

The New Localized Ray-Tracing-Maximum-Likelihood Method Estimates the Probability Distribution of the Field Strength

Mehdi Ardavan*

Abstract—The Ricean probability density function (pdf) is widely used to estimate the electromagnetic field distribution in indoor environments. The goal of using the Ricean or other pdfs is to evade the computational cost of deterministic field calculation. The parameters of the pdfs are usually obtained using the maximum-likelihood estimation which is here shown to fail in local areas close to the antenna where the direct field varies significantly. This paper presents the new localized maximum likelihood method which is valid in close regions as well. Moreover, the maximum-likelihood method requires a large number of field values within the local area to yield the parameters of the pdf. This paper presents the ray-tracing-maximum-likelihood (RTML) method where a much lower number of field values are required. These values are determined using ray-tracing and without the need to account for the computationally expensive higher-order reflections. The RTML fails in local areas close to the antenna, and thus the new localized RTML is presented to restore accuracy.

1. INTRODUCTION

Determining the electric field strength in indoor environments and at the ISM band frequencies or higher requires extravagantly large computational time if full-wave methods are used to solve Maxwell's equations. In some problems, such as electromagnetic interference (EMI) risk assessment studies [1], the location of the transmitter may vary, and thus the simulations must be repeated for many different source locations. Hence, even ray-tracing, from geometrical optics, may not satisfy a criterion for the computational speed.

The computational cost may be reduced if, instead of deterministic solutions, probabilistic estimations of the electromagnetic field are sought. The Nakagami, Weibul, Ricean, and Rayleigh distributions have been used to estimate the probability density function (pdf) of the electric field in indoor environments [2, 3]. The Ricean function has been shown to accurately predict the cumulative distribution function (cdf) of the field in a room similar to that of this paper in [4–6].

The parameters of the distribution functions are usually obtained by the maximum-likelihood estimation [7] and the method of moments [8]. To obtain these parameters, a large number of field values, over a dense grid, are required, and hence the computational efficacy of the probabilistic estimation of the field strength, using the maximum-likelihood estimation or the method of moments, is not considerable.

The objective of this paper is to reduce the computational time of estimating the Ricean parameters. The parameters are extracted from the electric field intensity values obtained by ray tracing. In ray-tracing, the computational time grows linearly by the number of observation points and exponentially by the highest reflection order accounted for. The goal of this paper is to reduce (i) the number of required values, and (ii) the highest reflection order accounted for.

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* Corresponding author: Mehdi Ardavan (m.ardavan@ieee.org).

The author is currently with the Ericsson Canada. At the time of conducting this research, he was a PhD student at Concordia University, Montreal, Canada.

In this paper, the minimum number of field values over an area of 10 by 10 wavelengths is found such that the Ricean cdf with the parameters determined using the maximum-likelihood method accurately approximates the cdf obtained statistically using the ray-tracing solutions at a very large number of points evenly distributed over the area.

Also we show that it is possible to reduce the maximum order of reflections accounted for in the ray-tracing simulation without degrading the accuracy of the Ricean parameters and cdf curve. The optimal value of the maximum reflection order is seen to be dependent on the distance between the transmitter and the observation area.

In this paper, we demonstrate that in regions close to the antenna, due to the large variations of the direct field, the Ricean distribution fails in accurately predicting the field distribution. To overcome this, we divide the region into smaller areas and present the *localized maximum-likelihood* method where the pdf parameters of the smaller areas are used to accurately estimate the field cdf in the larger region.

To further decrease the cost of computation, we introduce a new concept called the *residual field* which is the power-based sum of the field strengths of all rays arriving at the center of the room, each reflected two or more times. Using the residual field as a constant number over the entire room, we present a new method, the *ray-tracing-maximum-likelihood* (RTML) method, which significantly reduces the computational time. The RTML method fails in regions close the antenna. To overcome this, we present the *localized ray-tracing-maximum likelihood* (L-RTML) method.

Section 2 reviews some background theory and then presents the new methods, which are applied in Section 3 to the 15 by 5 m room of Fig. 1.

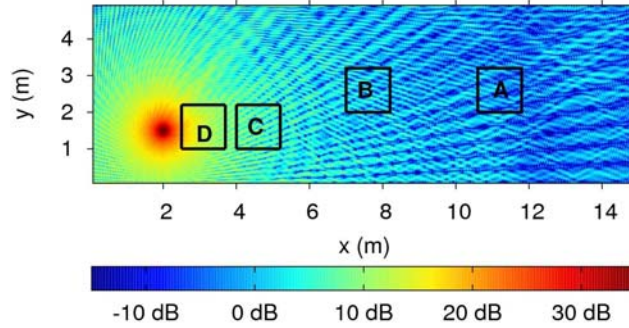


Figure 1. Electric field distribution over a large room measuring 15 m by 5.

2. METHODS

This section reviews the ray-tracing theory in geometrical optics and the Ricean distribution. Three new methods are presented for estimation of the Ricean parameters: the localized maximum-likelihood, the ray tracing-maximum-likelihood, and the localized ray tracing-maximum-likelihood methods.

