Synthesis of Planar Array Antenna for Wireless Power Transmission

Hua Guo*, **Huiliang Hao, Peng Song, Lijian Zhang, and Xiaodan Zhang**

Abstract—A newly synthesis method of planar array antenna for wireless power transmission (WPT) is introduced in this paper. The whole array aperture is divided into several subarrays which can reduce the complexity of the feed network and the cost of the array antenna. Invasive Weed Optimization (IWO) algorithm is used to optimize the subarray division and the excitation amplitude of each subarray. The maximum beam collection efficiency (BCE) and maximum sidelobe level outside the receiving area(CSL) are considered as the evaluation index. The synthesis results show that the proposed method can obtain higher BCE and lower CSL.

1. INTRODUCTION

Wireless power transfer technology (WPT) has a wide range of applications in many fields [1]. Long-range wireless power transmission is mainly applied to space solar satellites and remote desert areas for energy supply, while close-range wireless power transmission is mainly for wireless energy charging. Based on different energy transmission carriers, long-distance wireless energy transmission can be divided into microwave wireless energy transmission (MPT) and laser wireless energy transmission (LPT) [2]. As an energy transmission method, WPT system pursues higher energy transmission efficiency which generally requires BCE of more than 90% [3–5].

In order to improve the energy collection efficiency of WPT, the transmitting antenna needs to be designed to radiate as much energy as possible. BCE is defined as the ratio of the power collected by the receiving antenna to the power transmitted by the transmitting antenna. Many methods have been developed to improve the BEC of the transmitting array antenna. In [6], continuous aperture and array antennas have been extensively studied to maximize the BCE. The results show that uniformly distributed (UD) arrays are widely used due to their simple structure and easy maintenance. Amplitude tapering techniques and Gaussian amplitude distribution (GAD) have been used to obtain optimal distribution [7, 8]. In recent years, planar array antennas' maximum beam collection efficiency and the corresponding excitation coefficients can be obtained by solving the matrix's maximum generalized eigenvalues [9]. However, the array antenna is complex and expensive because each array element needs an amplifier. In order to reduce the complex of the antenna array, stepped amplitude distribution (SAD) is proposed [10–12]. Moreover, regular fully populated transmitting arrays need a large number of array elements which could increase the cost of WPT systems [13]. To reduce the number of array elements, the synthesis of a sparse uniform amplitude concentric ring array (SUACRA) for MPT is discussed in [14, 15]. Another new method based on subarray division is investigated in [16–18]. The complexity and cost of the array antennas are further reduced. The whole array aperture is divided into several subarrays, and each subarray has the same excitation amplitude which can greatly reduce the number of the amplifiers. Also, intelligent optimization algorithms, such as genetic algorithm (GA) and differential evolution algorithm (DE), are widely used in array antenna synthesis problems [19–21].

A synthesis method of planararray antenna for WPT is introduced in this paper. The whole array antenna is divided into several subarrays, and IWO is used to optimize the amplitude of each subarray to improve the BCE and reduce the CSL of radiation pattern of the antenna arrays. The rest of this paper is organized as follows. The optimization model of the array antenna is given in Section 2. The numerical analysis and discussion are provided in Section 3. Finally, conclusions are given in Section 4.

2. OPTIMIZATION MODEL

2.1. Array Structure

The planar array antenna of rectangular grid and circular boundary is depicted in Fig. 1. The radius of the array antenna is R. The array elements are located in the xoy plane, and the array elements are uniformly arranged in the x-axis and y-axis directions with the spacings of dx and dy. The position of array element is axisymmetric about x and y-axes. The number of array elements in the first quadrant is N. Since all the array elements are axisymmetric about the x and y-axes, the array factor of the array antenna can be given by [9]:

$$AF(u, v) = 4 \sum_{n=1}^{N} I_n \cos(kx_n u) \cos(ky_n v) \tag{1}$$

where I_n is the excitation coefficient of the nth arrayelement. The wave number is $k = 2\pi/\lambda$, λ the wavelength, and (x_n, y_n) the position of the nth element. $u = \sin\theta\cos\phi$, $v = \sin\theta\sin\phi$ are the direction cosines. $\theta \in [0, \pi/2]$ and $\phi \in [0, 2\pi]$ are the elevation angle and azimuth angle.
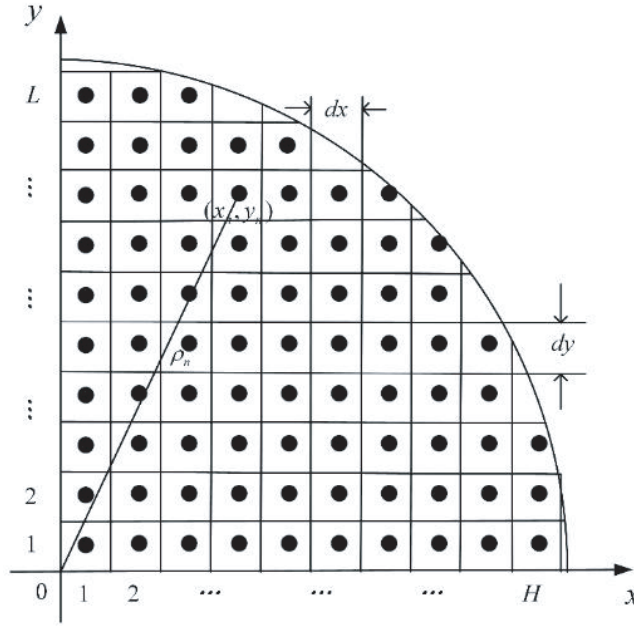

Figure 1. Structure of the array antenna.

Assume that the array antenna is divided into several subarrays and that each subarray has the same excitation coefficient. The subarray structure of the array antenna is shown at Fig. 2. Q is the number of subarrays. w_q and N_q are the excitation amplitude and element number of the qth subarray.

The relationship between the excitation amplitude of the array element and the excitation amplitude of each subarray can be formulated as:

$$I_n = \sum_{q=1}^{Q} \delta_{c_n q} w_q \tag{2}$$
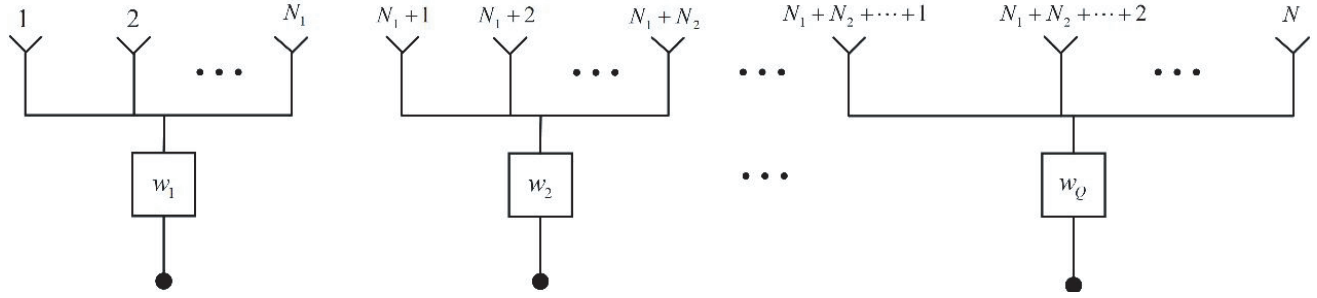
Figure 2. Configuration of sub-arrayed antenna array.

where $c_n \in [1, Q]$, $n = 1, 2, \ldots, N$ identifies the relationship between the nth array element and the qth subarray. $\delta_{c_n q}$ is the Kronecker delta function which can be given by:

$$\delta c_n q = \begin{cases} 1, & if \ c_n = q \\ 0, & otherwise \end{cases} \tag{3}$$

By substituting (2) into (1), the array factor can be rewritten as:

$$AF(u, v) = 4 \sum_{q=1}^{Q} w_q \sum_{n=1}^{N} \delta_{c_n q} \cos(k x_n u) \cos(k y_n v) \tag{4}$$

The corresponding power pattern can be formulated as:

$$P(u, v) = \left| AF(u, v)^2 \right| \tag{5}$$

BCE is used to evaluate the optimization effect. Assuming that the rectifying antenna is in the far-field region and has a circular shape, BCE can be written as [9]:

$$BCE = \frac{P_\psi}{P_\Omega} = \frac{\displaystyle\int_\psi |AF(u, v)|^2 \, dudv}{\displaystyle\int_\Omega |AF(u, v)|^2 \, dudv} \tag{6}$$

where P_ψ and P_Ω denote the power radiated over the angular region ψ and the total transmitted power over the visible region Ω, respectively. For a circular collection region, it can be expressed by:

$$\psi = \left\{ (u, v) : \sqrt{u^2 + v^2} = \sin \theta_0 \le r_0 \right\} \tag{7}$$

where r_0 denotes the radius of the rectenna in the u, v coordinate, and θ_0 is the inception angle of the rectenna.

