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Synthesis of In-Line Fully Canonical Response Filters with Frequency-Variant Couplings

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Abstract—A direct synthesis approach is presented to realize in-line topology filters with adjacent frequency-variant couplings implementing a transmission response with the same number of finite transmission zeros as poles. The proposed method starts with an N-order fully canonical filter response definition. A non-resonant node (NRN) is incorporated into the transversal network to make room for an extra coupling, and as a consequence of the extended similarity transformation applied, the NRN is transformed into a resonant node. The result is a network with N poles and N transmission zeros implemented with $N+1$ resonant nodes and N FVC, being able to describe a fully canonical response with an inline network without cross couplings.

1. INTRODUCTION

The most used and matured techniques to design filters with transmission zeros (TZs) are those that implement them with cross-couplings or extracted-pole techniques. The cross-coupled topology is the most widespread option despite the complexity in the design and tuning process due to multiple pathways. Alternatively, extracted-pole sections are useful because, among other advantages, it provides modularity to tune the transmission zeros. This topology, based on non-resonating nodes, requires extra circuit elements and, in general, a larger size. Recently, the use of Frequency-Variant Couplings (FVCs) has been exploited to introduce TZs to improve the design compactness and simplify the network complexity to implement.

In [1–3], several fundamental building blocks with FVCs for cross-coupled resonator networks have been studied. These blocks can be used in a cascaded structure to assemble high order filtering networks with finite TZs. Inline filters with quasi-elliptic filtering functions realized with mixed electric and magnetic coupling have been shown in [4, 5] for microstrip and [6] for coaxial filter technologies. In both contributions, the step-by-step design of non-fully canonical mixed-coupled filter prototypes is discussed.

In [7, 8], a complete synthesis methodology is published for inline filters designs with a quasi-elliptic filtering characteristic employing FVC. The proposed method only allows networks with up to $N/2$ TZs at finite frequencies, where N is the filter order. Other publications like [9, 10] present a direct matrix synthesis for in-line filter and diplexers with FVC. Alternatively, in [11] an optimization technique for synthesis of filters with constant and frequency-variant couplings is introduced.

More recently, [12] expands the synthesis to filtering functions up to $N-1$ TZs. Without incorporating a source-load coupling, it is known that it is the maximum achievable number of TZs. Although the method covers many cases flawlessly, a fully canonical transmission response is not considered in it nor the above-mentioned works, because these networks require a source-to-load coupling to match the degree of all the polynomials involved.

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In this work, we propose a general synthesis for fully canonical all-pole inline filter response with FVCs, complementing the general method introduced in [12]. With the developed technique, an N-order fully canonical filter requires $N+1$ resonant nodes and N FVCs to be implemented as an inline network without cross-couplings.

In Section 2, the basic theory is reviewed, while the proposed method is described in Section 3. An illustrative example with a general deterministic description and values for an external validation is described in Section 4. Finally, conclusions are drawn.

2. SYNTHESIS THEORY

The FVC generation is carried out by a similarity transformations sequence from a quadruplet following the procedure in [12], where the rotation affects both frequency-variant and frequency-invariant coefficients. The most basic structure to create an FVC is a four-node configuration with a cross-coupling between resonators $i+1$ and $i+2$ as shown in Fig. 1(a). The frequency-variant coupling is created between resonators i and $i+1$, resulting in the network depicted in Fig. 1(b). To synthesize whole inline filters with FVCs, the similarity transformations are applied to the transversal $N+2$ admittance coupling matrix [13], whose expression is:

$$[Y] = [M] + \Omega\,[W] - j\,[G]\,. \tag{1}$$

where $[M]$ is the frequency-invariant reactances (FIRs) coupling matrix. On the other hand, $[W]$ refers to the frequency-variant capacitance matrix multiplied by the lowpass frequency variable Ω. Meanwhile, matrix $[G]$ represents the terminal conductances of the network. Normally, only non-zero entries are the normalized source and load admittances G_{SS} and G_{LL}, respectively.

