

Compensation Effect in the Conductive Auroral Regions of the Terrestrial Atmosphere

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Abstract—Oblique incidence of small-amplitude electromagnetic wave on an anisotropic conductive collision semi-infinite turbulent plasma slab under the influence of a homogeneous magnetic field is considered. The conditions of both the ordinary and extraordinary waves' propagation along and transversal directions with respect to the external magnetic field in a homogeneous absorbing collisional magnetoplasma are obtained. Second order statistical moments of the spatial power spectrum of a scattered radiation in the polar ionosphere are calculated for the arbitrary correlation function of electron density fluctuations using the geometrical optics approximation. External magnetic field, oblique incidence of electromagnetic wave on a plasma slab, anisotropy of both ionospheric conductivity and dielectric permittivity, also elongated plasma irregularities in the auroral region of the terrestrial atmosphere are taken into account. The direction along which these asymmetric factors compensate each other is established. The conditions of the “Compensation Effect” are obtained: the spatial power spectrum not broadens, and its maximum is not displaced. Second order statistical moments of a scattered radiation: the shift of maximum and the broadening of the spatial power spectrum in the main and perpendicular planes are investigated analytically and numerically for the power law spectrum of the anisotropic ionospheric plasmonic structures using the experimental data.

1. INTRODUCTION

The analysis of the statistical characteristics of small-amplitude electromagnetic waves propagating in the turbulent anisotropic collision plasma is very important in many practical applications associated with both natural and laboratory plasmas [1, 2]. In a randomly inhomogeneous media strong absorption has a substantial influence on the statistical characteristics of waves, and first of all on the angular distribution of scattered radiation. Ionospheric irregularities cause rapid fluctuations of radio wave amplitude and phase that can degrade GPS positional accuracy and affect the performance of radio communication and navigation systems. Fluctuations of the phase difference cause the random variations of the direction fluctuations of the wave at different atmospheric conditions. They are connected with the angle-of-arrivals [3, 4].

Fluctuations of the parameters of electromagnetic waves propagating in randomly inhomogeneous media depend on features of scattering medium as well as on its absorption properties. The absorption attenuates spatial harmonics of the waves scattered by large angles decreasing fluctuations. Asymmetry of the problem is a necessary condition for anomalous growth in fluctuations in medium irregularities. Such an asymmetry is the oblique irradiation of a random medium boundary, as the waves penetrating in random medium become inhomogeneous. Anomalous growth in fluctuations can occur due to the asymmetry of damping of the scattered waves caused by the asymmetry of the imaginary part of the complex refractive index. In the anisotropic randomly inhomogeneous media absorption decreases the

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amplitude of scattered waves with distance from a slab boundary and leads to the significant distortion of the spatial power spectrum (SPS) of multiply scattered radiation [5].

It has been established that the energy loss due to collisions between plasma particles can lead not only to a decrease in the amplitude of the electromagnetic waves with distance from the slab boundary but also to a substantial distortion of the angular spectrum. It was shown [6, 7] that the absorption caused by collision frequency between plasma particles influences the SPS of scattered waves depends strongly on both the propagation direction of an incident wave and an inclined external magnetic field with respect to the plasma boundary. The nonmonotonic dependence of the variance and asymmetry coefficient on the distance from the plasma boundary was discovered.

The oblique incidence of a small-amplitude plane electromagnetic wave on a semi-infinite slab of the collisional turbulent plasma in an external homogeneous magnetic field was analyzed in the small-angle scattering approximation in [6, 8, 9]. The calculations have shown that the results obtained in the geometrical optics approximation agree well with the conclusions of a more rigorous theory of the multiple small-angle scattering. Polarimetric parameters of scattered electromagnetic waves in the conductive turbulent magneto-plasma were considered in [10].

Peculiarities of the angular power distribution of multiply scattered electromagnetic waves caused by both the anisotropy factors were investigated. It was shown that the extent of influence of absorption caused by the collision between plasma particles on the angular spectrum of scattered waves strongly depends on the direction of an incident wave, refractive angle, and strength of a homogeneous external magnetic field. The strong scattering anisotropy is observed in the multiple scattering theory for large-scale elongated irregularities. Spectrum width changes non-monotonically versus distance from a plasma boundary. Such asymmetry occurs not only in the case of oblique incidence of electromagnetic waves on a plasma-vacuum interface [4, 5, 12], it can also be an intrinsic property of the plasma in an external magnetic field. At strong absorption the width of the SPS of the received radiation is greater than that in the collisionless plasma; the spectral maximum is substantially displaced to the direction of the source.

The broadening and shift of maximum of the SPS of scattered electromagnetic waves in the turbulent collision magnetized plasma were analyzed for both power-law and anisotropic Gaussian correlation functions of electron density fluctuations [11] using the geometrical optics condition.

In this paper, the broadening of the SPS and shift of its maximum in the anisotropic conductive collision turbulent magnetized plasma are investigated using the geometrical optics approximation. Compensation condition has been obtained, i.e., the direction of wave propagation has been established along which four anisotropic factors: oblique incidence of wave, anisotropy caused by the homogeneous external magnetic field, anisotropy of conductivities, and elongated magnetoplasma irregularities compensate each other. This effect is analyzed for the first time. Numerical calculations have been carried out for a power law form for the wave number spectrum of electron density fluctuations applying the experimental data.

2. FORMULATION OF THE PROBLEM

Let a plane electromagnetic wave with the frequency ω_0 be incident from vacuum on a semi-infinite slab of an anisotropic conductive turbulent collision magnetized plasma. In the Cartesian coordinate system XOY plane is the vacuum-plasma boundary. Homogeneous external magnetic field is directed along the Z -axis ($\mathbf{H}_0 \parallel Z$), perpendicular to the plasma slab. The wave vector \mathbf{k}_0 of an incident wave and \mathbf{H}_0 are located in the main YOZ plane. The wave equation of the electric field is [3, 4]:

$$\nabla \times \nabla \times \mathbf{E}_i - k_0^2 \sum_{i=1}^3 \tilde{\epsilon}_{ij} \mathbf{E}_j = 0. \quad (1)$$

