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Direction Finding for Coherently Distributed Sources with Gain-Phase Errors

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Abstract—Affected by multipath propagation as well as the receiving conditions of an actual array, a distributed source model considering array uncertainties/errors is more consistent with the realistic scenarios. In this paper, a new direction finding method for coherently distributed (CD) sources in the presence of array gain-phase errors is proposed. By exploiting partly calibrated uniform linear arrays (ULA), the gain-phase errors are first estimated according to the relationship of elements of array covariance matrix, and then a two-step iterative procedure is introduced to achieve a joint estimation of nominal DOA and angular spread from sparse recovery perspective. Performance analysis and related Cramér-Rao bound (CRB) are also provided. Numerical examples show that the proposed method can provide improved resolution and estimation accuracy, and performs almost independently of gain-phase errors.

1. INTRODUCTION

Direction finding or direction of arrival (DOA) estimation is an important issue in many engineering fields including mobile communications, radar, sonar, as well as Internet of Vehicles (IoV) systems [1]. In most applications, a far-field point source model that corresponds to one-ray propagation is assumed to hold, and so far most of existing methods are presented based on this signal model. However, due to being blocked by buildings and vehicles in some practical applications, the effect of multipath propagation cannot be ignored. Under such circumstance, a distributed source model that contains both DOA and angular spread information would be more appropriate [2].

Depending on the correlation among different rays, the distributed source can be divided into incoherently distributed (ID) source and coherently distributed (CD) source [3]. For an ID source, the signal components from different rays are supposed uncorrelated, whereas for a CD source, these components are correlated. In recent years, several direction finding methods for distributed sources have also been proposed successively, examples including the multiple signal classification (MUSIC), multi-parameter estimation approach in sparse representation framework (MpECD-SR), generalized distributed signal parameter estimator (CCCS-DSPE) based algorithms for CD sources [4–6] and the covariance matching estimator (COMET), estimation of signal parameters via rotational invariance technique (ESPRIT), and Beamspace based algorithms for ID sources [7–9].

As demonstrated in the literature [10, 11], the performance of these methods mentioned above is critically dependent on the priori knowledge of the array manifold. However, there are often various array manifold errors (such as gain-phase errors, mutual coupling, and sensor location errors) in practice, which directly result in that the array manifold cannot be known precisely. As a result, the performance of these estimators could degrade seriously. So far, several direction finding methods in the presence

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of array manifold errors have also been presented, examples including the Hadamard product, rank-reduction (RARE) and ESPRIT based algorithms [12–14] for gain-phase errors, the MUSIC based algorithm [15] for mutual coupling, sparsity-iteration based algorithm [16] for sensor location errors, etc. However, most of them are established on point source model.

In this paper, we take the influence of array gain-phase errors into account and further propose a new direction finding method for CD sources. The method mainly comprises two steps: the first step is to achieve the estimation of gain-phase errors by exploiting the elements of array covariance matrix and simple algebraic operation, under the condition that part of sensors are well calibrated, and the second step is to construct a sparse iterative optimization problem to jointly estimate nominal DOAs as well as their corresponding angular spreads. By conducting theoretical analysis and numerical examples, we show that the proposed method has several salient advantages including increased resolution, improved estimation accuracy, and performing almost independently of gain-phase errors.

2. SIGNAL MODEL

Assume that K uncorrected CD source signals impinge on an M-element ULA. The distance between adjacent sensors is $d \leq \lambda/2$, where λ is the wavelength of the carrier. Taking the array gain-phase errors into account, the array output observed at time instant t, $t = 1, 2, \ldots, N$, can be written as

$$\mathbf{x}(t) = \mathbf{\Phi}\mathbf{A}(\theta, \sigma_\theta)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{C}^{M\times 1}$, $\mathbf{s}(t) \in \mathbb{C}^{K\times 1}$ and $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ are the array output vector, signal vector, and additive Gaussian white noise vector, respectively. $\mathbf{\Phi}$ denotes the array gain-phase errors matrix. Without loss of generality, it is assumed that the first $L \geq 2$ sensors of the array are well calibrated, whereas the remaining $M - L$ sensors are uncalibrated with errors modeled as unknown, direction-independent gains, and phases; therefore, $\mathbf{\Phi}$ is expressed as

$$\mathbf{\Phi} = \mathrm{diag}\left\{\mathbf{1}_L, \rho_L e^{j\psi_L}, \ldots, \rho_{M-1}e^{j\psi_{M-1}}\right\}, \tag{2}$$

where ρ_l and ψ_l, $l = L, \ldots, M-1$, are the unknown gains and phases of the uncalibrated sensors. $\mathbf{A}(\theta, \sigma_\theta) \in \mathbb{C}^{M\times K}$ is the array steering matrix with its kth column given by

$$\mathbf{a}\left(\theta_k, \sigma_{\theta_k}\right) = \int_{-\pi/2}^{\pi/2} \bar{\mathbf{a}}\left(\theta_k\right)\rho\left(\theta_k, \sigma_{\theta_k}\right)d\theta_k, \tag{3}$$