Ray tracing is applied in a long room which is shown in Fig. 1 and has N_w reflecting surfaces (walls, floor and ceiling). In the image-tree model of the ray-tracing method [9], N_w first-order images are determined regarding the transmitter location and the position and geometry of the N_w walls. Then each of these N_w is considered a *parent* image and will have N_w *child* images, forming a set of N_w^2 second-order images. Similarly in the k 'th order, we will have N_w^k images. If R is the maximum

order of reflection considered, the total number of images is $\sum_{k=1}^R N_w^k$. A valid ray is found tracing a potential path from the receiver to transmitter, starting from an image source and continuing through its parent, parent of parent and so forth. The path is discarded if a reflection point does not fall on the appropriate wall or if an image source and its parent are both obtained according to the same wall.

Hence the computational time is considered proportional to $N_T = \sum_{k=1}^R N_w^k$. The field vectors associated

with all valid rays yield the total electric field vector which is given by

$$\mathbf{E}_t(x, y, z) = \mathbf{E}_d(x, y, z) + \sum_{k=1}^R \sum_{i=1}^{N_k} \mathbf{E}_r^{(k)}(i, x, y, z) \quad (1)$$

where $\mathbf{E}_d$ is the direct field strength, and $\mathbf{E}_r^{(k)}$ is the field strength associated with the i 'th ray among N_k which have been reflected k times. Parallel and perpendicular coefficients of different multilayer walls [11] are used to determine each $\mathbf{E}_r^{(k)}$.

The field strength vector was computed by Eq. (1), using our source codes in Fortran, and its magnitude over the room is shown in Fig. 1. R is chosen to be R_X as accounting for reflections of orders higher than R_X does not add meaningful accuracy to the *point values* of the field. The closely-spaced local maxima and minima indicate the rapid variations of the field which is called fast fading. In communications or especially in EMI risk assessment studies, it is the statistics of the field and fast fading that is important. Using the ray-tracing solution at a large number of points, M , within a local area, the statistics of the field distribution within a local area can be summarized in the form of the cdf.

2.1. Reducing the Highest Reflection Order, R

In determining the cdf, it is possible to show that, in practice, there exists an R_0 for which the accuracy degradation is negligible if the maximum order of reflection decreases from R_X to R_0 , but considerable when the maximum order of reflection decreases from R_0 to $R_0 - 1$. The detailed explanation of the process of obtaining the cdf in the following is useful later in Section 2.3.

The field strength, $|\mathbf{E}_t|$, is determined using Eq. (1) at M points evenly distributed within a local area of 10λ by 10λ accounting for up to the R_X th order of reflection. These M values can be considered to form a one-dimension array of real numbers, denoted by AE . The likelihood or probability of having an electric field strength, E , in the local area, such that $E_0 < E < E_0 + \Delta E$ is given by

$$l(E_0) = \frac{n(AE, E_0, E_0 + \Delta E)}{M} \quad (2)$$

where $n(AE, E_0, E_0 + \Delta)$ is the number of the field values in the array AE which are between E_0 and $E_0 + \Delta E$. If ΔE is small enough, then the integral of Eq. (2) gives the cumulative distribution function of the electric field strength in the region.

The statistically obtained cdf, by integrating Eq. (2), is called the *reference curve* which provides a reference for comparison to evaluate the new methods developed in the paper. In the reference curve, ray-tracing is used and the highest reflection order accounted for is $R = R_X$.

Then the value of R is reduced to $R_X - 1$; the field strengths are recalculated by Eq. (1), and the cdf is obtained. Reducing R and obtaining a new cdf is repeated until the accuracy of the cdf compared to the reference cdf is not acceptable. The acceptability or the accuracy in comparing two curves is measured by the *error value* which is explained in Appendix A. We call an approximation of a cdf curve acceptable only if the two curves are visually almost indistinguishable at the first glance. According to our observations, this is usually satisfied by an error value of 0.01 or less for a cdf curve.

The lowest possible maximum order of reflection, denoted by R_0 , is found such that the approximate cdf yields an error value of less than 0.01. If the cdf is obtained using $R = R_0 - 1$, then the error value is larger than 0.01. Hence to obtain a cdf, one can use $R = R_0$ in order to save a considerable amount of computational cost.

2.1.1. Computational Time

The computational time of ray-tracing for a single observation point is proportional to $\sum_{k=1}^R N_w^k$. Our reference curve is obtained using $R = R_X$. If the required accuracy is obtained using $R = R_0 < R_X$,

then the factor by which the computational time is reduced is

$$\frac{\sum_{k=1}^{R_X} N_w^k}{\sum_{k=1}^{R_0} N_w^k}. \quad (3)$$

The computational time can also be reduced by using a pdf to estimate the field distribution.

2.2. The Maximum Likelihood Estimation and the Optimal Sample Size, M_0

In indoor environments, the probability distribution of the electric field strength, E , within a local area is widely predicted by the Ricean distribution [2, 7, 8, 10], given by

$$p(E; K, \Omega) = \frac{2(K+1)E}{\Omega} \exp\left(-K - \frac{(K+1)E^2}{\Omega}\right) \times I_0\left(2E\sqrt{\frac{K(K+1)}{\Omega}}\right), \quad E > 0 \quad (4)$$

where I_0 is the modified Bessel function of the first kind and of order 0. The Ricean K -factor and Ω are the two parameters of the Ricean function which are usually determined by the maximum likelihood estimation, according to the values of $|\mathbf{E}_t|$ at M points (samples) within a local area. The maximum-likelihood estimation of the two-parameter Ricean function is explained in [12] where the parameters are determined such that the likelihood of occurrence of a certain Ricean event, i.e., a number of Ricean random outcomes, is maximized.

