# Wideband Harmonic Suppressed Compact Rat-Race Coupler Using Triple Stub M-Shape Unit

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**Abstract**—A design of a compact wideband harmonic suppressed rat-race coupler (RRC) is presented in this paper. The present coupler is obtained by replacing each quarter wave length transmission line of a conventional double section rat-race coupler with a triple stub M-shape unit. The M-shape unit with 3 stubs is used to enhance the bandwidth, suppress the harmonics, and reduce the size of the coupler. Design guidelines are established using the lossless transmission line model. Theoretical predictions are verified by fabricating a prototype coupler. The proposed double section RRC provides harmonic suppression up to seventh of operating frequency and 62.4% size reduction with wide bandwidth, which is useful for wireless communication systems.

## 1. INTRODUCTION

Rat-race coupler (RRC) is prominent among number of segments in RF, microwave, and millimeter wave applications including in the design of feed networks for antennas, power dividers, power combiners, mixers, modulators, demodulators, and phase shifters. It provides a high level of segregation between the input ports compared with a quadrature hybrid coupler. Even though a conventional RRC has many advantages and applications, it also has fundamental limitations of relatively narrow bandwidth, large occupied area, and higher order harmonics due to periodic nature of the transmission line section.

The most prominent design aspect of RRC is an improvement in the bandwidth. A large amount of work has been done in this regard with increased design complexity and/or circuit size [5–13]. The various factors that influence design complexity include multilayer, via-holes, slots, tight coupling, lumped elements, etc. Existing techniques like left-handed transmission line sections [7], coupled lines, and impedance transformers [5] improve the bandwidth of RRCs to a maximum extent while occupying broad area and need multilayer realization. In [8], nonuniform microstrip transmission line sections are used as replacements for the branches of the conventional RRC for the use in wideband operation. The broadband RRC is realized using phase inverters as reported in [6, 10, 13]. It effectively incorporates via-holes and ground slots while the RRC is designed. Significant reduction in the size along with an improvement in bandwidth is achieved by avoiding the use of plated through holes and slots or bonding wires as observed in [11]. The presence of higher-order harmonics in existing wideband RRCs with distributed elements will have adverse impact. However among all the above RRCs, none have harmonic suppression capability. A solution to this problem can be obtained by designing the compact wide-band harmonic suppressed double-section RRC using series and shunt stepped impedance transmission lines, as presented in [12]. 20 dB Fractional Bandwidth (FBW) means range of frequencies where coupler metrics return loss ($S_{11}$) and isolation ($S_{41}$) are below $-20$ dB in the Y-axis as shown in Fig. 6. Reduction in size to a great extent as well as wider bandwidth is the current requirement in the ever developing

communication engineering. If these aspects can be achieved along with harmonic suppression, that design will best fit a given application.

In this paper, an M-shape unit is used to design a compact wideband harmonic suppressed RRC. A detailed study of bandwidth is carried out by varying impedances of the coupler and lengths of T, Pi, and M shapes. The size of the proposed two-section wide-band coupler is reduced by 62.4% with 66% bandwidth. The harmonic suppression ability up to seventh of operating frequency ($7f_0$) is observed from the EM simulated and measured responses.

## 2. DESIGN AND ANALYSIS

Figure 1 shows that single stub T [3], double stub Pi [4], triple stub M-shape units having line impedance $Z_A$ (electrical line length $\theta_A$) and stub impedance $Z_B$ (stub length $\theta_B$) which are equivalent to a traditional quarter wavelength transmission line having impedance $Z$ (for single section RRC $Z = 70.7\,\Omega$) are used in the conventional single section RRC [1] as depicted in Fig. 2. The conventional single section RRC is characterized by large size and higher order harmonics. Considering port1 as the input port, variability in 20 dB return loss FBW for each RRC is observed in Figs. 3(a) and 3(b) when each quarter wave section of single section RRC is replaced with T (1 open stub), Pi (2 open stubs), M (3 open stubs)-shape units, and here, simulations are done in Ansys circuit simulator. The fractional bandwidth (FBW) can be calculated using Eq. (1), where $f_2$ and $f_1$ are frequencies over which coupler metrics return loss ($S_{11}$), insertion loss ($S_{21}$, $S_{31}$), isolation ($S_{41}$), output imbalance (both phase and amplitude) are in the required limits, and $f_0$ is the mid-band frequency.