Components of the complex dielectric tensor $\tilde{\epsilon}_{ij}$ of a conductive magnetized plasma are [12]:

$$\begin{aligned} \tilde{\epsilon}_{xx} = \tilde{\epsilon}_{yy} = \varepsilon_{\perp} - i(sg + \tilde{\sigma}_{\perp}), \quad \tilde{\epsilon}_{xy} = -\tilde{\epsilon}_{yx} = -i(\varkappa + \tilde{\sigma}_H) + 2s\varkappa/(1-u), \\ \tilde{\epsilon}_{zz} = \varepsilon_{\parallel} - i(sv + \tilde{\sigma}_{\parallel}), \quad \tilde{\epsilon}_{xz} = \tilde{\epsilon}_{zx} = \tilde{\epsilon}_{yz} = \tilde{\epsilon}_{zy} = 0; \end{aligned} \quad (2)$$

where: $\varepsilon_{\perp} = 1 - v/(1-u)$, $\varkappa = v\sqrt{u}/(1-u)$, $\varepsilon_{\parallel} = 1 - v$, $g = v(1+u)/(1-u)^2$; $v(\mathbf{r}) = \omega_p^2(\mathbf{r})/\omega^2$ and $u = (eH_0/m_e c \omega)^2$ are non-dimensional magneto-ionic parameters of the ionospheric plasma;

$\omega_p(\mathbf{r}) = [4\pi N_e(\mathbf{r})e^2/m_e]^{1/2}$ is the plasma frequency; $\tilde{\sigma} = 4\pi\hat{\sigma}/k_0c$ is the normalized conductivity having components [12]:

$$\begin{aligned}\sigma_{\perp} &= e^2 n_e \left(\frac{\nu_e}{m_e(\nu_e^2 + \omega_e^2)} + \frac{\nu_i}{m_i(\nu_{in}^2 + \omega_i^2)} \right), & \sigma_H &= e^2 n_e \left(\frac{\omega_e}{m_e(\nu_e^2 + \omega_e^2)} - \frac{\omega_i}{m_i(\nu_{in}^2 + \omega_i^2)} \right), \\ \sigma_{\parallel} &= e^2 n_e \left(\frac{1}{m_e \nu_e} + \frac{1}{m_m \nu_{in}} \right),\end{aligned}$$

here: σ_{\perp} and σ_H are the Pedersen and Hall's conductivities, respectively; σ_{\parallel} is the longitudinal (parallel to the magnetic field) conductivity; $n_e(\mathbf{r}) = n_0 + n_1(\mathbf{r})$ is the electron density; $k_0 = \omega_0/c$, n_0 is a constant density component; $n_1(\mathbf{r})$ is a random function of the spatial coordinates describing electron density fluctuations; e and m_e are the charge and mass of an electron; $\nu_e = \nu_{en} + \nu_{in}$ is the effective collision frequency of electrons with other plasma particles

$$\nu_{ei} = N \left[59 + 4.18 \log \left(\frac{T_e^3}{N} \right) \right] \times 10^{-6} T_e^{-3/2} \text{ [m.k.s]},$$

and $\nu_{en} = 5.4 \times 10^{-16} N_n T_e^{1/2}$ [m.k.s] are the electron-ion and electron-neutral collision frequencies, respectively [13]; ω_e and ω_i are the angular gyrofrequencies of an electron and ion, respectively.

In the *YOZ* main plane (location of both the external magnetic field and the refracted wave vector) for a homogeneous plasma, the solution of Equation (1) at $s \ll \varepsilon_{ij}$, $\tilde{\sigma}_{ij}$ yields the biquadratic equation: $Ax^4 + Bx^2 + C = 0$ with the coefficients: $A = A_0 - iA'$; $A_0 = \varepsilon_{\perp} \sin^2 \theta + \varepsilon_{\parallel} \cos^2 \theta$, $A' = \sigma_{\perp} \sin^2 \theta + \sigma_{\parallel} \cos^2 \theta$; $B = -B_0 + iB'$; $B_0 = (\varepsilon_{\perp} \varepsilon_{\parallel} - \sigma_{\perp} \sigma_{\parallel})(1 + \cos^2 \theta) + [(\varepsilon_{\perp}^2 - \sigma_{\perp}^2) - (\varepsilon + \sigma_H)^2] \sin^2 \theta$, $B' = (\varepsilon_{\perp} \sigma_{\parallel} + \varepsilon_{\parallel} \sigma_{\perp}) \cdot (1 + \cos^2 \theta) + 2\varepsilon_{\perp} \sigma_{\perp} \sin^2 \theta$; $C = C_0 - iC'$; $C_0 = \varepsilon_{\parallel} [\varepsilon_{\perp}^2 - \sigma_{\perp}^2 - (\varepsilon + \sigma_H)^2] - 2\varepsilon_{\perp} \sigma_{\perp} \sigma_{\parallel}$, $C' = 2\varepsilon_{\parallel} \varepsilon_{\perp} \sigma_{\perp} + \sigma_{\parallel} [\varepsilon_{\perp}^2 - \sigma_{\perp}^2 - (\varepsilon + \sigma_H)^2]$; θ is an angle between the external magnetic field and the wave vector of a refracted wave in the conductive turbulent collision magnetized plasma. For homogeneous, nonconductive medium, we obtain the well-known result [12]. As a result, we obtain complex refractive index of the conductive homogeneous magnetoplasma in the polar region of the terrestrial atmosphere:

$$\tilde{N}^2 = T_0 + iT_1, \quad (3)$$

where: $T_0 = [A_0(B_0 \pm G_0) - A'(-B' \pm G')]$, $T_1 = A'(B_0 \pm G_0) + A_0(-B' \pm G')$, $G_0 = \sqrt{(r + D_0)/2}$, $G' = \sqrt{(r - D_0)/2}$, $r = \sqrt{D_0^2 + D'^2}$, $D_0 = B_0^2 - B'^2 - 4(A_0 C_0 - A' C')$, $D' = -2B_0 B' + 4(A_0 C' + C_0 A')$. Upper sign corresponds to the ordinary wave and lower sign to the extraordinary wave. For a nonconductive plasma, we obtain the well-known result [12].

