

Effect of Noise Jamming on Compressed Sensing SAR Imaging

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Abstract—Compressed sensing (CS) imaging radar can obtain higher resolution than the traditional synthetic aperture radar (SAR) with less data, which makes it important for military and civilian applications. However, noise, especially active noise jamming, will degrade its performance. This paper describes the signal model of a CS imaging radar under noise jamming. Through theoretical analysis and simulation experiments, the influences of different jamming patterns, jamming parameters, and reconstruction algorithms on the performance of CS imaging are compared. It can provide reference for the research of anti-jamming technology of CS imaging radar.

1. INTRODUCTION

Synthetic aperture radar (SAR) is useful in both military and civil applications, since it can obtain high-resolution images under all-weather conditions [1]. With the continuous improvement of radar resolution, SAR needs to collect the echo in a high sampling frequency, resulting in an increasing amount of data. Based on compressed sensing (CS) techniques, sparse scenes can be reconstructed in higher resolution, only with a small amount of sampled data [2]. Many works have focused on the study of CS imaging methods and applications [3, 4]. Because of the increasing importance, countermeasures such as active noise jamming will be generated inevitably to degrade the reconstruction image. However, very few researches have been done on the effect of active jamming on the performance of CS SAR imaging. In [5], the authors have pointed out that noise has a great impact on pulse compression, and it is difficult to reconstruct the target correctly in signal-to-noise ratio (SNR) 10 dB. An iteration algorithm based on basis pursuit denoising is proposed to overcome the impact of the perturbation in sensing matrix in [6]. However, the measurement noise has not been taken into consideration.

Based on the establishment of a signal model under noise jamming conditions, this paper uses a combination of theoretical analysis and simulation experiments to study the impact on reconstruction performance under different jamming patterns and different reconstruction algorithms.

2. SIGNAL MODEL UNDER NOISE JAMMING

A SAR image contains lots of pixel units, which correspond to the radar resolution. It means that the imaging scene can be represented by a discretized grid. The length and width of each grid are respectively equal to the range and azimuth resolution of the radar. In each grid, there exists only one point target whose radar cross section (RCS) is proportional to the pixel value of the image. In this way, the imaging scene can be modelled as:

$$\mathbf{X} = \begin{bmatrix} x(1,1) & \cdots & x(1,Q) \\ \vdots & \ddots & \vdots \\ x(P,1) & \cdots & x(P,Q) \end{bmatrix} \quad (1)$$

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where P and Q represent the grid number of the imaging scene in the range and azimuth directions, respectively. $\mathbf{X}$ can be rewritten as a $PQ \times 1$ vector as follows:

$$\mathbf{x} = [x(1, 1), \dots, x(P, 1), \dots, x(P, Q)]^T \quad (2)$$

The radar echo after digital sampling can be expressed as:

$$s(m, n) = \sum_{i=1}^{PQ} \mathbf{x}(i) a(m, n, i) = \sum_{i=1}^{PQ} \mathbf{x}(i) p\left(\tau_m - \frac{2R_i(\eta_n)}{c}\right) \exp\left(-j\frac{4\pi R_i(\eta_n)}{\lambda}\right) \quad (3)$$

where τ_m and η_n are the m th fast time and n th slow time samples, respectively. R_i denotes the distance between the i th grid and the radar. The above formula can be rewritten in a matrix form:

$$\mathbf{s} = \mathbf{\Phi} \mathbf{x} \quad (4)$$

where $\mathbf{s}$ is an $MN \times 1$ vector; M and N are the numbers of the range and azimuth samples, respectively; and the (p, q) th entry of $\mathbf{\Phi}$ can be expressed as following:

$$\Phi(p, q) = a\left(\text{rem}\left(\frac{p}{M}\right), \text{fix}\left(\frac{p}{M}\right), q\right) \quad (5)$$

where $\text{rem}(\cdot)$ and $\text{fix}(\cdot)$ are remainder and rounding operation, respectively.