$$\text{Fractional Bandwidth } FBW = \frac{f_2 - f_1}{f_0}; \quad f_0 = \frac{f_2 + f_1}{2} \tag{1}$$

Figures 3(a) and 3(b) show 20 dB $S_{11}$ FBW variation of the RRC with different stub units as a function of line length $\theta_A$ for stub length $\theta_B = 30°$, 15° and $\theta_B$ for $\theta_A = 60°$, 45°, respectively. It can be deduced from Figs. 3(a) and 3(b) that an improvement in FBW can be obtained by choosing more stubs, large line length, and small stub length. However, reduction in the size of RRC with simultaneous improvement
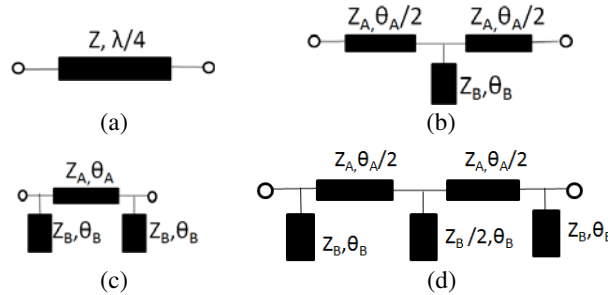


**Figure 1.** (a) Traditional quarter-wavelength transmission line and its equivalent, (b) single stub T-Shape unit, (c) double stub Pi-Shape unit, (d) triple stub M-Shape unit.
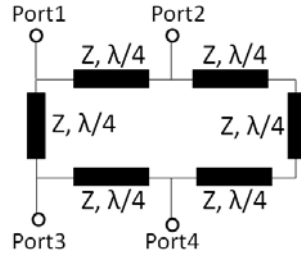


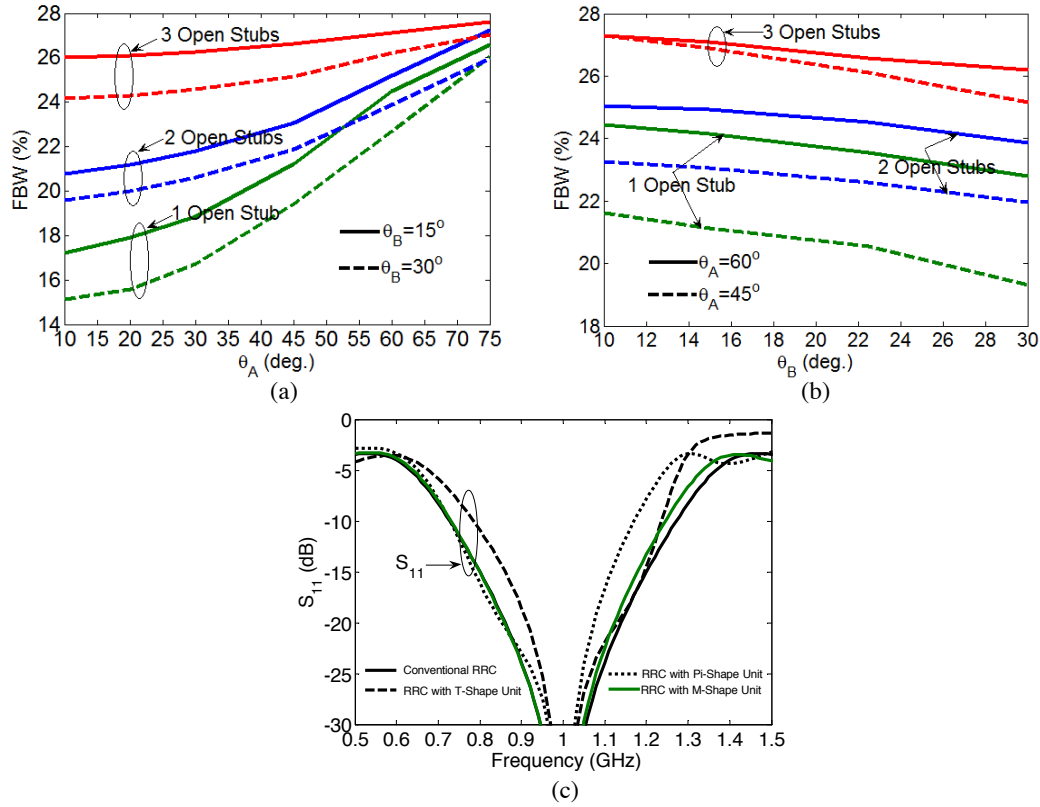**Figure 2.** Conventional Single section Rat-Race Coupler (RRC).