In the probability theory, a Ricean random outcome is given by

$$Ee^{j\phi} = E_0e^{j\phi_0} + \sum_{i=1}^N E_i e^{j\phi_i} \quad (5)$$

where E_0 , a real number, is the dominant component and in an ideal Ricean event all E_i 's are equal. Also ϕ_i 's (for $i = 0, 1, \dots, N$) are random numbers uniformly distributed between 0 and 2π .

The pdf of such an event is known to be given by Eq. (4) with $E_d = E_0$ and $E_m^2 = \sum_{i=1}^N E_i^2$, assuming $E_d^2 = K\Omega/(K+1)$ and $E_m^2 = \Omega/(K+1)$.

In ray-tracing, E_d is analogous to the LoS field (the direct field) and E_m to the field intensity associated with the power in all reflected fields.

The following explains how we can reduce the computational cost of estimating K and Ω by reducing M .

First the field strength, $|\mathbf{E}_t|$, is determined at M points evenly distributed within a local area of 10λ by 10λ , a mesh with the cell size of $\lambda/10$, and accounting for up to the R'_X th order of reflection. The reference cdf curve is obtained statistically using the M field values.

The MLE is applied to determine K and Ω and provide a close estimate of the reference curve.

Then M is reduced, and a smaller number of field values, $|\mathbf{E}_t|$, are used in the MLE method. The new K -factor and Ω and thus another estimate of the reference curve is obtained.

We continue to reduce M and obtain new approximations of the reference cdf until the accuracy becomes unacceptable that is the error value becomes larger than 0.01. Thus we have determined M_0 which is the smallest value of M for which the error value criterion is satisfied.

This process is again tested at different local areas to see whether M_0 varies by location.

2.2.1. Computational Time

The ray-tracing computational time is directly proportional to M . Hence, if the optimal M is found to be M_0 , the factor by which the computational time is reduced is

$$\frac{M}{M_0}. \quad (6)$$

One of the requirements that the maximum likelihood estimation of the Ricean parameters work well is that E_0 in Eq. (5), which is the direct field strength within a region, is constant. In regions with medium and large distances from the transmitter, the direct field is almost constant, and thus the Ricean distribution provides reliable estimates of the cdf. But in a region very close to the transmitter, the direct field varies considerably within the region, violating one of the assumptions of Eq. (5), and thus degrading the accuracy of the Ricean function in estimating the pdf of the field in this region. The localized maximum likelihood estimation method is presented here to accurately approximate the field strength distribution in near regions.

2.3. The Localized Maximum Likelihood Estimation

We divide the square region D into 9 smaller square regions. For every small region, D_i , where $i = 1, 2, \dots, 9$, we sample the electric field at M_0 evenly distributed points and thus obtain a one-dimensional array of real numbers AE_i . The length of each array is M_0 . The likelihood or probability of having an electric field strength $E_0 < E < E_0 + \Delta E$ in the i 'th region is given by

$$l_i(E_0) = \frac{n(AE_i, E_0, E_0 + \Delta E)}{M_0} \quad (7)$$

where $n(AE_i, E_0, E_0 + \Delta E)$ is the number of the field values between E_0 and $E_0 + \Delta E$ in the array AE_i . If ΔE is small enough, then the integral of Eq. (7) gives the cumulative distribution function of the electric field strength in the region D_i .

Then the MLE method is applied to the field values in AE_i and obtains the Ricean parameters, K_i and Ω_i . Using these parameters in Eq. (4), the Ricean function of the region D_i is obtained which is denoted by $p_i(E; K_i, \Omega_i)$.

The size of the small region is small enough to consider the direct field variations almost negligible. Hence p_i is a good approximation of l_i that is $l_i \approx p_i$.

To statistically obtain the cdf within the big region D, all AE_i 's are appended serially to make a larger AE_t which then has $9M_0$ field values. It is evident that the number of the field values between E_0 and $E_0 + \Delta E$ in the array AE_t is equal to

$$n(AE_t, E_0, E_0 + \Delta E) = \sum_{i=1}^9 n(AE_i, E_0, E_0 + \Delta E) \quad (8)$$

Then the likelihood or probability of having an electric field strength $E_0 < E < E_0 + \Delta E$ in the i 'th region is given by

$$l_t(E_0) = \frac{\sum_{i=1}^9 n(AE_i, E_0, E_0 + \Delta E)}{9M_0}. \quad (9)$$

Using Eq. (7), we have

$$l_t(E_0) = \frac{1}{9} \sum_{i=1}^9 \frac{n(AE_i, E_0, E_0 + \Delta E)}{M_0} = \frac{1}{9} \sum_{i=1}^9 l_i(E_0) \quad (10)$$

Considering $l_i \approx p_i$, we obtain

$$l_t(E) \approx \frac{1}{9} \sum_{i=1}^9 p_i(E; K_i, \Omega_i). \quad (11)$$

Hence the value of the probability density function at an electrical field intensity of E in the big region D is equal to the average of the values of the nine Ricean functions, at E , for all small regions. To obtain the cdf, Eq. (11) is integrated. We call this the localized maximum-likelihood estimation.