Considering the internal noise in the receiver and the jamming signal, Equation (4) becomes:

$$\mathbf{s} = \mathbf{\Phi} \mathbf{x} + \mathbf{e} \quad (6)$$

where $\mathbf{e} = \mathbf{n} + \mathbf{j}$, $\mathbf{n}$ and $\mathbf{j}$ denote the noise and jamming signal, respectively. Generally, the power of $\mathbf{j}$ is much larger than $\mathbf{n}$, so $\mathbf{n}$ is neglected in this paper.

Active jamming signal can be divided into two categories: unintentional jamming and intentional jamming. The unintentional jamming includes environmental electromagnetic noise, radiation sources from civilian installations. The intentional jamming is generated by military active jammers. This paper focuses on the description of the influence from noise jamming.

The noise jamming contains the following four categories [7]:

1) Radio frequency noise jamming (RFNJ)

RFNJ signal can be generated directly by amplifying microwave noise. It can be modelled as:

$$\mathbf{j}(n) = u_n(n) \exp\{j2\pi f_j n T_s\} \exp\{j\phi(n)\} \quad (7)$$

where $u_n(n)$ denotes the amplitude, obeying the Rayleigh distribution; f_j is the carrier frequency of the jamming signal; T_s is the sampling interval; $\phi(n)$ is the phase, obeying the uniform distribution and independent of $u_n(n)$.

2) Noise frequency modulation jamming (NFMJ)

The frequency of NFMJ is modulated by noise so that it has a large jamming bandwidth and a flat power spectrum. The jamming signal can be expressed as:

$$\mathbf{j}(n) = U_j \exp\left\{j2\pi f_j n T_s + j2\pi K_{FM} \int_0^t u_n(t) dt\right\} \exp\{j\phi\} \quad (8)$$

where U_j is the amplitude, K_{FM} the frequency modulation slope, and $u_n(t)$ the modulation noise.

3) Noise amplitude modulation jamming (NAMJ)

NAMJ signal can be described by:

$$\mathbf{j}(n) = (U_0 + u_n(n)) \exp\{j2\pi f_j n T_s\} \exp\{j\phi\} \quad (9)$$

where U_0 is the amplitude, and $u_n(n)$ is the modulation noise.

4) Noise phase modulation jamming (NPMJ)

NPMJ signal can be modelled as:

$$\mathbf{j}(n) = U_j \exp\{j2\pi f_j n T_s + jK_{PM} u_n(t)\} \exp\{j\phi\} \quad (10)$$

where K_{PM} is the phase modulation coefficient.

When the imaging scene $\mathbf{x}$ is sparse and the matrix Φ meets the Restricted Isometry Property (RIP) conditions, CS theory is often utilized to obtain higher resolution. In CS theory, the following optimization problem should be solved:

$$\hat{x} = \arg \min_x \|\mathbf{x}\|_1 \quad s.t. \quad \|\mathbf{s} - \Phi \mathbf{x}\|_2 \leq \sigma \quad (11)$$

where σ is the regularization parameter and determined by the energy of the internal Gaussian noise. The above problem can be solved by using Orthogonal Matching Pursuit (OMP), StOMP, BP, and other algorithms. It is worth noting that most of the noise jamming signals do not obey Gaussian distribution, and its energy is much larger than that of the internal noise. Thus, the jamming will bring bad influence to the final image reconstruction.

3. ANALYSIS ON THE IMPACT OF NOISE JAMMING

Theorem: Assume that the measurement matrix Φ satisfies the $2K$ -order constraint equidistance characteristic, and $\delta_{2K} < \sqrt{2} - 1$, $y = \Phi x + e$, where e is the error brought by the measurement process, $\|\Phi^T e\|_\infty \leq \lambda$. Then the reconstructed sparse solution $\hat{x}$ satisfies the following formula when $B(y) = \{z : \|\Phi^T(y - \Phi z)\|_\infty \leq \lambda\}$ [8]:

$$\|\hat{x} - x\|_2 \leq C_0 \frac{\sigma_K(x)_1}{\sqrt{K}} + C_3 \sqrt{K} \lambda \quad (12)$$

where $C_0 = 2 \frac{1-(1-\sqrt{2})\delta_{2K}}{1-(1+\sqrt{2})\delta_{2K}}$, $C_3 = \frac{4\sqrt{2}}{1-(1+\sqrt{2})\delta_{2K}}$.

