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DOA Estimation of Mixture Signals Based on the PSA

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Abstract—The problem of direction of arrival (DOA) estimation based on a polarization sensitive array (PSA) is considered in this paper. In the environment of the mixture signal, a novel DOA estimation for both the independent signals and coherent signals is proposed. The process of estimation is divided into two steps. First, the root-multiple signal classification algorithm is employed to estimate the DOAs of the independent signals. Then, the data covariance matrix which only contains the information of the coherent signals is estimated with improved vector reconstruction technique. Theoretical analysis and simulation results show that the proposed method can expand the array aperture and has small computation load as well as excellent estimation performance.

1. INTRODUCTION

Polarization-Sensitive Arrays (PSA) have become a new research hotspot in the field of array signal processing since the 1990s [1–7]. Polarization-sensitive array is a vector array in nature. Different from scalar array, the polarization selection characteristic of its internal components is to observe the wave field in a vector way, so that the microstructure information can be extracted. In this way, the polarization sensitive array can not only obtain polarization information, but also obtain spatial information by using the spatial distribution of array elements. Therefore, the polarization sensitive signal processing includes both polarization domain and spatial domain processing. In recent years, quaternion theory has been widely used in the field of signal processing, especially in polarization sensitive array signal model. Compared with the long vector model, the quaternion model has stronger orthogonality constraints and can reduce the computational complexity. Quaternion is a hypercomplex number that can fully mine and utilize the space, time, and polarization information of electromagnetic vector sensor, so the quaternion model with information in different positions can also be divided into quaternion [8] and double quaternion [9]. According to the physical characteristics of electromagnetic signal, the polarization information of electromagnetic signal includes three-dimensional electric field component and three-dimensional magnetic field component, and there is an orthogonal relationship among electric field component, magnetic field component, and Boynting vector. Therefore, in terms of physical nature, the mathematical properties of the quaternion model are more consistent with the inherent properties of electromagnetic signals. In addition, the data receiving model of polarized array can be converted into a quaternion form to efficiently complete the array signal processing, which has advantages that traditional complex modeling does not have. In the case of the same array element number, the number of antennas of the vector antenna array can be twice or more than that of the scalar array, so the data received will also be more. Therefore, combining with the characteristics of quaternion, how to greatly reduce the computational complexity with high-precision parameter estimation will become the focus of this paper.

In 1843, the Irish mathematician Hamilton extended a complex number to the four-dimensional space, and finally created a quaternion, but the real application of quaternion to the field of signal

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processing was proposed by Schutte and Wenzel in 1990 [11]. As mentioned above, an important reason to limit the polarization sensitive array is that the orthogonal polarization of the array elements increases the complexity of the operation, resulting in a large increase in the amount of operation. This is because the polarization sensitive array has increased the polarization amplitude angle and polarization phase angle besides the unknown parameters of pitch angle and azimuth angle in the traditional scalar array. The application of four parameters to the quaternion model can significantly reduce the amount of computation, in which the classic algorithm is quaternion MUSIC(Q-MUSIC) [10] proposed by Miron et al. in 2006. Although this method can greatly reduce the computational complexity, the whole process is carried out in the quaternion domain. Therefore, the effective information carried by the four positions is not equivalent, leading to a decrease in accuracy. Moreover, Q-MUSIC algorithm estimates unknown parameters through spectral peak search, which requires a lot of calculation, and the accuracy is affected by the search step size. The PAS algorithm proposed in [11] makes use of the polarization angle rotation invariant subarray of the electromagnetic vector sensor for decocoherence, and can be further extended to DOA estimation of mixture signals. On the basis of PAS, paper [12] proposed an improved algorithm based on spatial difference algorithm, called improved polarization array smooth (IPAS) algorithm, which is specialized in DOA estimation for mixture signals with coherent and incoherent signals coexisting. However, the computational complexity of these two algorithms is still very high. Till now, the estimation problem of mixture signals based on quaternion model has not been reported. Combined with quaternion model, the computational complexity of DOA estimation of mixture signals can be greatly reduced. Therefore, the research in this paper has practical significance.

The root MUSIC algorithm based on quaternion has high precision and can avoid searching for spectral peak. The algorithm is divided into three steps. In the first step, the semi-quaternion model is used to remove the redundant information of the received signal, and the dimension of the covariance matrix is reduced, so as to reduce the computation amount caused by the subsequent eigenvalue decomposition. In the second step, the root MUSIC algorithm is used to directly calculate the azimuth angle and pitch angle to avoid searching the spectral peak. The third step is to directly calculate the polarization amplitude and phase angle according to the principle of rank defect MUSIC method. In the conventional MUSIC algorithm, the computational complexity mainly comes from the decomposition of covariance and the search of spectral peak. However, the algorithm proposed in this section completely avoids the search of spectral peak while reducing the dimension of covariance matrix, and effectively reduces the calculation amount of DOA and polarization joint estimation. In practical engineering, due to the influence of various interferences and multipath effects, there will be a large number of coherent signals [13, 14], and even independent signals and coherent signals exist simultaneously. In this paper, we introduce a DOA estimation method based on quaternion theory, in which independent signals and coherent signals coexist. First of all, the method inherits the characteristics of low computational complexity of quaternion theory, and the amount of computation is small. Secondly, after constructing the quaternion model, the DOA estimation of independent signals and coherent signals is performed by using the spatial difference principle. The proposed algorithm can realize DOA estimation of mixture signals without loss of array aperture.