In the MLE and the localized MLE methods, we account for up to the R_X -th-order reflections. If the method of Section 2.1 is applied, the results may show that accounting up to a specific order of reflection is sufficient. In Section 2.1, the cdf is obtained statistically and thus requires a large number of field values. Also in the MLE and the new localized MLE methods, where lower number of observation points can be used, the used reflection order was R_X . However in the following, we present a new method which eliminates the need to compute the reflections of orders higher than one.

2.4. The Ray-Tracing-Maximum-Likelihood Estimation

In this section, we present a new concept called the *residual field* which enables us to estimate the field distribution with sufficient accuracy accounting *only* for the first order reflections. The idea is stemmed from an observation that, at a receiver point, the power in all the rays of a specific order both decreases and becomes rather constant over the room as the reflection order increases. The k' th order rays mean value is defined as

$$E_{rmv}^{(k)}(x, y, z) = \sqrt{\sum_{i=1}^{N_k} \left| \mathbf{E}_r^{(k)}(i, x, y, z) \right|^2} \quad (12)$$

and is plotted in Fig. 2 for the line $y = 2.5$ m which is along the x -axis. It is shown in Fig. 2 that the variations of $E_{rmv}^{(k)}$ from a point to another decrease as the reflection order increases. This suggests that, everywhere in the room, the power in the reflections of orders higher than a T can be approximated by a constant number which is determined only at a certain point, the center point, in the room. The way to determine T is demonstrated later in this section. This constant number is shown to be useful in improving the MLE estimate of K and Ω from the M_0 values of the electric field within a local area.

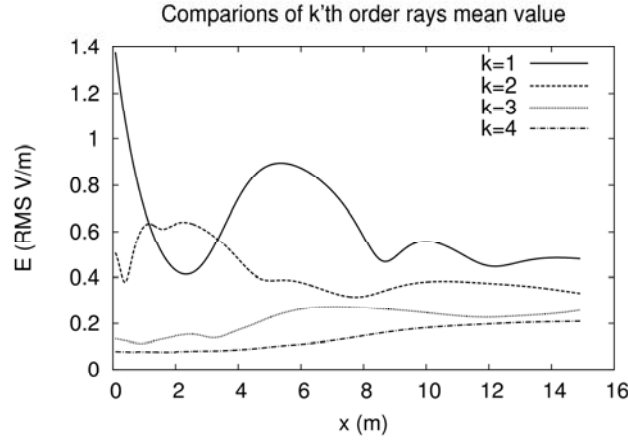


Figure 2. The k' th order rays mean value. The power in higher order reflections are smaller and less variant by location.

The RTML method requires only the T' th-order reflections in the M_0 values of the electric field, whereas in Eq. (4), $\Omega/(K+1)$, which is E_m^2 , represents the power in all reflected rays arriving at a point. But in the RTML method, instead of E_m , we define $E_M^{(T)}$ which is obtained by applying the simple maximum-likelihood estimation to the M_0 field values within the local area computed by Eq. (1) assuming $R = T$.

To account for an estimate of higher order reflections, we define E_{multi} which is given by

$$E_{\text{multi}} = \sqrt{E_M^{(1)} + E_{\text{res}}^2} \quad (13)$$

where E_{res} is the residual field defined by

$$E_{\text{res}} = \sqrt{\sum_{k=T+1}^{R_X} \sum_{i=1}^{N_k} \left| \mathbf{E}_r^{(k)}(i, x_c, y_c, z_c) \right|^2} = \sqrt{\sum_{k=T+1}^{R_X} \left| E_{rmv}^{(k)}(x, y, z) \right|^2} \quad (14)$$

in which (x_c, y_c, z_c) denotes the position of the center of the room. The residual field is computed only at the center point, hence it is a constant number.

Then E_d and E_{multi} are used to determine the two parameters of the Ricean function by

$$\begin{aligned} \Omega &= E_d^2 + E_{\text{multi}}^2, \\ K &= \frac{E_d^2}{E_{\text{multi}}^2}. \end{aligned} \quad (15)$$

How is the value of T defined? We applied the RTML method assuming $T = 1$, integrated the Ricean function with the parameters given by Eq. (15), which gave us an approximation of the reference cdf curve with an error value. We realized that the error value is acceptable. We increased the value of T to 2 and obtained another approximate cdf curve which had a slightly different shape but almost the same error value. We repeated the process and for $T = 3$ the error value did not improve considerably. Our intention is to reduce the computational cost of obtaining an approximate cdf while keeping the error value below 0.01. Therefore, $T = 1$ is the optimal choice. The results of the process of determining the optimal value of T are not presented.

2.4.1. Computational Time

The ray-tracing computational time is linearly dependent on the number of points, M , where $|\mathbf{E}_t|$ is determined. The computational time, however, grows exponentially with the highest order of reflections accounted for, R . The RTML method is devised to reduce both M and R . Hence the factor by which the computational time is reduced is in fact the product of the two factors in Eqs. (3) and (6) which is

$$\frac{M}{M_0} \times \frac{\sum_{k=1}^{R_X} N_w^k}{N_w}. \quad (16)$$

A question that arises is why, in the RTML method, we account only for the first order reflections everywhere in the room in determining $E_M^{(1)}$.

