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RRT-MWF-MVDR Algorithm for Space-Time Antijamming

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Abstract—Minimum variance distortionless response (MVDR) beamformer is one of the well-known space-time antijamming techniques for global navigation satellite system (GNSS). It can jointly utilize spatial filter and temporal filter to suppress interference signals. However, the computational complexity is usually so high that it is difficult to apply to engineering problems. In order to solve this problem, a novel MVDR algorithm based on rank-reducing transformation (RRT) and multistage wiener filter (MWF) is proposed for reducing the computational complexity, named as RRT-MWF-MVDR algorithm. Via the characteristics of the oppressive jamming environment and the steering vector of satellite signal, a rank-reducing transformation is given. By the rank-reducing transformation, a rank reduction is realized for the high dimensional received data. Taking these received data with reduced rank as the input of the MWF, the forward decomposition and backward iteration are accomplished. Then the equivalent reduced rank matrix and equivalent weight vector of MWF can be given, respectively. Finally, the space-time two-dimensional antijamming weight vector is given by the mathematical relationship between the reduced-rank matrix and the weight vector. The proposed method can effectively avoid the inverse of high-dimensional matrix. The proposed method offers a number of advantages over the existing algorithms. For example, (1) it has less computational load and is easier to be executed in practical application; (2) it can maintain higher output signal-to-interference-noise ratio (SINR). Simulation results verify the effectiveness of proposed method.

1. INTRODUCTION

Global navigation satellite system (GNSS) provides accurate location and velocity information in military and civilian fields, for instance, agriculture, aviation, land-vehicle navigation, and marine-navigation [1]. However, the GNSS receiver is easily disrupted by various interference signals since the power level of satellite signals is extremely weak after a long distance transmission. Thus, the antijamming performance has received significant attention for GNSS.

The space-time adaptive processor (STAP) is used to receive the satellite signals while suppressing the interfering signals, which add a temporal filtering on the basis of the spatial filtering [2, 3]. It is well known that the spatial degree of freedom (DOF) is equal to $M-1$ in the case of using an M-element antenna array. Compared with temporal and spatial filtering, STAP has lager antijamming DOF and can suppress more kinds of interferences. However, the received signal dimension of STAP increases from M to MP owing to the addition of temporal filtering (each antenna element is equipped with P delay taps). Therefore, the antijamming algorithm based on STAP has a large amount of computation due to calculating covariance matrix inverse, so that many low complexity methods are proposed, such as principal components (PC) [4], cross spectral metric (CSM) [5], multistage wiener filter (MWF) [6], and reduced-rank minimum variance beamformer (RRMVB) [7]. Unfortunately, the above algorithms may involve eigendecomposition (EVD) or covariance matrix inverse, which still require a large amount of computational load. In order to avoid the computation of covariance matrix inverse and EVD,

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dimensionality reduction and iterative algorithms attract much attention [8–12]. The householder MWF is improved to suppress interference and impulsive noise. It can gain good performance for narrow band and wideband signals [8]. An MWF with spatial blocking broadening and automatic rank selection is proposed [9]. It can ensure high angle estimation accuracy and robust reduced-rank beamforming. An adaptive reduced-rank method based on knowledge-aided joint iterative optimization is exploited [10]. Via joint optimization, the reduced-rank matrix and beamforming weight vector are constructed even under the condition of few samples. Besides, a space-time reduced-rank algorithm is presented for antijamming [11]. By combining the least squares (LS) and joint iterative optimization of parameter vectors, the best set of basis for reduced-rank processing can be automatically obtained. In [12], a weight vector is obtained via the Least Mean Square (LMS), which guarantees jamming nulling while declining the cost of hardware and computational load.

The signal feature is critical to improve the antijamming performance. Recently, several algorithms with lower complexity load have been proposed to improve the adaptive processing performance based on projection technique. A robust null broadening method based on projection technique is proposed to alleviate the computational load [13]. It can be robust even though the calibration errors exist. In [14], the received data are preprocessed by the projection technique. Via the diagonal loading (DL) method, the covariance matrix is obtained. Compared with [13], the method has not only a deeper null but also a lower computational complexity. Combining with projection transform and diagonal loading, a new adaptive beamforming algorithm is given [15]. It can gain a new sample covariance matrix via the subspace projection. This method can effectively improve the antijamming performance, such as broadening the width of jammer nulls and strengthen the null depth. Minimum variance distortionless response (MVDR) algorithm is one of the popular approaches for antijamming [16]. An improved MVDR is proposed to control the sidelobe level utilizing several constraints [17]. It enforces the direction-of-arrival (DOA) of the desired signal to be far away from the DOAs of the interfering signals. An idea of reconstructing the interference-plus-noise (IN) covariance matrix is presented [18]. The estimation error of the desired signal can be effectively eliminated since the reconstructed IN matrix does not contain the desired signal. To alleviate the computational burden caused by EVD, a fast reduced rank minimum variance beamformer (FRRMVB) is proposed [19], which estimates the interference subspace by using a set of the received data vectors.

In this paper, an RRT-MWF-MVDR algorithm is proposed for the antijamming problem. Via a rank-reducing transformation (RRT) and multistage wiener filter (MWF), the presented method can not only effectively save the computational cost, but also maintain a high array output power of the interesting satellite signal and a high output signal to interference-plus-noise-ratio (SINR). The rest of the paper is organized as follows. The data model is described in Section 2. Section 3 introduces the presented method. Section 4 shows some simulation results. Finally, the conclusion is given in Section 5.

