Progress In Electromagnetics Research M, Vol. 98, 183–192, 2020

Array Pattern Recovery under Amplitude Excitation Errors Using Clustered Elements

Jafar R. Mohammed[1, *], Ahmed J. Abdulqader[1, 2], and Raad H. Thaher[2]

Abstract—In practice, the amplitude and phase excitations of array elements undergo random errors that lead to unexpected variations in the array radiation patterns. In this paper, the technique of the clustered array elements with discretized amplitude excitations is used to minimize the effect of random amplitude excitation errors and restore the desired array patterns. The most important feature of the proposed technique is its implementation in the design stage which may instantly count for any errors in the amplitude excitations. The cost function of the used optimizer is constrained to prevent any undesirable increase in the sidelobe levels due to unexpected excitation errors. Moreover, the error occurrences on the element amplitude excitations are considered to be either randomly over the whole array aperture or regionally (i.e., error affecting only a part of the array elements that located in a particular quadrant of the array aperture). Simulation results fully verify the effectiveness of the proposed technique.

1. INTRODUCTION

Planar antenna arrays were effectively used in conventional communication systems, and they will be also used effectively in the modern and future communication systems such as massive MIMO in 5G applications due to their advantages for fulfilling the desired radiation characteristics. However, the good performances of such arrays cannot be maintained in practice due to unavoidable errors in the array design. Basically, there are two types of errors that may occur in the antenna arrays. The first type is mechanical nature that includes errors in the locations of the array elements. Examples of such a type of error are array imperfections [1, 2]. The second type is electrical nature that affects the feeding of the array elements in terms of amplitude and phase excitations [3]. The presence of these errors can cause many problems, including the variations in the field strength, damage in the radiation pattern, unexpected increase in the side lobe level, and deviations in null directions and depths. Therefore, the original radiation pattern must be restored, and these errors must be corrected.

In the literature, many techniques have been proposed by researches to solve these problems [4–16]. Rocca and others [4] used an analytical method based on interval analysis to predict unexpected variations in the array radiation pattern as a result of exposure of the excitation amplitudes to different changes due to random errors. In [5], a group of researchers proposed a statistical method based on Rician and Beckmann distributions to analyze the errors that could be exposed to the feed, as they used Monte Carlo simulation to study the damaged radiation pattern as a result of errors in the excitation of the amplitude and phase of the elements. In [6], the authors used Monte Carlo optimization to study the tolerance of error amplitude and its effect on the radiation pattern. Keizer in [7] used an iterative Fourier transform to restore the desired radiation pattern due to quantization errors and correct the failure elements. Tarek and Ahmed in [8] suggested different synthesis techniques for planar arrays

Received 19 October 2020, Accepted 12 November 2020, Scheduled 20 November 2020
* Corresponding author: Jafar R. Mohammed (jafar.mohammed@uoninevah.edu.iq).
[1] College of Electronic Engineering, Ninevah University, Mosul-41002, Iraq. [2] College of Engineering, Mustansiriyah University, Baghdad-10052, Iraq.

under random errors. In [9], Peters suggested the method of conjugate gradient to reconfigure the amplitude and phase of the non-failed elements to reduce the sidelobe level in the sum and difference patterns of the planar arrays. In [10], the sensitivity of the adaptive nulling under the effect of random excitation errors was studied. The effect of selecting a small number of the controllable elements on the array performance was also investigated in [11–15]. Interestingly, it is found that the arrays become more robust against errors when using only limited number of controllable elements instead of all of them.

In [16], the authors proposed two new array architectures based on clustered elements. The clustered elements were chosen to be either regular or irregular. These clustered architectures were applied to linear arrays. In this paper, the method that was presented in [16] is extended to address the problem of random amplitude errors in the planar arrays where the clusters are formed in the shape of tiles. An initial planar array is first divided into a number of small clusters; each cluster is excited with a common discrete amplitude weighting control. The range of the amplitude weighting is constrained to be within a specified bound of the available digital attenuators. Thus, by optimizing the clustered element weights under these constraints, the effect of the random amplitude excitation errors due to quantization errors can be minimized. Also, much smaller number of discrete attenuators will be needed in practice. Moreover, the error could affect the element amplitude excitations randomly or regionally (i.e., error affecting only a part of the array elements that located in a particular quadrant of the array aperture). The genetic algorithm (GA) in [17] was used to construct the clustered arrays and minimize the effect of the random errors.

2. THE PROPOSED TECHNIQUE

The illustration of the idea of this study can be divided into two parts. In the first part, the array factor of the clustered planar array is formulated. The amplitude excitations of the initial fully planar array (before clustering) could be chosen according to the Dolph-Chebyshev distribution, and its corresponding radiation pattern is assumed to be the desired one (i.e., reference pattern without error). Then, the effects of the amplitude excitation errors on the array patterns are examined. The second part explains the mechanism of using the genetic algorithm to restore the array pattern under error exposure in the amplitudes of the array elements. The errors are assumed to be either random or regional.