The accuracy of the RTML method is at best equal to the accuracy of the cdf curve obtained by integrating the Ricean function whose parameters are determined using M field strength values within a local area. Hence, as in Section 2.3, the accuracy of the RTML method may not be reliable in the near region.

2.5. The Localized Ray-Tracing-Maximum-Likelihood Method

The localized RTML method is presented to improve the accuracy of the RTML model in regions close to the transmitter. In the localized RTML method, the region is divided into nine smaller areas as described in Section 2.3. The field values are determined at M_0 points at each small region. However these field values are obtained by ray-tracing accounting only for the direct field and the first-order reflections. Using the maximum-likelihood method, the Ricean parameters, K_i and Ω_i , are found for each small region and thus nine Ricean functions, $p_i(E; K_i, \Omega_i)$, are formed.

In each region, K_i and Ω_i are converted to $E_d^{(i)}$ and $E_m^{(i)}$. Then the residual field, already computed by Eq. (14), is used to determine $E_{\text{multi}}^{(i)}$ by Eq. (13). The new Ricean parameters, in each of the nine regions, are determined by Eq. (15).

Then the field distribution in the large area is given by Eq. (11) where the Ricean parameters, in each small area, are determined as explained in the foregoing paragraph.

3. APPLICATION TO THE LONG ROOM

In this section, the methods explained in Section 2 are applied to the room shown in Fig. 1 which measures 15 m by 5 m. There are four walls, each consisting of five layers (1.5 centimeters of concrete, 0.8 cm of brick, 9.4 cm of space, 0.8 cm of brick and 1.5 cm of concrete). The angle-averaged reflection coefficients for these walls are 0.353 and 0.618 for parallel and perpendicular polarizations respectively. The floor and ceiling are two 30-cm thick concrete slabs separated by a 3-m vertical distance. The angle-averaged reflection coefficients are 0.325 and 0.594 the parallel and perpendicular polarizations respectively. Complex reflection coefficients, dependent on angle of incidence, are used in Eq. (1) to compute $\mathbf{E}_r^{(k)}$ and $|\mathbf{E}_t|$, and thus obtain the cdf of the field strength at the local areas, each measuring 120 cm by 120 cm that is 10λ by 10λ . The transmitter is a half-wave dipole located at (2 m, 1.5 m, 1.6 m), oriented vertically, and radiating 500 mW at 2.45 GHz. The cell size of approximately $\lambda/10$ results in $M = 10,000$.

In computing the electric field strength using Eq. (1), we observed that increasing R beyond 6 does not add meaning contributions to the point values of the field. Hence the maximum order of reflection is chosen to be $R_X = 6$.

3.1. The Optimal Highest Reflection Order is $R = 3$

How does the cdf computed using the 10,000 samples and the MLE method change as R is reduced from 6 to 1? Using the method explained in Section 2.1, the cdf curves are obtained using different values of $R = 1, 2, \dots, 6$. Fig. 3 shows these cdfs for region A. When only the first order reflections are accounted for, that is $R = 1$, the cdf curve shows a conspicuous difference from the reference curve which accounts for reflections of up to the 6th order, $R = 6$. Curves with values of R higher than unity are closer to each other and the visual comparison is cumbersome. To compare the cdfs, the error value is obtained as explained in Appendix A. The error values, for $R = 1, 2, \dots, 5$, are presented in Table 1.

Table 1. The error values of the cdfs in regions A and D compared to their reference curves. The reference curve is the statistically obtained cdf curve using 10,000 field strengths determined by ray-tracing accounting for up to the 6th order of reflection. The same cdf is then approximated by reducing the maximum order of reflection. This cdf is compared to the reference cdf and the error value is obtained using the method explained in Appendix A.

	Error in Regions:	
R	A	D
5	0.0030	0.0007
4	0.0063	0.0005
3	0.0115	0.0009
2	0.0181	0.0013
1	0.0486	0.0025
LoS Only	0.1832	0.0155

It is seen in Table 1 that when $R = 5, 4$, or 3 , the error value, in region A, is almost acceptable, but when $R \leq 2$, the error criterion of being less than almost 0.01 is not satisfied. Therefore in region A, it is sufficient to account for up to only the third-order reflections.

Similar observation is made for regions B and C, and the optimal value of R is seen to be 3. Due to the similarity to the previous case, the results are not shown. However, it is interesting to observe that, in region D, even the direct field values (without accounting for any reflections) can provide an unexpectedly close estimate of the reference curve. The cdfs are shown in Fig. 4. The curve accounting for the first order reflections is a good estimate and the error value for the $R = 1$ curve is, as shown in Table 1, is 0.0025 which is well below the criterion of 0.01. Hence when we are close to the antenna, the residual field and accounting for only the first order reflections are sufficient to get an accurate estimate of the field strength statistics.

If R is chosen to be 1, the ray-tracing computational time, compared to the reference cdf, is reduced by a factor of 66430 which is computed by Eq. (3).

Also using the Ricean function to estimate the cdf together with an optimal sample size can reduce the computational time compared to the reference curve.