Another evaluation index used in this paper is CSL, which is defined as the maximum side-lobe level outside the receiving area ψ. According to [10], CSL (dB) can be written as:

$$CSL(dB) = 10 \lg \frac{\max_{\theta, \phi \notin \psi} |F(\theta, \phi)|^2}{\max_{\theta, \phi \in \Omega} |F(\theta, \phi)|^2} \tag{8}$$

Taking maximizing BCE and minimizing CSL as optimization objective, the optimization model can be expressed as:

$$\begin{cases} \max\{BCE\} \ and \ \min\{CSL\} \\ \text{subject to} \ 0 < w_q \le 1, \ w_q \in R, \ q = 1, 2, \ldots, Q \\ \qquad\qquad 1 \le c_n \le Q, \ c_n \in Z, \ n = 1, 2, \ldots, N \end{cases} \tag{9}$$

So, the fitness function for this optimization problem can be given by:

$$f(w_1, \ldots, w_q, c_1, \ldots, c_N) = \frac{\beta_1}{BCE} + \frac{\beta_2}{|CSL|} \tag{10}$$

where β_1 and β_2 are weight coefficients.

2.2. Optimistic Method

As given in Fig. 1, the distance between the nth element and the original point can be calculated by:

$$\rho_n = \sqrt{x_n^2 + y_n^2} \tag{11}$$

As given in Fig. 3, the array elements that have the same ρ_n are placed on the same circle. The number of circles is M, and the radius of mth circle is R_m, $m = 1, 2, \ldots, M$. Then, M circles are divided into Q subarrays. The boundary of the subarray and the excitation amplitude of each subarray are taken as the optimization parameters. As $w_1 = 1$ is determined, $Q - 1$ excitation amplitudes should be optimized. As shown in Fig. 4, the boundary of the subarray is depicted as ρ_q', $q = 0, 1, \ldots, Q$. As
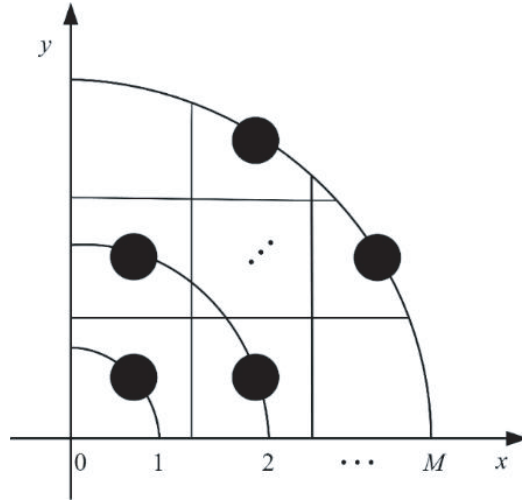
Figure 3. Subarray structure of the array antenna.
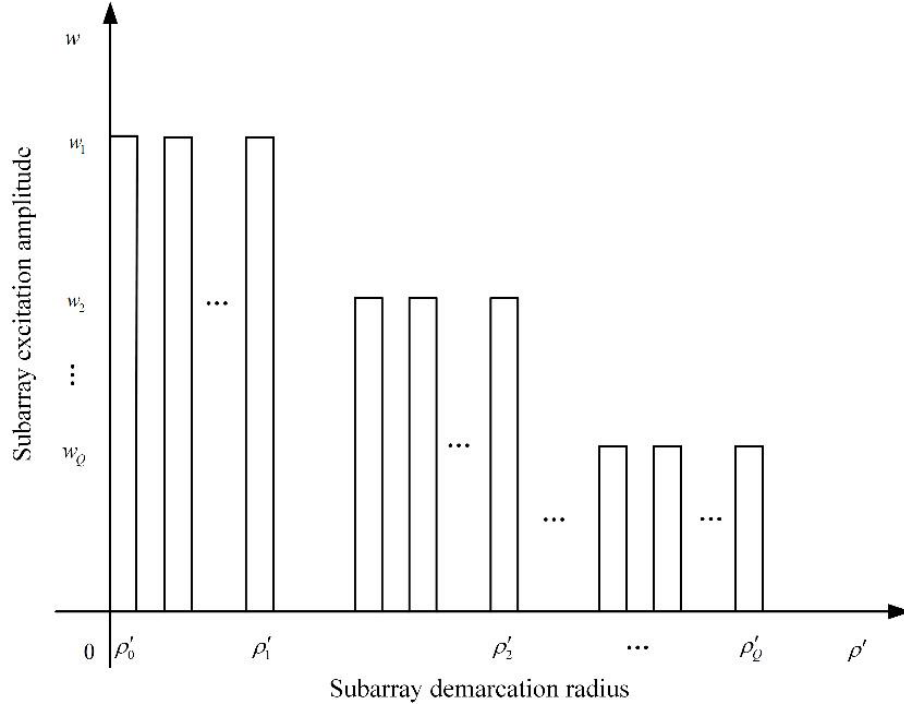
Figure 4. Excitation amplitude of the subarray.