2.1. The Clustered Planar Array

Consider a two-dimensional planar array with size $N_x \times N_y$, as shown in Fig. 1, and the separation distance between any two adjacent elements in x or y axes is set to $d_x = d_y = \frac{\lambda}{2}$. The elements are symmetrically divided into a number of clusters equal to $C_x \times C_y$, and each cluster consists of a certain number of elements equal to $M_x \times M_y$. Note that the size of the cluster $M_x \times M_y$ is much smaller than that of the original planar array $N_x \times N_y$. The radiation pattern of the clustered arrays in both elevation,

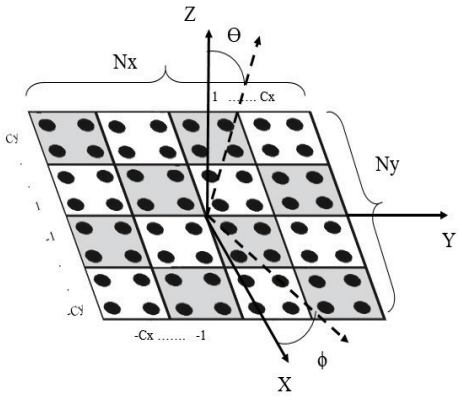


Figure 1. Clustered planar array.

θ, and azimuth, $\emptyset$, angles can be written as:

$$AF(\theta,\emptyset) = 4\sum_{c_x=1}^{C_x/2}\sum_{c_y=1}^{C_y/2} A_{c_xc_y} F_{c_x,c_y} \sum_{m_x=1}^{M_x/2}\sum_{m_y=1}^{M_y/2} W_{m_xm_y} F_{m_x,m_y} \tag{1}$$

where $A_{c_xc_y}$ is the common amplitude weighting of a group of elements within a specific cluster, $W_{m_xm_y}$ the complex (amplitude and phase) weighting of each individual radiating element in the original planar array, and

$$F_{c_x,c_y} = \{\cos[(c_y-0.5)\,d_{c_y}]\cos[(c_x-0.5)\,d_{c_x}]\}\frac{2\pi}{\lambda}\sin\theta\cos\emptyset \tag{2}$$

$$F_{m_x,m_y} = \{\cos[(m_y-0.5)\,d_y]\cos[(m_x-0.5)\,d_x]\}\frac{2\pi}{\lambda}\sin\theta\sin\emptyset \tag{3}$$

where $d_{c_x}=d_{c_y}$ is the spacing between clusters in x and y axes. For amplitude-only control where the phases of the array elements are set to zero, the complex weighting at both the individual element level and clustered element level becomes $W_{m_xm_y}=\left|W_{m_xm_y}\right|$ and $A_{c_xc_y}=\left|A_{c_xc_y}\right|$, respectively.

The complex weighting of the individual array elements, $W_{m_xm_y}$, may be chosen according to Dolph distribution, and its corresponding radiation pattern will be used as a reference pattern for recovering the damaged pattern.

2.2. Error Affect

To consider the effect of random errors, the amplitude weighting errors, $\Delta A_{c_xc_y}$, are included in the clustered levels. As a result, the array pattern in Eq. (1) will undergo deviations from its initially ideal condition (i.e., error free) as shown below

$$AF(\theta,\emptyset)_{error} = 4\sum_{c_x=1}^{C_x/2}\sum_{c_y=1}^{C_y/2} \left|A_{c_xc_y}-\Delta A_{c_xc_y}\right| F_{c_x,c_y} \sum_{m_x=1}^{M_x/2}\sum_{m_y=1}^{M_y/2} \left|W_{m_xm_y}\right| F_{m_x,m_y} \tag{4}$$

The term $A_{c_xc_y}-\Delta A_{c_xc_y}$ can be represented as $A_{c_xc_y}\delta_{c_xc_y}$, where $\delta_{c_xc_y}$ is the error factor

$$\delta_{c_xc_y} = 1-\frac{\Delta A_{c_xc_y}}{A_{c_xc_y}} \tag{5}$$

Its mean can be calculated by $\bar{\delta}=\frac{1}{2\gamma}\int_{-\gamma}^{\gamma}\delta_{c_xc_y}d\delta$, where $\gamma=\frac{\gamma_r}{2(2^a-1)}$, γ_r is the range of values used in the digital attenuator, and a is the number of bits. If γ_r is in decibels, then γ can also be computed in decibels, then the range of error factor in decibels is $-\frac{\gamma_r}{2(2^a-1)}\le\delta_{\mathrm{dB}}\le\frac{\gamma_r}{2(2^a-1)}$.

The variance of the clustered weight error can be obtained from

$$\sigma_\delta^2 = \frac{1}{2\gamma}\int_{-\gamma}^{\gamma}\left(10^{\frac{\delta_{\mathrm{dB}}}{20}}-\bar{\delta}\right)^2 d\delta_{\mathrm{dB}} \tag{6}$$

This can be determined as

$$\sigma_\delta^2 = 5\frac{10^{-\gamma/10}}{\gamma\,\ln(10)}\left[\left(10^{\gamma/5}-1\right)-20\frac{\left(10^{\gamma/10}-1\right)^2}{\gamma\,\ln(10)}\right] \tag{7}$$

Clearly, the limited ranges of the digital attenuators are causing significant errors in the array weights. Generally, the amplitude's range can be computed by the ratio of the weight at the edge to the weight at the center of the array elements. The use of amplitude weighting at the clustered elements level instead of individual elements level reduces the quantization errors.

