Progress In Electromagnetics Research Letters, Vol. 102, 167–175, 2022

An Efficient ADBF Algorithm Based on Keystone Transform for Wideband Array System

Yiyang Jiang[1, *], Mingwei Shen[1], and Guodong Han[2]

Abstract—In this paper, an efficient wideband array adaptive beamforming (ADBF) approach based on keystone transform is presented. In order to eliminate the aperture effect of the wideband signal, the modified keystone transform is applied to remove the time delay between different array elements. Thus, the wideband array is equivalent to a narrowband array, and the orthogonal projection matrix of the target steering vector can be used to filter the desired signal in the training samples, which avoids the signal cancellation caused in the estimation of ADBF covariance matrix. Compared with the established algorithm of sliding window, this approach can significantly reduce the computational burden. The feasibility and effectiveness of the proposed method are validated through numerical simulations.

1. INTRODUCTION

With the development of radar technology, the capability of high-resolution measurement in target recognition and identification is a requirement for modern radar, requiring a relatively wide frequency bandwidth to achieve richer information of the target. Thus, the signal processing technology combining wideband phased array has become a research focus in modern radar technology. Meanwhile, beamforming technique, which can suppress unwanted clutter received from external sources, plays an important role in the research of wideband phased array radar. However, the envelope and phase of wideband signals received by each antenna are not aligned because of aperture fill time, which will bring a serious influence on pulse compression results [1–3]. Therefore, some effective methods have been proposed to solve this problem.

One approach is the tapped delay line which is performed by inserting a transversal filter in each channel to derive the adaptive weights under different standards and enhance the capability of jamming suppression [4]. Investigations indicate that as the bandwidth increases, a large number of filter orders, which increase the computational burden, are required to maintain an acceptable SINR performance for ADBF. The other approach is wideband beamforming based on subband decomposition which is processed in the frequency domain. This method weights each subband of the wideband signals by a narrowband processor structure and aligns their beam directions [5]. Since the performance of the algorithm is limited by the number of subbands and its jamming cancellation capability should be weighed with more calculations brought by the increase in bandwidth. Meanwhile, desired signals with high amplitude are prone to the problem of signal cancellation [6, 7] and cause beam distortion.

In this paper, considering the performance loss caused by the aperture effect in wideband arrays, keystone transform [8–12] is applied for the aperture effect of the received wideband array signal model for its capability to achieve envelope alignment. Besides, the performance of ADBF is severely degraded, in the case that the target signal is present in the training samples since the target signal is considered as

Received 26 November 2021, Accepted 28 January 2022, Scheduled 16 February 2022
* Corresponding author: Yiyang Jiang (jyy_hhu1997@163.com).
[1] College of Computer and Information Engineering, Hohai University, Nanjing, China. [2] The 54th Research Institute of CETC, Shijiazhuang 050081, China.

interference to be suppressed. Therefore, the orthogonal projection known as a fast subspace projection algorithm has been applied to filter out the target signal [13, 14].

The content of this paper is organized as follows. The received wideband array signal model is established, and the modified keystone transform is introduced in Section 2. The principle of the orthogonal projection matrix is indicated in Section 3. Some simulation experiments are presented in Section 4. The conclusion of this paper is given in Section 5.

2. CORRECTION OF APERTURE EFFECT USING MODIFIED KEYSTONE TRANSFORM

2.1. The Principle of Keystone Transform

Consider that a far-field wideband plane waveform $d_0(t)$ and P independent wideband jamming signals whose angles are $\theta_0, \theta_1, \dots, \theta_P$ impinging on a uniform linear array (ULA) constituted by M elements. The signal wavelength is λ, and the distance between adjacent array elements is $d = \lambda/2$. There is no correlation between the received signal sources $\mathbf{D}(t) = [d_0(t), d_1(t), \dots, d_i(t)]$ and the channel noise $n(t)$, and $\mathbf{A} = [a(\theta_0), a(\theta_1), \dots, a(\theta_i)]$ is the array manifold matrix of the received signal. Then, the echo signal received by the array antenna at the range time t can be expressed as