2. MATHEMATICAL MODEL

Consider completely polarized transverse electric and magnetic (TEM)[†] waves impinging on the polarization linear array (PLA), with M sensors. Assume that K signals impinge on an uniform linear array as shown Fig. 1. These K signals are composed of K_u independent signals and K_c coherent signals, and the K_c coherent signals are produced by N independent sources. Assuming that the nth independent sources produce K_n signals, we have

$$K = K_u + K_c \tag{1}$$

$$K_c = \sum_{n=1}^{N} K_n \tag{2}$$

[†] It is indicated that the electric and magnetic fields of the electromagnetic wave are in the plane perpendicular to the propagation direction.

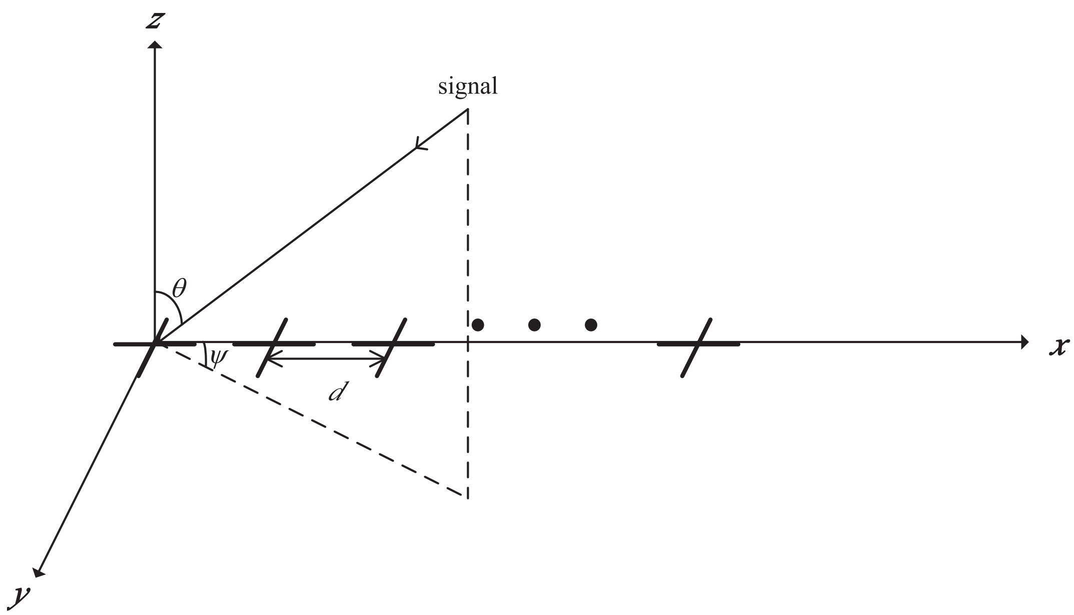

Figure 1. Uniform linear array configuration of crossed dipoles.

Each sensor can receive two electric field components ($x_m^{[1]}(t)$ and $x_m^{[2]}(t)$). Here we assume azimuth angle $\varphi = 0°$, then we can get the output of the mth array element at time t as follows

$$x_m^{[1]}(t) = -\sum_{k=1}^{K} s_k(t) e_{1,k} \mathrm{e}^{-\mathrm{j}2\pi(m-1)d\sin\theta_k/\lambda} + n_m^{[1]}(t) \tag{3}$$

$$x_m^{[2]}(t) = -\sum_{k=1}^{K} s_k(t) e_{2,k} \mathrm{e}^{\mathrm{j}\eta_k} \mathrm{e}^{-\mathrm{j}2\pi(m-1)d\sin\theta_k/\lambda} + n_m^{[2]}(t) \tag{4}$$

where $m = 1, 2, \ldots, M$ and $k = 1, 2, \ldots, K$. $n_m^{[1]}(t)$ and $n_m^{[2]}(t)$ are the corresponding noise vectors of antennas parallel to x-axis and y-axis, respectively. λ denotes the wavelength of signal. e_1 and e_2 represent two electric field components. From [3], we can know

$$\begin{bmatrix} e_{1,k} \\ e_{2,k} \end{bmatrix} = \begin{bmatrix} -\sin\varphi_k & \cos\theta_k\cos\varphi_k \\ \cos\varphi_k & \cos\theta_k\sin\varphi_k \end{bmatrix} \begin{bmatrix} \cos\gamma_k \\ \sin\gamma_k e^{j\eta_k} \end{bmatrix} \tag{5}$$

where $0 \le \gamma_k < \pi/2$ is the auxiliary polarization angle, and $-\pi \le \eta_k < \pi$ is the polarization phase difference. Then Equations (3) and (4) can be further rewritten as

$$\mathbf{X}(t) = \begin{bmatrix} \mathbf{X}_1(t) \\ \mathbf{X}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{AP}_1\mathbf{S}(t) + \mathbf{N}_1(t) \\ \mathbf{AP}_2\mathbf{S}(t) + \mathbf{N}_2(t) \end{bmatrix} \tag{6}$$

where $\mathbf{S(t)}$ and $\mathbf{N(t)}$ are the source and noise vectors, respectively. $\mathbf{A}$ is the $M \times K$ steering matrix, and $\mathbf{P_1}$, $\mathbf{P_2}$ are given by

$$\begin{aligned} \mathbf{P}_1 &= \begin{bmatrix} e_{1,1} & & \\ & \ddots & \\ & & e_{1,K} \end{bmatrix} \\ \mathbf{P}_2 &= \begin{bmatrix} e_{2,1} & & \\ & \ddots & \\ & & e_{2,K} \end{bmatrix} \end{aligned} \tag{7}$$