According to the above theorem, the influence of the noise jamming signal on the image reconstruction is mainly determined by the orthogonality $\|\Phi^T e\|_\infty$ between the observation matrix Φ and the jamming signal $e = \mathbf{j}$. The lower $\|\Phi^T e\|_\infty$ will bring smaller reconstruction error $\|\hat{x} - x\|_2$. In other words, when the projection component of the noise jamming in the subspace formed by Φ is smaller, the reconstruction error will be smaller too. It is worth mentioning that the larger the energy and the more similar the jamming signal is to the echo, the larger the reconstruction error is.

Different parameters of the jamming signal and different reconstruction algorithms will bring different reconstruction errors. In this paper, simulation experiments are carried out for analysis of the reconstruction performance. In these experiments, the average relative error (ARE) is used to measure the size of the reconstruction error, which is defined as follows:

$$\Delta_{\text{ARE}} = \frac{\|\mathbf{x} - \hat{x}\|_2}{\|\mathbf{x}\|_2} \quad (13)$$

4. SIMULATION RESULTS

The simulation parameters are listed as follows: the speed of SAR platform 100 m/s, the slant distance between SAR and the illuminated area 50 km, carrier frequency 10 GHz, bandwidth 100 MHz, pulse width 5 μ s, sampling frequency 150 MHz, the azimuth resolution 1.5 m. There are 10 point targets in the illuminated scene. The locations of the targets are randomly generated. Three types of noise jamming (RFNJ, NAMJ, and NFMJ) are added to the echo in different Jamming-to-Signal Ratio (JSR). Four CS algorithms (Orthogonal Matching Pursuit (OMP), Stagewise OMP (StOMP), Compressive Sampling Matching Pursuit (CoSaMP), and SL algorithm) are used to reconstruct the final image.

As can be seen from Fig. 1, the reconstruction accuracy of StOMP and SL0 algorithms is lower than OMP and CoSaMP. Otherwise, at the same JSR, the impact of RFNJ is smaller than NFMJ and NAMJ. The reason is that StOMP and SL0 algorithms obtain the sparse solution by obtaining the projection of the observation data on the subspace composed of Φ in multiple times, and it makes the errors of each iteration overlap each other. OMP algorithm calculates the maximum projection of the observation data on the column vectors of Φ , and the error has not accumulated. The CoSaMP algorithm adjusts the observation matrix during the solution process to improve the orthogonality with the observation data, so that the error caused by the jamming can be reduced.