**Figure 3.** For single section RRC with T, Pi, M-Shape units, (a) 20 dB Matching ($S_{11}$) FBW versus line length ($\theta_A$) with given length of open stubs ($\theta_B$), (b) 20 dB Matching ($S_{11}$) FBW versus stub length ($\theta_B$) with given line lengths ($\theta_A$), (c) $S_{11}$ (dB).

in harmonic suppression is feasible only when line and stub lengths are small [2]. Thus, a three-stub M-shape unit is the best choice in terms of FBW, miniaturization, and harmonic suppression. Fig. 3(c) presents in band performance of RRC, where single section RRC has an M-shape unit getting FBW on par with conventional RRC.

The required parameters of the M-shape unit can be derived with the help of ABCD matrices of the M-shape unit and conventional RRC quarter wavelength arm of $Z$ at any operating frequency ($f_0$), as given below. For conventional quarter wavelength transmission line (QWTL), the ABCD matrix is:

$$\left[\begin{array}{cc} A & B \\ C & D \end{array}\right]_{QWTL} = \left[\begin{array}{cc} 0 & jZ \\ \dfrac{j}{Z} & 0 \end{array}\right] \tag{2a}$$

The ABCD matrix of the M-shape unit, obtained by multiplying the individual units of two series arms and three shunt open stub units.

$$\left[\begin{array}{cc} A & B \\ C & D \end{array}\right]_M = \left[\begin{array}{cc} 1 & 0 \\ \dfrac{j}{Z_B}\tan\theta_B & 1 \end{array}\right] \times \left[\begin{array}{cc} \cos(\theta_A/2) & jZ_A\sin(\theta_A/2) \\ \dfrac{j}{Z_A}\sin(\theta_A/2) & \cos(\theta_A/2) \end{array}\right] \times \left[\begin{array}{cc} 1 & 0 \\ \dfrac{2j}{Z_B}\tan\theta_B & 1 \end{array}\right]$$
$$\times \left[\begin{array}{cc} \cos(\theta_A/2) & jZ_A\sin(\theta_A/2) \\ \dfrac{j}{Z_A}\sin(\theta_A/2) & \cos(\theta_A/2) \end{array}\right] \times \left[\begin{array}{cc} 1 & 0 \\ \dfrac{j}{Z_B}\tan\theta_B & 1 \end{array}\right] \tag{2b}$$

From Eq. (2b) $A_M = \cos\theta_A - 4\dfrac{Z_A}{Z_B}\sin(\theta_A/2)\cos(\theta_A/2)\tan\theta_B + 2(\dfrac{Z_A}{Z_B})^2\sin^2(\theta_A/2)\tan^2\theta_B$ and

$$B_M = 2Z_A\cos(\theta_A/2)\sin(\theta_A/2) - 2\left(\dfrac{Z_A}{Z_B}\right)^2\sin^2(\theta_A/2)\tan\theta_B \tag{2c}$$

The final impedances of the M-shape unit are obtained by equating Eqs. (2a), (2c) and solving them, as

$$Z_A = \frac{0.707Z}{\sin \dfrac{\theta_A}{2}} \tag{3}$$

$$Z_B = \frac{0.707Z \tan \theta_B}{\cos \dfrac{\theta_A}{2} - 0.707} \tag{4}$$

Thus, it can be observed from Fig. 3 that the three-stub M-shape unit with $\theta_A = 45°$, $\theta_B = 15°$ is a better replacement while designing wideband miniaturized RRC with better harmonic suppression.

Furthermore, the improvement in FBW is possible with the help of a double section RRC as presented in Fig. 4 [11], where the impedances of the coupler are optimized using a linear analysis tool. In the current work, the impedances used in [11] are modified to get more FBW.