Equation (6) can be rewritten as

$$\begin{bmatrix} \mathbf{AP}_1\mathbf{S}(t)+\mathbf{N}_1(t) \\ \mathbf{AP}_2\mathbf{S}(t)+\mathbf{N}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_u\mathbf{P}_{u1}\mathbf{S}_u(t)+\mathbf{A}_c\mathbf{P}_{c1}\Gamma\mathbf{S}_c(t)+\mathbf{N}_1(t) \\ \mathbf{A}_u\mathbf{P}_{u2}\mathbf{S}_u(t)+\mathbf{A}_c\mathbf{P}_{c2}\Gamma\mathbf{S}_c(t)+\mathbf{N}_2(t) \end{bmatrix} \tag{8}$$

where $\mathbf{A_u}$ is the $M \times K_u$ steering matrix of independent signals, and $\mathbf{A_c}$ is the $M \times K_c$ steering matrix of coherent signals. $\mathbf{S_u}$ is the signal vector of independent signals, and $\mathbf{S_c}$ is the signal vector of coherent signals. Γ is a block diagonal matrix with $\{\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_N\}$ on its diagonal, and $\mathbf{b}_i$ is a $K_n \times 1$ vector composed of the fading factor of the ith group of signals.

The quaternion model can be written as

$$\mathbf{X}_q = \mathbf{X}_1 + \mathbf{X}_2 \cdot \mathrm{j} \tag{9}$$

Then we can get the covariance of $\mathbf{X}_q$ as follows

$$\begin{aligned} \mathbf{R} &= E\left\{\mathbf{X}_q\mathbf{X}_q^H\right\} = E\left\{(\mathbf{X}_1+\mathbf{X}_2\cdot \mathrm{j})\left(\mathbf{X}_1^H - \mathrm{j}\cdot\mathbf{X}_2^{\mathrm{H}}\right)\right\} \\ &= E\left\{\mathbf{X}_1\mathbf{X}_1^H + \mathbf{X}_2\mathbf{X}_2^H\right\} + \Big(\mathbf{AP}_2E\left\{\mathbf{SS}^T\right\}\mathbf{P}_1^T\mathbf{A}^T + \mathbf{AP}_2E\left\{\mathbf{SN}_1^T\right\} \\ &\quad +E\left\{\mathbf{N}_2\mathbf{S}^T\right\}\mathbf{P}_1^T\mathbf{A}^T + E\left\{\mathbf{N}_2\mathbf{N}_1^T\right\} - \mathbf{AP}_1E\left\{\mathbf{SS}^T\right\}\mathbf{P}_2^T\mathbf{A}^T \\ &\quad -\mathbf{AP}_1E\left\{\mathbf{SN}_2^T\right\} - E\left\{\mathbf{N}_1\mathbf{S}^T\right\}\mathbf{P}_2^T\mathbf{A}^T - E\left\{\mathbf{N}_1\mathbf{N}_2^T\right\}\Big)\cdot \mathrm{j} \end{aligned} \tag{10}$$

where $(\cdot)^H$ denotes the conjugate transpose matrix and $(\cdot)^T$ the transposed matrix. Quaternion has the following characters.

$$\begin{aligned} h\cdot \mathrm{j} &= \mathrm{j}\cdot \mathrm{h}^* \\ q^*q &= qq^* \\ q_1q_2 &\neq q_2q_1 \\ (q_1q_2)^* &= q_2^*q_1^* \end{aligned} \tag{11}$$

where $h \in \mathbf{C}$ and $q_1, q_2 \in \mathbf{Q}$ ($\mathbf{Q}$ is the quaternion field). The basic assumptions utilized throughout this paper are listed as follows.

(1). The K incoherent arriving signals $\mathbf{S}(t)$ are narrow band and circular signals, which means $E\{\mathbf{SS}^T\} = 0$.

(2). The entries of $\mathbf{N}(t)$ are white Gaussian noise and uncorrelated with each other. Noises from different sensors are independent, which means $E\{\mathbf{N}_1\mathbf{N}_2^T\} = E\{\mathbf{SN}_1^T\} = E\{\mathbf{N}_2\mathbf{S}^T\} = E\{\mathbf{N}_1\mathbf{N}_2^T\} = 0$.

Hence, Equation (10) can be written as

$$\mathbf{R} = \mathbf{R}_1 + \mathbf{R}_2 \tag{12}$$

where $\mathbf{R}_1 = E\{\mathbf{X}_1\mathbf{X}_1^H\}$ and $\mathbf{R}_2 = E\{\mathbf{X}_2\mathbf{X}_2^H\}$.