$\bar{\mathbf{a}}(\theta_k) = [1, e^{-j\omega_k}, \ldots, e^{-j(M-1)\omega_k}]^T$ with superscript T representing the transpose operation, $\omega_k = 2\pi d\sin\theta_k/\lambda$. $\rho(\theta_k, \sigma_{\theta_k})$ is the probability density function of angular spread, where θ_k and σ_{θ_k} are the nominal DOA and angular spread of the kth CD source, respectively. It is further assumed that the CD source is located in small angle scattering environment (i.e., $\{\sigma_{\theta_k}\}_{k=1}^K$ are sufficiently small) [4–9] and follows Gaussian distribution, hence $\rho(\theta_k, \sigma_{\theta_k})$ is given by

$$\rho(\theta_k, \sigma_{\theta_k}) = \frac{1}{\sqrt{2\pi\sigma_{\theta_k}^2}}\exp\left(-\frac{\theta_k - \bar{\theta}_k}{2\sigma_{\theta_k}^2}\right), \tag{4}$$

which directly yields that $\mathbf{a}(\theta_k, \sigma_{\theta_k})$ can be approximately expressed as [4]

$$\mathbf{a}\left(\theta_k, \sigma_{\theta_k}\right) \approx \left[1, e^{-\left(j\omega_k + \sigma_{\theta_k}^2/2\right)}, \ldots, e^{-(M-1)\left(j\omega_k + \sigma_{\theta_k}^2/2\right)}\right]^T. \tag{5}$$

According to the array received data vector $\mathbf{x}(t)$, the array covariance matrix can be expressed as

$$\mathbf{R} = \mathbb{E}\left\{\mathbf{x}\left(t\right)\mathbf{x}^H\left(t\right)\right\} = \mathbf{\Phi}\mathbf{A}\left(\theta, \sigma_\theta\right)\mathbf{\Lambda}_S\mathbf{A}^H\left(\theta, \sigma_\theta\right)\mathbf{\Phi}^H + \sigma_n^2\mathbf{I}_M, \tag{6}$$

where $\mathbf{\Lambda}_S = \mathbb{E}\{\mathbf{s}(t)\mathbf{s}^H(t)\} = \mathrm{diag}\{P_1, \ldots, P_K\}$ with P_k representing the power of kth source signal; σ_n^2 is the noise variance; and $\mathbf{I}_M$ stands for an $M \times M$ identity matrix. $\mathbb{E}\{\cdot\}$ denotes expectation, superscript H the conjugate transpose, and $\mathrm{diag}\{\cdot\}$ the diagonal operation. In practice, $\mathbf{R}$ is replaced by its sampled value, i.e.,

$$\mathbf{R} \approx \hat{\mathbf{R}} = N^{-1}\sum\nolimits_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t) \tag{7}$$

and we utilize $\hat{\mathbf{R}}$ for DOA estimation of CD sources.

3. PROPOSED METHOD

3.1. Gain-phase Errors Estimation

In this paper, we assume that the noise variance σ_n^2 is known *in prior*, or simply estimated according to [17]. Thus, we have

$$\mathbf{R}_0 = \mathbf{R} - \sigma_n^2 \mathbf{I}_M = \mathbf{\Phi}\mathbf{A}(\theta, \sigma_\theta)\mathbf{\Lambda}_S \mathbf{A}^H(\theta, \sigma_\theta)\mathbf{\Phi}^H. \tag{8}$$