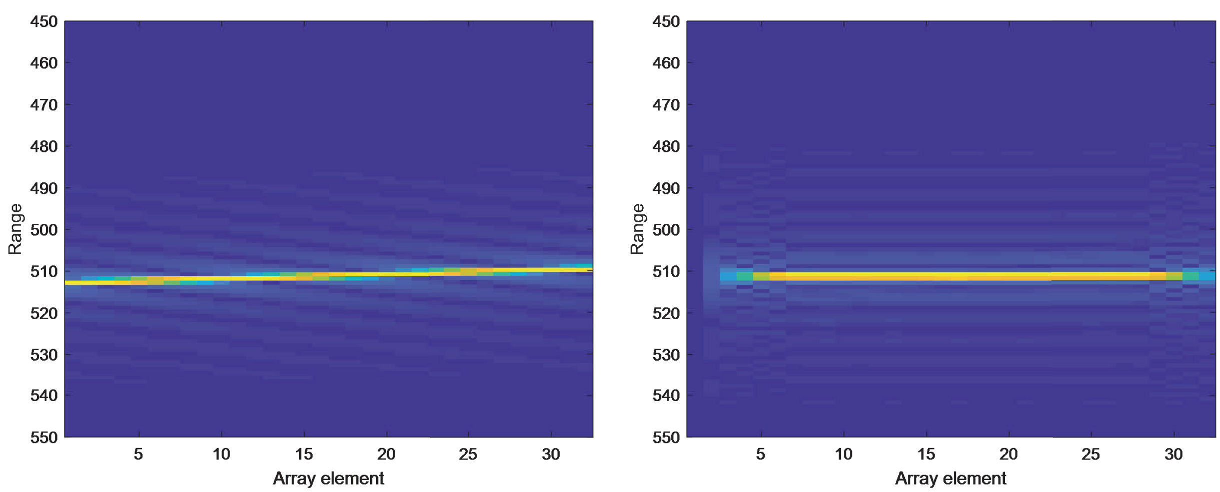
$$\mathbf{x}\left(t\right) = \mathbf{AD}\left(t\right) + n\left(t\right) = a\left(\theta_0\right) d_0\left(t\right) + \sum_{i=1}^{P} a\left(\theta_i\right) d_i\left(t\right) + n\left(t\right) \tag{1}$$

Suppose that there is only one target signal in the received signal, and the range-compressed signal can be formulated by a time-element function as

$$\mathbf{y}\left(t, n\right) = \mathbf{x}\left(t - \frac{r_n}{c}, n\right) \exp\left(-j\frac{2\pi f_0}{c} r_n\right) \tag{2}$$

where c is the speed of light, and f_0 is the radar center frequency. Referring to the first element, $r_n/c = (n-1)d\sin\theta_0/c(n \in [1, M])$ represents the time delay of each element.

From Eq. (2), instantaneous data of r_n cause a relative delay in the envelope of the received signal of each array element. As shown in Fig. 1(a), a range-compressed signal with a bandwidth of 200 MHz is formed in the spatial domain as a diagonal line and spans multiple distance cells, which is the phenomenon of aperture fill time.



(a) (b)

Figure 1. Simulation data image after range compression. (a) Before keystone transform. (b) After keystone transform.

Performing FFT transform on Eq. (2), its Fourier spectrum is

$$\mathbf{y}\left(f,n\right)=\mathbf{x}(f,n)\exp\left(-j\frac{2\pi f_0}{c}r_n\right)\exp\left(-j\frac{2\pi f}{c}r_n\right) \tag{3}$$

where the range frequency variable $f \in [-B/2, B/2]$ and array element variable $n \in [1, M]$ couple with each other in the exponential term $\exp(-j\frac{2\pi f}{c}r_n)$ which makes the array signal steering vector continuously change with the instantaneous frequency. Therefore, keystone transform is applied to correct the aperture effect to ensure that the wideband steering vector is not connected with the instantaneous frequency. From Eq. (4), the scale transformation is performed on the coordinate axis of the array element

$$\hat{n}=\frac{f_0+f}{f_0}n \tag{4}$$

Rewrite Eq. (3) as follows

$$\mathbf{y}\left(f,\hat{n}\right)=\mathbf{x}(f,\hat{n})\exp\left(-j\frac{2\pi f_0}{c}r_{\hat{n}}\right)=\mathbf{x}(f,\hat{n})\exp\left(-j\frac{2\pi f_0}{c}d\left(\hat{n}-1\right)\sin\left(\theta_0\right)\right) \tag{5}$$

From Eq. (5), it is clear that the wideband steering vector after keystone transform is only related to the center frequency f_0. As shown in Fig. 1(b), the target signal is transformed into the time-element dimension by using IFFT, and the range-compressed target signal has been modified into a horizontal straight line. Therefore, the keystone transform can effectively decouple the range frequency and element time to correct the aperture effect of the wideband array.

2.2. The Modified Keystone Transform

A commonly used method for keystone transform is the convolution of these signals with sinc-interpolation function [15, 16], and the signal in Eq. (5) can be formulated as

$$\mathbf{y}\left(f,\hat{n}\right)=\mathbf{y}\left(f,\frac{f_c}{f_c+f}\hat{n}\right)=\sum_{n=-N/2}^{N/2}\mathbf{y}\left(f,n\right)\mathrm{sinc}\left(\frac{f_c}{f_c+f}\hat{n}-n\right) \tag{6}$$

where $\hat{n}$ is the sampling point after interpolation.