3. DOA ESTIMATION OF INDEPENDENT SIGNALS

The covariance matrix of independent signals is a non-singular matrix, and the the covariance matrix of coherent signals is a dissatisfied rank matrix. Note that the rank of $\mathbf{R}$ is $K_u + N$, then the covariance matrix $\mathbf{R}$ can be further written as follows

$$\mathbf{R} = \mathbf{U}_s\mathbf{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_N\mathbf{\Lambda}_N\mathbf{U}_N^H \tag{13}$$

where $\mathbf{\Lambda}_s$ is the diagonal matrix which is composed of $K_u + N$ larger eigenvalues, and $\mathbf{U}_s$ is the eigenvectors of larger eigenvalues. $\mathbf{\Lambda}_n$ is the diagonal matrix which is composed of $M - K_u - N$ smaller eigenvalues, and $\mathbf{U}_n$ is the eigenvectors of smaller eigenvalues.

Invoking the MUSIC algorithm, we know

$$\hat{\theta} = \arg\min_{\theta} \mathbf{a}^H(\theta)\mathbf{U}_N\mathbf{U}_N^H\mathbf{a}(\theta) \tag{14}$$

where $a(\theta)$ is the steering vector

$$\mathbf{a}(\theta) = \left[1, e^{j2\pi d\sin\theta/\lambda}, \dots, e^{j2\pi Md\sin\theta/\lambda}\right]^T \tag{15}$$

Substituting Equation (15) in Equation (14), we have

$$\begin{aligned}\mathbf{a}^H(\theta)\mathbf{U}_N\mathbf{U}_N^H\mathbf{a}(\theta) &= \sum_{m=0}^{M-1}\sum_{n=0}^{M-1} e^{2\pi md\sin\theta/\lambda}\mathbf{R}_N(m,n)e^{-j2\pi nd\sin\theta/\lambda} \\ &= \sum_{l=-(M-1)}^{M-1} e^{j2\pi ld\sin\theta/\lambda}\cdot \mathbf{r}_l \end{aligned} \tag{16}$$

where $\mathbf{R}_N = \mathbf{U}_N\mathbf{U}_N^H$ is a $M \times M$ matrix, and $\mathbf{r}_l$ is given by

$$\mathbf{r}_l = \sum_{m-n=l} \mathbf{R}_N(m,n) \tag{17}$$

Let $z = e^{j2\pi d\sin\theta/\lambda}$, then Equation (16) can be written as

$$\mathbf{D}(z) = \sum_{m-n=l} \mathbf{r}_l z^l \tag{18}$$

Thus the estimation of Equation (14) is changed into the solution of $\mathbf{D}(z) = 0$. Notice that the steering matrix $\mathbf{A_u}$ which is composed of K_u eigenvectors is Vandermonde matrix. As $z = e^{j2\pi d\sin\theta/\lambda}$, K_u roots are supposed to lie on the unit circle. Then, the estimation of elevation angle is given by

$$\hat{\theta} = \arcsin\left(\frac{\lambda}{2\pi d}\arg(z)\right) \tag{19}$$

4. DOA ESTIMATION OF COHERENT SIGNALS

The covariance matrix can be written as

$$\begin{aligned}\mathbf{A}\mathbf{R}_S\mathbf{A}^H &= \mathbf{A}_u\mathbf{R}_{Su}\mathbf{A}_u^H + \mathbf{A}_c\mathbf{\Gamma}\mathbf{R}_{Sc}\mathbf{\Gamma}^H\mathbf{A}_c^H \\ &= \mathbf{R}_u + \mathbf{R}_c\end{aligned} \tag{20}$$

where R_u is the covariance matrix of independent signals, and R_c is the covariance matrix of coherent signals. The **Toeplitz** matrix satisfies the formula:

$$\mathbf{R} = \mathbf{J}\mathbf{R}^T\mathbf{J} \tag{21}$$

where $\mathbf{R}$ is an $M \times M$ matrix; $\mathbf{J}$ is the anti-angular identity matrix; and then $\mathbf{R}$ is called **Toeplitz** matrix. Since $\mathbf{R}_u$ and $\mathbf{R}_{Su}$ in Equation (21) are real diagonal matrices, then:

$$\begin{aligned}\mathbf{J}\mathbf{R}_u^{\mathrm{T}}\mathbf{J} &= \mathbf{J}\mathbf{A}_u^*\mathbf{R}_{Su}\mathbf{A}_u^{\mathrm{T}}\mathbf{J} \\ &= \mathbf{A}_u\mathbf{\Phi}_u^{-(M-1)}\mathbf{R}_{Su}\mathbf{\Phi}_u^{-(M-1)}\mathbf{A}_u^{\mathrm{H}} \\ &= \mathbf{A}_u\mathbf{R}_{Su}\mathbf{A}_u^{\mathrm{H}} \\ &= \mathbf{R}_u\end{aligned} \tag{22}$$

where $\mathbf{\Phi}_u = \mathrm{diag}[e^{-j\pi\sin\theta_1}, \ldots, e^{-j\pi\sin\theta_{Ku}}]$. It satisfies the definition of the **Toeplitz** matrix, so $\mathbf{R}_u$ is the **Toeplitz** matrix. Similarly, $\mathbf{I}_M$ also satisfies the definition of **Toeplitz** matrix, so the same transformation is performed for the received data covariance matrix $\mathbf{R}$:

$$\begin{aligned}\mathbf{J}\mathbf{R}^{\mathrm{T}}\mathbf{J} &= \mathbf{J}\mathbf{R}_u^{\mathrm{T}}\mathbf{J} + \mathbf{J}\mathbf{R}_c^{\mathrm{T}}\mathbf{J} + \sigma_N^2\mathbf{J}\mathbf{I}_M\mathbf{J} \\ &= \mathbf{R}_u + \mathbf{J}\mathbf{R}_c^{\mathrm{T}}\mathbf{J} + \sigma_N^2\mathbf{I}_M\end{aligned} \tag{23}$$

where $\sigma_N^2 = \sigma_1^2 + \sigma_2^2$. According to Equations (20) and (23), the data matrix without independent signal information and noise information can be obtained:

$$\begin{aligned}\mathbf{R}_{c1} &= \mathbf{R} - \mathbf{J}\mathbf{R}^{\mathrm{T}}\mathbf{J} \\ &= \mathbf{R}_u + \mathbf{R}_c + \sigma_N^2\mathbf{I}_M - \mathbf{J}\mathbf{R}_u^{\mathrm{T}}\mathbf{J} - \mathbf{J}\mathbf{R}_c^{\mathrm{T}}\mathbf{J} - \sigma_N^2\mathbf{J}\mathbf{I}_M\mathbf{J} \\ &= \mathbf{R}_c - \mathbf{J}\mathbf{R}_c^{\mathrm{T}}\mathbf{J}\end{aligned} \tag{24}$$

4.1. DOA Estimation of Coherent Signals

The data covariance matrix $\mathbf{R}_{c1}$ obtained after matrix transformation only contains the information of K_c coherent signals, which are generated by N independent signal sources, that is to say, $K_c = \Sigma_{n=1}^{N} K_n$. At this point, perform eigenvalue decomposition on $\mathbf{R}_{c1}$:

$$\mathbf{R}_{c1} = \mathbf{U}_{S1}\mathbf{\Lambda}_{S1}\mathbf{U}_{S1}^{\mathrm{H}} + \mathbf{U}_{N1}\mathbf{\Lambda}_{N1}\mathbf{U}_{N1}^{\mathrm{H}} \tag{25}$$

where $\mathbf{\Lambda}_{S1}$ is a diagonal matrix composed of large eigenvalues, and $\mathbf{U}_{S1}$ is the corresponding eigenvector matrix. $\mathbf{\Lambda}_{N1}$ is the diagonal matrix composed of small eigenvalues, and $\mathbf{U}_{N1}$ is the corresponding eigenvector matrix. When $M \geq K_c$, $\mathbf{\Lambda}_{S1}$ contains N pairs of large eigenvalues, and each pair of eigenvalues is negative to each other. At this time, there is a $K_c \times 2N$ dimensional column full-rank matrix $\mathbf{T}$ that satisfies:

$$\mathbf{U}_{S1} = \mathbf{A}_c\mathbf{T} \tag{26}$$

In order to obtain the signal subspace, a new data covariance matrix can be constructed:

$$\begin{aligned}\mathbf{R}_{c2} &= \mathbf{U}_{S1}\mathbf{U}_{S1}^{\mathrm{H}} = \mathbf{A}_c\tilde{\mathbf{T}}\mathbf{A}_c^{\mathrm{H}} \\ &= \mathbf{U}_{S2}\mathbf{\Lambda}_{S2}\mathbf{U}_{S2}^{\mathrm{H}} + \mathbf{U}_{N2}\mathbf{\Lambda}_{N2}\mathbf{U}_{N2}^{\mathrm{H}}\end{aligned} \tag{27}$$

where $\widetilde{\mathbf{T}} = \mathbf{T}\mathbf{T}^H$, $\mathbf{U}_{S2}$ is the subspace spanned by the eigenvectors corresponding to $2N$ large eigenvalues, that is, the signal subspace.

According to the definition of eigenvectors, there exists a $K_c \times 1$ dimensional column vector $\mathbf{\Psi}_i$ that satisfies:

$$\mathbf{A}_c\mathbf{\Psi}_i = \mathbf{u}_i \tag{28}$$

Let's take the eigenvectors corresponding to $2N$ large eigenvalues, and call them $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{2N}\}$. At the same time, we define $L_{fi} = \lfloor (K_i + 1)/2 \rfloor$, $L_{bi} = K_i - L_{fi}$, $D = M + 1 - \lfloor(\max(K_i) + 1)/2\rfloor$, and $\lfloor \cdot \rfloor$ is rounded down. Under the circumstance, according to the EVM method, the $D \times L_{fi}$ dimensional forward matrix $\mathbf{R}_i^f$ can be constructed:

$$\mathbf{R}_i^f = \left[\mathbf{u}_{i1}, \mathbf{u}_{i2}, \ldots, \mathbf{u}_{iL_{fi}}\right] = \mathbf{A}_D\mathbf{D}_{\mathbf{\Psi}_i}\mathbf{A}_{L_{fi}}^{\mathrm{T}} \tag{29}$$