The (p, q)th element of $\mathbf{R}_0$ is expressed as

$$[\mathbf{R}_0]_{p,q} = \rho_{p-1}\rho_{q-1}e^{j(\psi_{p-1}-\psi_{q-1})}\sum_{k=1}^{K} P_k e^{-(p-q)\left(j\omega_k+\sigma_{\theta_k}^2/2\right)}, \tag{9}$$

where $p, q \in [1, M]$. Define $p = q$ and $p = q + 1$, respectively, then we have

$$[\mathbf{R}_0]_{q,q} = \rho_{p-1}^2 \sum_{k=1}^{K} P_k, \tag{10}$$

$$[\mathbf{R}_0]_{q+1,q} = \rho_q\rho_{q-1}e^{j(\psi_q-\psi_{q-1})}\sum_{k=1}^{K} P_k e^{-\left(j\omega_k+\sigma_{\theta_k}^2/2\right)}. \tag{11}$$

Since the first L sensors of the array are well calibrated, the following relationships hold

$$\sum_{k=1}^{K} P_k = [\mathbf{R}_0]_{1,1} = \ldots = [\mathbf{R}_0]_{L,L} = \kappa_1, \tag{12}$$

$$\sum_{k=1}^{K} P_k e^{-\left(j\omega_k+\sigma_{\theta_k}^2/2\right)} = [\mathbf{R}_0]_{2,1} = \ldots = [\mathbf{R}_0]_{L,L-1} = \kappa_2. \tag{13}$$

To estimate array gain-phase errors, we further define

$$f_m = \prod_{i=1}^{m-1} [\mathbf{R}_0]_{i+1,i} \Big/ \prod_{i=1}^{m-1} [\mathbf{R}_0]_{i,i} = \rho_m e^{j\psi_m}(\kappa_2/\kappa_1)^{m-1}, \tag{14}$$

where $\prod_{i=1}^{m-1}(\cdot)$ denotes the continuous multiplication operation of $m-1$ elements, $m \in [L, M-1]$.

According to Eq. (14), the gain errors and phase errors can be estimated as

$$\hat{\rho}_m = \left|\hat{f}_m(\hat{\kappa}_1/\hat{\kappa}_2)^{m-1}\right|,\ m \in [L, M-1], \tag{15}$$

$$\hat{\psi}_m = \angle \hat{f}_m(\hat{\kappa}_1/\hat{\kappa}_2)^{m-1},\ m \in [L, M-1], \tag{16}$$

where a certain value $\hat{\upsilon}$ denotes the estimation result of its sampled one, and $\angle[\cdot]$ is the phase of a complex number.

3.2. DOA Estimation of CD Sources

As the array gain-phase errors have been estimated, we can compensate them as

$$\mathbf{R}_1 = \mathbf{\Phi}^{-1}\mathbf{R}_0\mathbf{\Phi} = \mathbf{A}(\theta, \sigma_\theta)\mathbf{\Lambda}_S\mathbf{A}^H(\theta, \sigma_\theta). \tag{17}$$

By applying vectorizing operation on $\mathbf{R}_1$, we have

$$\mathbf{y} = \mathrm{vec}(\mathbf{R}_1) = \bar{\mathbf{A}}\mathbf{p}, \tag{18}$$

where $\bar{\mathbf{A}} = [\bar{\mathbf{a}}(\theta_1, \sigma_{\theta_1}), \ldots, \bar{\mathbf{a}}(\theta_K, \sigma_{\theta_K})]$, $\bar{\mathbf{a}}(\theta_k, \sigma_{\theta_k}) = \mathbf{a}^*(\theta_k, \sigma_{\theta_k}) \otimes \mathbf{a}(\theta_k, \sigma_{\theta_k})$ with $\otimes$ and superscript $*$ representing the Kronecker product and conjugate operation, respectively, $\mathrm{vec}(\cdot)$ is the vectorization operation, and $\mathbf{p} = [P_1, \ldots, P_K]^T$.

Now we consider the impact of angular spreads as the small array perturbations. Consequently, $\mathbf{y}$ can be rewritten as

$$\mathbf{y} = \left(\tilde{\mathbf{A}} + \mathbf{E}\right)\mathbf{p}, \tag{19}$$

where $\tilde{\mathbf{A}} = [\tilde{\mathbf{a}}(\theta_1), \ldots, \tilde{\mathbf{a}}(\theta_K)]$, $\tilde{\mathbf{a}}(\theta_k) = \bar{\mathbf{a}}^*(\theta_k) \otimes \bar{\mathbf{a}}(\theta_k)$, $\mathbf{E} = \mathbf{G} \odot \tilde{\mathbf{A}}$, $\mathbf{G} = [\mathbf{g}(\sigma_{\theta_1}), \ldots, \mathbf{g}(\sigma_{\theta_K})]$, and the kth column of $\mathbf{G}$ is $\mathbf{g}(\sigma_{\theta_k}) = \mathbf{w}(\sigma_{\theta_k}) \otimes \mathbf{w}(\sigma_{\theta_k}) - \mathbf{1}_{M^2}$, $\mathbf{w}(\sigma_{\theta_k}) = [1, e^{-\sigma_{\theta_k}^2/2}, \ldots, e^{-(M-1)\sigma_{\theta_k}^2/2}]^T$.