The method of increasing the interpolation order N is usually applied to improve the accuracy of the interpolated data and $N/2$ array elements before and after each interpolation point is used for the calculation of this point, thus aggravating the problem of the insufficient data at both ends of the range-compressed signal. From Eq. (6), considering that the number of array elements is Na, and the central array element is the reference array element so that the range of interpolation calculation points is $[-(Na-N)/2, (Na-N)/2]$. Therefore, there are at least $N/2$ interpolation points lacking enough data for interpolation calculation at both ends of the array. As shown in Fig. 1(b) in Section 2.1, fewer array elements will result in a large amount of insufficient data and a reduction in the amplitude of the desired signal after ADBF.

In order to improve this situation and ensure the accurate reconstruction of echo data, the interpolation order of the array elements with insufficient data can be changed, and the requirement for calculation points in the interpolation process is relatively reduced. The specific process is as follows

(1) From the number of array elements Na and interpolation order N, the number of missing elements m can be obtained. Then the range is $m \in [-Na/2, -(Na+N)/2] \cup [(Na-N)/2, Na/2]$.

(2) The principle of decreasing interpolation order is applied in this algorithm. If the interpolation point advances one element towards the edge, the two-point interpolation order will be reduced correspondingly. Suppose that $\tilde{n}$ is the sampling point after the modified interpolation. Locating each element of insufficient data and improving the interpolation, Eq. (6) can be rewritten as

$$\mathbf{y}\left(f,\tilde{n}\right)=y\left(f,\frac{f_0}{f_0+f}\tilde{n}\right)=\sum_{n=|\tilde{n}|-Na/2}^{Na/2-|\tilde{n}|}\mathbf{y}\left(f,n\right)\mathrm{sinc}\left(\frac{f_0}{f_0+f}\tilde{n}-n\right)=\mathbf{x}(f,\tilde{n})\exp\left(-j\frac{2\pi f_0}{c}r_{\tilde{n}}\right) \tag{7}$$

From Eq. (7), the data of elements at both ends of the array can be basically filled. According to the derivation in this section, the phase change between different elements is no longer shifted and only related to the center frequency after keystone transform, so the aperture fill phenomenon is effectively eliminated. Meanwhile, the steering vector of the wideband signal has been transformed to the form of narrowband so that the narrowband adaptive beamforming can be directly applied in the wideband system.

3. EFFICIENT ADBF ALGORITHM BASED ON ORTHOGONAL PROJECTION MATRIX

Suppose that the received signal now contains the target signal, wideband jamming signals, and channel noise as shown in Eq. (1). When calculating the interference sample covariance matrix, the sliding window algorithm [17] processed in the time domain is usually applied for ADBF, which sets the protection cells on both sides of the test cell, and the adaptive weight can be calculated for the test cell independently. Thus, the range IFFT is performed on $\mathbf{y}(f,\tilde{n})$, and the time-domain function can be formulated as

$$\mathbf{y}\left(t,\tilde{n}\right)=\mathbf{x}\left(t,\tilde{n}\right)\exp\left(-j\frac{2\pi f_0}{c}d\left(\tilde{n}-1\right)\sin\left(\theta_0\right)\right) \tag{8}$$

In the simulation experiment, to avoid the performance loss of signal cancellation in training samples, the number of protection cells around the test cell u is set to 5, and the covariance matrix can be estimated by the data of remaining cells as

$$R=\frac{1}{Nr-11}\left[\sum_{t=1}^{u-6}y\left(t,\tilde{n}\right)y^H\left(t,\tilde{n}\right)+\sum_{t=u+6}^{Nr}y\left(t,\tilde{n}\right)y^H\left(t,\tilde{n}\right)\right] \tag{9}$$

The adaptive weight W_{sw} for the test cell can be written as

$$W_{sw}=\frac{R^{-1}a\left(\theta_0\right)}{a^H\left(\theta_0\right)R^{-1}a\left(\theta_0\right)} \tag{10}$$

From Eq. (9), it is clear that the optimal weight vector has been obtained under the requirement of at least Nr times of calculations. However, the sliding window algorithm has excellent performance, especially in the areas with dense targets, but the computational burden will increase significantly.