FBW increases as $Z_2$ and $Z_4$ increase while maintaining $Z_1 = 66.1\,\Omega$, $Z_3 = 30.8\,\Omega$, $Z_5 = 52.9\,\Omega$ as shown in Fig. 5(a). It is preferable to take $Z_2 = Z_4 = 75\,\Omega$ for simplicity of design. Hence in the proposed work, $Z_2 = Z_4 = 75\,\Omega$ is taken which results in the improvement of FBW. FBW decreases with the increase in $Z_3$ for fixed values of $Z_2 = Z_4 = 75\,\Omega$, $Z_1 = 66.1\,\Omega$, $Z_5 = 52.9\,\Omega$ as shown in Fig. 5(b). Even though an increase in $Z_3$ will result in the decrease in FBW, a slight increase is required for design purpose. Hence in the proposed work, $Z_3 = 35\,\Omega$ is taken. No significant improvement in the response of RRC is observed when $Z_5$ increases for given values of $Z_1 = 66.1\,\Omega$, $Z_2 = Z_4 = 75\,\Omega$, $Z_3 = 35\,\Omega$. So $Z_5$ is kept almost unchanged. 20 dB return loss FBW decreases, with simultaneous increase in $S_{11}$ magnitude at $f_0$ as $Z_1$ increases observed in Figs. 5(c) and 5(d). However, isolation
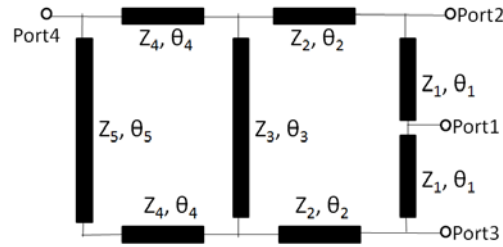


**Figure 4.** Conventional wideband double section Rat-Race Coupler (RRC).

**Table 1.** FBW variation with various impedances.

| Step | Fixed impedances ($\Omega$) | Variable impedances ($\Omega$) | FBW (%) | Finalized impedances ($\Omega$) after each step |
|------|------|------|------|------|
| 1 | $Z_1 = 66.1$, $Z_3 = 30.8$, $Z_5 = 52.9$ | $Z_2, Z_4$; ($Z_2 = Z_4$) | Increases with increase of $Z_2$, $Z_4$ | $Z_2 = Z_4 = 75$ |
| 2 | $Z_1 = 66.1$, $Z_2 = Z_4 = 75$, $Z_5 = 52.9$ | $Z_3$ | Decreases with increase of $Z_3$ | $Z_3 = 35$ |
| 3 | $Z_1 = 66.1$, $Z_2 = Z_4 = 75$, $Z_3 = 35$ | $Z_5$ | No change | $Z_5 = 50$ |
| 4 | $Z_1 = 66.1$, $Z_2 = Z_4 = 75$, $Z_3 = 35$ | $Z_1$ | Decreases with increase of $Z_1$ | $Z_1 = 67$ |

FBW remains unchanged. Thus, selecting $Z_1 = 67\,\Omega$ will satisfy the above mentioned conditions. From Fig. 6 it is found that the present double section RRC with modified impedances gives more FBW than [11], conventional single section RRC. In addition, all these observations are shown in a tabular form. In Table 1, fixed impedances in step1 are taken directly from [11]. Modified impedances are updated in
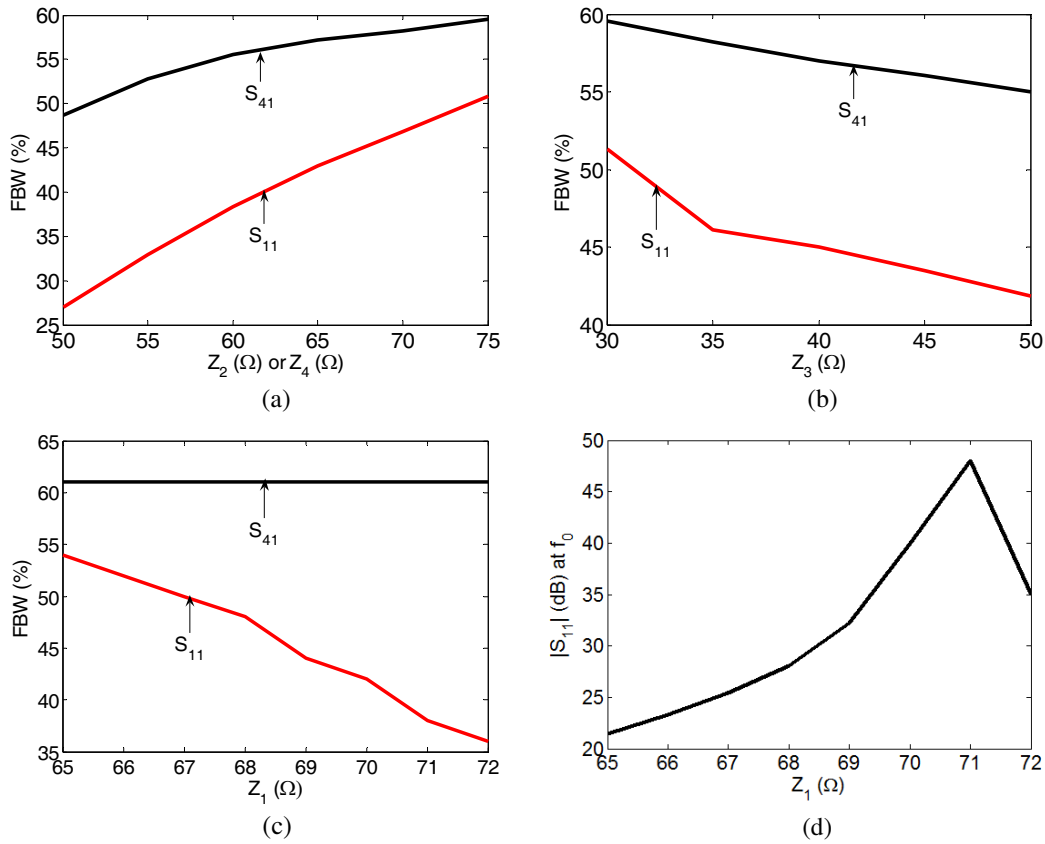