Suppose that the range of spatial directions is quantized into G grids, where $G \gg K, M$. Hence, we have

$$\mathbf{y} = \left(\tilde{\mathbf{A}} + \mathbf{E}\right)\mathbf{p} = \left(\mathbf{\Psi} + \breve{\mathbf{E}}\right)\breve{\mathbf{p}}, \tag{20}$$

where $\mathbf{\Psi}$, $\breve{\mathbf{E}}$ and $\breve{\mathbf{p}}$ are sparse representations of $\tilde{\mathbf{A}}$, $\mathbf{E}$ and $\mathbf{p}$, respectively. Subsequently, the direction finding of CD sources can be realized by solving the following optimization problem:

$$\min \left\|\breve{\mathbf{p}}\right\|_1 + \tau\left\|\breve{\mathbf{E}}\right\|_1, \ s.\,t. \ \left\|\mathbf{y} - (\mathbf{\Psi} + \breve{\mathbf{E}})\breve{\mathbf{p}}\right\|_2^2 < \varepsilon, \tag{21}$$

where $\|\cdot\|_1$ and $\|\cdot\|_2$ denote the ℓ_1 norm and ℓ_2 norm, respectively; τ is the regularization constant; and ε is the precision parameter, which can be well ensured by the cross validation scheme [18].

It can be seen that the optimization problem in Eq. (21) is not a convex one under both variables $\breve{\mathbf{p}}$ and $\breve{\mathbf{E}}$. In order to efficiently solve Eq. (21), a two-step iterative procedure with initialized value $\breve{\mathbf{E}} = \mathbf{0}$ is exploited in this paper:

- Step 1: Solve the following convex optimization problem with $\breve{\mathbf{E}}$ to achieve DOA estimation.

$$\min \left\|\breve{\mathbf{p}}\right\|_1, \ s.\,t. \ \left\|\mathbf{y} - \left(\mathbf{\Psi} + \breve{\mathbf{E}}\right)\breve{\mathbf{p}}\right\|_2^2 < \varepsilon.$$

- Step 2: With estimated $\breve{\mathbf{p}}$ from Step 1, solve another convex optimization problem as shown below to produce the estimation of $\breve{\mathbf{E}}$.

$$\min \tau\left\|\breve{\mathbf{E}}\right\|_1, \ s.\,t. \ \left\|\mathbf{y} - \left(\mathbf{\Psi} + \breve{\mathbf{E}}\right)\breve{\mathbf{p}}\right\|_2^2 < \varepsilon.$$

- Iterate alternately between Steps 1 and 2 until the estimated deviation of the adjacent two iterations is less than a predefined threshold.

Notice that each step or iteration belongs to convex optimization framework; therefore, the convergence of this two-step iterative procedure can be efficiently guaranteed. Finally, by finding the indexes of non-zeros components in $\breve{\mathbf{p}}$ as well as corresponding elements in $\breve{\mathbf{E}}$, the DOAs and angular spreads of all CD sources are estimated.

3.3. Performance Analysis and Cramér-Rao Bound

3.3.1. Performance Analysis

The performance of the proposed method depends on the initial value of $\breve{\mathbf{E}}$ greatly. According to the analysis of perturbations in sparse recovery or compressed sensing theory [19], the estimation error bound of $\breve{\mathbf{p}}$ in Step 1 with given $\breve{\mathbf{E}}$ obeys

$$\left\|\breve{\mathbf{p}}' - \breve{\mathbf{p}}\right\|_2^2 \leq \beta\|\mathbf{\Psi}\|_2 \left\|\breve{\mathbf{E}}\right\|_2^{(S)} \Big/ \|\mathbf{\Psi}\|_2^{(S)}, \tag{22}$$

where $\breve{\mathbf{p}}'$ is the estimate of $\breve{\mathbf{p}}$, β a constant, and superscript S the extremal value of ℓ_2 norm of a matrix. Note that we have assumed that $\{\sigma_{\theta_k}\}_{k=1}^K$ are sufficiently small, which implies that $\breve{\mathbf{E}} \to \mathbf{0}$ and $||\breve{\mathbf{E}}||_2^{(S)}$ is a small value. As a result, it can be deduced that if $\breve{\mathbf{E}}$ is initialized with $\breve{\mathbf{E}} = \mathbf{0}$, a good DOA estimation performance can be guaranteed.