Finally, we can find the deviation in the directivity due to the error

$$D_{deviation} = \frac{4\pi}{\sum_{c_x,c_y}\left|A_{c_x,c_y}\right|^2}E\left[\left|AF_{error}\right|^2\right] \tag{8}$$

where E is the average (or expectation) operator, then Eq. (8) can be further simplified to

$$D_{deviation} = \bar{\delta}^2 D_{original} + 4\pi\sigma_\delta^2 \tag{9}$$

The deviation in the directivity, $D_{deviation}$, is different from the error-free directivity, $D_{original}$, by $\bar{\delta}^2$ which is small in the clustered array. Thus, the directivity is not much affected by the errors. On the other hand, the error in the sidelobe level pattern can be found by

$$SLL_{error} = \frac{\sigma_\delta^2 \sum_{c_x,c_y} \left|A_{c_x,c_y}\right|^2}{\bar{\delta}^2 \left|\sum_{c_x,c_y} A_{c_x,c_y}\right|^2} \tag{10}$$

From Eq. (10), it is clear that the clustered arrays are more tolerant of amplitude errors due to the dependence of SLL_{error} on the clustered amplitudes which is less than that of the individual elements amplitudes.

3. SIMULATION RESULTS

To evaluate the effectiveness of the clustered arrays in minimizing the effects of amplitude quantization and at the same time simplifying the array complexity, extensive computer simulations are carried out under different scenarios. The original fully planar array is made up of 30×30 isotropic elements with a separation distance $d_x{=}d_y{=}\frac{\lambda}{2}$. The errors that added to the clustered array weights are real random numbers of zero average value. The errors are assumed to be either randomly affecting the element excitations over the whole array aperture or just regionally (i.e., affecting only a part of the array elements that located in a particular quadrant of the array aperture). The genetic algorithm with single point crossover, population size of 20, and mutation rate equal to 0.15 is used to optimize the clustered weights according to the following cost function

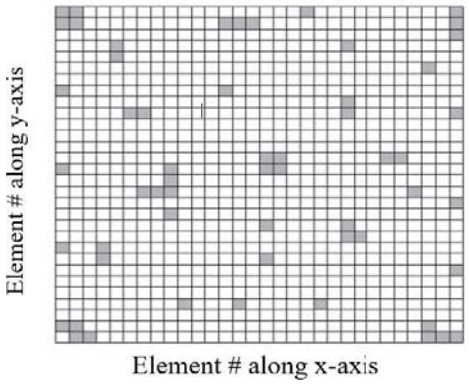
$$CF = \sum \left|AF_{error}\left(\theta, \emptyset\right) - Constraints\right|^2 \tag{11}$$

where $Constraints$ represent the desired limits on the clustered array pattern which is used to minimize the sidelobe error pattern, SLL_{error}. The lower and upper bounds of the cluster amplitude weights are set between 0 and 1. Both regular and irregular clustered arrays are constructed and examined in this paper. In the regular clustered arrays, all the clusters compose the same number of the elements with size 2×2, or 3×3, or any other size. On the other hand, in irregular clustered arrays, the numbers of elements are non-uniformly but evenly distributed among the clusters, and thus the size of a certain cluster is different from other clusters. The used genetic algorithm has $C_x{\times}C_y{+}M_x{\times}M_y$ parameters to optimize. The following scenarios were considered to investigate the performance of the proposed array.

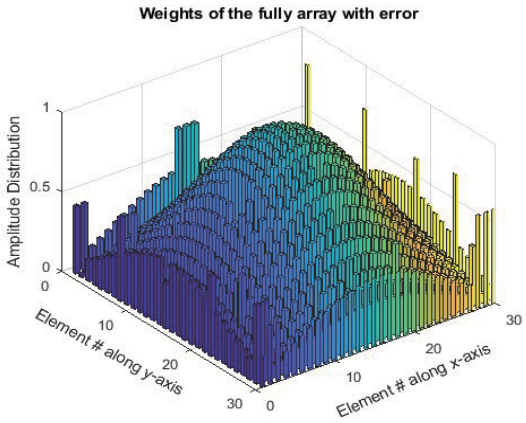
3.1. Scenario 1: Random Errors with Regular Clustered Arrays

Figures 2(a) and 2(b) show the layout and weights of the effected elements, while Figs. 2(c) and 2(d) show the results of applying the regular clustered arrays with size $M_x{\times}M_y{=}\,2 \times 2$ to minimize the sidelobe error pattern. For comparison purpose, the patterns of the fully planar array (i.e., without clusters) and the planar array with errors are also shown in Fig. 2(c). The sidelobe level constraint is chosen to not exceed -30 dB. From this figure, it can be seen that the proposed clustered array is capable to reduce the error and maintain the sidelobe level below a constraint limit.

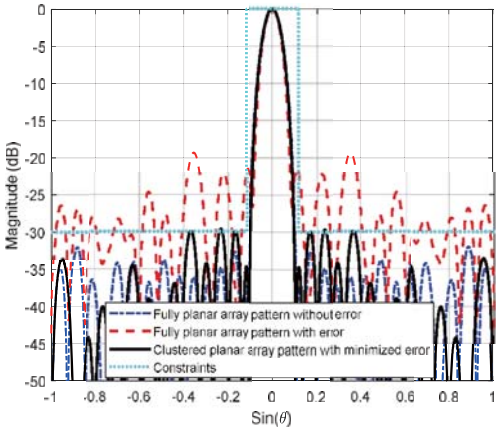
The effect of the random errors on the placed nulls is studied in the next example. Fig. 3 shows the results of the optimized clustered array with a wide null centered at $u = 0.75$. In this case, the regular clustered arrays with uniform size equal to $M_x{\times}M_y{=}\,2 \times 2$ are considered. The obtained results verify the robustness of the placed nulls against the random errors.