After keystone transform, the wideband signal has been converted into the form of narrowband so that the accurate beam pointing can be obtained because of the cancellation of aperture fill phenomenon. Meanwhile, the beam pointing angle of the desired signal can be predicted as it is consistent with the transmitted beam. Therefore, the projection matrix is designed according to the pointing angle of the transmitting signal directly to suppress the target information. The steering vector $C=a(\theta_0)$ is orthogonally complemented to obtain the orthogonal projection matrix

$$B=I-C\left(C^HC\right)^{-1}C^H \tag{11}$$

where $B^HC=0$ and matrix B is applied to block the target signal. After $y(t,\tilde{n})$ is processed by the projection matrix, the information of the desired signal is set to zero

$$\mathbf{y_1}\left(t,\tilde{n}\right)=B^H\mathbf{y}\left(t,\tilde{n}\right)=B^H\left(\sum_{i=1}^{P}a\left(\theta_i\right)d_i\left(t\right)+n\left(t\right)\right) \tag{12}$$

Rewrite the covariance matrix as follows

$$R_1=E\left[\mathbf{y_1}\left(t,\tilde{n}\right)\mathbf{y_1^H}\left(t,\tilde{n}\right)\right]=E\left[B\mathbf{y}\left(t,\tilde{n}\right)\mathbf{y^H}\left(t,\tilde{n}\right)B^H\right] \tag{13}$$

Substituting Eq. (13) into Eq. (10), the optimal adaptive weight vector is therefore expressed as

$$W_{PM}=\frac{R_1^{-1}a\left(\theta_0\right)}{a^H\left(\theta_0\right)R_1^{-1}a\left(\theta_0\right)}=\frac{E\left(B\mathbf{y}\left(t,\tilde{n}\right)\mathbf{y^H}\left(t,\tilde{n}\right)B^H\right)^{-1}a\left(\theta_0\right)}{a^H\left(\theta_0\right)E\left(B\mathbf{y}\left(t,\tilde{n}\right)\mathbf{y^H}\left(t,\tilde{n}\right)B^H\right)^{-1}a\left(\theta_0\right)} \tag{14}$$

Finally, the LCMV narrowband processing method is enabled to complete the ADBF process.

Compared with the sliding window method, the method of preprocessing by orthogonal projection matrix has a much lower computational burden as it only adds some calculation amount in the process of constructing matrix with no need to achieve the location of the desired signal through a large amount of calculation. Therefore, the performance of the ADBF based on the projection matrix can properly improve the efficiency and accuracy of beamforming.

The flowchart of the processing chain is shown in Fig. 2.

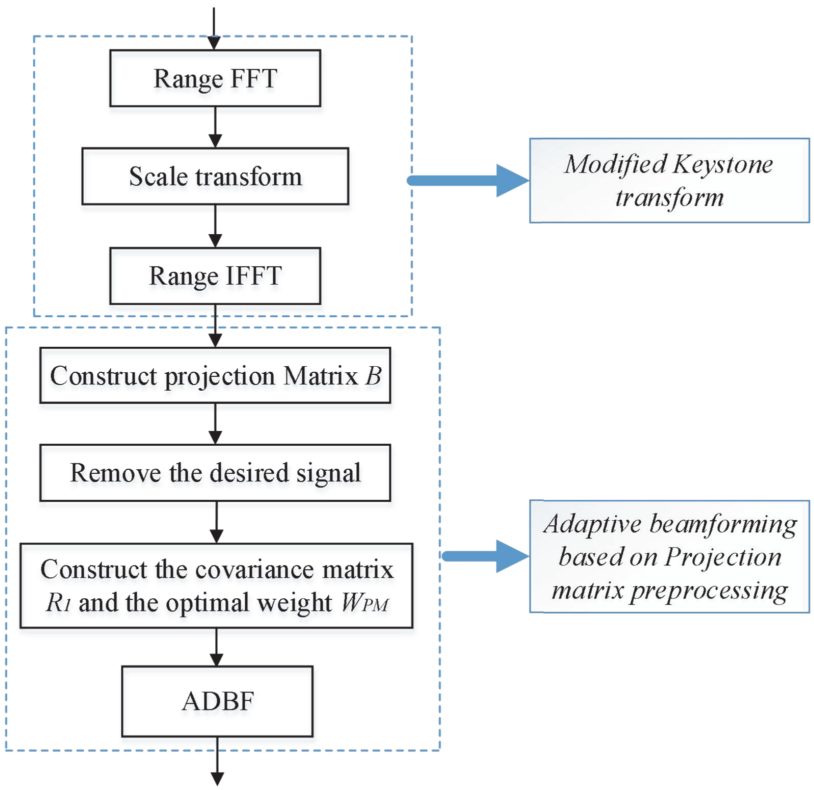


Figure 2. Flow chart of the proposed algorithm.