3.2. The Optimal Number of Sample Points Is $M = 100$

What is the limit in reducing the sampling size so that the maximum likelihood estimation of the Ricean parameters remains valid? Here we apply the method described in Section 2.2. In region A, the field strengths are obtained at 10,000 points using ray-tracing and accounting for up to the 6th order of reflection. Then M evenly distributed points are chosen. Application of the MLE method to these M values yield the Ricean parameters. The cdf is obtained and compared to the reference cdf. Fig. 5 shows

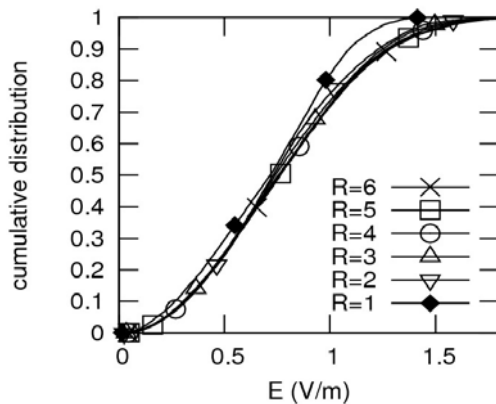


Figure 3. The cdf curves for the electric field intensity are obtained statistically using ray-tracing results at 10,000 points within region A. The highest order of reflection, R , is different in each curve.

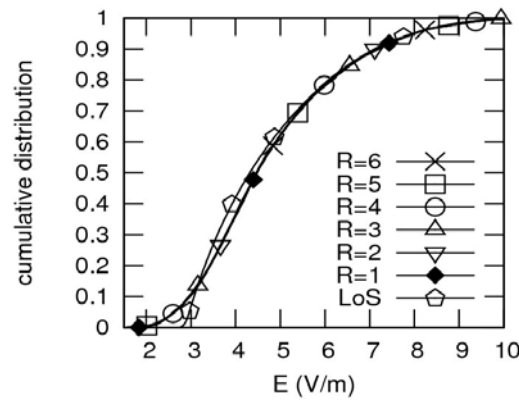


Figure 4. The cdf curves for the electric field intensity are obtained statistically using ray-tracing results at 10,000 points within region D. The highest order of reflection, R , is different in each curve. A cdf accounting for only the direct field, LoS, is also presented.

this comparison while three different values of M (100, 64 and 36) are considered. Fig. 5 shows that the MLE curve with the sample size of 100 follows the reference curve very closely. When the sample size is reduced to 64 or 36, the difference between the MLE curve and the reference curve is increased. The differences for these and more values of M are interpreted into the error value, and are summarized in Table 2.

Table 2. Arithmetic mean of absolute values of errors in regions A, B, C, and D for difference values of N in the MLE method.

	Arithmetic Mean of Error in Regions:			
N	A	B	C	D
10,000	0.0029	0.0085	0.0122	0.027
400	0.0108	0.0092	0.0128	0.0320
100	0.0162	0.0158	0.0146	0.0369
64	0.0112	0.0214	0.0151	0.0365
36	0.0284	0.0395	0.0210	0.0438

The error value for different values of M in regions B, C, and D are also presented in Table 2. In regions A, B, and C, the sample size of 400 is required if the error mean must be 1 percent or less. However, the error for the sample size of 100 is almost 1.6 percent or less. Accepting a 1.6 percent error value does not degrade the accuracy compared to an error value of 1 percent. But performing ray-tracing at 100 points is 4 times faster than at 400 points, so $M = 100$ is chosen as the optimal sample size.

Another criterion for choosing an optimal sample size is that the error values do not change much when the observation location varies that is when the local area is slightly shifted to the right or left. Our results, not shown here, indicate that reducing the sample size below 100 gives rise to a considerable sensitivity of the error value to the exact location of the local area, and hence 100 is a suitable choice.

Computing the field strengths at only 100 points reduces the ray-tracing computational time, compared to the reference cdf by a factor of 100 computed by Eq. (6).

The comparison of the cdf curves in the region D is presented in Fig. 6. It is seen that the MLE method is not as accurate as it was in region A. From Table 2, it is seen that even a sample size of

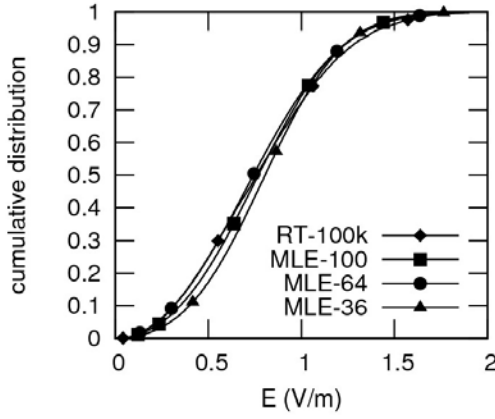


Figure 5. The cdf curves for the electric field intensity are obtained for region A using the Ricean parameters obtained by the MLE method. For each curves different sample size is used ($M = 100, 64, 36$). These curves are compared to the cdf curve obtained statistically using the full ray-tracing solution at 10,000 points ('MLE-10k').

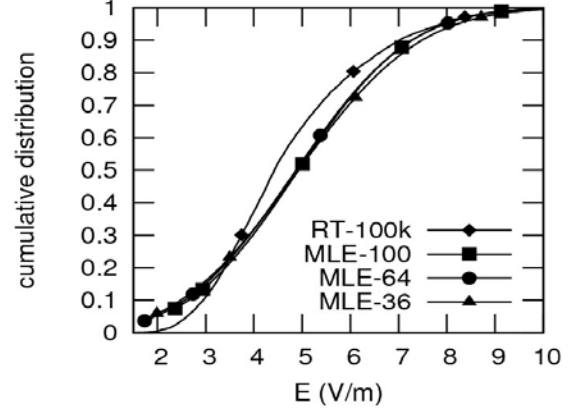


Figure 6. The cdf curves for the electric field intensity are obtained for region D using the Ricean parameters obtained by the MLE method. For each curves different sample size is used ($M = 100, 64, 36$). These curves are compared to the cdf curve obtained statistically using the full ray-tracing solution at 10,000 points ('MLE-10k').