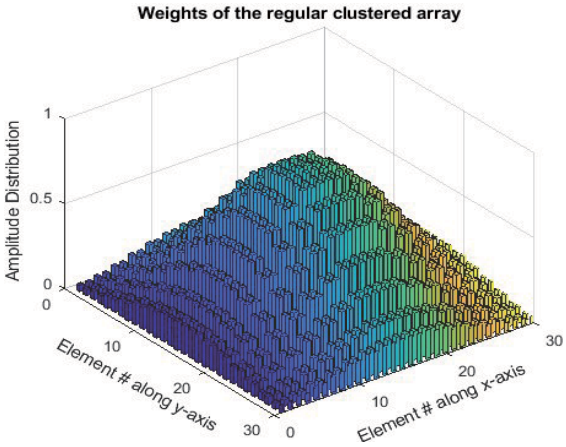
(a) Elements layout



(b) Weights of the fully array with errors

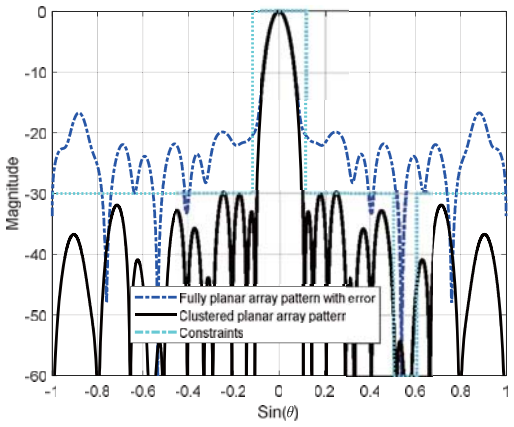


(c) Radiation patterns



(d) Weights of the 2×2 regular clustered array

Figure 2. Results of applying regular clustered arrays with $M_x \times M_y = 2 \times 2$ and for an original planar array with $N_x \times N_y = 30 \times 30$.



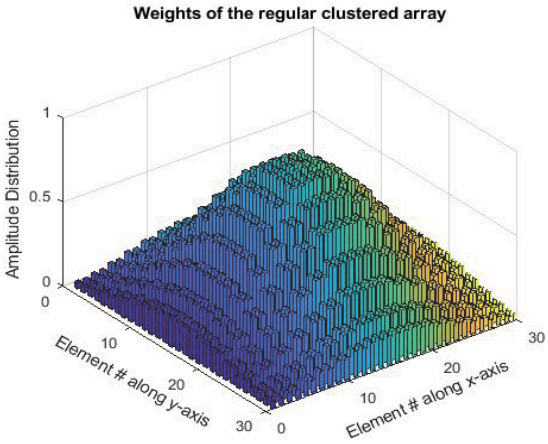
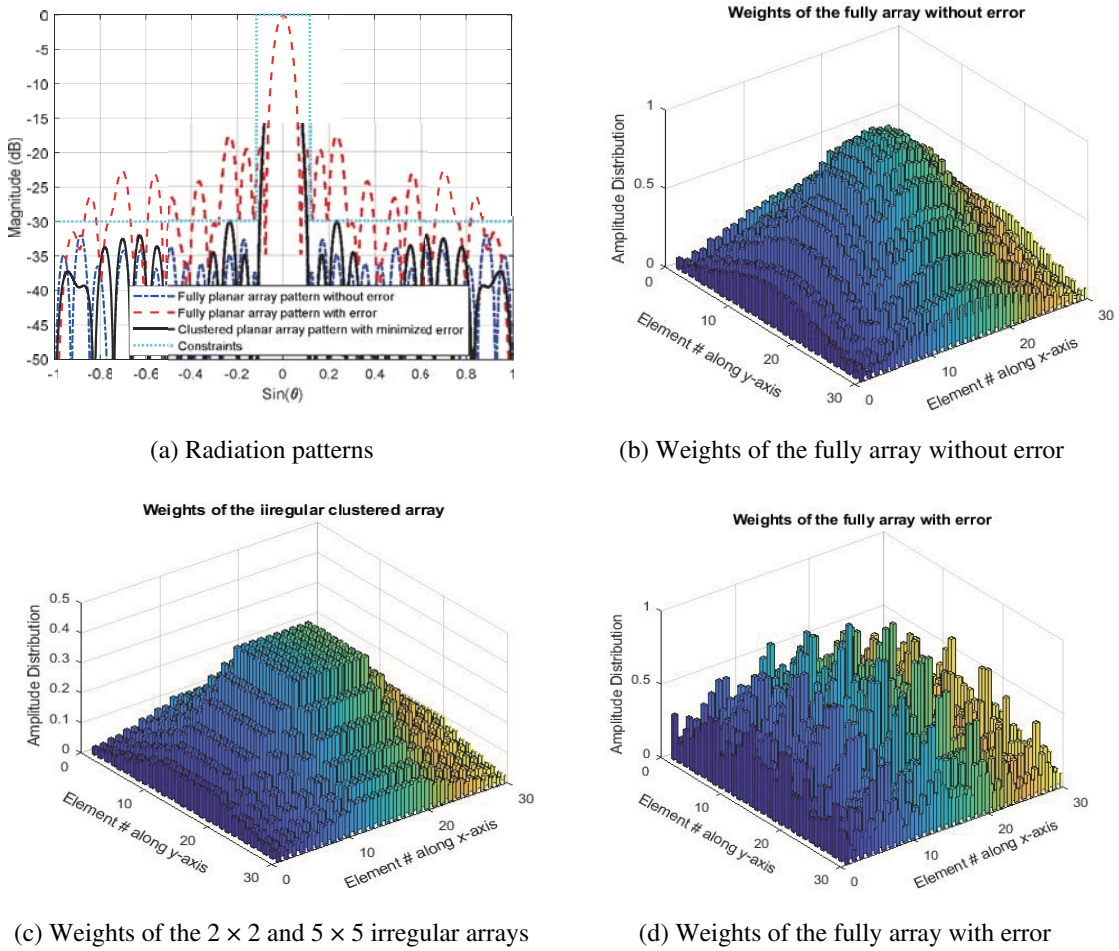