**Figure 5.** For double section RRC, (a) 20 dB Return loss ($S_{11}$), Isolation ($S_{41}$) FBW versus $Z_2$ or $Z_4$ ($Z_2 = Z_4$) when $Z_1 = 66.1\,\Omega$, $Z_3 = 30.8\,\Omega$, $Z_5 = 52.9\,\Omega$, (b) 20 dB Return loss ($S_{11}$), Isolation ($S_{41}$) FBW versus $Z_3$ when $Z_1 = 66.1\,\Omega$, $Z_2 = Z_4 = 75\,\Omega$, $Z_5 = 52.9\,\Omega$, (c) 20 dB Return loss ($S_{11}$), Isolation ($S_{41}$) FBW versus $Z_1$ when $Z_1 = 35\,\Omega$, $Z_2 = Z_4 = 75\,\Omega$, $Z_5 = 50\,\Omega$, (d) $|S_{11}|$ (dB) at $f_0$ versus $Z_1$.
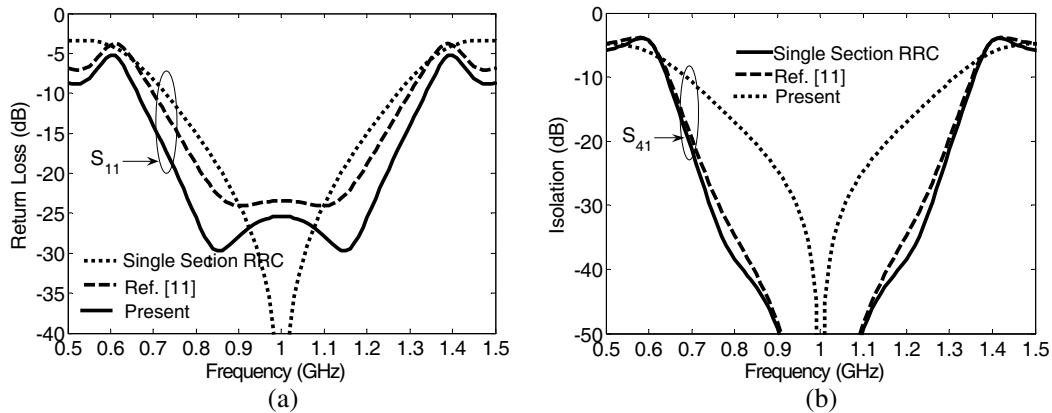


**Figure 6.** Comparison of single section RRC, double section RRC used in Ref. [11] and present modified double section RRC with respect to (a) $S_{11}$ (dB), (b) $S_{41}$ (dB).

the remaining steps. Here, all these effects are observed with circuit simulation of double section RRC. Further in the finalization of impedances fabrication limit of FR4 substrate is also taken into account.