3.3.2. Cramér-Rao Bound

The Cramér-Rao Bound (CRB) is obtained by taking the inverse of the Fisher information matrix (FIM), which is established on the approximate covariance matrix $\mathbf{R}$ of the array output in this paper. The (l,h)th element of FIM is given by [14]

$$\mathbf{F}(l,h) = N\mathbf{Tr}\left\{\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \varsigma_l}\mathbf{R}^{-1}\frac{\partial \mathbf{R}}{\partial \varsigma_h}\right\}, \tag{23}$$

where ς_l is the lth unknown parameter. Define the unknown parameter vectors $\mathbf{z} = [\boldsymbol{\theta}^T\ \boldsymbol{\sigma}_\theta^T\ \ \mathbf{g}^T\ \boldsymbol{\psi}^T]^T \in \mathbb{R}^{2(K+M-L)\times 1}$, where $\boldsymbol{\theta} = [\theta_1,\ldots,\theta_K]^T$, $\boldsymbol{\sigma}_\theta = [\sigma_{\theta_1}^2,\ldots,\sigma_{\theta_K}^2]^T$, $\mathbf{g} = [\rho_L,\ldots,g_{M-1}]^T$, and $\boldsymbol{\psi} = [\psi_L,\ldots,\psi_{M-1}]^T$. Then, the various blocks of FIM can be respectively given by

$$\mathbf{F}_{\theta\theta} = 2N\cdot \mathrm{Re}\left\{(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S)\odot\left(\dot{\mathbf{A}}_\theta^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)\odot\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T\right\}$$

$$\mathbf{F}_{\theta\sigma_\theta} = 2N\cdot \mathrm{Re}\left\{\left[(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S)\odot\left(\dot{\mathbf{A}}_{\sigma_\theta}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)\odot\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T\right]\right\}$$

$$\mathbf{F}_{\theta\rho} = 2N\cdot \mathrm{Re}\left\{\left[\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\rho^H\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\rho^H\right)\odot\left(\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\theta\psi} = 2N\cdot \mathrm{Re}\left\{\left[\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\psi^H\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\psi^H\right)\odot\left(\mathbf{R}^{-1}\dot{\mathbf{A}}_\theta\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\sigma_\theta\sigma_\theta} = 2N\cdot \mathrm{Re}\left\{(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S)\odot\left(\dot{\mathbf{A}}_{\sigma_\theta}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)\odot\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T\right\}$$

$$\mathbf{F}_{\sigma_\theta\rho} = 2N\cdot \mathrm{Re}\left\{\left[\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\rho^H\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\rho^H\right)\odot\left(\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\sigma_\theta\psi} = 2N\cdot \mathrm{Re}\left\{\left[\left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\psi^H\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T + \left(\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\psi^H\right)\odot\left(\mathbf{R}^{-1}\dot{\mathbf{A}}_{\sigma_\theta}\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\rho\rho} = 2N\cdot \mathrm{Re}\left\{\mathbf{H}\left[\left(\dot{\mathbf{A}}_\rho\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\rho\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)^T + \left(\dot{\mathbf{A}}_\rho\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\rho^H\right)\odot\left(\mathbf{R}^{-1}\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\rho\psi} = 2N\cdot \mathrm{Re}\left\{\mathbf{H}\left[\left(\dot{\mathbf{A}}_\psi\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\rho\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)^T + \left(\dot{\mathbf{A}}_\psi\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\rho^H\right)\odot\left(\mathbf{R}^{-1}\right)^T\right]\mathbf{H}^T\right\}$$

$$\mathbf{F}_{\psi\psi} = 2N\cdot \mathrm{Re}\left\{\mathbf{H}\left[\left(\dot{\mathbf{A}}_\psi\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)\odot\left(\dot{\mathbf{A}}_\psi\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\right)^T + \left(\dot{\mathbf{A}}_\psi\boldsymbol{\Lambda}_S\mathbf{A}^H\mathbf{R}^{-1}\mathbf{A}\boldsymbol{\Lambda}_S\dot{\mathbf{A}}_\psi^H\right)\odot\left(\mathbf{R}^{-1}\right)^T\right]\mathbf{H}^T\right\}$$