For a homogeneous medium seeking the solution of Equation (1) in the form of plane waves $\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \exp(-i\mathbf{k}\mathbf{r})$, we receive a system of the equations for a component of electric field. Equating the determinant to zero, after transformations we will receive:

$$\text{ctg}^2 \theta = - \frac{(N^2 - \varepsilon_{zz}) (\varepsilon_{xx} N^2 - D_1 D_2)}{\varepsilon_{zz} (N^2 - D_1) (N^2 - D_2)}, \quad (4)$$

where: $D_1 = \varepsilon_{xx} + i\varepsilon_{xy}$, $D_2 = \varepsilon_{xx} - i\varepsilon_{xy}$. The right part of this equation has zero and two poles, i.e., in the homogeneous conductive collisional plasma we have four main waves. Let's consider particular cases at $s^2 \ll 1$.

1. $\theta = \pi/2$, i.e., the direction of wave propagation perpendicular to the impose magnetic field. For the ordinary wave from Equation (4) we obtain:

$$\text{Re} N_O^2 = 1 - v, \quad -\text{Im} N_O^2 = sv + \tilde{\sigma}_{\parallel}, \quad (5)$$

for the extraordinary wave we have:

$$\begin{aligned}\text{Re} N_X^2 &= \varepsilon_{\perp} - \frac{\varepsilon + \tilde{\sigma}_H}{(\varepsilon_{\perp}^2 + \tilde{\sigma}_{\perp}^2) + 2s\tilde{\sigma}_{\perp}g} \left[\varepsilon_{\perp}(\varepsilon + \tilde{\sigma}_H) - s \frac{2\varepsilon}{1-u} \right], \\ -\text{Im} N_X^2 &= (sg + \tilde{\sigma}_{\perp}) - \frac{\varepsilon + \tilde{\sigma}_H}{(\varepsilon_{\perp}^2 + \tilde{\sigma}_{\perp}^2) + 2s\tilde{\sigma}_{\perp}g} \left\{ \tilde{\sigma}_{\perp} \varepsilon_{\perp} (\varepsilon + \tilde{\sigma}_H) + s \left[\frac{4\varepsilon_{\perp} \varepsilon}{1-u} + g(\varepsilon + \tilde{\sigma}_H) \right] \right\},\end{aligned} \quad (6)$$

Negative sign in the imaginary parts points to the attenuation of both waves at propagation in the terrestrial atmosphere. Notice that the longitudinal conductivity enters only in the attenuation of an ordinary wave, and the other conductivities (Hall and Pedersen) cause attenuation of an extraordinary wave.

2. $\theta = 0^\circ$, i.e., we consider the propagation of phase along the imposed magnetic field in the radio approximation ($\omega \gg \omega_i$); the ions do not vibrate significantly, so that only the electrons are involved [14]. For the longitudinal propagation of electromagnetic waves in a magnetoplasma, all electromagnetic vectors vibrate perpendicular to the direction of propagation. We obtain:

$$\begin{aligned} N_L^2 &= \left[\left(1 - \frac{v}{1 + \sqrt{u}} \right) + \tilde{\sigma}_H \right] - i \left[\tilde{\sigma}_\perp + s \frac{v(1 - \sqrt{u})^2}{(1 - u)^2} \right], \\ N_R^2 &= \left[\left(1 - \frac{v}{1 - \sqrt{u}} \right) - \tilde{\sigma}_H \right] - i \left[\tilde{\sigma}_\perp + s \frac{v(1 + \sqrt{u})^2}{(1 - u)^2} \right] \end{aligned} \quad (7)$$

here: N_L^2 is the left-handed circular polarization wave, and N_R^2 is the right-handed circular polarization wave. The designations L and R refer to the left-handed and right-handed senses of rotation of the electromagnetic vectors about the imposed magnetic field, not about the direction of propagation. In the absence of external magnetic field, there is only an ordinary wave. If $s = \tilde{\sigma}_{ij} = 0$, we obtain the well-known result [12].

If a linearly polarized pulse of electromagnetic radiation with an arbitrary oriented magnetic vector is radiated from the ground at different latitudes in the vertical direction, then the terrestrial ionosphere splits it on ordinary and extraordinary waves. These waves have different frequencies ($N^2 = 0$) and reflected on different altitudes. Therefore, both components return to the earth at different times, and at registration of echo-signal by ground-based radar systems on an oscillograph two impulses instead of one are visible. Particularly, for extraordinary wave from Equation (7) we obtain:

$$\begin{aligned} \omega_L &= (1 + \tilde{\sigma}_H) \left\{ \omega_p \sqrt{1 + \frac{1 + \tilde{\sigma}_H}{4} \frac{\Omega_e^2}{\omega_p^2}} - \frac{\Omega_e}{2} \right\} < \omega_p, \\ \omega_R &= (1 - \tilde{\sigma}_H) \left\{ \omega_p \sqrt{1 + \frac{1 - \tilde{\sigma}_H}{4} \frac{\Omega_e^2}{\omega_p^2}} + \frac{\Omega_e}{2} \right\} > \omega_p. \end{aligned} \quad (8)$$

It is evident that the Hall's conductivity plays an essential role in reflection (or propagation) of a signal at various altitudes. Electronic plasma frequency ω_p is the critical frequency of the ordinary wave.

3. FLUCTUATION OF THE PHASE OF SCATTERED ELECTROMAGNETIC WAVES IN THE CONDUCTIVE TURBULENT COLLISION MAGNETOPLASMA

We will use the complex geometrical optics approximation as the characteristic linear scale of the ionospheric irregularities substantially exceeds the wavelength λ of an incident wave. Turbulence in the conductive magnetoplasma is caused by the electron density fluctuations: $n(\mathbf{r}) = n_0 + n_1(\mathbf{r})$, $n_1(\mathbf{r}) \ll n_0$, where the second term is a random function of the spatial coordinates. Using the stochastic eikonal equation $k^2 = k_0^2 \tilde{N}^2$, $k_0 = \omega/c$, expand the wave vector \mathbf{k} and the phase of the scattered electromagnetic waves in the series, using the perturbation method: $\mathbf{k}(\mathbf{r}) = \mathbf{k}_0 + \mathbf{k}_1(\mathbf{r}) + \dots$, $\varphi(\mathbf{r}) = \varphi_0 + \varphi_1(\mathbf{r}) + \dots$. In a vacuum: $k_y = k_0 \sin \theta_i$, $k_z = k_0 \cos \theta_i$ are real quantities, and θ_i is the angle of an incident plane electromagnetic wave from vacuum on a semi-infinite slab of a conductive turbulent plasma. Penetrating wave vector in a conductive collision magnetoplasma becomes complex: $k_{zp} = \sqrt{k_0^2 \tilde{N}^2 - k_y^2}$. In the geometrical optics condition ignoring the interaction between the normal waves, the eikonal equation of the complex phase φ is [3, 4, 12]: $\mathbf{k}(\mathbf{r}) = -\nabla \varphi$. Small electron density fluctuations lead to the wave field fluctuations at the observation point. In a linear approximation at $k_{0x} = 0$ we obtain [6]:

$$k_{0z} \frac{\partial \varphi_1}{\partial z} + \left(k_{0y} - \frac{1}{2} k_0^2 \frac{\partial \tilde{N}_0^2}{\partial k_{0y}} \right) \frac{\partial \varphi_1}{\partial y} = g k_0^2 n_1, \quad (9)$$

where: φ_1 and n_1 are the phase and electron density fluctuations, respectively; \tilde{N}_0^2 is the unperturbed refracting index; $g = -(\partial\tilde{N}_0^2/\partial n_0)/2$.

Integrating Equation (8) along the complex characteristics using the boundary condition $\varphi_1(z = 0) = 0$ phase fluctuation can be written in the form:

$$\varphi_1(x, y, L) = \frac{gk_0^2}{k_{zp}} \int_{-\infty}^{\infty} d\mathbf{x}_x \int_{-\infty}^{\infty} d\mathbf{x}_y \exp(i\mathbf{x}_x x + i\mathbf{x}_y y) \int_0^L d\xi n_1(\mathbf{x}_x, \mathbf{x}_y, \xi) \exp \left[i\mathbf{x}_x \frac{\partial k_{zp}}{\partial k_{0y}} (L - \xi) \right]. \quad (10)$$

where: $n_1(\mathbf{x}_x, \mathbf{x}_y, \xi)$ is the two-dimensional Fourier transform of electron density fluctuations, and L is the distance from a plasma boundary. Applying the Snell's law, we get:

$$\begin{aligned} \frac{\partial k_{zp}}{\partial k_{0y}} &= -\frac{k_0 \tilde{N}_0}{k_{zp}} \left(\sin \theta - \frac{1}{2\tilde{N}_0^2 \cos \theta} \frac{\partial \tilde{N}_0^2}{\partial \theta} \right) = \beta + i\gamma, \\ \beta &= -\tan \theta + \frac{1}{2 \cos^2 \theta (T_0^2 + T_1^2)} \left(T_0 \frac{\partial T_0}{\partial \theta} + T_1 \frac{\partial T_1}{\partial \theta} \right), \\ \gamma &= \frac{1}{2 \cos^2 \theta (T_0^2 + T_1^2)} \left(T_0 \frac{\partial T_1}{\partial \theta} - T_1 \frac{\partial T_0}{\partial \theta} \right) \end{aligned} \quad (11)$$

θ is the angle between the vector \mathbf{H}_0 and the wave vector of a refracted wave \mathbf{k} .

In the zeroth approximation, neglecting electron density fluctuations, only this nonuniform refracted wave gives the contribution in a scattered radiation. Varying the angle of incidence or the inclination angle of an external magnetic field, parameter γ varies, i.e., the amplitude of a refracted wave decreases from a plasma boundary. At normal incidence ($\theta = 0^\circ$, $\mathbf{k}_0 \parallel \mathbf{H}_0$) of wave on a turbulent magnetized plasma $\gamma = 0^\circ$ [4].

4. STATISTICAL CHARACTERISTICS OF A SCATTERED RADIATION

The transverse correlation function of the phase fluctuation can be written as:

$$\begin{aligned} W_\varphi(\rho_x, \rho_y, L) &= \langle \varphi_1(x + \rho_x, y + \rho_y, L) \varphi_1^*(x, y, L) \rangle = \left(\frac{gk_0^2}{k_{zp}} \right)^2 \int_{-\infty}^{\infty} d\mathbf{x}_x \int_{-\infty}^{\infty} d\mathbf{x}_y V_n(\mathbf{x}_x, \mathbf{x}_y, \beta \mathbf{x}_y) \\ &\quad \frac{1}{2\gamma \mathbf{x}_y} [1 - \exp(-2\gamma \mathbf{x}_y L)] \exp(i\mathbf{x}_x \rho_x + i\mathbf{x}_y \rho_y), \end{aligned} \quad (12)$$

where: $V_n(\mathbf{x}_x, \mathbf{x}_y, \mathbf{x}_z)$ is the arbitrary three-dimensional spectral correlation function of electron density fluctuations, and ρ_x and ρ_y are distances between two observation points locating normal to the layer. This expression is derived in the small-angle scattering approximation and is valid for the finite distances from a plasma boundary; it contains important wave dumping parameter γ including both dielectric permittivity and conductivity tensors. Angular brackets denote the ensemble average.

The knowledge of this statistical characteristic allows to calculate the second order moment of the complex field. Correlation function of the complex field (spatial power spectrum) can be written as [3, 4]:

$$\begin{aligned} W_E(\rho_x, \rho_y, L) &= \langle E(x + \rho_x, y + \rho_y, L) E^*(x, y, L) \rangle = E_0^2 \langle \exp[i\varphi_1(x + \rho_x, y + \rho_y, L) - i\varphi_1^*(x, y, L)] \rangle \\ &\quad \cdot \exp[ik_{0y}\rho_y - 2(\text{Im}k_{zp})L]. \end{aligned} \quad (13)$$

Contribution of the imaginary part of the wavenumber $k_{zp} = \sqrt{k_0^2 \tilde{N}^2 - k_y^2}$ in Equation (12) increases in proportion to the distance L . In the most interesting case at strong fluctuation of the phase $\langle \varphi_1 \varphi_1^* \rangle \gg 1$ as usual [4], the phase is normally distributed. In this case correlation function decreases sharply as ρ_x and ρ_y increase. We can transform Equation (13) expanding the argument of the second exponential in a series [6, 8, 9, 15]:

$$W_E(\rho_x, \rho_y, L) = E_0^2 \exp[ik_{0y}\rho_y - 2(\text{Im}k_{zp})L] \exp \left\{ \frac{\partial W_\varphi}{\partial \rho_y} \rho_y + \frac{1}{2} \frac{\partial^2 W_\varphi}{\partial \rho_y^2} \rho_y^2 + \frac{1}{2} \frac{\partial^2 W_\varphi}{\partial \rho_x^2} \rho_x^2 \right\}, \quad (14)$$

The derivatives of the phase correlation function are taken at the point $\rho_x = \rho_y = 0$.