4. SIMULATION RESULTS

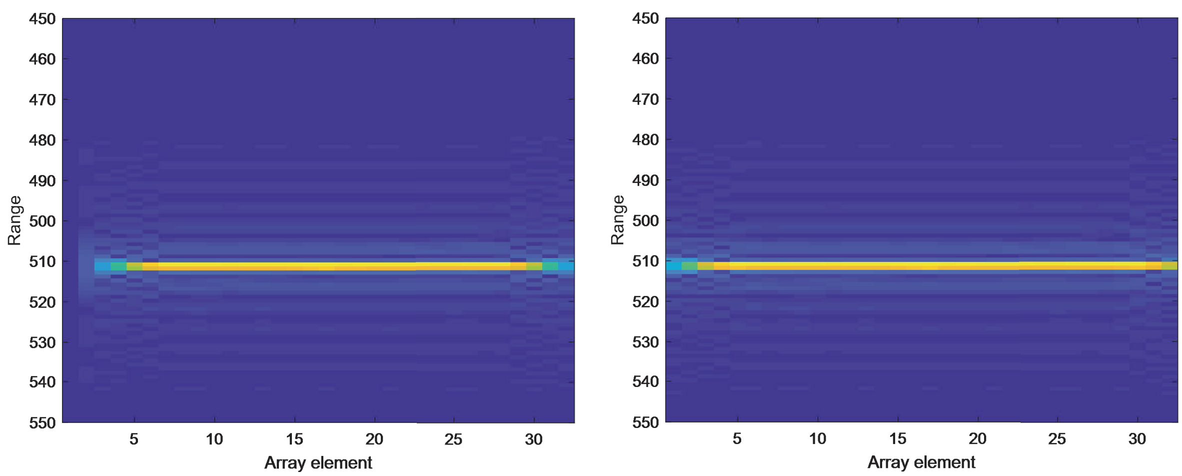
In this section, comparative simulation results are provided to verify the performance of the proposed algorithm. The ULA model and LFM signal as target signal are simulated in the experiment. The simulation parameters are listed as shown in Table 1.

The first experiment demonstrates the benefits of the modified Keystone transform. As illustrated in Fig. 3(a), the target signal is corrected into a horizontal straight line in the range-element dimension with at least 3 elements missing data at both ends of the array, but this situation is optimized in Fig. 3(b). Meanwhile, the signal amplitudes of the 3rd, 16th, and 30th elements are observed in Fig. 4. After the improved interpolation, the amplitude of the improved algorithm has been significantly increased. It is clear that the modified keystone transform can not only solve the problem of the aperture effect but also improve the phenomenon of insufficient data.

Based on the above simulation, the next experiment sets three wideband jamming signals as shown in Table 1. After the received signal is preprocessed by keystone transform following the processing chain descripted by Fig. 2, the covariance matrix is constructed in the time domain, and then the LCMV algorithm [18] is used to accomplish the process of beamforming. In order to verify the performance of the proposed method, LCMV algorithm and sliding window algorithm are also simulated to obtain the comparison results.

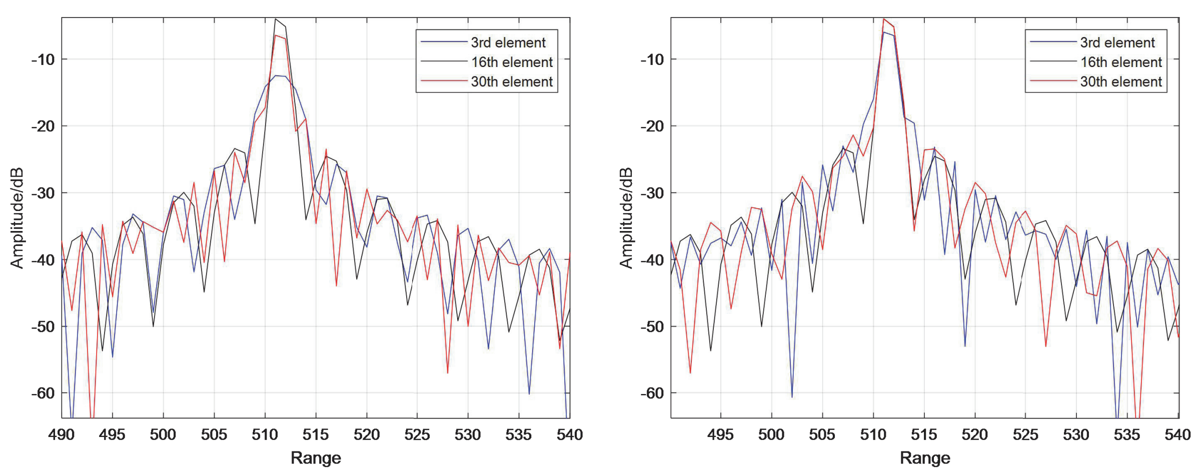
Table 1. Simulation parameter table.