Figure 3. Results of regular clustered arrays with a wide null.

3.2. Scenario 2: Random Errors with Irregular Clustered Arrays

Figure 4 shows the results of applying the irregular clustered arrays with two different sizes, $M_x \times M_y = 2 \times 2$ for the outer array elements and $M_x \times M_y = 5 \times 5$ for the inner array elements. As



(a) Radiation patterns

(b) Weights of the fully array without error

(c) Weights of the 2×2 and 5×5 irregular arrays

(d) Weights of the fully array with error

Figure 4. Results of irregular clustered arrays with two different sizes $M_x \times M_y = 2 \times 2$ and $M_x \times M_y = 5 \times 5$ for an original planar array with $N_x \times N_y = 30 \times 30$.

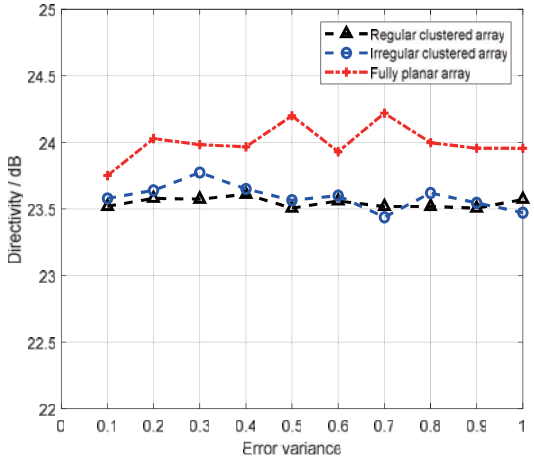


Figure 5. Directivity versus error variance.