Spatial (angular) power spectrum (SPS) of a scattered radiation is of greatest practical importance and is the Fourier transform of the correlation function (13) [4]:

$$S(\mathfrak{x}_x, \mathfrak{x}_y, L) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\rho_x \int_{-\infty}^{\infty} d\rho_y W_E(\rho_x, \rho_y, L) \exp(-i\mathfrak{x}_x \rho_x - i\mathfrak{x}_y \rho_y). \quad (15)$$

This characteristic is equivalent to the ray intensity (brightness) in radiation transport equation [15, 16]. In the most interesting case of strong fluctuation of the phase $\langle \varphi_1 \varphi_1^* \rangle \gg 1$, APS is expressed as follows [6, 8, 9, 11].

At strong fluctuations of the phase this spectral function has the Gaussian form:

$$S(\mathfrak{x}_x, \mathfrak{x}_y, L) = S_0 \exp \left\{ -\frac{(\mathfrak{x}_y - \Delta \mathfrak{x}_y)^2}{2\langle \mathfrak{x}_y^2 \rangle} - \frac{\mathfrak{x}_x^2}{2\langle \mathfrak{x}_x^2 \rangle} \right\}, \quad (16)$$

where: $\Delta \mathfrak{x}_y$ determines the shift of maximum of the SPS caused by random electron density irregularities in the main plane; $\langle \mathfrak{x}_y^2 \rangle$ and $\langle \mathfrak{x}_x^2 \rangle$ describe the widths of the spectrum in the main and perpendicular planes, respectively; S_0 is the maximum value of a spectral curve. These expressions can be defined directly applying formula (14):

$$\Delta \mathfrak{x}_y = \frac{1}{i} \frac{\partial W_\varphi}{\partial \rho_y}, \quad \langle \mathfrak{x}_y^2 \rangle = -\frac{\partial^2 W_\varphi}{\partial \rho_y^2}, \quad \langle \mathfrak{x}_x^2 \rangle = -\frac{\partial^2 W_\varphi}{\partial \rho_x^2}, \quad (17)$$

all derivatives are taken at the point $\rho_x = \rho_y = 0$.

Second order statistical moments (12) and (17) correctly describe that the SPS of scattered radiation is determined by the following inequalities [9]: $|\Delta \mathfrak{x}_y| \ll 2\pi/\lambda$, $\sqrt{\langle \mathfrak{x}_x^2 \rangle} \ll 2\pi/\lambda$, $\sqrt{\langle \mathfrak{x}_y^2 \rangle} \ll 2\pi/\lambda$. These conditions are not in contravention to the assumption of strong phase fluctuations because in a smoothly inhomogeneous medium mean spatial scale of plasma inhomogeneities of the phase correlation function substantially exceeds the wavelength of scattered electromagnetic waves, $l \gg \lambda$ [4], and the angle of the normal to the random wave front $\Delta \theta \propto \lambda \sqrt{\langle \varphi_1^2 \rangle}/l$ can remain small even at $\langle \varphi_1^2 \rangle \gg 1$.

Substituting Equation (12) into (17) we obtain:

$$\Delta \mathfrak{x}_y = \frac{2\pi}{\gamma} \left(\frac{gk_0^2}{k_{zp}} \right)^2 \int_{-\infty}^{\infty} d\mathfrak{x}_x \int_{-\infty}^{\infty} d\mathfrak{x}_y V_n(\mathfrak{x}_x, \mathfrak{x}_y, \beta \mathfrak{x}_y) [1 - \exp(-2\gamma \mathfrak{x}_y L)], \quad (18)$$

$$\langle \mathfrak{x}_y^2 \rangle = \frac{\pi}{\gamma} \left(\frac{gk_0^2}{k_{zp}} \right)^2 \int_{-\infty}^{\infty} d\mathfrak{x}_x \int_{-\infty}^{\infty} d\mathfrak{x}_y \mathfrak{x}_y V_n(\mathfrak{x}_x, \mathfrak{x}_y, \beta \mathfrak{x}_y) [1 - \exp(-2\gamma \mathfrak{x}_y L)], \quad (19)$$

$$\langle \mathfrak{x}_x^2 \rangle = \frac{\pi}{\gamma} \left(\frac{gk_0^2}{k_{zp}} \right)^2 \int_{-\infty}^{\infty} d\mathfrak{x}_x \int_{-\infty}^{\infty} d\mathfrak{x}_y \frac{\mathfrak{x}_x^2}{\mathfrak{x}_y} V_n(\mathfrak{x}_x, \mathfrak{x}_y, \beta \mathfrak{x}_y) [1 - \exp(-2\gamma \mathfrak{x}_y L)]. \quad (20)$$

In some cases, characteristic spatial scales of electron density fluctuations are quite big, and we can use the geometrical optics approximation. We will consider the distances satisfying the condition $(L/\xi l_{||}) \ll 1$. Hence, the obtained formulas are valid for the arbitrary distances propagating by waves in the turbulent magnetized plasma. From these formulas follow that the effects of anomalous broadening of the SPS and shift of its maximum are determined by imaginary part of the derivative $\partial k_{zp}/\partial k_{0y}$. This term contains oblique incident of the wave and anisotropy of the electron density fluctuations. Investigation shows that the preferable direction exists determined by equation $\gamma = \text{Im}(\partial k_{zp}/\partial k_{0y}) = 0$ at which either shift or the anomalous broadening does not take place, and the oblique incidence of the wave and medium anisotropy compensate each other. This is so called compensation direction. At waves propagation on another angle, the SPS will broadens.

5. NUMERICAL CALCULATIONS

The incident electromagnetic wave having the frequency of 3 MHz ($k_0 = 6.28 \cdot 10^{-2} \text{ m}^{-1}$) propagates along the Z -axis. Plasma parameters at the altitude of 300 km are: $u_0 = 0.22$, $v_0 = 0.28$ [17, 18].