2. DATA MODEL

Consider that GNSS receiver with a uniform linear array (ULA) which consisting of M antennas. Each antenna is equally spaced with P taps, and the space-time filter structure is given in Figure 1.

Assume that there are an interesting satellite signal $s(t)$ and q narrow-band oppressive jamming signals $j_k(t)$ $(k = 1, \cdots, q)$ impacting on the ULA. Then, the $MP \times 1$ received signals $\mathbf{x}(t)$ at time instant t can be expressed as [20]

$$\mathbf{x}(t) = \mathbf{a}_0 s(t) + \sum_{k=1}^{q} \mathbf{a}_k j_k(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{x}(t) = [x_{11}, x_{12}, \cdots, x_{1P}, \cdots, x_{M1}, \cdots, x_{MP}]^T$, $s(t)$ and $j_k(t)\,\{k = 1, 2, \cdots, q\}$ represent the interesting satellite signal and the kth oppressive jamming signal, respectively. $\mathbf{n}(t)$ denotes the white Gaussian noise of the space-time filter structure. $\mathbf{a}_0$ and $\mathbf{a}_k\,\{k = 1, 2, \cdots, q\}$ represent steering vectors of satellite and interference signals, respectively, which have the following forms

$$\begin{aligned}
\mathbf{a}_l &= \mathbf{a}_s(\theta_l) \otimes \mathbf{a}_t(f_l) \text{ for } l = 0, 1, 2, \cdots, q \\
\mathbf{a}_s(\theta_l) &= [1, \exp\{-j2\pi f_l d \sin(\theta_l)/c\}, \cdots, \exp\{-j2\pi f_l (M-1) d \sin(\theta_l)/c\}]^T \\
\mathbf{a}_t(f_l) &= [1, \exp\{-j2\pi f_l \tau\}, \cdots, \exp\{-j2\pi f_l (P-1)\tau\}]^T
\end{aligned} \tag{2}$$

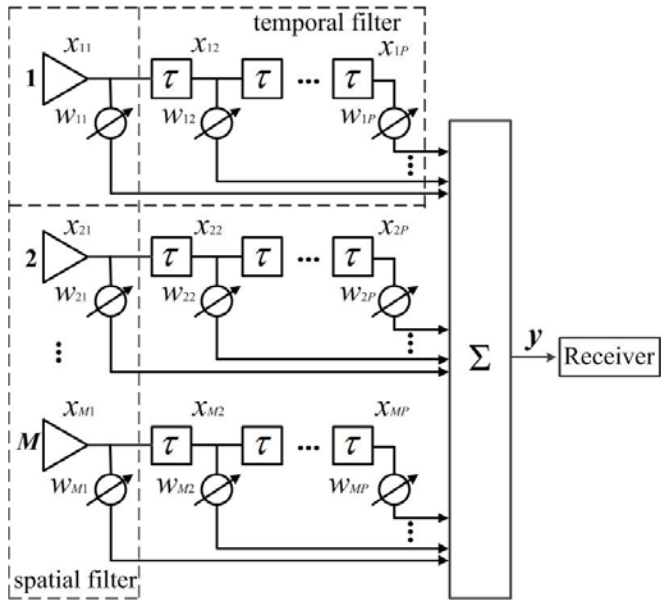

Figure 1. Space-time antijamming model.

where θ_l and f_l denote the DOA and frequency for the lth signal, respectively. τ stands for the delay time. The superscript $(\cdot)^T$ denotes the transpose operation.

The covariance matrix of $\mathbf{x}(t)$ can be expressed as

$$\mathbf{R}_x = E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \sigma_s^2\mathbf{a}_0\mathbf{a}_0^H + \mathbf{A}_j\mathbf{R}_b\mathbf{A}_j^H + \sigma_n^2\mathbf{I} = \mathbf{R}_d + \mathbf{R}_{j+n} \tag{3}$$

where $E\{\cdot\}$ represents the statistical average operation, and the superscript $(\cdot)^H$ denotes the conjugate transpose. $\sigma_s^2 = E\{\mathbf{s}(t)\mathbf{s}^H(t)\}$ is the power of the interesting satellite signal. $\mathbf{R}_b = E\{\mathbf{j}(t)\mathbf{j}^H(t)\}$, where $\mathbf{j}(t) = [j_1(t), j_2(t), \cdots, j_k(t), \cdots, j_q(t)]$ indicates q interference signals. $\mathbf{R}_b$ is a diagonal matrix whose kth diagonal element is the power of the kth interference signal for $k = 1, 2, \cdots, q$. The $MP \times q$ steering vector matrix $\mathbf{A}_j = [\mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_k, \cdots, \mathbf{a}_q]$. σ_n^2 and $\mathbf{I}$ indicate Gaussian noise power and identity matrix, respectively. $\mathbf{R}_d = \sigma_s^2\mathbf{a}_0\mathbf{a}_0^H$ is the interesting satellite signal covariance matrix. $\mathbf{R}_{j+n} = \mathbf{A}_j\mathbf{R}_b\mathbf{A}_j^H + \sigma_n^2\mathbf{I}$ stands for the interference-plus-noise (IN) covariance matrix.