Parameter	Value
Carrier frequency	100 MHz
Bandwidth	400 MHz
Sampling frequency	500 MHz
Number of array elements	32
Element spacing	0.125 m
Direction of the target source	-10°
Direction of the interference sources	-40° 20° 50°
Input SNR of each frequency point	$15 \sim 30$ dB
Input INR of each frequency point	30 dB



(a) (b)

Figure 3. Range-compressed target. (a) Conventional keystone transform. (b) Modified keystone transform.



(a) (b)

Figure 4. Amplitudes of different array elements after keystone transform. (a) Conventional keystone. (b) Modified keystone.

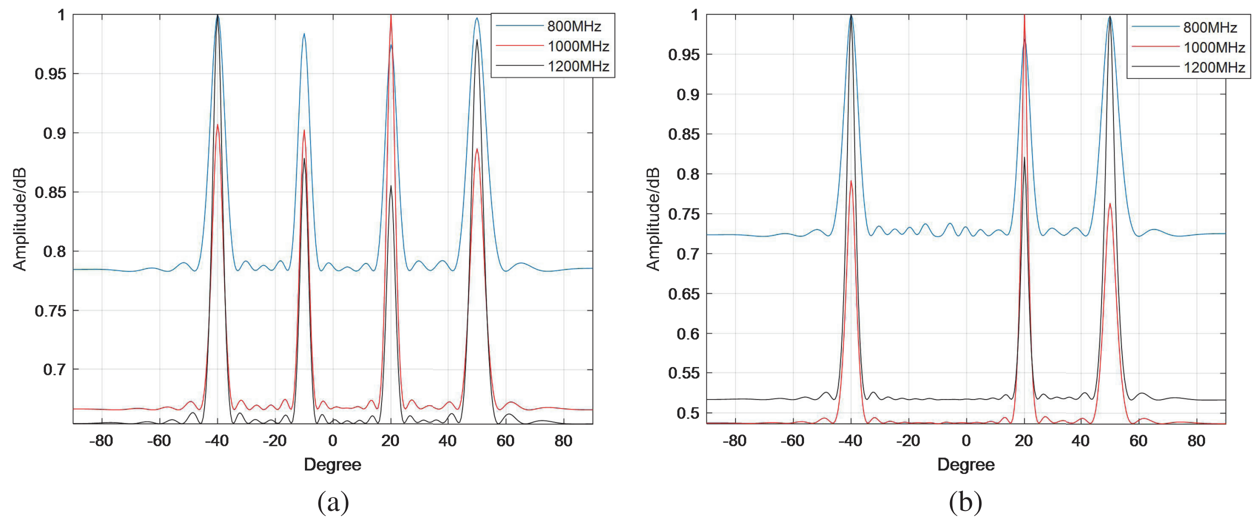


Figure 5. Estimated angles of the received signal. (a) Before being processed by projection matrix. (b) After being processed by projection matrix.

As illustrated in Fig. 5, it is clear that, after keystone transform, the angles of different frequency points can be correctly estimated. Meanwhile, since the DOA of the desired signal is not estimated in Fig. 5(b), the information of the target is eliminated by projection matrix effectively. Moreover, due to the random signals applied in the construction of jamming signals, the amplitudes of the DOA estimation are unequal in the simulations.

In order to obtain comparative results and demonstrate the validity of the proposed approach, the sample covariance matrices are estimated by using 300 range cells around the test cell in the time domain. Fig. 6 shows the results obtained by different ADBF schemes after being preprocessed by keystone transform, and Table 2 shows the output SINR of the three algorithms based on different input SNRs. When the interference covariance matrix in the method of sliding window is calculated, 20 range cells around the test cell are removed to eliminate the phenomenon of signal cancellation. As the input SNR increases shown in Fig. 6, it is clear that due to the problem of signal cancellation, the output SINR of the conventional LCMV algorithm does not increase, but even has a decreasing trend, hence the target signal with high amplitude will exacerbate the problem of signal cancellation. The output SINR of the sliding window algorithm, which has the best performance, theoretically, is linearly related with input SNR. As illustrated in Table 2, the method based on orthogonal projection matrix always has a similar output SINR to the sliding window. Therefore, the proposed algorithm can effectively remove the target information in training samples and achieve the best jamming suppression performance.

Table 2. Output SINR with input INR fixed at 30 dB.