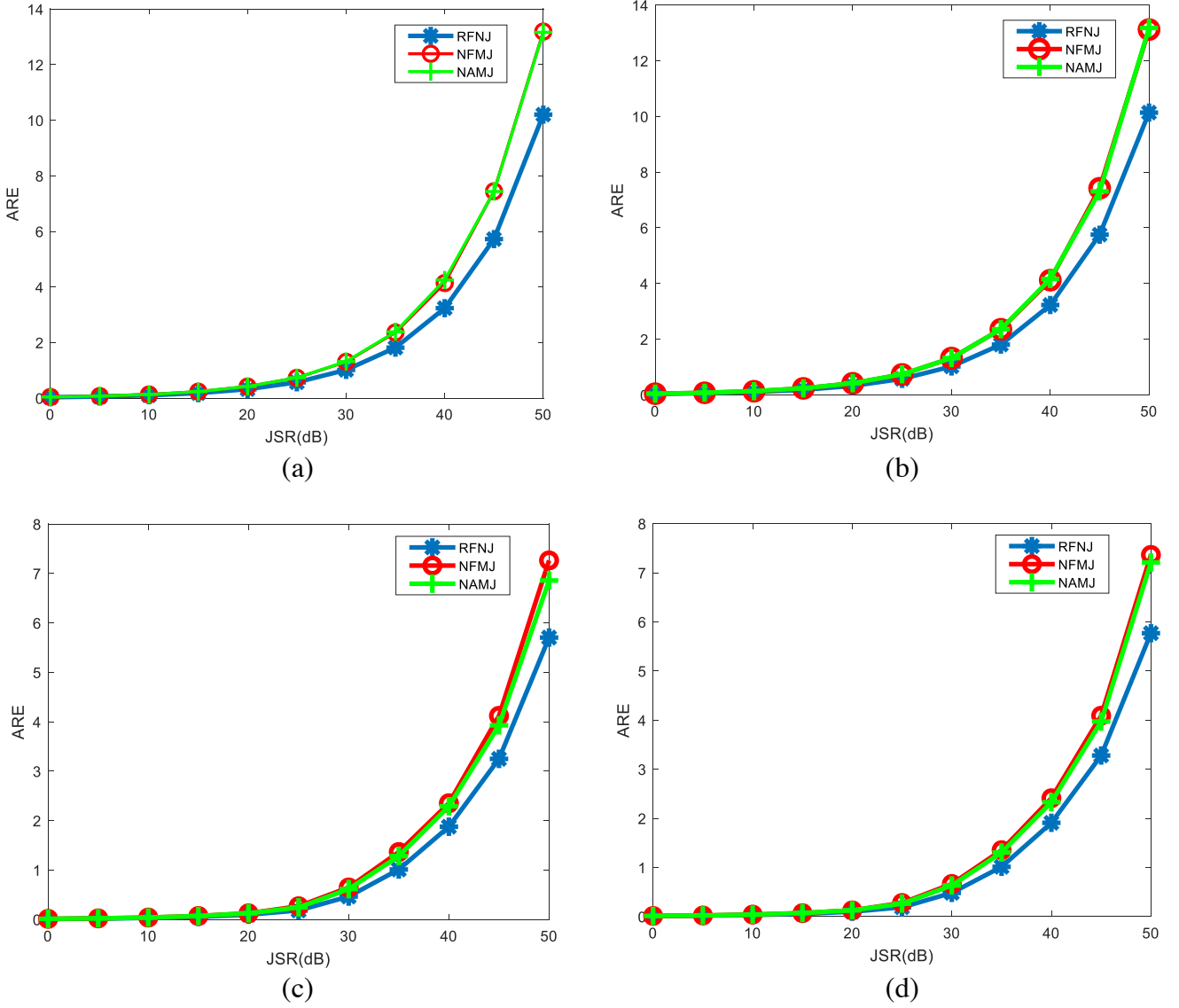


Figure 1. The impact of RFNJ, NAMJ and NFMJ on the imaging performance with different reconstruction algorithms. (a) StOMP. (b) SL0. (c) OMP. (d) CoSaMP.

StOMP is a reconstruction algorithm commonly used in CS imaging radars. The next experiment shows the influence of the threshold coefficient in StOMP algorithm with $\text{JSR} = 50 \text{ dB}$.

From Fig. 2, it can be seen that the threshold coefficient should be appropriately increased with a large jamming power. Large threshold coefficient reduces number of the vectors corresponding to the point target, and it makes the projection component of the jamming not be superposed in different iterations. However, the threshold coefficient should not be set too large; otherwise, the solution corresponding to the point target will not be obtained correctly.

In the remaining simulation experiments, we will study the influence of the key jamming parameters on the reconstruction accuracy of StOMP algorithm.

From Fig. 3, it can be seen that different FM slopes have a small impact on the reconstruction accuracy, and larger FM slope brings slightly increased reconstruction error.

Figure 4 shows that when the center frequency of NAMJ is aligned with the carrier frequency of the radar, the reconstruction error is the largest. As the center frequency of NAMJ deviates from the radar frequency, the impact becomes smaller. This is because the larger frequency deviation is, the lower the