10,000 is not sufficient. The reason is the considerable variations of the direct field within the region D. To improve the accuracy of the MLE method in region D, the new localized maximum likelihood estimation is applied.

3.3. The Localized Maximum Likelihood Estimation

Region D is divided into 9 smaller regions as shown in Fig. 7. The electric field is sampled using 100 points within each small region. The MLE method in each small region obtains K_i and Ω_i . The pdf is obtained using Eq. (11), and the cdf curve is presented in Fig. 8. The cdf curve for this region obtained by the MLE method using 100 points is also presented in Fig. 8 and compared to the reference curve.

D7	D8	D9
D4	D5	D6
D1	D2	D3

Figure 7. The region D is divided into 9 smaller regions in order to apply the new localized maximum-likelihood estimation. The direct field strength variations within each small region does not affect the accuracy of the MLE method within that small region.

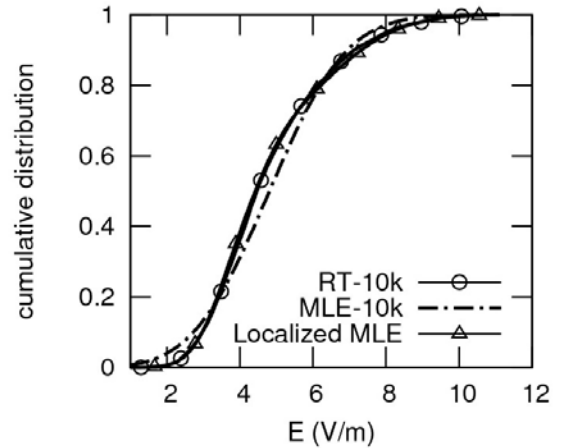


Figure 8. The new "localized MLE" method estimates the cdf of the field strength distribution in a near region.

This is the same comparison presented in Fig. 6. The agreement is poor with an error value of 0.027. For the localized maximum likelihood estimation, the error value is 0.007.

Hence the localized MLE method results in a smaller error value and is useful in regions close to the antenna.

3.4. The Ray-Tracing-Maximum-Likelihood Estimation

The method described in Section 2.4 is applied. A residual field is computed only at the center of the room using Eq. (14). The field strengths at 100 points within the region A are obtained using ray-tracing and accounting only up to the first-order reflections. The MLE method is used to estimate the Ricean parameters. The residual field is used to improve the Ricean parameters by Eq. (15). The cdf of this Ricean function, marked as 'RTML', is compared, in Fig. 9, to the reference cdf. The error value as shown in Table 3 is 0.005 and acceptable. Hence the L-RTML method is useful in region not very close the antenna.

The factor by which the ray-tracing computational time is reduced using the RTML method is 6643000 which is computed by Eq. (16).

In Table 3, it is seen that the error value of the cdf curve obtained using the RTML method is 0.0303 in the near region, and hence the method does not provide the required accuracy in this region. This shortcoming is overcome using the localized RTML method.

Table 3. Error values of the RTML, L-MLE and L-RTML methods in regions A and D.

R	Error in Regions:	
	A	D
RTML	0.005	0.0303
Localized MLE	-	0.0068
Localized RTML	-	0.0078

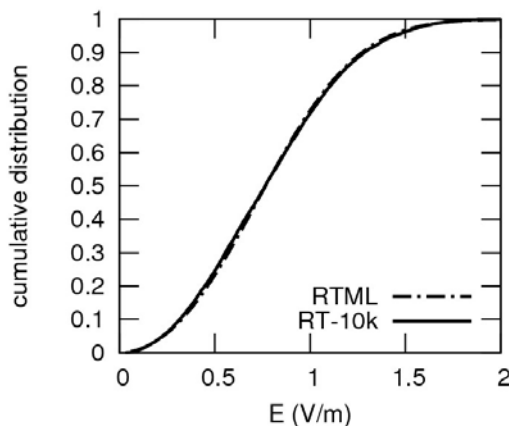


Figure 9. The MLE method in region A.

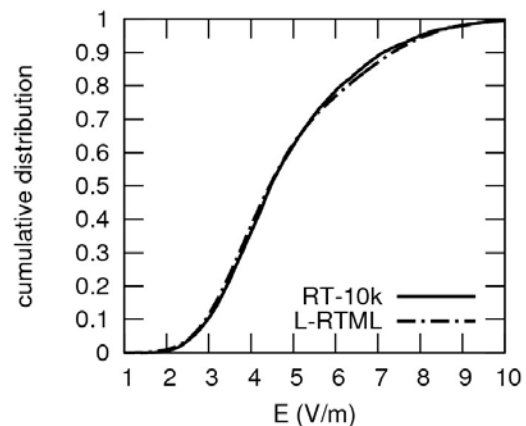


Figure 10. The L-RTML method in region D.