$\rho'_0=0$ and $\rho'_Q = R$ are determined, there are $Q - 1$ boundaries to be determined. So, the number of the optimization parameters is $2Q - 2$.

Let $r_i \in [0, 1]$, $i = 1, 2, \ldots, 2Q - 2$ as optimization variables. The first $Q - 1$ optimization variables are used to optimize the excitation amplitude. The excitation amplitude of each subarray is $w_1 = 1$, $w_{q+1} = r_q$, $q = 1, 2, \ldots, Q - 1$. The rest $Q - 1$ optimization variables are used to determine the boundary of each subarray. The boundary of each subarray can be defined by:

$$\rho'_q = \begin{cases} 0, & if \quad q = 0 \\ R_{M_q}, & if \quad q = 1, 2, \ldots, Q - 1 \\ R, & if \quad q = Q \end{cases} \tag{12}$$

$M_q \in [1, M)$, $q = 1, 2, \ldots, Q - 1$ is an integer that depicts the boundary is the M_qth circle which can be defined in Fig. 3. The boundary can be determined when M_q is optimized.

Two different boundaries cannot overlap which means that $M_q - M_{q-1} \geq 1$, $q = 2, 3, \ldots, Q$, then the circle number that occupies is $Q - 1$. The remaining number of circles available for optimization is:

$$K = M - Q + 1 \tag{13}$$

Let $r'_i = r_{i+q-1}$, then $Q - 1$ random integer number that in the range of $[0, K]$ can be calculated by:

$$c'_i = round\left\{(M - Q + 1) \times r'_i\right\}, \quad i = 1, 2, \ldots, Q - 1 \tag{14}$$

where $round$ is a rounding function. Then, c'_i is arranged from small to large to get a new random vector $c_i = [c_1, c_2, \ldots, c_{Q-1}]^T$, where $c_1 \leq c_2 \leq, \ldots, \leq c_{Q-1}$, then, each subarray demarcation number can be calculated by:

$$\begin{bmatrix} M_1 \\ M_2 \\ \ldots \\ M_{Q-1} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ \ldots \\ Q - 1 \end{bmatrix} + \begin{bmatrix} c_1 \\ c_2 \\ \ldots \\ c_{Q-1} \end{bmatrix} = \begin{bmatrix} c_1 + 1 \\ c_2 + 2 \\ \ldots \\ c_{Q-1} + Q - 1 \end{bmatrix} \tag{15}$$

After each subarray demarcation number is determined, the boundary of each subarray can be calculated by (12). The subarray division and the weight of each subarray can be determined by:

$$\begin{cases} \delta_{c_n q} = 1, \quad w_q = r_q \\ if \quad \rho'_{q-1} < \rho_n \leq \rho'_q \\ \quad q = 1, 2, \ldots, Q \\ \quad n = 1, 2, \ldots, N \end{cases} \tag{16}$$

2.3. Optimization Steps

IWO is an evolutionary algorithm which has been widely used in the synthesis of array antenna. The synthesis problem of maximizing BCE and minimizing CSL by using IWO algorithm is introduced in this paper. The algorithm procedure is shown as follows.

Step 1. A $(2Q - 2) \times P$ matrix r among the range of $[0, 1]$ is generated as the initial population which can be given by:

$$r = \begin{bmatrix} r_1^1 & r_2^1 & \cdots & r_{2Q-2}^1 \\ r_1^2 & r_2^2 & \cdots & r_{2Q-2}^2 \\ \cdots & \cdots & \cdots & \cdots \\ r_1^P & r_2^P & \cdots & r_{2Q-2}^P \end{bmatrix} \tag{17}$$

The matrix r is depicted by $r = [r^1, r^2, \ldots, r^P]$ and $r^p = [r_1^p, r_2^p, \ldots, r_{2Q-2}^p]^T$. The element of the matrix is given by $r_i^p \in [0, 1]$, $i = 1, 2, \ldots, 2Q - 2$, $p = 1, 2 \ldots, P$.

Step 2. The excitation amplitude and demarcation radius of the subarray are determined by (12) and (16), respectively.

Step 3. Calculate the fitness value by using (10). The population that can lead to the maximum BCE and the minimum CSL is maintained as the final result.