The electrical lengths of double section RRC are $\theta_1 = \theta_2 = \theta_4 = 90°$ and $\theta_3 = \theta_5 = 180°$. The fundamental limitation of this coupler is that it requires a large area. Replacing the conventional quarter wave transmission line with the M-shape unit will result in considerable reduction in the circuit size. Impedances of M-shape unit according to [11] and present modified impedances to get more FBW for $\theta_A = 45°$, $\theta_B = 15°$ are shown in Table 2 and Table 3. Fig. 7 shows the final schematic diagram of proposed RRC. Fig. 8 visualizes circuit simulated magnitude response of conventional double section RRC with modified impedances and proposed double section RRC with M-shape unit. In Fig. 8, it is observed that the proposed double section RRC gives harmonic suppression compared with conventional double section RRC due to insertion of the M-shape unit.

**Table 2.** Impedances used in Ref. [11] for $\theta_A = 45°$, $\theta_B = \theta_C = 15°$.

|            | $Z_1 = 66.1\,\Omega$ | $Z_2 = 56.7\,\Omega$ | $Z_3 = 30.8\,\Omega$ | $Z_4 = 74.1\,\Omega$ | $Z_5 = 52.9\,\Omega$ |
|------------|------|------|------|------|------|
| $Z_A(\Omega)$ | 122.13 | 104.76 | 56.9 | 136.9 | 97.74 |
| $Z_B(\Omega)$ | 57.74 | 49.53 | 26.9 | 64.73 | 46.21 |

**Table 3.** Modified impedances in the proposed work using linear analysis tool for $\theta_A = 45°$, $\theta_B = \theta_C = 15°$.

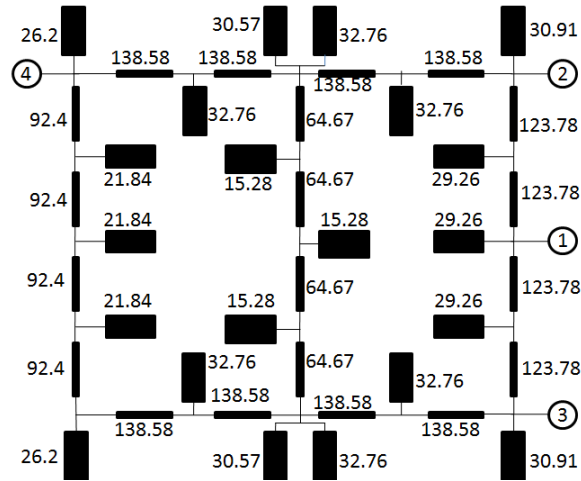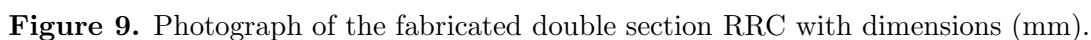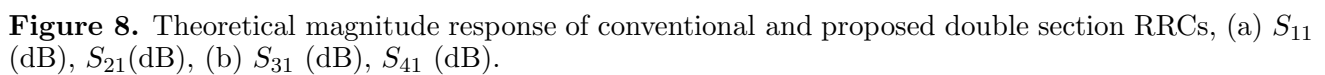|            | $Z_1 = 67\,\Omega$ | $Z_2 = 75\,\Omega$ | $Z_3 = 35\,\Omega$ | $Z_4 = 75\,\Omega$ | $Z_5 = 50\,\Omega$ |
|------------|------|------|------|------|------|
| $Z_A(\Omega)$ | 123.78 | 138.58 | 64.67 | 138.58 | 92.4 |
| $Z_B(\Omega)$ | 58.52 | 65.52 | 30.57 | 65.52 | 43.68 |



**Figure 7.** Final layout of the proposed wideband RRC with impedance values in ohms.

## 3. FABRICATION AND MEASUREMENT

The fabrication of microstrip wideband rat race coupler using an M-shape unit and operating at 0.6 GHz is carried out on a low cost FR4 substrate having a dielectric constant $\varepsilon_r = 4.3$, loss tangent 0.022, and thickness 1.58 mm. The parameters of the basic unit are obtained from Eq. (3) to Eq. (4) as shown in Table 3 and $\theta_A = 45°$, $\theta_B = 15°$. The proposed wide-band rat race coupler is obtained by cascading ten

**Figure 8.** Theoretical magnitude response of conventional and proposed double section RRCs, (a) $S_{11}$ (dB), $S_{21}$(dB), (b) $S_{31}$ (dB), $S_{41}$ (dB).