where $\odot$ denotes the Hadamard-Schur product, and

$$\widehat{\mathbf{A}} = \boldsymbol{\Phi}\mathbf{A}(\theta,\sigma_\theta), \quad \dot{\mathbf{A}}_\theta = \sum\nolimits_{k=1}^K \frac{\partial\widehat{\mathbf{A}}}{\partial\theta_k}, \quad \dot{\mathbf{A}}_{\sigma_\theta} = \sum\nolimits_{k=1}^K \frac{\partial\widehat{\mathbf{A}}}{\partial\sigma_{\theta_k}^2},$$

$$\dot{\mathbf{A}}_\rho = \sum\nolimits_{l=L}^M \frac{\partial\widehat{\mathbf{A}}}{\partial\rho_l}, \quad \dot{\mathbf{A}}_\psi = \sum\nolimits_{l=L}^M \frac{\partial\widehat{\mathbf{A}}}{\partial\psi_l}, \quad \mathbf{H} = \left[\mathbf{0}_{(M-L)\times L}\ \ \mathbf{I}_{M-L}\right].$$

Finally, by taking the inverse of $\mathbf{F}$, the CRB of nominal DOA estimation is given by

$$\mathrm{CRB} = \sqrt{\frac{1}{K}\sum\nolimits_{k=1}^K [\mathbf{F}^{-1}]_{kk}}. \tag{24}$$

4. NUMERICAL EXAMPLES

In this section, some numerical examples are carried out to demonstrate the effectiveness of the proposed method. The MUSIC in [4] and the MpECD-SR in [5] are selected as compared methods. The number of ULA sensors is 10, and two BPSK modulated CD source signals are considered. The gain errors and phase errors are generated by [11]

$$\rho_l = 1 + \sqrt{12}\sigma_\rho \xi_l, \quad \psi_l = \sqrt{12}\sigma_\psi \zeta_l, \tag{25}$$

where ξ_l and ζ_l are independent and identically distributed random variables distributed uniformly over $[-0.5, 0.5]$, and σ_ρ and σ_ψ are the standard deviations of ξ_l and ζ_l, respectively. In all examples, both the root mean square error (RMSE) and probability of separation of DOA estimation are obtained by 300 independent Monte-Carlo trials.

In the first example, we examine the performance of the proposed method with respect to SNR and the number of snapshots. The simulation result is shown in Fig. 1, $\sigma_\rho = 0.1, \sigma_\psi = 20°$, $\{\theta_1, \sigma_{\theta_1}\} = \{-10°, 1°\}, \{\theta_2, \sigma_{\theta_2}\} = \{20°, 1°\}$, and $L = 5$. In Fig. 1(a), the number of snapshots is fixed at 500, and SNR varies from 0 dB to 15 dB, whereas in Fig. 1(b), SNR is set to 6 dB, and the number of snapshots varies from 100 to 600. Note that the MUSIC method in [4] assumes that the PDF of angular spread is known in priori, hence we take the same assumption for the simulations of the MUSIC method here. It can be seen that the proposed method outperforms the compared methods, and its RMSE of DOA estimation decreases rapidly as SNR and the number of snapshots increase. Meanwhile, a good angular spread estimation shown in Table 1 can also be provided. As a comparison, there is little performance improvement with the increase of SNR and the number of snapshots for the compared methods.