The output signal $y(t)$ by the STAP can be expressed as

$$y(t) = \mathbf{w}^H\mathbf{x}(t) \tag{4}$$

where $\mathbf{w} \in \mathbf{C}^{MP\times 1}$ represents the space-time antijamming weight vector.

For a space-time two-dimensional antijamming weight vector $\mathbf{w}$, the power of the output signals can be expressed as follows

$$P_o(\mathbf{w}) = E\left\{y(t)y^H(t)\right\} = \mathbf{w}^H\mathbf{R}_d\mathbf{w} + \mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w} = \sigma_s^2\left|\mathbf{w}^H\mathbf{a}_0\right|^2 + \mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w} \tag{5}$$

where $\sigma_s^2|\mathbf{w}^H\mathbf{a}_0|^2$ stands for the output power of the interesting satellite signal. $\mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w}$ represents the output power of the interferences plus noise.

Similarly, the output SINR is obtained as follows

$$\gamma_{\mathrm{SINR}}(\mathbf{w}) = \frac{\mathbf{w}^H\mathbf{R}_d\mathbf{w}}{\mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w}} = \frac{\sigma_s^2|\mathbf{w}^H\mathbf{a}_0|^2}{\mathbf{w}^H\mathbf{R}_{j+n}\mathbf{w}} = \sigma_s^2\mathbf{a}_0^H\mathbf{R}_{j+n}^{-1}\mathbf{a}_0 \tag{6}$$

where $\gamma_{\mathrm{SINR}}(\mathbf{w})$ stands for the array output SINR.

The common formulation of the space-time two-dimensional antijamming weight vector $\mathbf{w}$ can be solved by MVDR algorithm, which can effectively suppress the interference signals while ensuring distortionless response of the interesting satellite signal. The optimal solution of MVDR weight vector can be expressed as

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_{j+n} \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1 \tag{7}$$

where $\mathbf{R}_{j+n}$ is the IN covariance matrix given by Eq. (3).

Since $\mathbf{R}_{j+n}$ cannot be directly obtained, the problem in Eq. (7) is usually rewritten as the following optimization problem

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R}_x \mathbf{w} \quad \text{s.t.} \quad \mathbf{w}^H \mathbf{a}_0 = 1 \tag{8}$$

where the IN covariance matrix $\mathbf{R}_{j+n}$ is replaced by the covariance matrix $\mathbf{R}_x$.

The optimal solution of the problem in Eq. (8) can be given by $\mathbf{w} = \frac{\mathbf{R}_x^{-1}\mathbf{a}_0}{\mathbf{a}_0^H \mathbf{R}_x^{-1} \mathbf{a}_0}$, where the superscript $(\cdot)^{-1}$ denotes the matrix inverse. In practical applications, $\mathbf{R}_x$ is replaced by a finite sample covariance matrix $\hat{\mathbf{R}}_x = \frac{1}{N}\sum_{k=1}^{N} \mathbf{x}(k)\mathbf{x}^H(k)$, where N denotes the snapshot number.

3. PROPOSED ALGORITHM

In this section, RRT-MWF-MVDR algorithm is described in detail. The structure of the proposed algorithm is shown as in Figure 2. A rank-reducing transformation and array output SINR are carried out. Via MWF, a closed-form solution of the space-time weight vector is given.

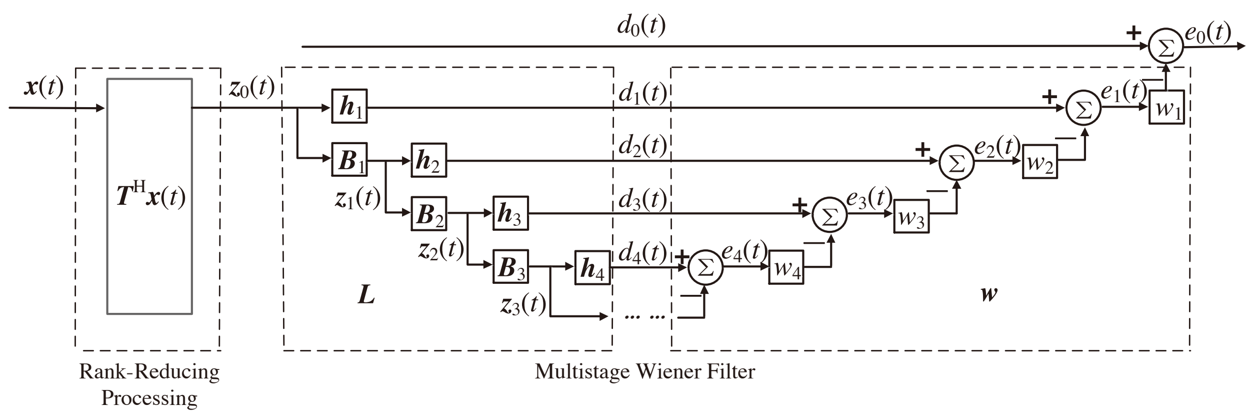

Figure 2. The structure of SSP-MWF-MVDR algorithm.