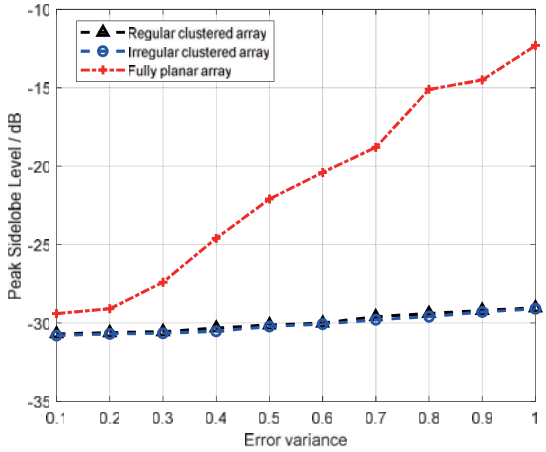
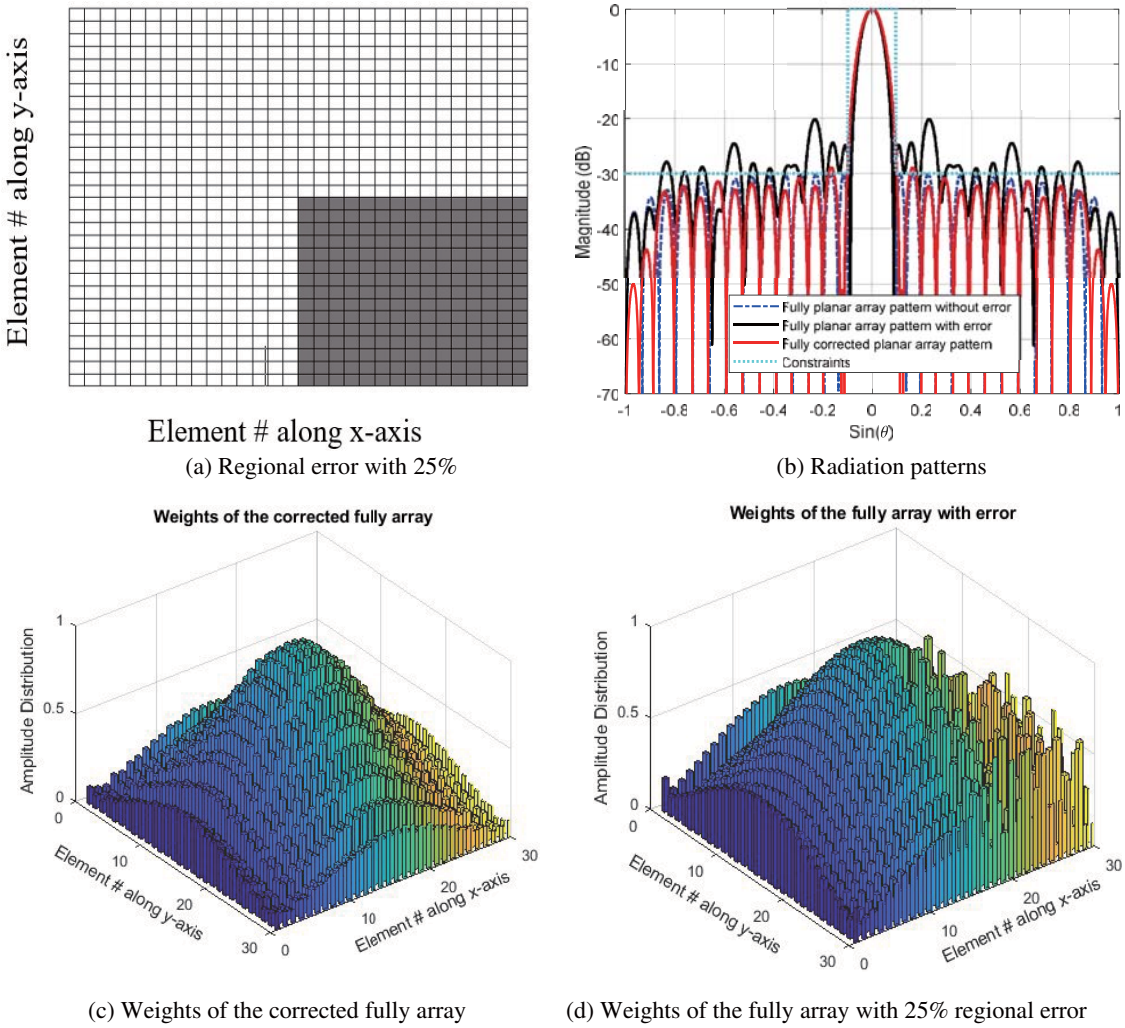


Figure 6. SLL versus error variance.



(a) Regional error with 25%

(b) Radiation patterns

(c) Weights of the corrected fully array

(d) Weights of the fully array with 25% regional error

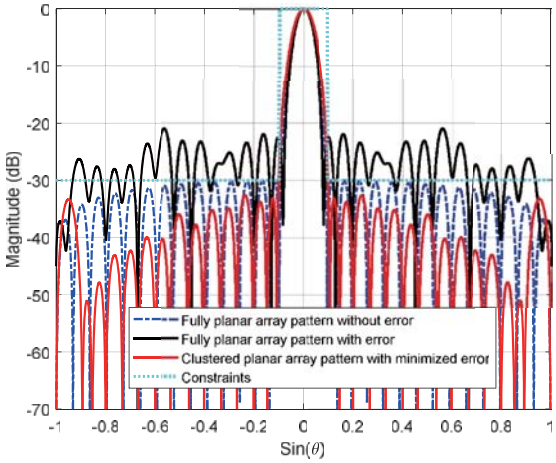
Figure 7. Results of regular clustered arrays with 25% regional error for an original Dolph planar array with $N_x \times N_y = 30 \times 30$.

in the previous example, the patterns of the initially ideal planar array without clusters and the fully planar array with error are also shown in this figure. Again, the clustered array is capable to effectively reduce the sidelobe error pattern.

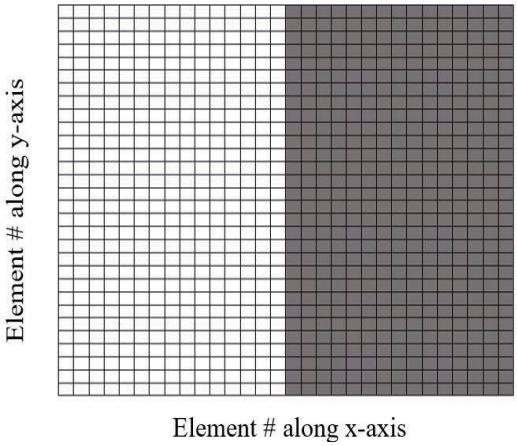
Next, the variations of the directivity and the sidelobe level as a function of the error variance are investigated for both regular and irregular clustered arrays. The sizes of the clusters are as in the previous examples. The results of the fully planar array without clusters are also shown for comparison purpose. Fig. 5 shows the variations of the directivity, while Fig. 6 shows the variations of the sidelobe error pattern. It can be seen that the directivities of the clustered arrays are almost same as that of the original planar array, whereas the peak sidelobe level of the original fully planar array is found to be more changeable than that of the clustered array. These results fully confirm the effectiveness of the proposed regular and irregular clustered arrays.