The spectrum form of electron density irregularities of the auroral ionosphere along the direction of the geomagnetic field \mathbf{H} has been measured for the first time by a radio-scintillation method using orbital satellites signals. It is shown that the spectral density has the characteristic scale of “cutoff” $l_{\parallel} \sim 5\text{--}15 \text{ km}$ along \mathbf{H} [16]. In the plane orthogonal to \mathbf{H} for measured scales of $100 \text{ m} \leq l_{\perp} \leq 1 \text{ km}$ the turbulence is isotropic, and the spectral density decreases according to the power law.

RH-560 rocket flight observations from Sriharikota rocket range (SHAR), India (14°N , 80°E , dip latitude 5.5°N ; apogee was 348 km) show [19] that the intermediate range irregularities (100 m–2 km) were observed in abundance in altitude regions 220–250 km and 290–320 km. Irregularities of a range of scale sizes starting from a few hundred meters to a few tens of kilometers are observed in these patches, and the spectral index of intermediate scale irregularities in 160–170 km region was found to be -3.78 .

Reflected signals from small-scale irregularities located on the altitudes from 200 km up to 400 km were registered by radar system “Sura” in 2006 [20]. Cross-sections of back-scattered radio waves were measured in the same area of the ionospheric plasma at vertical and oblique soundings of radio waves having frequency 9 MHz. Anisotropy factor has been estimated by the ratio of these cross sections, which is $\sim \chi^{-(p-1)}$. The relation of measured back-scattered cross-sections of pulse signals at vertical sounding of an ionosphere by different carrier frequencies allows to define the spectral index p of small-scale ionospheric turbulence. The knowledge of parameter p allows to define anisotropic factor of elongated small-scale irregularities of electron density fluctuations in the ionospheric plasma. It was established that for the Kolmogorov spectrum ($p = 11/3$) for strong anisotropic small-scale ionospheric irregularities $\chi \approx 10 \div 100$, the variance of electron density irregularities $\sigma_n^2 \approx 10^{-5} \div 10^{-2}$.

An experiment in satellite radio wave probing of the ionosphere, modified by powerful waves from the HF heating facility at Tromsø (Norway) in May 1995 has been described in [21]. The observations show that both large-scale irregularities (several tens of kilometers in size) and small-scale ones (from hundreds of meters to kilometers) are generated in the high-latitude ionosphere. A distinctive property of the polar ionosphere is that the magnetic field lines are oriented almost vertically promoting an easy plasma transport along the field lines and, as a result, the formation of elongated vertical structures. A strong elongation of the irregularities in the direction of the magnetic field makes it possible to observe their features in the corresponding sector using the aspect-angle effect. Using the model of a power-law spectrum it was shown that the spectral power indices $p \approx 3.8$.

Measurements of satellite’s signal parameters moving in the ionosphere show that in the F-region of the ionosphere irregularities have power-law spectrum. Satellite observations show that the ionospheric irregularities of electron density fluctuations in the ionospheric F-region have the power-law spectrum [11]:

$$V_n(\mathbf{k}) = \frac{\sigma_n^2}{\pi^{5/2}} \Gamma\left(\frac{p}{2}\right) \Gamma\left(\frac{5-p}{2}\right) \sin\left[\frac{(p-3)\pi}{2}\right] \frac{l_{\parallel}^3}{\chi^2 [1 + l_{\perp}^2 (k_{\perp}^2 + \chi^2 k_{\parallel}^2)]^{p/2}}, \quad (21)$$

where: σ_n^2 is the mean-square fractional deviation of electron density; $\Gamma(p)$ is the gamma function [22]; $\chi = l_{\parallel}/l_{\perp}$ is the anisotropy factor — the ratio of longitudinal and transverse characteristic linear sizes of plasma irregularities. The shape of electron density irregularities has a spheroidal form. Anisotropy of the shape of irregularities is connected with the difference of the diffusion coefficients in the field aligned and field perpendicular directions.

Figure 1 illustrates the displacement of maximum of the SPS of scattered extraordinary wave as a function of non-dimensional spatial parameter L/l_{\parallel} for fixed anisotropy factor $\chi = 10$ and different penetration angles in the anisotropic conductive magnetized plasma. Curve 1 corresponds to the angle $\theta = 2^\circ$, curve 2 to $\theta = 5^\circ$, curves 3 and 4 to $\theta = 6^\circ$ and $\theta = 15^\circ$, respectively. Numerical calculations show that curve 1 reaches its maximum at (0.62; 0.57), curve 2 (0.53; 0.54), curves 3 and 4: (0.66; 0.42) and (1.16; 0.42), respectively. Analyses show that increasing angle θ the maximum of the SPS is displaced to the opposite side with respect to the compensation direction. At fixed anisotropy factor χ , decreasing angle θ the curves tend to the compensation direction faster. In the polar region of the

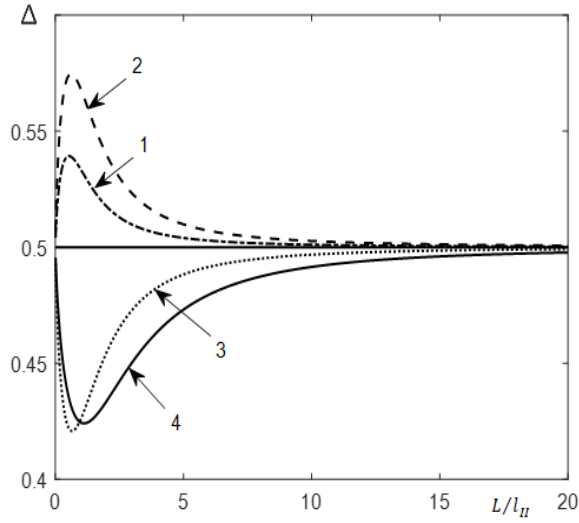


Figure 1. The dependence of the shift of maximum of the SPS of the extraordinary wave versus nondimensional spatial parameter $L/l_{||}$ at fixed anisotropy factor χ and different penetration angle of wave in the conductive collision magnetized plasma.