3.1. Rank-Reducing Transformation and Output SINR

Assume that the IN covariance matrix $\mathbf{R}_{j+n}$ can be estimated. Starting from the eigenvalue decomposition (EVD) of the IN covariance matrix, $\mathbf{R}_{j+n}$ can be expressed as

$$\mathbf{R}_{j+n} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^H = \mathbf{U}_j \boldsymbol{\Lambda}_j \mathbf{U}_j^H + \sigma_n^2 \mathbf{U}_n \mathbf{U}_n^H \tag{9}$$

where $\boldsymbol{\Lambda}$ and $\boldsymbol{\Lambda}_j$ are diagonal matrixes. The diagonal elements of matrix $\boldsymbol{\Lambda}_j \in \mathbf{C}^{q\times q}$ consist of q principal eigenvalues of $\mathbf{R}_x$, that is, $\boldsymbol{\Lambda}_j = \text{diag}(\lambda_1, \lambda_2, \cdots, \lambda_q), \lambda_1 \geq \lambda_2 \cdots \geq \lambda_q$, and the columns of $\mathbf{U}_j \in \mathbf{C}^{MP\times q}$ are the corresponding eigenvectors which form interference signal subspace. The columns of $\mathbf{U}_n$ are the remaining eigenvectors of $\mathbf{R}_x$ which construct noise subspace. In addition, $\mathbf{U}_j^H \mathbf{U}_j = \mathbf{I} \in \mathbf{C}^{q\times q}$, and $\mathbf{U}_j^H \mathbf{U}_n = \mathbf{0}$.

Therefore, the signal subspace $\mathbf{U}_s$ can be written as $\mathbf{U}_s = [\mathbf{U}_j \ \mathbf{a}_0]$, where $\mathbf{a}_0$ is the known steering vector of interesting satellite. Let the rank-reducing transformation $\mathbf{T} = \mathbf{U}_s = [\mathbf{U}_j \ \mathbf{a}_0]$. Then, via the rank-reducing transformation $\mathbf{T}$, the $MP \times 1$ data vector $\mathbf{x}(t)$ can be written as $\mathbf{T}^H\mathbf{x}(t)$. Therefore, the problem in Eq. (7) can be rewritten as

$$\min_{\mathbf{w}_r} \mathbf{w}_r^H \mathbf{T}^H \mathbf{R}_{j+n} \mathbf{T} \mathbf{w}_r \quad \text{s.t.} \quad \mathbf{w}_r^H \mathbf{T}^H \mathbf{a}_0 = 1 \tag{10}$$

where $\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T}$ and $\mathbf{w}_r$ are the $r \times r$ rank-reduced matrix and corresponding $r \times 1$ antijamming weight vector, respectively.

For the problem in Eq. (10), the weight can be expressed as

$$\mathbf{w}_r = \frac{(\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T})^{-1}\mathbf{a}_{0r}}{\mathbf{a}_{0r}^H(\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T})^{-1}\mathbf{a}_{0r}} \tag{11}$$

where $\mathbf{a}_{0r} = \mathbf{T}^H\mathbf{a}_0$.

Similarly, according to Eq. (6), the output SINR with rank reducing processing can be given as

$$\gamma_{\mathrm{SINR}r} = \frac{\sigma_s^2|\mathbf{w}_r^H\mathbf{a}_0|^2}{\mathbf{w}_r^H\mathbf{R}_z\mathbf{w}_r} = \sigma_s^2\mathbf{a}_0^H\mathbf{T}(\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T})^{-1}\mathbf{T}^H\mathbf{a}_0 \tag{12}$$

where $\mathbf{R}_z = \mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T}$, it has the following forms

$$\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T} = \begin{bmatrix}\mathbf{U}_j^H \\ \mathbf{a}_0^H\end{bmatrix}(\mathbf{U}_j\mathbf{\Lambda}_j\mathbf{U}_j^H + \mathbf{U}_n\mathbf{\Lambda}_j\mathbf{U}_n^H)\begin{bmatrix}\mathbf{U}_j & \mathbf{a}_0\end{bmatrix} = \begin{bmatrix}\mathbf{\Lambda}_j & \mathbf{\Lambda}_j\mathbf{U}_j\mathbf{a}_0 \\ \mathbf{a}_0^H\mathbf{U}_j\mathbf{\Lambda}_j & \mathbf{a}_0^H(\mathbf{U}_j\mathbf{\Lambda}_j\mathbf{U}_j^H + \mathbf{U}_n\mathbf{\Lambda}_j\mathbf{U}_n^H)\mathbf{a}_0\end{bmatrix} \tag{13}$$

In terms of the matrix inversion lemma, the inverse matrix of $\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T}$ can be given as

$$(\mathbf{T}^H\mathbf{R}_{j+n}\mathbf{T})^{-1} = \begin{bmatrix}\mathbf{\Lambda}_j^{-1} + \mathbf{U}_j\mathbf{a}_0(\mathbf{a}_0^H\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H\mathbf{a}_0)^{-1}\mathbf{a}_0^H\mathbf{U}_j & -\mathbf{U}_j\mathbf{a}_0(\mathbf{a}_0^H\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H\mathbf{a}_0)^{-1} \\ -(\mathbf{a}_0^H\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H\mathbf{a}_0)^{-1}\mathbf{a}_0^H\mathbf{U}_j & (\mathbf{a}_0^H\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H\mathbf{a}_0)^{-1}\end{bmatrix} \tag{14}$$