where $\mathbf{u}_{ij} = [\mathbf{u}_i(j), \ldots, \mathbf{u}_i(j + D - 1)]^T$; $\mathbf{A}_D$ is the matrix formed by the first D rows of array flow pattern matrix $\mathbf{A}$; $\mathbf{A}_{L_{fi}}^{\mathrm{T}}$ is the matrix formed by the first ${L_{fi}}^{\mathrm{T}}$ rows of array manifold matrix $\mathbf{A}$; and $\mathbf{D}_{\mathbf{\Psi}_i}$ is the diagonal matrix with the elements in $\mathbf{\Psi}_i$ as the diagonal. Similarly, construct an $D \times L_{bi}$ dimensional backward matrix $\mathbf{R}_i^b$:

$$\mathbf{R}_i^b = \left[\mathbf{u}_{i1}^b, \mathbf{u}_{i2}^b, \ldots, \mathbf{u}_{iL_{bi}}^b\right] = \mathbf{A}_D\mathbf{\Phi}^{-(M-1)}\mathbf{D}_{\mathbf{\Psi}_i^*}\mathbf{A}_{L_{bi}}^{\mathrm{T}} \tag{30}$$

where $\mathbf{u}_i^b = \mathbf{J}\mathbf{u}_i^*$, $\mathbf{\Phi}_i = \mathrm{diag}\{e^{-j\pi\sin\theta_1}, \ldots, e^{-j\pi\sin\theta_{K_c}}\}$, $\mathbf{u}_{ij}^b = [\mathbf{u}_{ij}^b(j), \ldots, \mathbf{u}_{ij}^b(j + D - 1)]^{\mathrm{T}}$. Then, construct a $D \times K_i$ dimensional backward matrix $\mathbf{R}_i^{f/b}$:

$$\mathbf{R}_i^{f/b} = \left[\mathbf{R}_i^f, \mathbf{R}_i^b\right] \tag{31}$$

where $\mathbf{R}_i^{f/b}$ has the number of columns $L = L_{bi} + L_{fi}$. By substituting Equations (29) and (30) into Equation (31), we can get:

$$\mathbf{R}_i^{f/b} = \mathbf{A}_D\mathbf{D}_{\mathbf{\Psi}_i}\mathbf{G}_i \tag{32}$$

where,

$$\mathbf{G}_i = \left[\mathbf{A}_{L_{fi}}^{\mathrm{T}}, \mathbf{D}_{\mathbf{\Psi}_i}^{-1}\mathbf{\Phi}^{-(M-1)}\mathbf{D}_{\mathbf{\Psi}_i^*}\mathbf{A}_{bi}^{\mathrm{T}}\right] \tag{33}$$

According to [15], the probability that $\mathbf{G}_i$ is a full-rank matrix is 1. Moreover, because the rank of matrix $\mathbf{A}_D$ and matrix $\mathbf{D}_{\mathbf{\Psi}_i}$ is k_c, $\mathbf{R}_i^{f/b}$ is a column full rank matrix with rank k_i. To restore the rank of the received data covariance matrix, construct $\mathbf{R}^{f/b}$:

$$\mathbf{R}^{f/b} = \left[\mathbf{R}_1^{f/b}, \mathbf{R}_2^{f/b}, \ldots, \mathbf{R}_N^{f/b}\right] \tag{34}$$

By substituting Equation (32) into Equation (34), the following equation can be obtained:

$$\mathbf{R}^{f/b} = [\mathbf{A}_D\mathbf{D}_{\mathbf{\Psi}_1}\mathbf{G}_1, \mathbf{A}_D\mathbf{D}_{\mathbf{\Psi}_2}\mathbf{G}_2, \ldots, \mathbf{A}_D\mathbf{D}_{\mathbf{\Psi}_N}\mathbf{G}_N] \tag{35}$$

Since $\mathbf{R}_i^{f/b}$ is composed of independent eigenvectors $\mathbf{u}_i$, $\mathbf{R}_i^{f/b}$ is also independent of each other. In summary, the rank of $\mathbf{R}^{f/b}$ is $K_c = \sum_{i=1}^{K} K_i$.

At this point, the restored full-rank data covariance matrix $\mathbf{R}^{f/b}$ is obtained, and the Q-Root-MUSIC algorithm proposed in Section 3 can continue to perform DOA estimation of coherent signals. It is worth noting that the quaternion based hybrid signal parameter estimation algorithm proposed in this section performs DOA estimation of signals, while the polarization parameters need to be estimated using the original data covariance matrix.

5. SUMMARY OF THE ALGORITHM

The quaternion based method proposed in this section can estimate $M-1$ signals at most, while the spatial smoothing class method can only estimate $\lfloor 2M/3 \rfloor$ signals at most. The polarization smoothing method can only estimate 3 coherent signals at most, but does not lose the array aperture, and has better array utilization and higher estimation performance. The quaternion based Root-MUSIC algorithm eliminates the need for spectral peak searching and reduces the dimension of the covariance matrix, so it has a lower computational complexity.

To sum up, the proposed algorithm uses eigenvector method (EVM) under the quaternion model to solve the coherence, realizes the DOA estimation of the mixture signal of uniformly polarized linear array, and has the characteristics of low computation and good estimation performance. The proposed algorithm firstly under the quaternion model uses the Q-Root-MUSIC algorithm for independent signal DOA estimation, then according to the nature of the Toeplitz matrix, a new covariance matrix of coherent signal information is obtained. Finally, the solution of the coherent covariance matrix still uses Q-Root-MUSIC algorithm to estimate DOA of coherent signals.