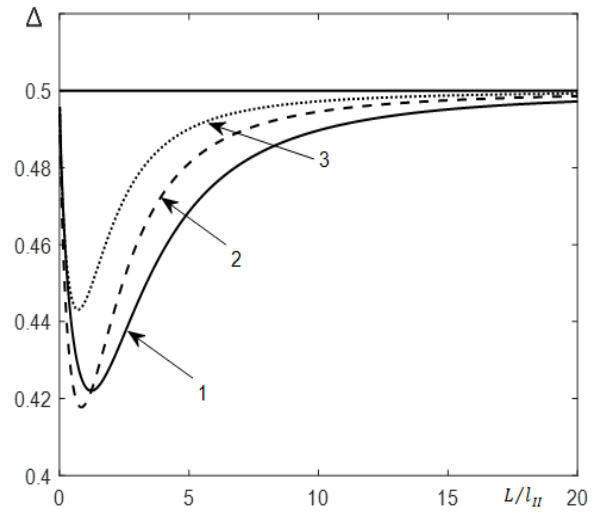


Figure 2. Displacement of maximum of the SPS of the extraordinary wave at fixed angle θ and different anisotropy factor χ .

terrestrial ionosphere, taking into account anisotropy of an external magnetic field, conductivity and electron density irregularities $\chi = 10$, decreasing oblique incidence angle of an extraordinary wave on a plasma boundary, “Compensation Effect” takes place at $L/l_{||} = 60$.

Figure 2 depicts the evaluation of the shift of maximum of the SPS of the extraordinary wave at fixed oblique incidence angle $\theta = 10^\circ$ and different anisotropy factors. Curve 1 corresponds to $\chi = 5$, curve 2 to $\chi = 10$, and curve 3 to $\chi = 20$. Numerical calculations show that increasing anisotropy factor of elongated plasma irregularities maximum of the SPS quickly tends to the compensation direction, and in the polar ionosphere, the “Compensation Effect” takes place at $L/l_{||} = 60$.

Figure 3 demonstrates the displacement of maximum of the SPS of scattered ordinary wave as a function of spatial parameter $L/l_{||}$ at fixed anisotropy factor $\chi = 15$ and different refractive angles. Curve 1 corresponds to the angle $\theta = 2^\circ$, curve 2 to $\theta = 5^\circ$, curve 3 and 4 to the angles $\theta = 10^\circ$ and $\theta = 20^\circ$. Each maximum of the SPS has the coordinates: curve 1 (0.52; 0.43), curve 2 (0.39; 0.45), curves 3 and 4: (0.76; 0.57) and (1.44; 0.54), respectively. Maximum of the SPS is displaced into opposite direction with respect to the compensation direction increasing angle θ . At small refractive angles the curves approach the compensation direction faster, and the “Compensation Effect” takes place at $L/l_{||} = 20$.

Figure 4 represents the shift of maximum of the SPS at the penetration angle $\theta = 15^\circ$ and different anisotropy factors of elongated plasma irregularities. Curve 1 corresponds to $\chi = 5$, curve 2 to $\chi = 10$, and curve 3 to $\chi = 20$. Numerical calculations show that the maxima of curves 1 and 2 have coordinates (1.45; 0.58) and (1.12; 0.57), respectively, curve 3 coordinate (1.04, 0.54). Numerical calculations show that maximum of the SPS fast tends to the compensation direction in proportion to the parameter χ . In this case “Compensation Effect” takes place at $L/l_{||} = 20$.

Figure 5 shows plots of the broadening of the SPS for an extraordinary wave scattered in the conductive collisional magnetoplasma (polar region of the terrestrial atmosphere) for different anisotropic factors of elongated ionospheric electron density irregularities. Curves 1 and 2 correspond to the anisotropic factor $\chi = 15$ at different penetration angles $\theta = 5^\circ$ and $\theta = 30^\circ$, respectively. Curves 3 and 4 are devoted to the $\chi = 5$ at the same penetration angles. Curves describe asymmetric aspiration for the SPS to the compensation direction from left and right sides. Coordinates of these curves: 1 — (1.41, 0.57), 2 — (2.27, -0.17), 3 — (1.07, 0.29), 4 — (2.39, -0.37). Compensation direction is at $L/l_{||} = 60$.

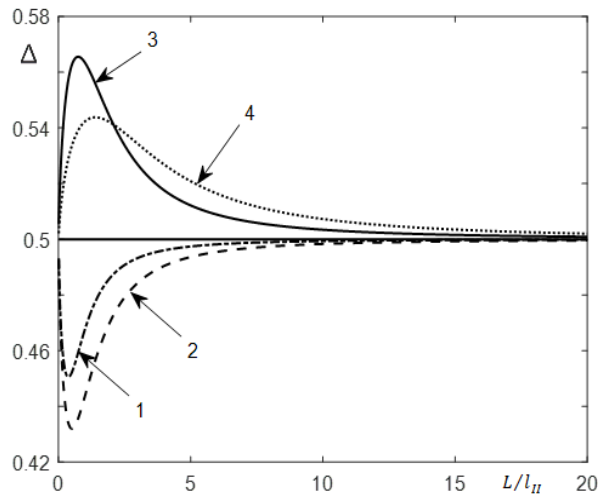


Figure 3. Illustrate the dependence of shift of maximum of the SPS of scattered ordinary wave in the conductive magnetized plasma at fixed anisotropy factor χ and different penetration angle θ .

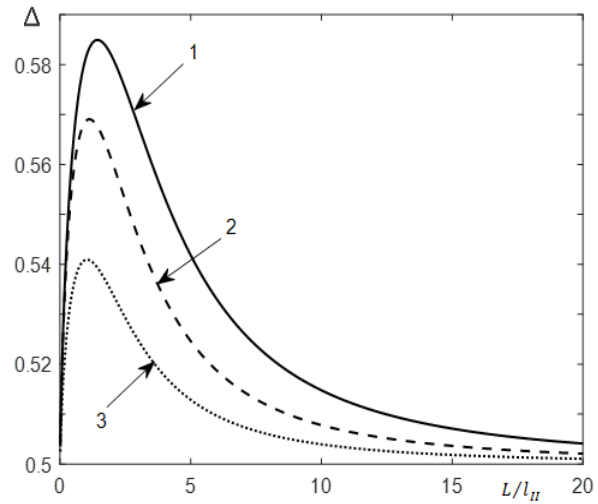


Figure 4. Represents displacement of maximum of the SPS at fixed inclination angle θ and different anisotropy factor of elongated electron density irregularities.