Substituting Eq. (14) into Eq. (12), the output SINR can be rewritten as

$$\begin{aligned}\gamma_{\mathrm{SINR}r} &= \sigma_s^2\mathbf{a}_0^H\mathbf{T}(\mathbf{T}^H\mathbf{R}_x\mathbf{T})^{-1}\mathbf{T}^H\mathbf{a}_0 = \sigma_s^2\left(\mathbf{a}_0^H\mathbf{U}_j\mathbf{\Lambda}_j^{-1}\mathbf{U}_j^H\mathbf{a}_0 + \frac{(\mathbf{a}_0^H\mathbf{a}_0 - \mathbf{a}_0^H\mathbf{U}_j\mathbf{U}_j^H\mathbf{a}_0)^2}{\mathbf{a}_0^H\mathbf{U}_n\mathbf{\Lambda}_n\mathbf{U}_n^H\mathbf{a}_0}\right) \\ &= \sigma_s^2\left(\mathbf{a}_0^H\mathbf{U}_j\mathbf{\Lambda}_j^{-1}\mathbf{U}_j^H\mathbf{a}_0 + \frac{(\mathbf{a}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}_0)^2}{\sigma_n^2\mathbf{a}_0^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}_0}\right) = \sigma_s^2\mathbf{a}_0^H(\mathbf{U}_j\mathbf{\Lambda}_j^{-1}\mathbf{U}_j^H + \frac{1}{\sigma_n^2}\mathbf{U}_n\mathbf{U}_n^H)\mathbf{a}_0 \\ &= \sigma_s^2\mathbf{a}_0^H\mathbf{R}_{j+n}^{-1}\mathbf{a}_0\end{aligned} \tag{15}$$

From Eqs. (6) and (15), it is clear that $\gamma_{\mathrm{SINR}} = \gamma_{\mathrm{SINR}r}$, namely, the weight vector $\mathbf{w}_r$ given by the problem (10) can maintain the same output SINR as the weight vector $\mathbf{w}$ given by the problem in Eq. (7).

Therefore, the rank-reducing transformation can be chosen as $\mathbf{T} = [\hat{\mathbf{U}}_j, \mathbf{a}_0]$, where $\hat{\mathbf{U}}_j$ is the estimated interference subspace. Furthermore, in order to alleviate the computational load, $\hat{\mathbf{U}}_j$ is estimated roughly by the $(r-1)$ snapshot vector $\mathbf{X} = [\mathbf{x}(t_1), \cdots, \mathbf{x}(t_{r-1})]$, that is, the rank reducing transformation can be given by [21]

$$\mathbf{T} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \cdots, \mathbf{x}(t_{r-1}), \mathbf{a}_0] \tag{16}$$

where r denotes the rank-reducing number with $r \geq \sqrt{q(M+1)} - 1$.

3.2. Multistage Wiener Filter

To avoid matrix inversion, this subsection gives a computation method based on MWF for the antijamming weight vector. The structure of MWF is shown as in Figure 2. The input signal $\mathbf{z}_0(t)$ of the MWF can be given by $\mathbf{z}_0(t) = \mathbf{T}^H\mathbf{x}(t)$, and the interesting satellite signal $s(t)$ is regarded as the reference signal $d_0(t)$. $\mathbf{z}_i(t) \in \mathbf{C}^{(r-i)\times 1}$ $(i = 1, 2, \cdots, r-1)$ is the input signal for the ith stage filter. The signal $d_i(t)$ is defined as

$$d_i(t) = \mathbf{h}_i^H\mathbf{z}_{i-1}(t) \tag{17}$$

where $\mathbf{h}_i = \frac{\mathbf{r}_{z_{i-1}d_{i-1}}}{\left\|\mathbf{r}_{z_{i-1}d_{i-1}}\right\|} = \frac{\mathbf{r}_{z_{i-1}d_{i-1}}}{\sqrt{\mathbf{r}_{z_{i-1}d_{i-1}}^H\mathbf{r}_{z_{i-1}d_{i-1}}}}$ guarantees the correlation between $d_i(t)$ and $d_{i-1}(t)$. $\mathbf{r}_{z_{i-1}d_{i-1}} = E\{\mathbf{z}_{i-1}(t)d_{i-1}^*(t)\}$. $(\cdot)^*$ represents the complex conjugation.

Let $\mathbf{B}_i$ be an $(r-i)\times(r-i+1)$ row full rank blocking matrix with $\mathbf{B}_i\mathbf{h}_i = \mathbf{0}$. The input signal of the filter can be expressed as

$$\mathbf{z}_i(t) = \mathbf{B}_i^H \mathbf{z}_{i-1}(t) \tag{18}$$

$\mathbf{z}_{i-1}(t)$ is projected on a subspace orthogonal to the cross-correlation vector $\mathbf{r}_{z_{i-1}d_{i-1}}$, so that $\mathbf{z}_{i-1}(t)$ does not contain any information in $d_i(t)$. In terms of Eqs. (17) and (18), the upper branch signal of each stage of the filter can be further expressed as

$$d_i(t) = \mathbf{h}_i^H \left(\prod_{j=1}^{i-1} \mathbf{B}_j\right) \mathbf{z}_0(t) \tag{19}$$