Input SNR/dB	Output SINR/dB		
	LCMV	Sliding Window	The Proposed algorithm
15	27.6540	28.4549	28.3621
20	29.2531	33.2078	33.2021
25	29.0062	38.1033	38.0912
30	27.7780	43.0591	43.3524

Furthermore, when we perform the sliding window algorithm, due to the unknown distance of the target, it is necessary to perform sliding window processing on all range cells to obtain the location of the detection cell for the optimal weight vector. Suppose that the number of the array elements is Na and that the number of range cells is Nr, the conventional LCMV method

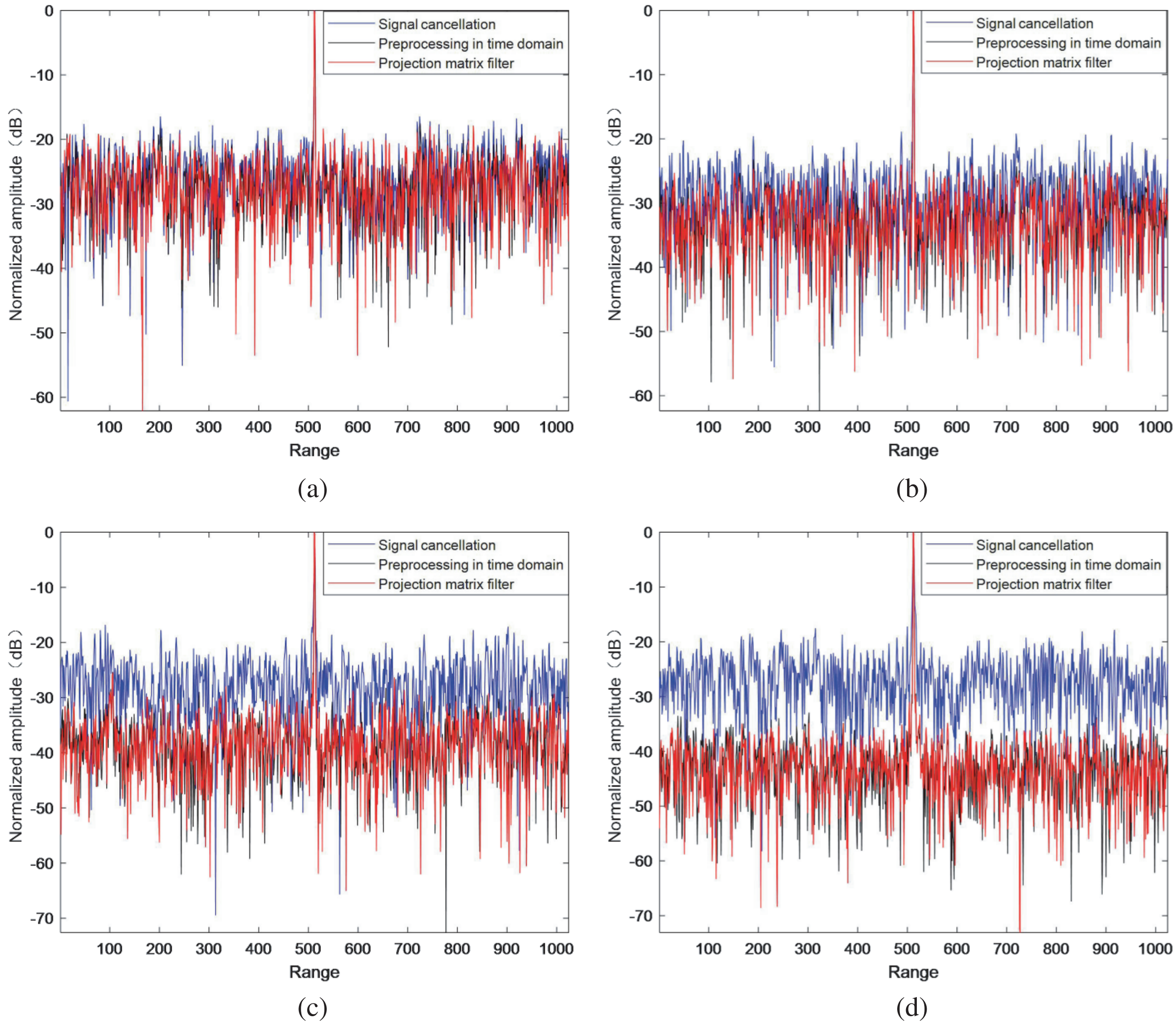


Figure 6. Normalized amplitude of different algorithms. (a) Input SNR = 15 dB. (b) Input SNR = 20 dB. (c) Input SNR = 25 dB. (d) Input SNR = 30 dB.

has a requirement of $O(Na^3 + Na^2 + 2Na)$ multiplication operations to obtain the adaptive weight. Thus the sliding window requires $O(Nr(Na^3 + Na^2 + 2Na))$ multiplication operations. By contrast, the proposed algorithm after being preprocessed by orthogonal projection matrix only requires $O(Na^3 + 2Na^2 + 4Na)$ multiplication operations. It is clear that the implementation of the approach based on orthogonal projection matrix yields the capabilities of suppressing jamming comparable to those of the sliding window algorithm, while the computational burden is effectively reduced.

