C., "Matrix analysis and applied linear algebra," *Society for Industrial and Applied Mathematics*, 2000.

20. Horst, R. and P. M. Pardalos, *Introduction to Global Optimization*, Springer-Verlag New York, Inc., 2002.

21. Fazel, M., H. Hindi, and S. P. Boyd, "Log-det heuristic for matrix rank minimization with applications to Hankel and Euclidean distance matrices," *American Control Conference*, Vol. 3, 2156–2162, 2003.

22. Grant, M., "CVX: Matlab software for disciplined convex programming," *Version 1.21*, 2008.