**Figure 9.** Photograph of the fabricated double section RRC with dimensions (mm).

basic units. The EM simulations are carried out by the full wave simulator Ansys HFSS. The coupler developed in the proposed manner along with the physical dimensions is shown in Fig. 9. Here, all the stub lengths are same, and not-mentioned values can be deduced from symmetry assumptions. The wide-band rat race coupler occupies a compact rectangular area of $89.04 \times 89\,\text{mm}^2$ excluding the ports, which is 37.6% of a rectangular area of conventional double section coupler.

Measured, EM simulated, and computed magnitudes of the $S$-parameters are shown in Fig. 10. The phase responses at port1 (sum port) and port4 (difference port) are shown in Fig. 11. The wide-band double section rat race coupler is tested using precision network analyzer. It was observed that operating frequency $f_0$ is at 0.6 GHz. After considering connector losses, $S_{21}$ and $S_{31}$ at the operating frequency are 3.26 dB and 3.31 dB, respectively. The return loss $S_{11}$ and isolation $S_{41}$ are 21.99 dB and 30.09 dB, respectively at operating frequency. The 20 dB return loss (isolation) is from 0.446 (0.41) to 0.714 (0.772) GHz, and the fractional bandwidth (FBW) is 46.13% (61.23%). The magnitude balance within $0 \pm 0.5$ dB is from 0.427 to 0.755 GHz, and FBW is 55.39%. The measured in phase (out of phase) with $0° \pm 5°$ ($180° \pm 5°$) is from 0.384 (0.436) to 0.781 (0.746) GHz, and FBW is 68.05% (52.48%). The phase difference at sum and difference ports are $1.7°$ and $181.5°$, respectively. Moreover, the proposed
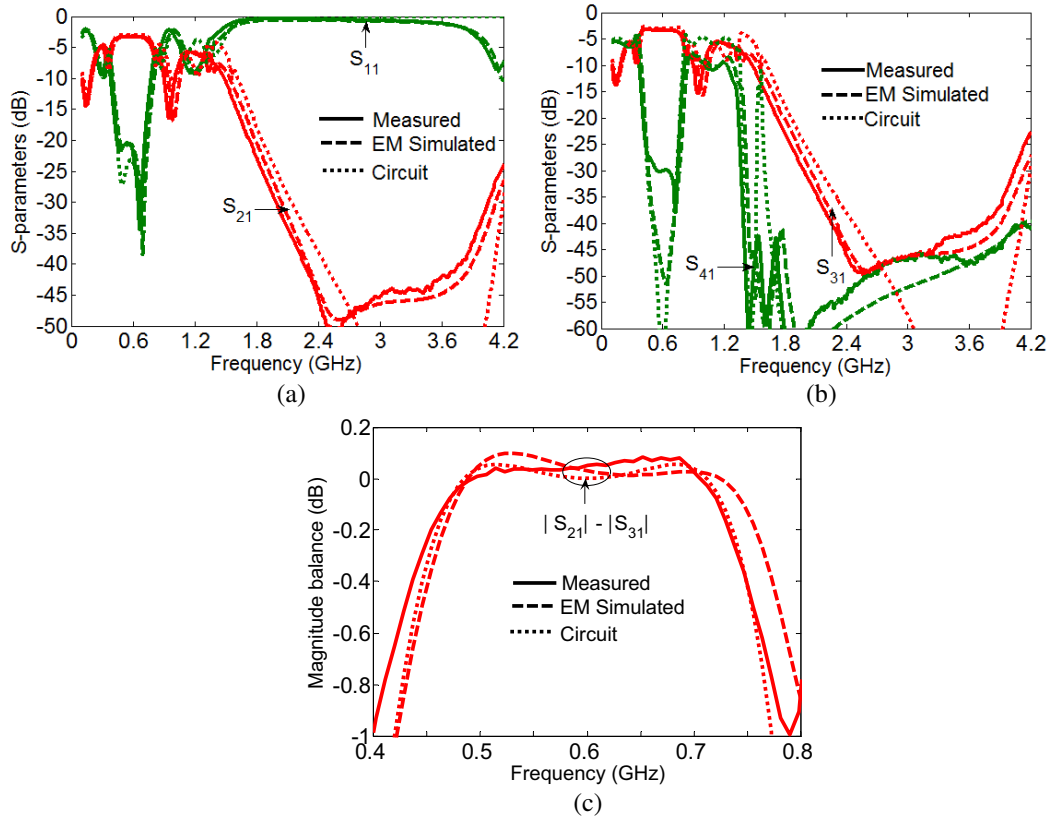
**Figure 10.** Measured, EM Simulated, Computed $S$-Parameters, (a) $S_{11}$ and $S_{21}$, (b) $S_{31}$ and $S_{41}$, (c) Magnitude difference.
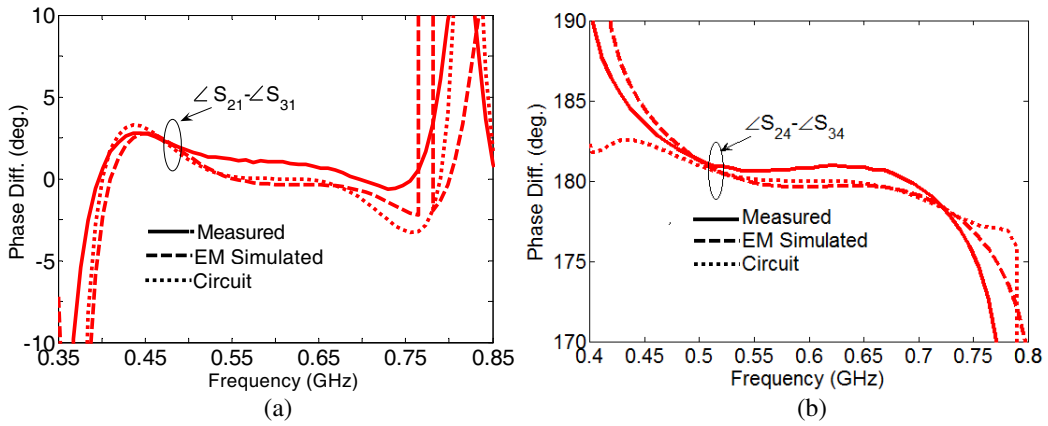