3.3. Scenario 3: Regional Errors with Regular Clustered Arrays

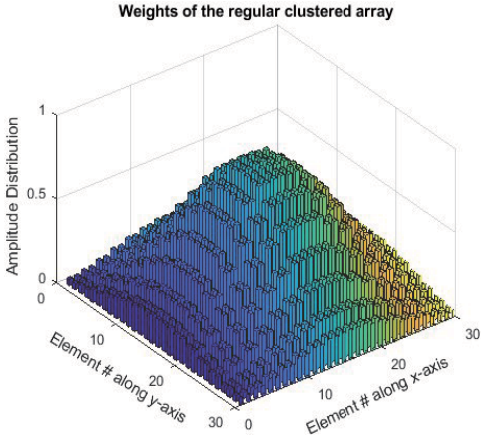
In this scenario, a 30×30 Dolph-Chebyshev excited array with SLL $= -30$ dB is considered as the initial planar array where its corresponding radiation pattern is assumed to be the desired error-free one. Then, the clustered elements along with the genetic algorithm are used to restore the array pattern under regional error.



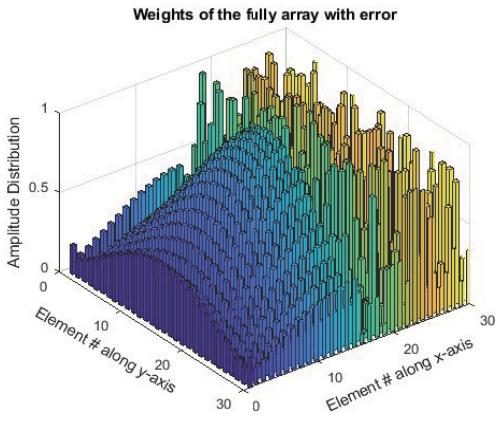
(a) Radiation patterns



(b) Regional error with 50%



(c) Weights of the 2×2 regular clustered array



(d) Weights of the fully array with regular error

Figure 8. Results of regular clustered arrays with 50% regional error.

In the first example of this scenario, we assumed that the error affected the amplitude elements in the first quadrant of the array aperture (i.e., the error percentage was 25% as shown in Fig. 7(a) with grey color). Fig. 7(b) shows the comparison between the original Dolph, fully planar arrays with and without errors, and the restored pattern. Figs. 7(c) and 7(d) show the corresponding amplitude excitations for each case. From this figure, it is observed that the peak SLL of the damaged pattern was at level -26 dB.

In the second example of this scenario, the error percentage was chosen to be 50%, and regular clustered with size 2×2 was used for each cluster. Fig. 8 shows the results.

Finally, the error percentage was chosen to be 75%, and irregular clustered arrays with two different sizes 2×2 and 3×3 were used (see Fig. 9).