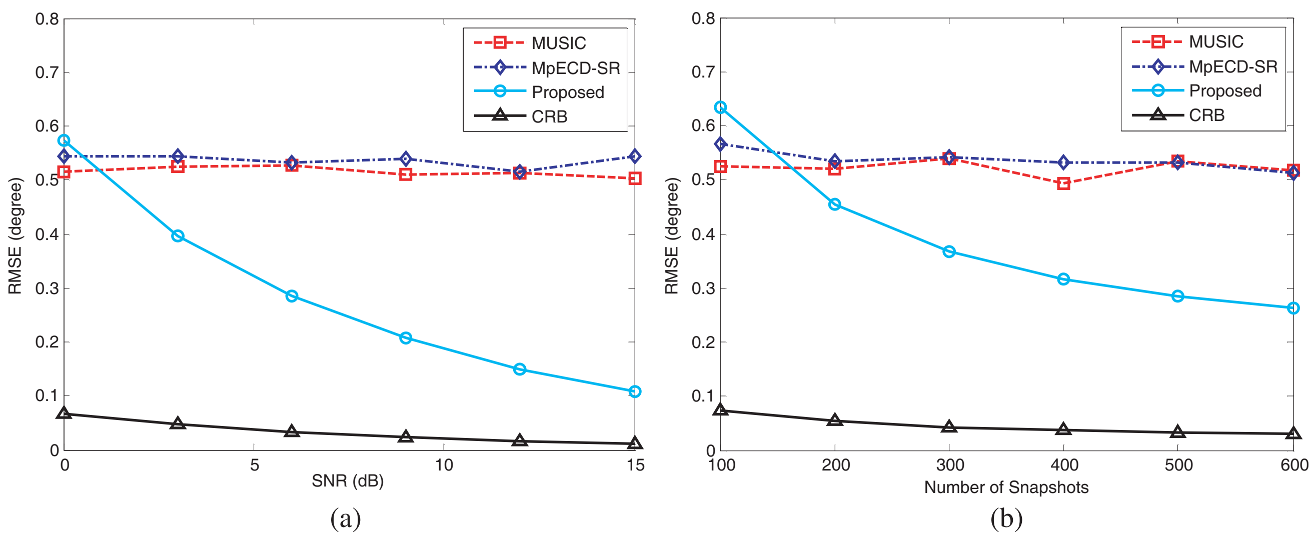

Figure 1. RMSE of DOA estimates versus SNR and the number of snapshots. (a) Versus SNR. (b) Versus the number of snapshots.

Table 1. RMSE of σ_θ^2 estimates in different SNRs.

SNR	0 dB	3 dB	6 dB	9 dB	12 dB	15 dB
Proposed	0.0725	0.0676	0.0563	0.0445	0.0293	0.0197
MpECD-SR	0.1448	0.1345	0.1300	0.1265	0.1258	0.1249

In the second example, we evaluate the performance of the proposed method in terms of different gain errors and phase errors. The RMSE curves are depicted in Fig. 2. The SNR and the number of snapshots are set to 6 dB and 500, respectively. In Fig. 2(a), $\sigma_\psi = 20°$, σ_ρ varies from 0.05 to 0.3, whereas in Fig. 2(b), $\sigma_\rho = 0.1$ and σ_ψ varies from 5° to 30°. Other simulation conditions are the same as those in the first example. It can be observed from Fig. 2 that the proposed method performs almost

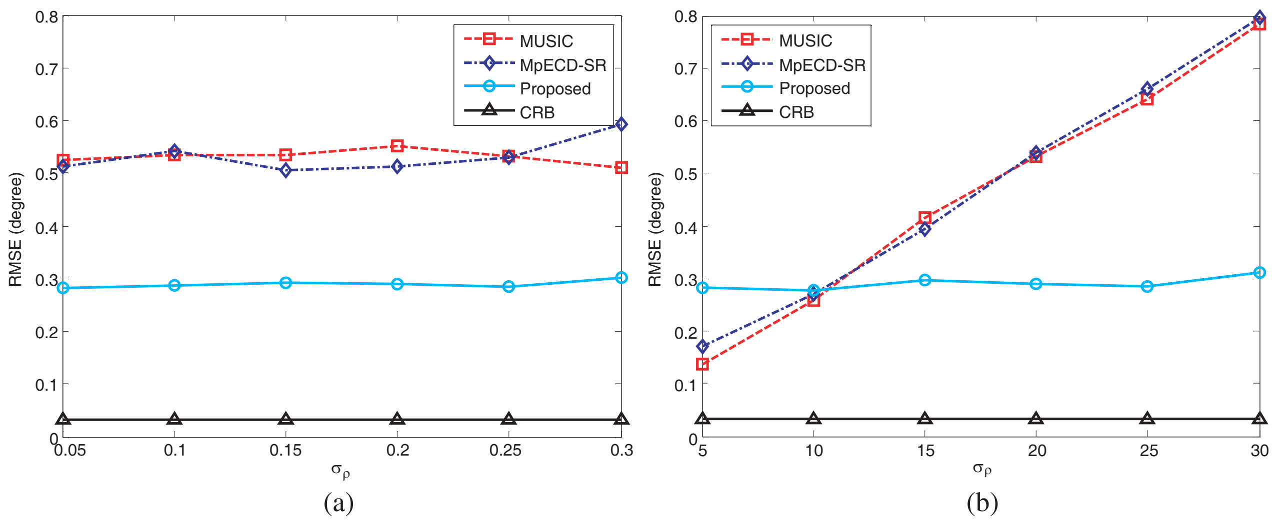

Figure 2. RMSE of DOA estimates in different gain errors and phase errors. (a) In different gain errors. (b) In different phase errors.