**Figure 11.** Measured, EM Simulated, Computed phase response of proposed RRC at the (a) sum port, (b) difference port.

RRC has no spurious response up to 4.2 GHz, which is about $7f_0$, where a deep suppression (at least 20 dB) is observed from the third to seventh harmonics.

The performance of the proposed wideband double section rat race coupler using an M-shape unit is compared with existing work as shown in Table 4. An improvement in FBW and harmonic suppression is noticed compared to [11, 12].

**Table 4.** Performance comparison with existing wideband harmonic suppressed compact double section RRC.

| Parameter/Ref. | Ref. [11] | Ref. [12] | Proposed |
|---|---|---|---|
| $(f_0)$ | 1.4 GHz | 0.9 GHz | 0.6 GHz |
| FBW ($S_{11}$) | 50% (10 dB) | 50% (10 dB) | 46.13% (20 dB) |
| FBW ($S_{41}$) | 50% (20 dB) | 50% (16 dB) | 61.23% (20 dB) |
| $S_{11}$ at $f_0$ | < 20 dB | −21.02 dB | −21.99 dB |
| $S_{41}$ at $f_0$ | > 30 dB | −21.43 dB | −30.09 dB |
| $S_{21}$ at $f_0$ | - | −3.5 dB | −3.26 dB |
| $S_{31}$ at $f_0$ | - | −3.42 dB | −3.31 dB |
| Magnitude balance_FBW | 50% ($0 \pm 0.5$ dB) | 51.1% ($0 \pm 1$ dB) | 55.39% ($0 \pm 0.5$ dB) |
| Magnitude balance at $f_0$ | - | 0.08 dB | 0.05 dB |
| Sum port FBW | 50% ($0 \pm 5°$) | 49.18% ($0 \pm 1.5°$) | 68.05% ($0 \pm 5°$) |
| Difference port FBW | 50% ($180 \pm 5°$) | 48.64% ($180 \pm 5°$) | 52.48% ($180 \pm 5°$) |
| Relative circuit size | 31% | 14% (With meandering) | 37.6% |
| Harmonic Suppression | Nil | Up to $5f_0$ | Up to $7f_0$ |

## 4. CONCLUSION

The design and implementation of a microstrip compact wide-band harmonic suppressed RRC is presented using an M-shape unit. The proposed double section RRC provides good input matching, isolation, magnitude balance over a wideband along with harmonic suppression up to seventh of operating frequency $f_0$ and occupies 37.6% area of conventional double section RRC. Variation of FBW against line, stub lengths, and impedances is presented graphically using transmission line model, which is very easy to follow. 20 dB FBW is improved well compared with other existing double section wideband RRCs.

## ACKNOWLEDGMENT

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