where $\prod$ represents the multiplicative symbol, and $\prod_{j=1}^{i-1}\mathbf{B}_j = \mathbf{B}_{i-1}\mathbf{B}_{i-2}\cdots\mathbf{B}_1$. The Wiener filter is decomposed step by step until $\mathbf{z}_i(t)$ is transformed into a scalar, i.e.,

$$\mathbf{z}_{r-1}(t) = d_r(t) = e_r(t) = \mathbf{B}_{r-1}\mathbf{z}_{r-2}(t) = \cdots = \left(\prod_{j=1}^{r-1}\mathbf{B}_j\right)\mathbf{z}_0(t) \tag{20}$$

where $e_i(t)$ is the iterative output error of the ith stage filter and can be expressed as

$$e_i(t) = d_i(t) - w_{i+1}^* e_{i+1}(t) \quad (i = 1, 2, \cdots, r-1) \tag{21}$$

where $w_i = \xi_i^{-1}\delta_i$, and $\xi_i = \mathbf{h}_i^H(\prod_{j=1}^{i-1}\mathbf{B}_j)\mathbf{R}_{z0}(\prod_{j=1}^{i-1}\mathbf{B}_j)^H\mathbf{h}_i$, $\delta_i = \sqrt{\mathbf{r}_{z_{i-1}d_{i-1}}^H \mathbf{r}_{z_{i-1}d_{i-1}}}$. Thus, the final iteration output of the MWF is obtained as follows

$$e_0(t) = d_0(t) - w_1^*\{d_1(t) - w_2^*[d_2(t) - \cdots - w_{r-1}^*(d_r(t) - w_r^* e_r(t))]\} = d_0(t) - \mathbf{w}_d^H\mathbf{d}(t) \tag{22}$$

where $\mathbf{w}_d = [w_1, -w_1w_2, w_1w_2w_3, \cdots, (-1)^{r+1}\prod_{i=1}^{r}\mathbf{w}_i]^T$, $\mathbf{d}(t) = [d_1(t), d_2(t), \cdots, d_r(t)]^T$ which can be further denoted as follows

$$\mathbf{d}(t) = \begin{bmatrix} \mathbf{h}_i^H \\ \mathbf{h}_2^H\mathbf{B}_1 \\ \vdots \\ \mathbf{h}_{r-1}^H \prod_{j=1}^{r-2}\mathbf{B}_j \\ \prod_{j=1}^{r-1}\mathbf{B}_j \end{bmatrix} \mathbf{z}_0(t) = \mathbf{L}\mathbf{z}_0(t) \tag{23}$$

The above $\mathbf{d}(t)$ is brought back to Eq. (22), which can be obtained as follows

$$e_0(t) = d_0(t) - \mathbf{w}_d^H\mathbf{L}\mathbf{z}_0(t) \tag{24}$$

Finally, the adaptive weight vector of the proposed algorithm can be given as

$$\mathbf{w} = \mathbf{L}^H\mathbf{w}_d \tag{25}$$

where $\mathbf{L}$ and $\mathbf{w}_d$ are the equivalent reduced rank matrix and weight of MWF, respectively.

Summary of SSP-MWF-MVDR Algorithm

Step 1 Collect data $\mathbf{X} = [\mathbf{x}(1), \cdots, \mathbf{x}(N)]$ where $\mathbf{x}(n)$ denotes the nth snapshot of the array.

Step 2 Estimate the signal subspace $\mathbf{T} = [\mathbf{x}(t_1), \mathbf{x}(t_2), \cdots, \mathbf{x}(t_{r-1}), \mathbf{a}_0]$, where $r \geq \sqrt{q(M+1)} - 1$.

Step 3 Take $\mathbf{z}_0(k) = \mathbf{T}^H\mathbf{x}(k)$ as the input vector and calculate the equivalent reduced rank matrix $\mathbf{L}$ and weight $\mathbf{w}_d$ of MWF, respectively.

Step 4 Compute the space-time 2-D antijamming weight vector $\mathbf{w}$, according to Eq. (25).

3.3. Computational Complexity Analysis

The computational complexity of the proposed algorithm is briefly investigated. The computational complexity of the proposed algorithm mainly includes: (1) the rank-reducing transformation of the $MP \times 1$ snapshot vector $\mathbf{x}(k)$, of order $\mathcal{O}(MPr)$, where r denotes the rank-reducing number; (2) the equivalent reduced rank matrix and weight estimation of MWF of order $\mathcal{O}(r^2)$. Therefore, the computational complexity of the proposed algorithm is of order $\mathcal{O}(MPr)$.

Remarks. (1) It is clear that $\sqrt{q(M+1)}-1 \leq r < MP$, where q denotes the number of interference signals, and M stands for the number of antennas. In other words, the determination of r depends on the number of interference signals and the number of antennas. (2) The rank-reducing number r affects the computational complexity of the proposed algorithm, and the computational complexity of the proposed algorithm decreases with the decrease of r. The lower bound of r is equal to $\sqrt{q(M+1)}-1$.

Table 1 presents the computational complexity of the proposed method and several relevant algorithms including as PC [4], CSM [5], MWF-D [6], and FRRMVB [19]. From Table 1, it can be seen that the proposed method has lower computational costs than other algorithms.

Table 1. Comparison of computational complexities.