6. EXPERIMENTAL ANALYSIS

This section verifies the effectiveness of the proposed algorithm through simulation experiments and compares it with the polarized array spatial smoothing (SS) [16] algorithm mentioned. In the simulation, a uniform linear array composed of orthogonal dipoles is used, and the array element spacing is half wavelength. If no special instructions are given, the statistical results of 500 Monte Carlo experiments are used in this section, and root mean square error (RMSE) is used to evaluate the estimation accuracy of the algorithm.

Experiment 1 Estimation accuracy under different signal to noise ratios (SNRs). The number of array elements is 11, and there are six far-field narrowband signals incident on the uniformly polarized linear array, including two independent signals and two groups of coherent signals. The pitch angle, polarization auxiliary angle and polarization phase difference of the two independent signals are $\{5°, 5°, 11°\}$ and $\{16°, 22°, 34°\}$, respectively. And the pitch angle, polarization auxiliary angle, and polarization phase difference of the first group of coherent signals are $\{-28°, 26°, 56°\}$ and $\{33°, 35°, 74°\}$, respectively. Similarly, $\{-40°, 46°, 90°\}$ and $\{42°, 66°, 124°\}$ are the pitch angle, polarization auxiliary angle, and polarization phase difference of the second group of coherent signals, respectively. The three algorithms can estimate all independent signals and coherent signals, but the proposed algorithm and IPAS algorithm estimate independent signals and coherent signals separately, so the advantages and disadvantages of the three algorithms can be more clearly seen by comparing the estimation accuracy of independent signals and coherent signals separately. The number of snapshots is fixed at 1000. Fig. 2 shows the variation curve of the estimation accuracy with the SNR when the three algorithms estimate independent signals, and Fig. 3 shows the variation curve of the estimation accuracy with the SNR when the three algorithms estimate coherent signals.

It can be seen from Fig. 2 and Fig. 3 that the estimation accuracy of the two algorithms is improved with the increase of SNR, but the proposed algorithm obviously has better estimation performance. This is because the SS method of polarized array needs to extract all signals first, and the array aperture is lost.

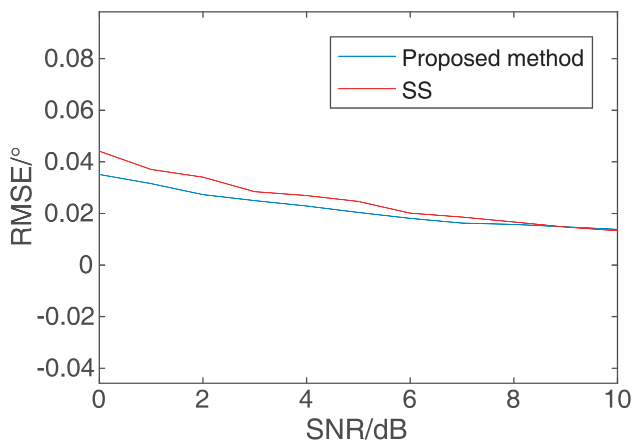

Figure 2. The RMSE of independent signals versus SNR.

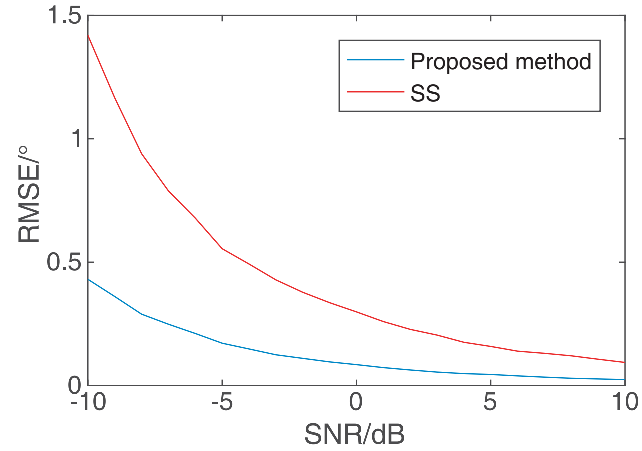

Figure 3. The RMSE of coherent signals versus SNR.

Experiment 2 Estimated accuracy at different snapshots. The pitch angle and polarization angle of the incident signal are the same as in Experiment 1. In this experiment, the SNR is fixed at 10 dB. Fig. 4 shows the variation curve of the estimation accuracy of the two algorithms with different numbers of snapshots when independent signals are estimated, and Fig. 5 shows the variation curve of the estimation accuracy of three algorithms with the different numbers of snapshots. The advantages of the proposed algorithm and SS algorithm in aperture expansion can be clearly seen by comparing the independent signal and coherent signal separately.