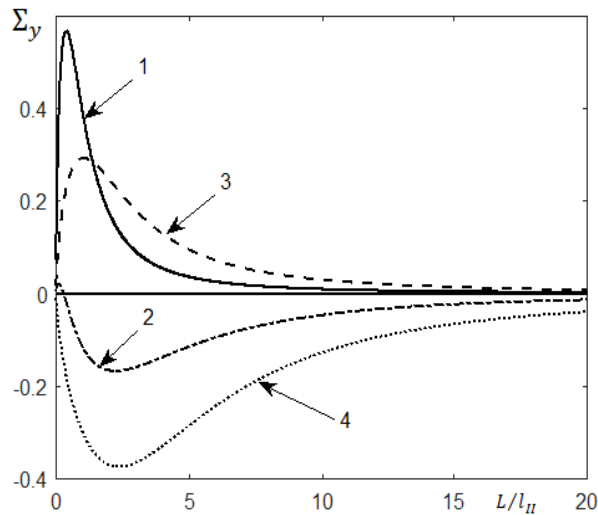


Figure 5. The broadening of the SPS of a scattered extraordinary wave in the main plane.

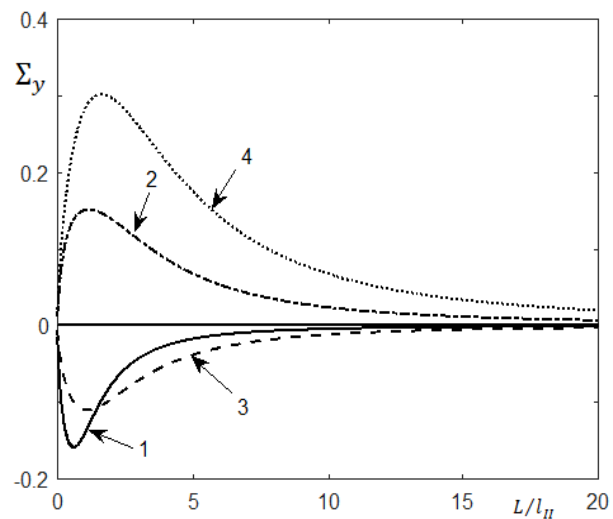


Figure 6. The broadening of the SPS of a scattered ordinary wave in the main plane.

Figure 6 shows plots of the broadening of the SPS for an ordinary wave scattered in the conductive collision magnetized plasma (polar region of the terrestrial atmosphere) for different anisotropic factors of elongated ionospheric electron density irregularities. Curves 1 and 2 correspond to the anisotropic factor $\chi = 15$ at different penetration angles $\theta = 5^\circ$ and $\theta = 20^\circ$, respectively. Curves 3 and 4 are devoted to $\chi = 5$ at the same penetration angles. Curves describes asymmetric aspiration for the SPS to the compensation direction from left and right sides. Coordinates of these curves: 1 — (0.59, -0.16), 2 — (1.15, 0.15), 3 — (1.14, -0.11), 4 — (1.62, 0.30). Both ordinary and extraordinary waves tend to the compensation direction at small anisotropy factors traveling small distances from a plasma boundary.

Numerical analyses show that the “Compensation Effect” does not take place for both electromagnetic waves in the XOZ plane — perpendicular to the main plane (location of an external

homogeneous magnetic field) at different refraction angles. At small distances from the boundary, the maximums of the ordinary waves exceed the maximums of the extraordinary wave. The broadening of the SPS for both waves is approximately the same.

6. CONCLUSION

The obtained results show that the evaluation of the SPS and the “Compensation Effect” of scattered small-amplitude ordinary and extraordinary electromagnetic waves strongly depend on the following anisotropic factors: oblique incidence of the wave on a turbulent anisotropic conductive collision semi-infinite plasma slab containing anisotropic electron density irregularities and the direction of a homogeneous external magnetic field.

Wave equation has been solved for homogeneous absorbing collisional magnetoplasma. Propagation conditions of the ordinary and extraordinary waves along and transversal directions with respect to the external magnetic field are obtained. At longitudinal propagation in the auroral regions of the terrestrial ionosphere only Hall’s conductivity gives contribution in both left-handed and right-handed circular polarized extraordinary waves, at transversal propagation — Pedersen and Hall’s conductivities.

Second order statistical moments of the SPS are calculated in the geometrical optics approximation taking into account all above mentioned anisotropic factors. It is important that separately all these factors lead to the asymmetry of the task, while joint action of these effects leads to the “Compensation Effect” in the conductive auroral region of the terrestrial atmosphere; along compensation direction these asymmetric factors compensate each other. In this case, neither the SPS broadens nor its maximum shifts. Evaluation of the spectrum is investigated analytically and numerically for the power law spectrum of electron density fluctuations using experimental data. Numerical calculations show that at fixed anisotropy factor χ , decreasing refraction angle θ the curves tend to the compensation direction faster. Increasing the anisotropy factor of elongated plasma irregularities at fixed refraction angle, maximum of the SPS quickly tends to the compensation direction in the polar ionosphere.

Measuring cross sections of back-scattered radio waves at vertical and oblique soundings in the upper ionosphere, it is possible to investigate generation, relaxation, and transfer processes of both natural and artificial ionospheric turbulence on different altitudes of the ionosphere. With radar sounding of the same area of the ionosphere by radio waves with different frequencies and at various incidence angles, “Compensation Effect” allows to determine all main parameters of the anisotropic spectrum of large- and small-scale ionospheric turbulence. We can conclude that the anisotropy of the longitudinal, Pedersen and Hall’s conductivity plays an important role on the evaluation of the SPS compared to anisotropy of elongated plasmonic structures and the spectrum of electron density fluctuations in the polar ionosphere of the terrestrial atmosphere. Let’s note that the results of numerical simulations are valid in wider interval of input parameters of a task, than that received by method of geometrical optics [6].

The obtained results can be of interest to the experts developing systems of a long-distance radio communication, and also system of sounding of artificial and natural plasma formations (an ionosphere, a solar crown, semiconductor plasma, etc.) radio translucence method.

New features of the “Compensation Effect” will be revealed at different inclination angles of an external magnetic field on different altitudes and in various regions of the terrestrial atmosphere. It should be useful in longwave and shortwave communication systems, longwave navigation systems, and shortwave surveillance systems.

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