Algorithm	Computational complexities
PC	$\mathcal{O}((MP)^3)$
CSM	$\mathcal{O}((MP)^3)$
MWF-D	$\mathcal{O}((MP)^2)$
FRRMVB	$\mathcal{O}(\max(r^3, MPr))$
Proposed Algorithm	$\mathcal{O}(MPr)$

4. SIMULATION RESULTS

In this section, several simulation results are constructed to evaluate the proposed algorithm. Consider a ULA with $M = 8$ antenna elements whose separation distances are half-wavelength, and each element is equally spaced with $P = 5$ taps. Assume that there are four far field signals impinging on the array. Among them, the DOA of satellite signal is set to $\theta_1 = 0°$, and its normalized center frequency is $f_1 = 1.0\,\text{GHz}$. The DOAs of three interference signals are $\theta_2 = -10°$, $\theta_3 = 5°$, $\theta_4 = 0°$, respectively. Their corresponding center frequencies are $f_2 = 0.9\,\text{GHz}$, $f_3 = 1.1\,\text{GHz}$, $f_4 = 1.2\,\text{GHz}$. The SNR of satellite signal is equal to $-20\,\text{dB}$, and the interference-to-noise ratio (INR) of the three interfering signals is equal to $40\,\text{dB}$.

Figure 3(a) shows the space-time 2-D response diagram of the proposed algorithm. As can be seen in the pattern, it has good mainlobe for the interesting satellite signal at $\theta_1 = 0°$ and $f_1 = 1.0\,\text{GHz}$, and the three interference signals are effectively suppressed. Figure 3(b) is a contour plot of the response shown in Fig. 3(a). It is easy to know that the coordinates of the three interference signals are located at $(\theta_2, f_2) = (-10°, 0.9\,\text{GHz})$, $(\theta_3, f_3) = (5°, 1.1\,\text{GHz})$, $(\theta_4, f_4) = (0°, 1.2\,\text{GHz})$, respectively. As shown in Figure 3, the proposed algorithm not only effectively blankets jammings, but also enhances the power of the satellite signal.

Figure 4 gives the array output power curves of the interesting satellite signal and the array output SINR curves. Five algorithms are carried out for comparison, including PC [4], CSM [5], MWF-D [6], FRRMVB [19], and the proposed algorithm with the reduced-rank ranging from 4 to 20. From Figure 4, we can see that the proposed method can provide better array output power and output SINR than the aforementioned four algorithms, even under the condition of different rank-reduced numbers. This is because the rank of the received data is reduced by projecting the received data on the signal subspace in the proposed algorithm, and then takes the reduced rank data as the input data of multi-stage Wiener filtering, so that the proposed approach can not only effectively reduce the computational complexity, but also enhance the interesting satellite signal.

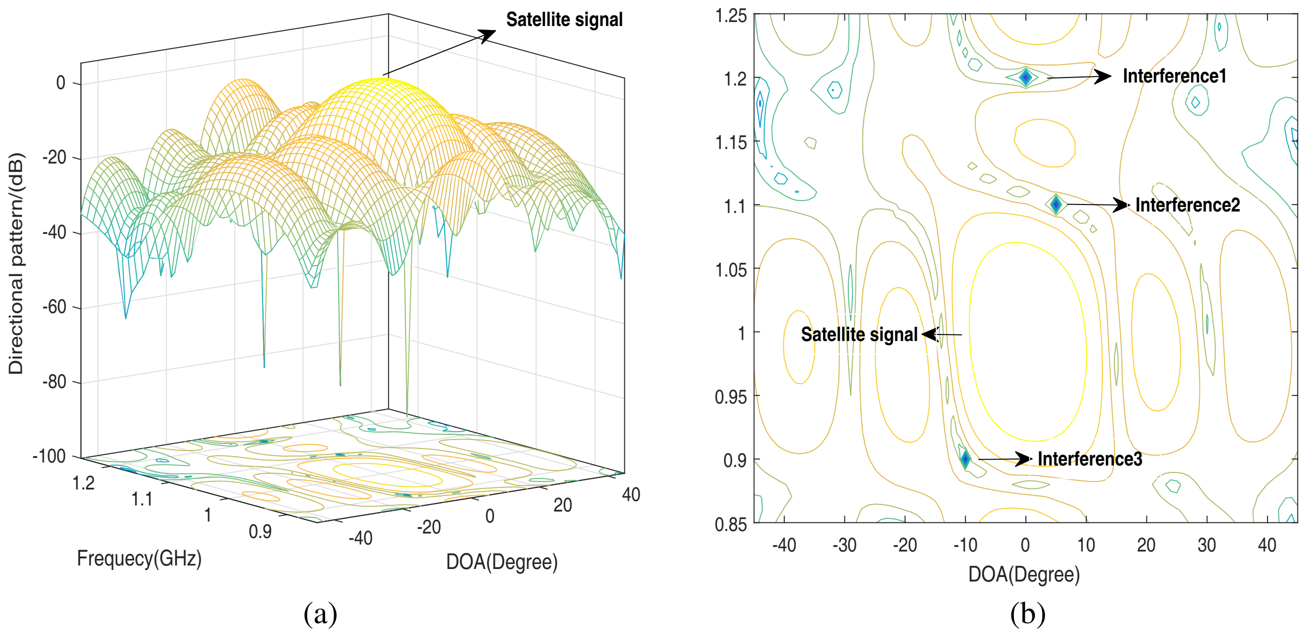

Figure 3. Three-dimensional beam pattern and contour plot of the proposed algorithm. (a) Beam pattern. (b) Contour plot.