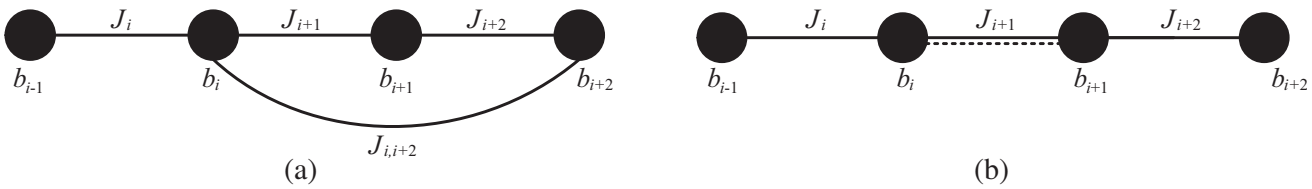

Figure 1. (a) Scheme of a four resonating nodes structure with a coupling between resonator i and $i+2$, and (b) its equivalent network after the similarity transformation with a FVC between resonators i and $i+1$. The FVC is represented by a dashed line.

The transformation procedure comprises four steps. Firstly, the transversal coupling matrix is obtained from the characteristics polynomials [13]. Next, the network is transformed into arrow topology through rotation as shown in Fig. 2(a). This configuration is the previous stage before the generation of the FVCs. The last step comprises an iterative method to remove every cross-coupling from each resonator to load. With further manipulation of the coupling matrix, a triplet is forced into the network by creating a cross-coupling between resonators 1 and 3, [13], as Fig. 2(b) illustrates. The basic four-node structure to perform the transformation is configured by the first three resonators and the source node.

The FVC creation is the fourth step. It is obtained by a similarity rotation with pivot $[i, i+1]$. The rotation is carried out by the transformation matrix $[T]$ as follows:

$$\left[Y'\right] = [T_i]\,[Y]\,[T_i]^T\,. \tag{2}$$

The transformation matrix is $[T_i] = [R_{i1}][U_i][R_{i2}]$, where $[R_{i1}]$ is a matrix created with a rotation angle θ_{i1}, while $[R_{i2}]$ is a matrix created with a rotation angle θ_{i2}. Finally, $[U_i]$ is an escalating matrix whose factor is α_i. Angle θ_{i1} can be chosen arbitrarily while α_{i2} is determined by it, and the θ_{i2} is related to the other two. The expressions to calculate them are described with depth and accuracy in [12].

As a result of this manipulation, we achieve both, an FVC between resonators 1 and 2, and the annihilation of the cross-couplings J_{13} and J_{1L} as shown in Fig. 2(c). This part of the procedure,

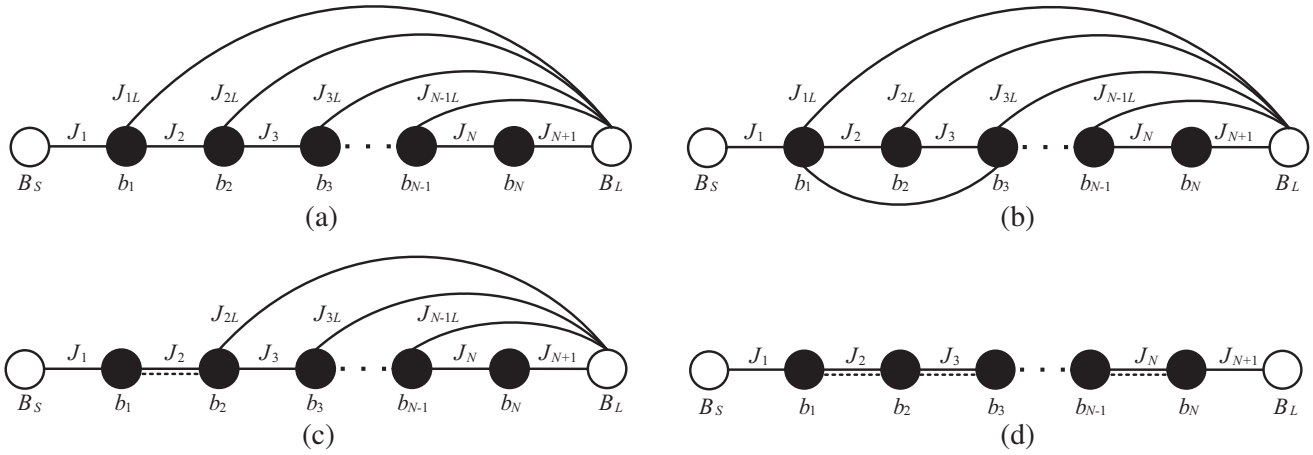

Figure 2. Synthesis procedure for an $N-1$ TZs filter of N-order. (a) Arrow filter configuration which is the initial prototype. (b) The arrow prototype with a cross-coupling J_{13} between resonators 1 and 3, forming a four-node structure together with the source node. (c) After its transformation, an FVC appears between resonators 1 and 2. (d) The resulting topology is an inline network with $N-1$ FVCs.