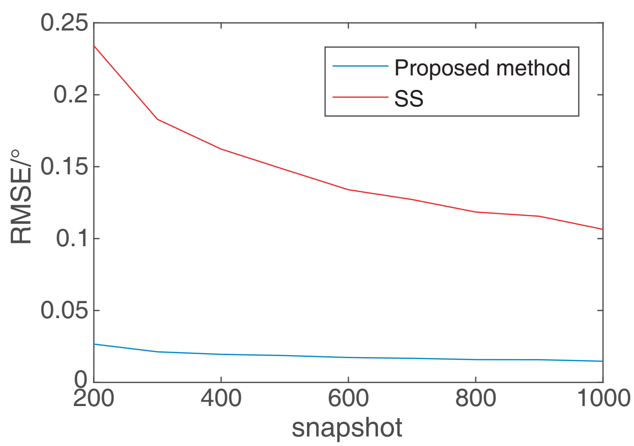

Figure 4. The RMSE of independent signals versus snapshot.

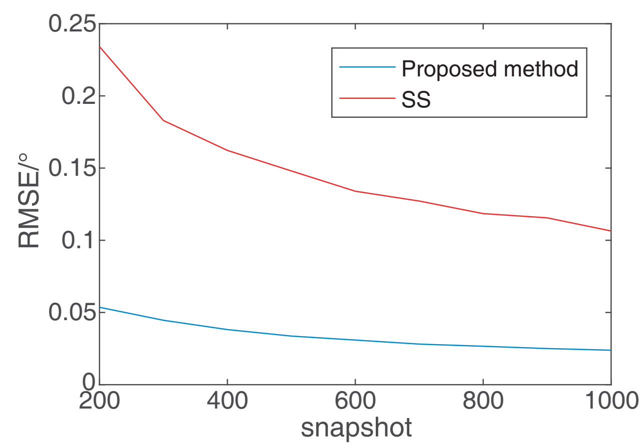

Figure 5. The RMSE of coherent signals versus snapshot.

It can be seen from Fig. 4 and Fig. 5 that the estimation accuracies of the two algorithms improve with the increase of the number of snapshots, and the proposed algorithm has a higher estimation accuracy. This is because the proposed algorithm not only increases the array aperture, but also uses the quaternion method to remove the redundant information and improve the utilization rate of the received data.

Experiment 3 Expansion of array aperture by algorithm. As in Experiment 1, there are six far-field narrow-band signals incident on the uniformly polarized array, including two independent signals and two sets of coherent signals. The difference is that the number of array elements in this experiment is 6, and the SNR is 10 dB. Fig. 6 shows the estimated results of 50 independent experiments.

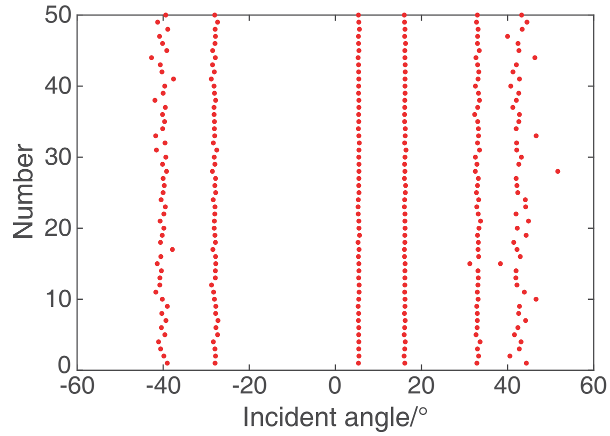

Figure 6. The estimated results of the proposed method.

It can be seen from Fig. 6 that the proposed algorithm can successfully estimate the angle value of mixture signals with good performance. Moreover, six signals can be successfully estimated in the case of six array elements, which proves that the proposed algorithm has the ability to expand the array aperture. This is because the proposed algorithm estimates independent signals and coherent signals separately, which is also the premise of the proposed algorithm to improve the utilization of array elements. As you can see, the smaller the angle is, the smaller the deviation is, and the larger the angle is, the smaller the deviation is from the true value. The smaller the incident angle is, the higher the estimation accuracy is, which is determined by the algorithm itself.

7. CONCLUSION

In this paper, the definition and properties of quaternion are introduced first, and then the quaternion receiving model is reconstructed by using the characteristics of orthogonal dipole in polarization sensitive array, which reduces the dimension of the received data matrix and reduces the operational complexity. Based on the quaternion receiving model and uniform linear array, the DOA parameters were estimated by Root-MUSIC algorithm, and the polarization parameters were estimated by the generalized eigenvalue method to avoid the parameter search process. Finally, the Q-Root-MUSIC method was proposed. This method has low computational complexity and can also take advantage of the orthogonality of quaternion itself to remove the redundant information in the covariance matrix, so as to improve the utilization of data. Further, the EVM method is used for decoherence in the quaternion model, and Root-MUSIC algorithm is used for decoherence signals. The performance of this method is still good in the case of the coexistence of independent signals and coherent signals. Because the independent signals and coherent signals are estimated separately, this method realizes the expansion of the array aperture. For the uniform linear array composed of biorthogonal dipoles, this chapter studies the quaternion method in the case of independent signal and mixture signal. The simulation results show that Q-Root-MUSIC algorithm has lower computational complexity than the traditional polarized Root-MUSIC algorithm and higher estimation accuracy in the estimation of DOA and polarization parameters. In the mixture signals case, compared with IPAS and SS algorithms, the proposed algorithm has lower computational complexity and higher precision, while retaining the advantages of IPAS algorithm for array aperture expansion.

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