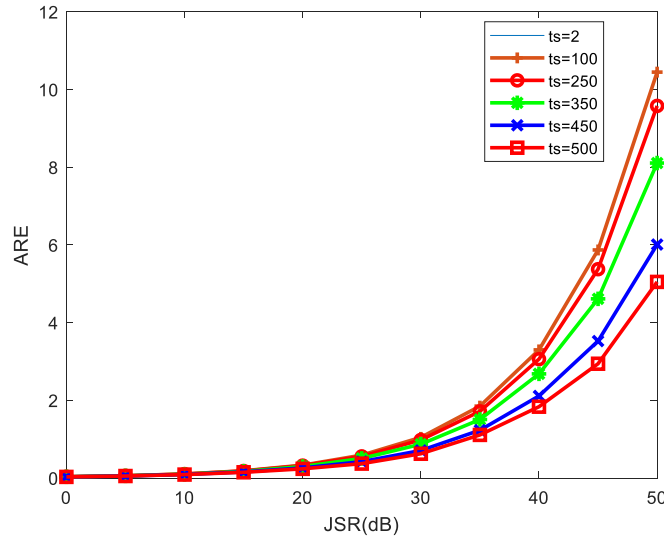


Figure 2. The influence of different threshold coefficients (ts) on the reconstruction performance.

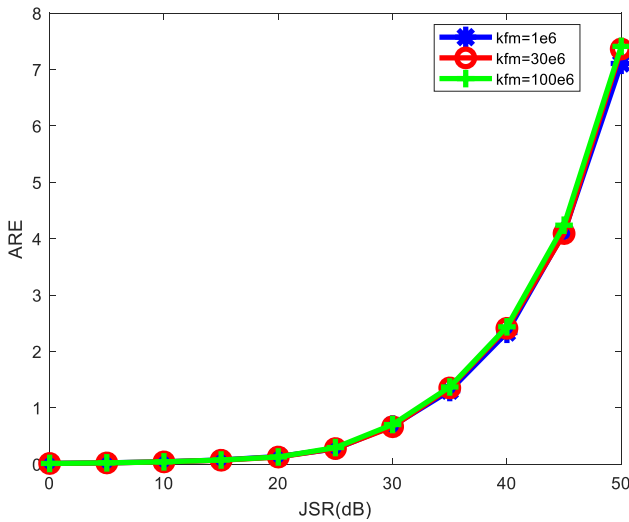


Figure 3. The impact of NFMJ on the reconstruction performance under different FM slopes (kfm).

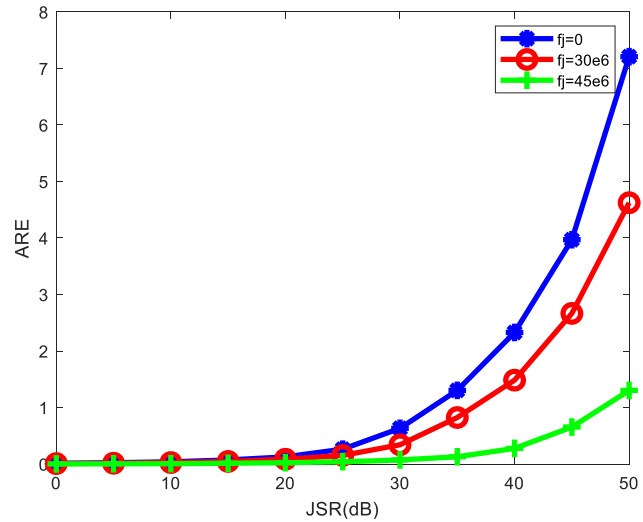


Figure 4. The impact of NAMJ on the reconstruction performance under different frequency offsets (fj).

orthogonality is between the jamming and the subspace formed by Φ , and the jamming energy is also reduced.

5. CONCLUSION

The paper studies the influence of different noise jamming styles on the performance of compressed sensing imaging with different reconstruction algorithms. Theoretical analysis shows that the better the orthogonality is between the observation matrix and the noise jamming, the smaller the influence of noise jamming is. The simulation experiments show that the influences of NFMJ and NAMJ are relatively close, and both are greater than RFNJ. OMP and CoSaMP algorithms have better anti-jamming performance than StOMP and SL0. The key parameters of the reconstruction algorithm are also important for the reconstruction performance with noise jamming.

Future work will conduct two real-data experiments to validate the noise jamming effects. The first experiment will be carried out in an anechoic chamber, where the noise jamming signal is generated by an arbitrary waveform generator, and a vector analyzer is placed on the slide rail to irradiate several metal balls and collect echoes and jamming signals. The second experiment will be conducted on the RADARSAT-1 data set and a real jamming data set generated by an arbitrary waveform generator. Based on the above two experiments, the noise jamming effects in different sparsities of the targets will be further studied.

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