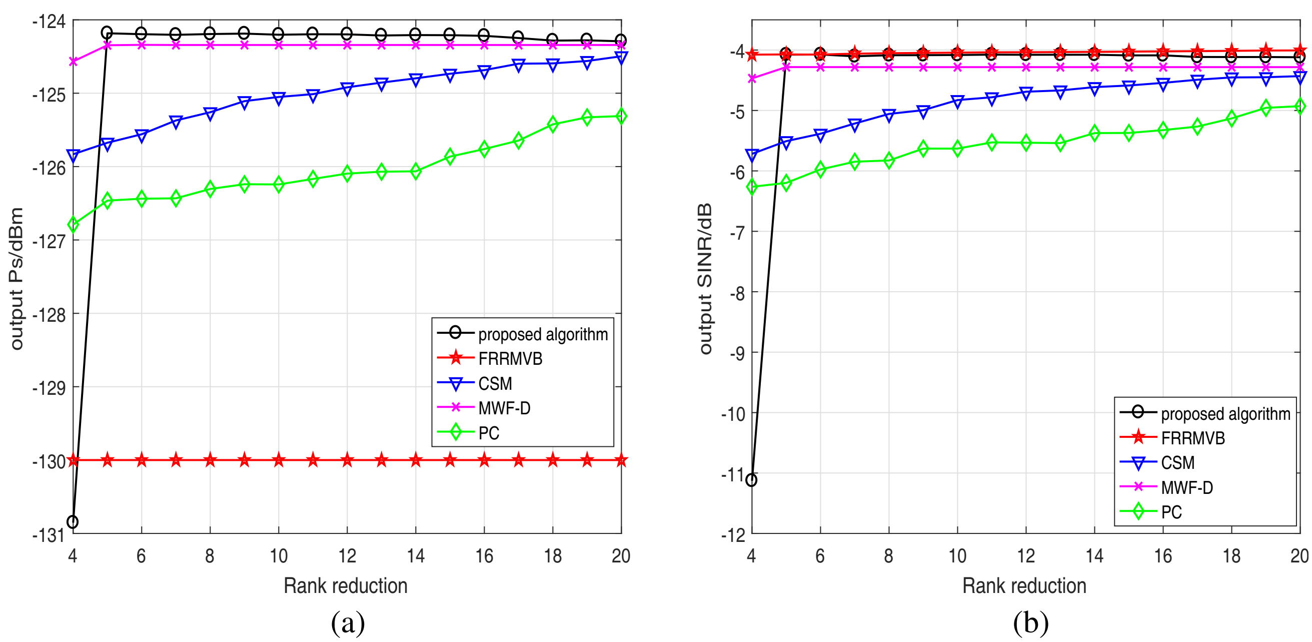

Figure 4. Performance curves for array output power and SINR. (a) Output power of satellite signal. (b) Output SINR.

Figure 5 shows the output SINR curves versus number of snapshots for the aforementioned algorithms. As can be seen, the proposed algorithm and FRRMVB algorithm have similar properties. They can obtain better array output SINR than PC, MWF-D, and CSM algorithms. The simulation result can show that the proposed algorithm can reduce the processing dimension of data by using a small number of snapshots, so as to ensure that the space-time anti-jamming weight vector can be obtained with a lower complexity and improve the real-time performance of interference suppression.

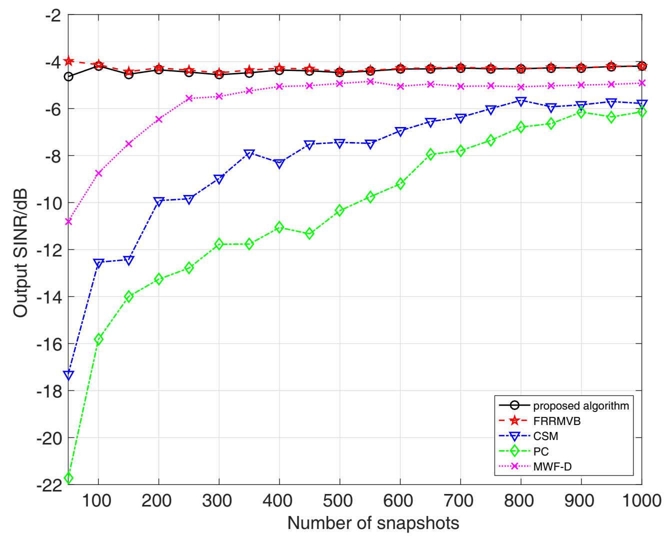

Figure 5. Output SINR curves versus number of snapshots.

5. CONCLUSIONS

Most of the existing space-time antijamming methods require computing the inverse of a high dimensional covariance matrix, which incurs high computational complexity. Therefore, this paper presents a lower complexity algorithm based on rank-reducing transformation and MWF for the space-time antijamming problem. Via the rank-reducing transformation, the rank reduction of the received data is accomplished. For the reduced rank data, MWF is used to avoid computing the matrix inversion. Finally, the antijamming weight vector is calculated by the equivalent reduced rank matrix and weight of MWF. Simulation results show that the proposed method can maintain a high array output power of the interesting satellite signal and a high output SINR.

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