5. CONCLUSION

This paper presents a new wideband array adaptive beamforming method based on modified keystone transform and orthogonal projection matrix to improve the performance of ADBF. According to the inherent characteristics of the two-dimensional data received by the wideband array, we first use the modified keystone transform to make the wideband equivalent to the narrowband thus eliminating the effect of aperture fill time. Furthermore, in order to solve the problem of signal cancellation, we construct the orthogonal projection matrix to remove the desired signal in the training samples which can reduce computational complexity significantly. The validity of the proposed algorithm is demonstrated by simulation results.

ACKNOWLEDGMENT

This work was supported in part by National Natural Science Foundation of China (No. 41830110, No. 61771182), and fundamental research funds for the central universities (No. B210202076).

REFERENCES

1. Wang, J., D.-D. Cai, and F. Yang, "Aperture effect influence and analysis of wideband phased array radar," *Procedia Engineering*, 2012.
2. Zhu, X. and Z. Kai, "A study on compensation of aperture fill time based onfrequency-shifting," *International Radar Conference*, IET, 2013.
3. Zhang, C. and Q. Lai, "Research on phased array radar affected by aperture fill time," *Journal of Microwave Science*, Vol. 33, No. 04, 67–69, 2017.
4. Frost, III, O. L., "An algorithm for linearly constrained adaptive array processing," *Proc. IEEE*, Vol. 60, No. 8, 926–935, 1972.
5. Hoffman, A. and S. M. Kogon, "Subband STAP in wideband radar systems," *Proceedings of the 2000 IEEE Sensor Array and Multichannel Signal Processing Workshop*, IEEE, 2000.
6. Zhang, P., Z. Yang, G. Jing, et al., "Adaptive beamforming via desired signal robust removal for interference-plus-noise covariance matrix reconstruction," *Circuits Systems* & *Signal Processing*, Vol. 40, No. 3, 2021.
7. Hu, P., M. Shen, C. Liang, et al., "An efficient broadband adaptive beamforming algorithm based on frequency-space cascade processing," *Circuits Systems* & *Signal Processing*, Vol. 37, No. 1, 1–12, 2017.
8. Bao, Z., M. Xing, and T. Wang, *Radar Imaging Technology*, Beijing Publishing House of Electronics Industry, 2005
9. Yi, H., C. Y. Fan, J. G. Yang, et al., "Imaging and locating multiple ground moving targets based on keystone transform and FrFT for single channel SAR system," *2nd Asian-Pacific Conference onSynthetic Aperture Radar, 2009, APSAR 2009*, 2009.
10. Jiao, Z. and Z. Wei, "A novel detection method based on generalized keystone transform and RFT for high-speed maneuvering target," *International Symposium on Computational Intelligence* & *Design*, IEEE, 2016.
11. Wang, N., M. Zhou, B. Zhong, et al., "Wideband signal DOA estimation method based on keystone transform," *The Journal of Engineering*, Vol. 4, 2019.
12. Zhang, N., X. Tang, and J. Tang, "Broadband beamforming method based on keystone transform pre-processing," *Journal of Tsinghua University (Science and Technology)*, Vol. 53, No. 7, 991–994, 2013.
13. Subbaram, H. and K. Abend, "Interference suppression via orthogonal projections: A performance analysis," *IEEE Sixth Sp Workshop on Statistical Signal* & *Array Processing*, IEEE Xplore, 1993.
14. Wang, Y., W. Sheng, and X. Chen, "A fast orthogonal projection beamforming algorithm for planar antenna arrays," *Radar Science and Technology*, Vol. 17, No. 3, 339–344, 2019.
15. Wang, J. and Y. Zhao, "Research on keystone transform implementation method," *Fire Control Radar Technology*, Vol. 40, No. 1, 45–51, 2011.
16. Chen, S., L. Huang, and L. Yu, "Variable PRF sampling spotlight SAR imaging based onimproved sinc interpolation," *Journal of Radars*, Vol. 8, No. 4, 527–536, 2019.
17. Shackelford, A. K., K. Gerlach, and S. D. Blunt, "Partially adaptive STAP using the FRACTA Algorithm," *IEEE Transactions on Aerospace* & *Electronic Systems*, Vol. 45, No. 1, 58–69, 2009.
18. Xu, Z. H., M. Z. Chen, and B. Rao, "The optimal LCMV beamformer under multiple desired signals case," *International Radar Conference*, IET, 2013.