generating a triplet with a cross-coupling and the four-node structure transformation within the filter prototype, is repeated until the last resonator. The reconfigured coupling matrix yields the topology depicted in Fig. 2(d), where an N-order filter with $N-1$ TZs requires the implementation of $N-1$ FVCs.

As stated in [12], this technique is valid only for filtering functions with up to $N-1$ TZs. In fully canonical filter cases, the arrow topology includes a source-to-load coupling that cannot be removed with this method. In the following section, it will be explained how to overcome this limitation, complementing the above-described synthesis methodology.

3. EXPANSION TO FULLY CANONICAL FILTERS

The proposed method starts with a generalized Chebyschev N-order fully canonical filter response definition. A non-resonant node is incorporated into the transversal network to make room for an extra coupling, and as a result of the extended similarity transformation, the NRN is transformed into a resonant node. The result is a network with N poles and N transmission zeros implemented with $N+1$ resonant nodes and N FVC, being able to describe a fully canonical response with an inline network without cross-couplings.

Consider again the four-node structure as a prerequisite to remove all cross-coupling. As commented, the proposed solution in this work is the implementation of an extra FIR and a mainline coupling aside from the arrow topology.

This can be achieved by rebuilding the coupling matrix introducing two ideal phase shifters with phases ϕ and $-\phi$ to the source port. The first phase shift is applied to the transversal network, while the negative one is used to implement the extra nodes necessary for the proposed topology. The opposite sign in phase shifts is required to keep the filter response unchanged.

The phase shifter can be applied to the admittance matrix prior to the generation of the transversal coupling matrix as in Fig. 3(a) with the techniques described in [14]. Meanwhile, the complementary phase shifter $-\phi$ can be implemented with two FIRs coupled by an admittance inverter as shown in Fig. 3(b).

After reconfiguring the prototype into an arrow topology, the network presents the nodal diagram depicted in Fig. 4(a). Notice that, as a consequence of the transformation of the ideal phase shifter, the first node just after source terminal is an FIR instead of a resonator.

Once reaching the arrow topology, the process continues as described in the previous Section [12]. However, there is a remarkable characteristic with the generation of the FVC concerning the NRN B_x

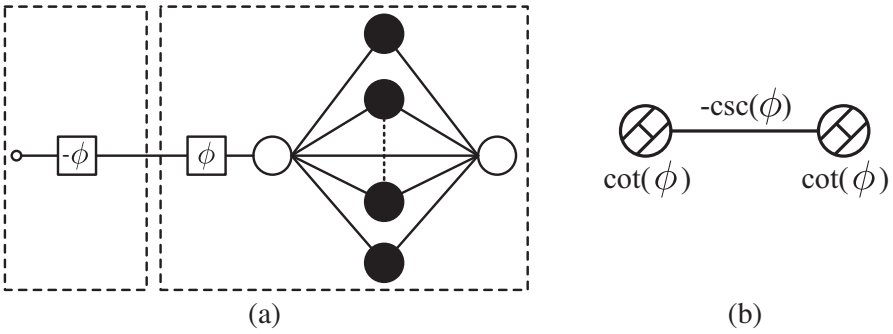

Figure 3. (a) Transversal network with cascaded ideal phase shifters with opposing signs, and (b) the equivalent network of the ideal phase shifter $-\phi$. The nodes are the symbol for FIRs.