3.4.1. The Localized RTML Method

The accuracy of the ray tracing-maximum-likelihood method is, at best, equal to the accuracy of the maximum-likelihood estimation. It is seen in Fig. 6 that the MLE method does not provide the desired accuracy in the region D due to the variations of the direct field. Hence the RTML method, too, will be inaccurate and thus is not applied in this region.

Instead, the localized ray-tracing-maximum-likelihood, presented in Section 3.5, is applied. We obtained the field values accounting for only the first-order reflections. In each of the nine small regions, K_i and Ω_i are determined using 100 field values. These parameters are then improved using the residual field which is computed for the center point of the room. The obtained cdf curve is compared to the reference curve in Fig. 10. The error value as seen in Table 3 is 0.008.

Hence the L-RTML method is applicable in near regions.

In the case of propagation between the rooms, the direct field will not be as strong as in region D of Fig. 1. Depending on the position of the antenna and where in the adjacent room the field is estimated, the distribution may resemble any of those in regions A, B, or C.

4. CONCLUSION

This paper shows that the Ricean function is useful in estimating the probability distribution of the field strength. The Ricean parameters are found using the maximum-likelihood method. The new localized maximum-likelihood, new ray-tracing-maximum-likelihood, and new localized ray-tracing-maximum-likelihood methods are presented in this paper. They immensely reduce the ray-tracing computational time in estimating the cdf of the field distribution.

The maximum likelihood method was used to estimate the Ricean parameters to approximate the empirical cdf of field distribution. It was shown that the accuracy of the Ricean function is preserved even if the size of the sample set and hence the computational time of ray-tracing are decreased by a factor 100 compared to a dense grid with a cell size of $\lambda/10$ by $\lambda/10$.

It was demonstrated that in regions with far and medium distances to the transmitter, the Ricean function satisfied the desired accuracy where the error was below 1%. However, in the close region, due to the large variations of the direct field strength, the Ricean function is not as accurate.

To overcome this inaccuracy, the new localized maximum-likelihood estimation was devised and presented. In the new method, the close region is divided into nine smaller regions where the MLE method is applied separately to determine nine different Ricean functions. It is shown in this paper that the integral of average of these Ricean functions accurately estimate the field distribution in the larger region.

The ray-tracing-maximum-likelihood method was developed and introduced to show that in regions with far and medium distances to the antenna, the maximum reflection order can be reduced to one. The desired accuracy was obtained using a new parameter called the residual field, a constant number for the room. The RTML method reduced the ray-tracing computational time of by a factor of 6643000.

The RTML method expectedly fails in close regions due to the variations of the direct field strength. The new localized ray-tracing-maximum likelihood is presented to restore the accuracy which is demonstrated to be useful.

Hence it is strongly recommended that the new ray-tracing-maximum likelihood method be used to estimate the field distribution in local areas with medium and large distances from the antenna. In closer regions, the new localized ray-tracing maximum-likelihood method is suggested.

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APPENDIX A.

The *error value* is defined to compare two cdf curves, the reference curve, $\text{cdf}_{\text{reference}}$, and its estimate, $\text{cdf}_{\text{estimate}}$. The domain of a cdf function in this paper is an interval two positive real numbers which indicate the minimum and maximum electric field strengths, respectively u_s and u_f , within the region of interest. The range of the function is ideally between zero and one.

Numerically, the domain of the reference cumulative distribution function is a partition, $u_s, u_1, u_2, \dots, u_P, u_f$, of the interval (u_s, u_f) . The domain of the estimate curve is the partition $v_s, v_1, v_2, \dots, v_Q, v_f$.

To compare the two curves, a common refinement of the two partitions is obtained as follows. In cdf comparisons, it is the rising section of the curve which is of interest. For example, a cdf is almost equal to unity for all real numbers greater than a specific value. This part of the function does not contain much information and is not included in our comparison process.

First, the curves are truncated from left and right. The part of the domain for which the value of the function is very close to zero is trimmed. The part of the domain for which the value of the function is very close to unity is truncated as well. Let $u_{\min}$ be the greatest u_i such that $\text{cdf}_{\text{reference}}(u_i)$ is less than 0.001. Let $u_{\max}$ be the least u_i such that $\text{cdf}_{\text{estimate}}(u_i)$ is greater than 0.999. Similarly, $v_{\min}$ and $v_{\max}$ are obtained for the estimate cdf curve. Let $e_{\min}$ be the minimum of $u_{\min}$ and $v_{\min}$. Let $e_{\max}$ be the minimum of $u_{\max}$ and $v_{\max}$. The partition $e_{\min}, e_1, e_2, \dots, e_{200}, e_{\max}$ is obtained such that $e_i - e_{i-1}$ does not vary with i . The values of $\text{cdf}_{\text{reference}}$ and $\text{cdf}_{\text{estimate}}$ are obtained over the e partition using interpolation regarding the u and v partitions, respectively.

The error array, $|\text{cdf}_{\text{reference}}(e_i) - \text{cdf}_{\text{estimate}}(e_i)|$, is determined for all points over the partition e . The maximum value and arithmetic mean of the 202 elements in the error array are obtained using a means of comparing the reference and the estimate cdf curves.

We have observed in numerous comparisons that the two cdf curves appear visually identical if the arithmetic mean of the error array, the *error value*, is less than 0.01.

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