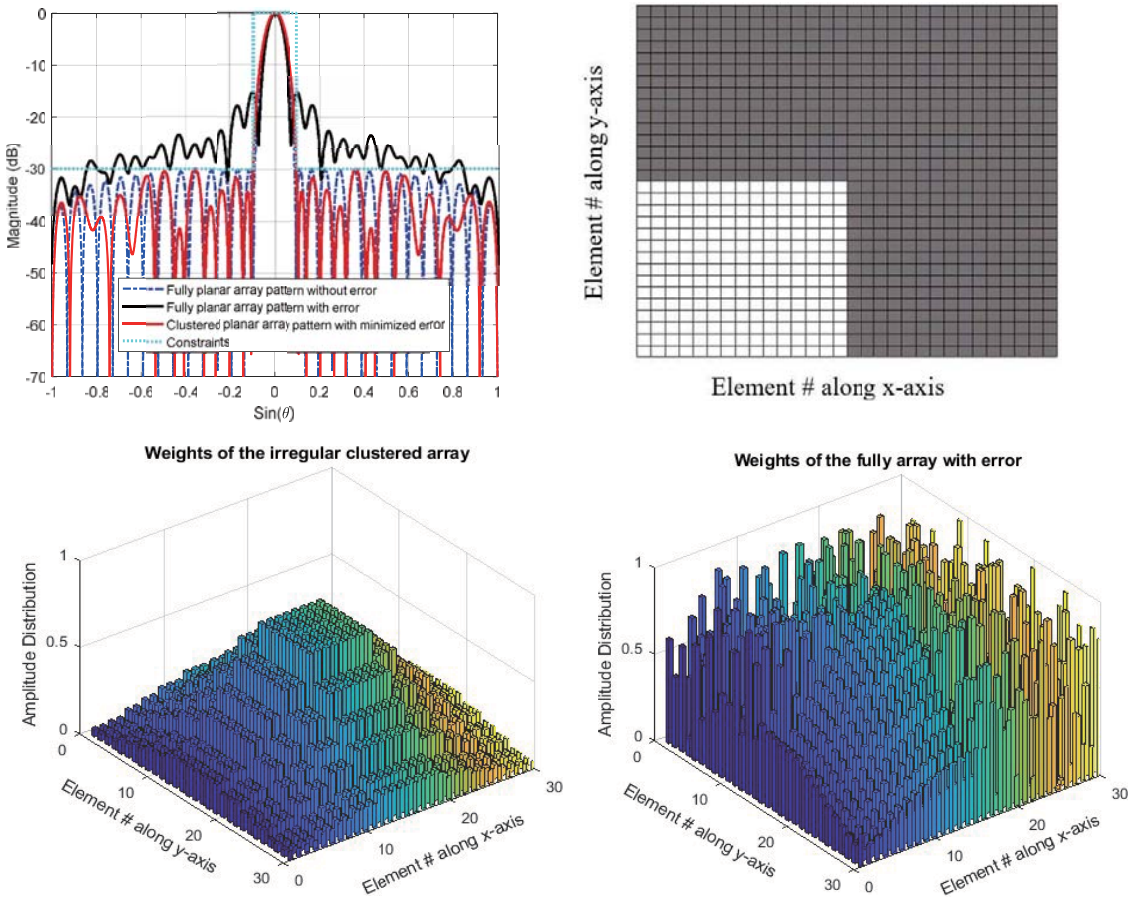


Figure 9. Results of regular clustered arrays with 75% regional error.

4. CONCLUSIONS

It is found from the presented results that the directivity and sidelobe pattern of the conventional fully planar arrays were greatly affected under the presence of random or regional amplitude errors. This problem was effectively solved with the proposed clustered arrays where the sidelobe error pattern was maintained at an acceptable limit, and the error effects were minimized. For an array with size 30×30 of elements, the directivities of the regular and irregular clustered arrays were found approximately between the values 23.5 dB and 23.6 dB for a range of error variance between 0.1 and 1, while the peak sidelobe variations were between -29 and -31 dBs which are very near the desired limit -30 dB. In view of the above, the clustered arrays represent a good solution for both error minimization and complexity reduction.

REFERENCES

1. Wang, H. S. C., "Performance of phased-array antennas with mechanical errors," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. 28, No. 2, 535–545, Apr. 1992.
2. Mohammed, J. R., "Obtaining wide steered nulls in linear array patterns by controlling the locations of two edge elements," *AEÜ International Journal of Electronics and Communications*, Vol. 101, 145–151, Mar. 2019.
3. Amir, Z. and A. R. Bijan, "GA-based approach to phase compensation of large phased array antennas," *Journal of Systems Engineering and Electronics*, Vol. 29, No. 4, 723–730, Aug. 2018.
4. Rocca, P., L. Manica, N. Anselmi, and A. Massa, "Analysis of the pattern tolerances in linear arrays with arbitrary amplitude errors," *IEEE antennas and wireless propagation letters*, Vol. 12, 2013.
5. Van den Biggelaar, A. J., U. Johannsen, P. Mattheijssen, and A. B. Smolders, "Improved statistical model on the effect of random errors in the phase and amplitude of element excitations on the array radiation pattern," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 5, 2309–2317, 2018.
6. Lee, J., Y. Lee, and H. Kim, "Decision of error tolerance in array element by the Monte Carlo method," *IEEE Transaction on Antennas and Propagation*, Vol. 53, No. 4, Apr. 2005.
7. Keizer, W. P. M. N., "Low sidelobe phased array pattern synthesis with compensation for errors due to quantized tapering," *IEEE Trans on Antennas and Propagation*, Vol. 59, No. 12, Dec. 2011.
8. Tarek, S. and M. A. Ahmed, "Different array synthesis techniques for planar antenna array," *The Applied Computational Electromagnetics Society Journal*, Vol. 34, No. 5, May 2019.
9. Peters, T. J. "A conjugate gradient-based algorithm to minimize the sidelobe level of planar arrays with element failures," *IEEE Trans. Antennas Propag.*, Vol. 39, 1497–1504, Oct. 1991.
10. Mohammed, J. R. and K. H. Sayidmarie, "Sensitivity of the adaptive nulling to random errors in amplitude and phase excitations in array elements," *International Journal of Telecommunication, Electronics, and Computer Engineering*, Vol. 10, No. 1, 51-56, Jan.–Mar. 2018.
11. Zhang, W., L. Li, and F. Li, "Reducing the number of elements in linear and planar antenna arrays with sparseness constrained optimization," *IEEE Trans on Antennas and Propagation*, Vol. 59, No. 8, Aug. 2011.
12. Mohammed, J. R., "Optimal null steering method in uniformly excited equally spaced linear array by optimizing two edge elements," *Electronics Letters*, Vol. 53, No. 13, 835–837, June 2017.
13. Mohammed, J. R. and K. H. Sayidmarie, "Synthesizing asymmetric sidelobe pattern with steered nulling in non-uniformly excited linear arrays by controlling edge elements," *International Journal of Antennas and Propagation*, Vol. 2017, Article ID 9293031, 8 pages, 2017.
14. Sayidmarie, K. H. and J. R. Mohammed, "Performance of a wide angle and wideband nulling method for phased arrays," *Progress in Electromagnetics Research M*, Vol. 33, 239-249, 2013.
15. Abdulkader, A. J., J. R. Mohammed, and R. H. Thaher,, "Phase-only nulling with limited number of controllable side elements," *Progress In Electromagnetics Research C*, Vol. 99, 167–178, 2020.
16. Mohammed, J. R., A. J. Abdulqader, and R. Hamdan, "Antenna pattern optimization via clustered arrays," *Progress In Electromagnetics Research M*, Vol. 95, 177–187, 2020.
17. Haupt, R. L. and D. Werner, *Genetic Algorithms in Electromagnetics*, John Wiley & Sons, New York, 2007.