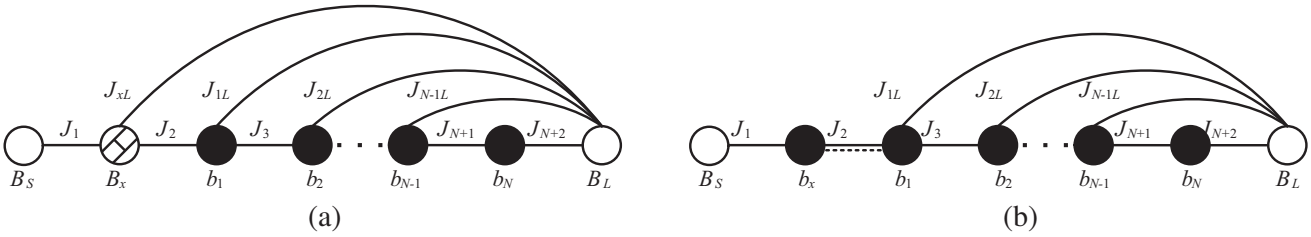

Figure 4. (a) Arrow configuration of a fully canonical network without direct source-to-load coupling. The network posses N resonators plus an extra NRN. (b) The same prototype after the generation of the first FVC, where the NRN is turned into a resonator node, increasing to $N+1$ the total resonator number.

in Fig. 4(a). The enforcement of the transformation matrices in Eq. (2) not only makes an FVC appear, but also provides a frequency dependency to b_x, Fig. 4(b). Although this change in the element nature was not considered a priori, the FIR is transformed into a resonant node ($B_x \rightarrow b_x$).

Then, with the proposed method, an N-order fully canonical filter response requires $N+1$ resonant nodes. To build a network having $N+1$ resonators with only N poles might seem, to say the least, inefficient since the out of band rejection is lower, but having a response with fully canonical characteristics without a source-load coupling is an interesting insight with myriad of applications. The result does not violate the source-to-load coupling rule for fully canonical filters since the network definition has changed when the resonator comes up.

4. ILLUSTRATIVE EXAMPLE

The illustrative example in [12] is a 5th-order filter with 4 TZs $\Omega_{tz} = \{-1.7, 1.5, 2, 3\}$ rads/s and $RL = 26$ dB. To offer a direct comparison with it, we decided to implement a Chebyshev filtering function with the same TZs and RL but reducing the filter order one degree to have a 4-th order fully canonical network.

The characteristic polynomials are $F(s) = s^4 - 0.4965js^3 + 0.9643s^2 - 0.392js - 0.0996$, $E(s) = s^4 + (2.6854 - 0.7085j)s^3 + (3.9682 - 1.6079j)s^2 + (4.6538 - 2.4422j)s + (1.5085 - 3.0431j)$, and $P(s) = 0.2219js^4 + 1.0653s^3 - 0.5437js^2 - 0.3955j$. The phase shifter $\phi = -15^\circ$ is applied to the network prototype. The opposite phase term $-\phi$ is employed to generate the FIR-inverter-FIR structure in Fig. 3(b) that yields $\cot(\phi)) = -3.7321$ and $\csc(\phi) = -3.8637$.

Next, the coupling matrix is assembled and reconfigured to form the arrow topology depicted in Fig. 5(a). To introduce the FVC, the rotation angles are $\theta_{i1} = \{-20^\circ - 20^\circ 10^\circ - 30^\circ\}$.

The generation of the first FVC is carried out with $\Omega = \{-1.7\}$. The similarity transformation turns the NRN into a resonator node, whose result is shown in Fig. 5(b). The TZs used for the rest of FVC follow the order established in vector Ω_{tz}.