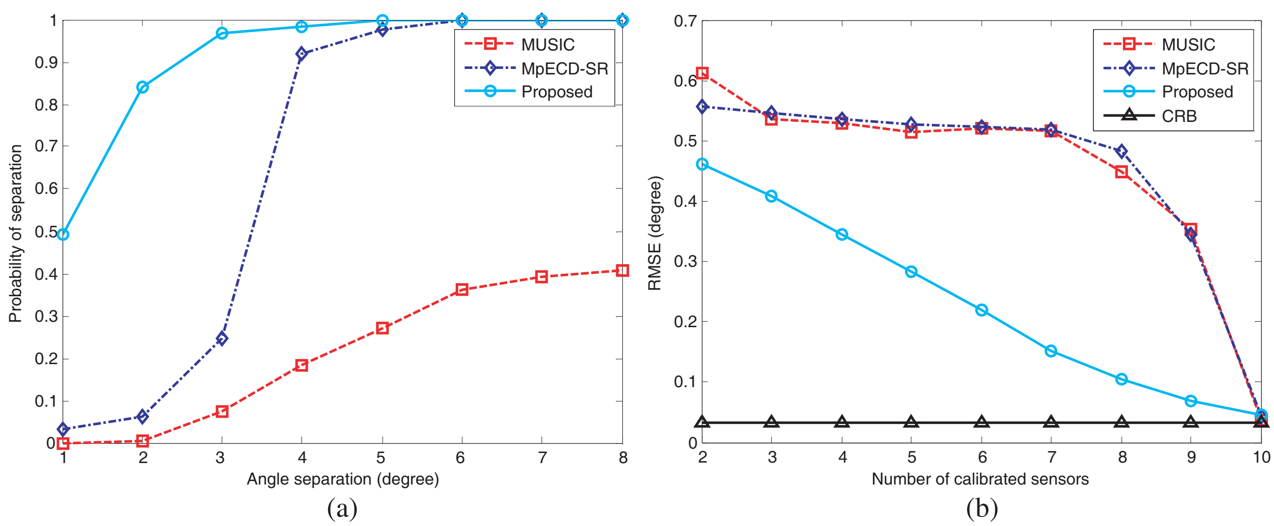

Figure 3. Performance of the proposed method against angle separation and the number of calibrated sensors L. (a) Probability of separation against angle separation. (b) RMSE of DOA estimates versus L.

independently of gain-phase errors. In contrast, the compared methods are affected by the phase errors tremendously, whose RMSEs increase monotonically with phase errors.

In the last example, we test the probability of separation of the proposed method as well as the impact of the number of calibrated sensors L on its performance. The number of snapshots, SNR, σ_ρ, and σ_ψ are set to 500, 6 dB, 0.1, and 20°, respectively. In Fig. 3(a), θ_1 is fixed at $-10°$, whereas θ_2 varies from $-9°$ to $-2°$. By definition, two CD sources are resolved in a given run if both the bias of two directions are smaller than 0.5°. From Fig. 3(a), we can conclude that the resolution performance of the proposed method outperforms the compared methods. In Fig. 3(b), the parameters of two CD sources are same as those in the first examples, and the number of calibrated sensors L varies from 2 to 10, from which we can see that the performance of all methods increases as L increases. In particular, when the array is fully calibrated (i.e., $L = 10$), the performance of all methods is satisfactory and follows CRB very well. When only part of the array is calibrated, the proposed method performs better than the compared methods.

5. CONCLUSION

In this paper, a new direction finding method for CD sources considering array gain-phase errors is proposed. Under the condition of partly calibrated ULA, the gain-phase errors are first estimated using the elements of array covariance matrix. Then, with the aid of the estimated gain-phase errors as well as a two-step iterative sparse recovery approach, an improved direction finding result is achieved. By performance analysis as well as numerical examples, we show that the proposed method can provide an increased resolution and estimation accuracy in the presence of array gain-phase errors.

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