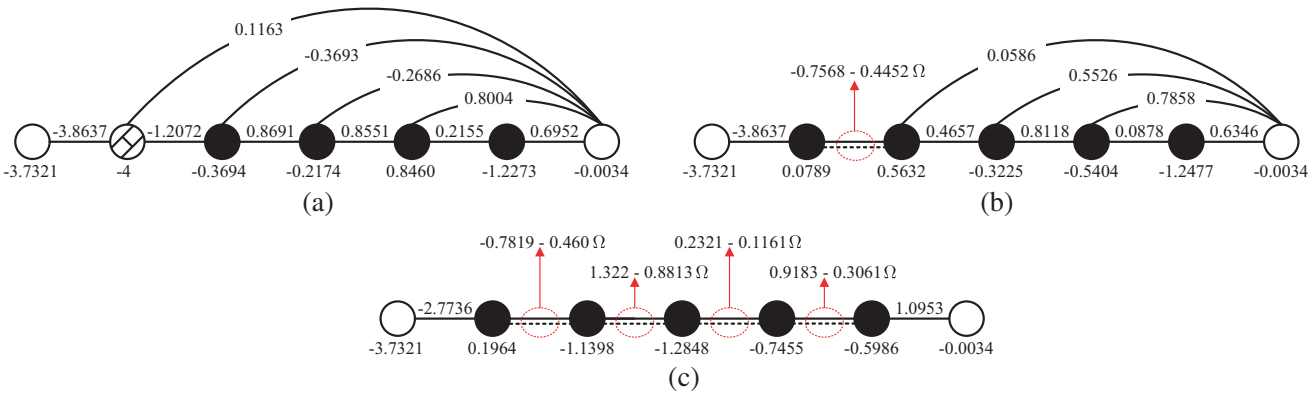

Figure 5. (a) Arrow configuration. (b) The same prototype after the generation of the first FVC. (c) The ultimate inline network with 4 FVC, where the resonators capacitances have been scaled to 1.

Finally, the network is scaled to force all resonator's capacitances to be 1. The final inline prototype is presented in Fig. 5(c). The filter response is shown in Fig. 6(a). The inline filter is denormalized to bandpass domain with $BW = 85\,\text{MHz}$ and $f_0 = 1.7825\,\text{GHz}$ using the method described in [2, 12], whose response is plotted in Fig. 6(b).

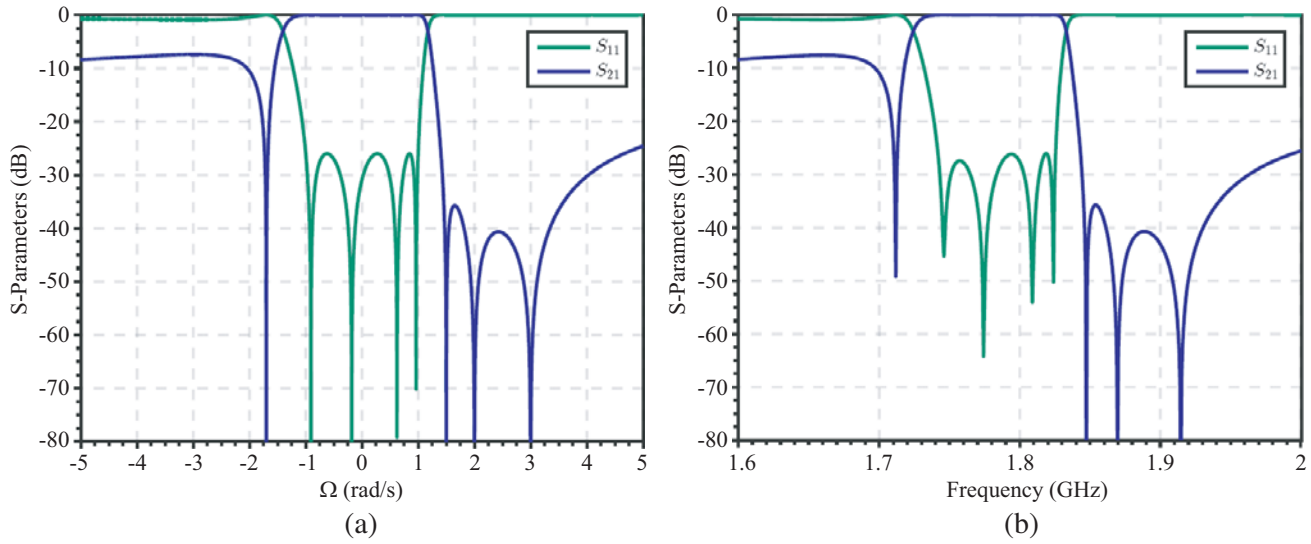

Figure 6. (a) Lowpass response and (b) bandpass response of the network prototype.

5. CONCLUSION

Based on previous works, which provide a robust synthesis method to implement inline filters with up to $N - 1$ TZs with frequency-variant coupling, this paper shows a new approach to expand synthesis techniques to include fully canonical filters with frequency-variant coupling. The implementation of transmission zeros with FVC in fully canonical cases requires the addition of an extra node to the network to make the similarity transformation possible. During the process, the new node (an NRN) is transformed into a resonating node. The result states that an N-order fully canonical response can be implemented without cross-couplings requiring $N + 1$ resonators and N FVC.

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