Step 4. Updating optimization variable r^p by IWO algorithm.

Step 5. Let $iter = iter+1$, if $iter < iter\_\max$, return to **step 2**, otherwise, terminate the iteration and output the result.

In order to describe the algorithm more clearly, the optimization process of the proposed algorithm is shown in Fig. 5.
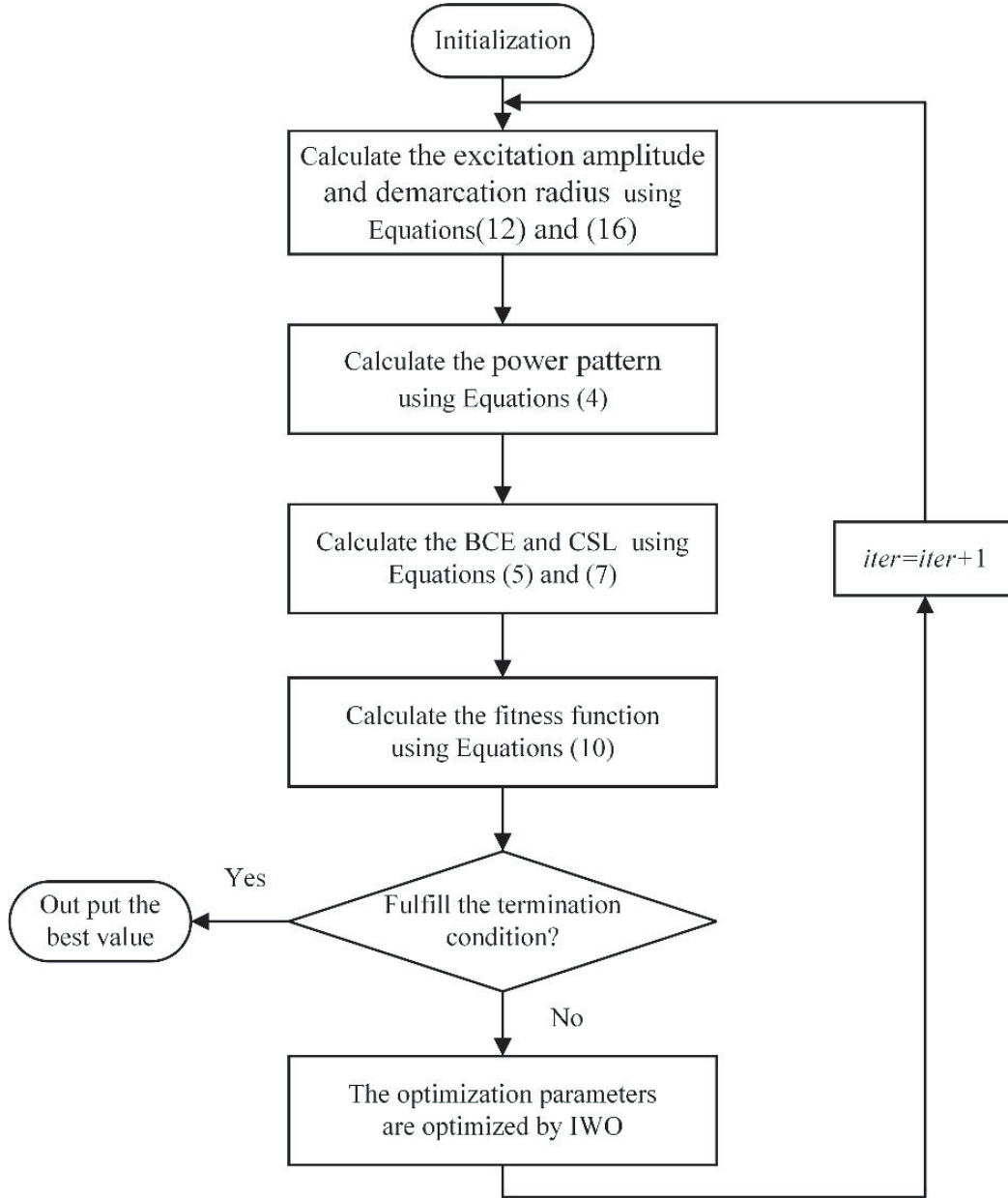
Figure 5. Algorithm processes of the proposed algorithm.

3. NUMERICAL ANALYSIS AND DISCUSSION

In this section, several optimization results are given. The array elements are symmetrical about x and y-axes. The array size in the first quadrant is $R = 5\lambda$. The uniform spacing along the x and y-axes is $dx = dy = 0.5\lambda$. The parameter values of IWO are given in Table 1.
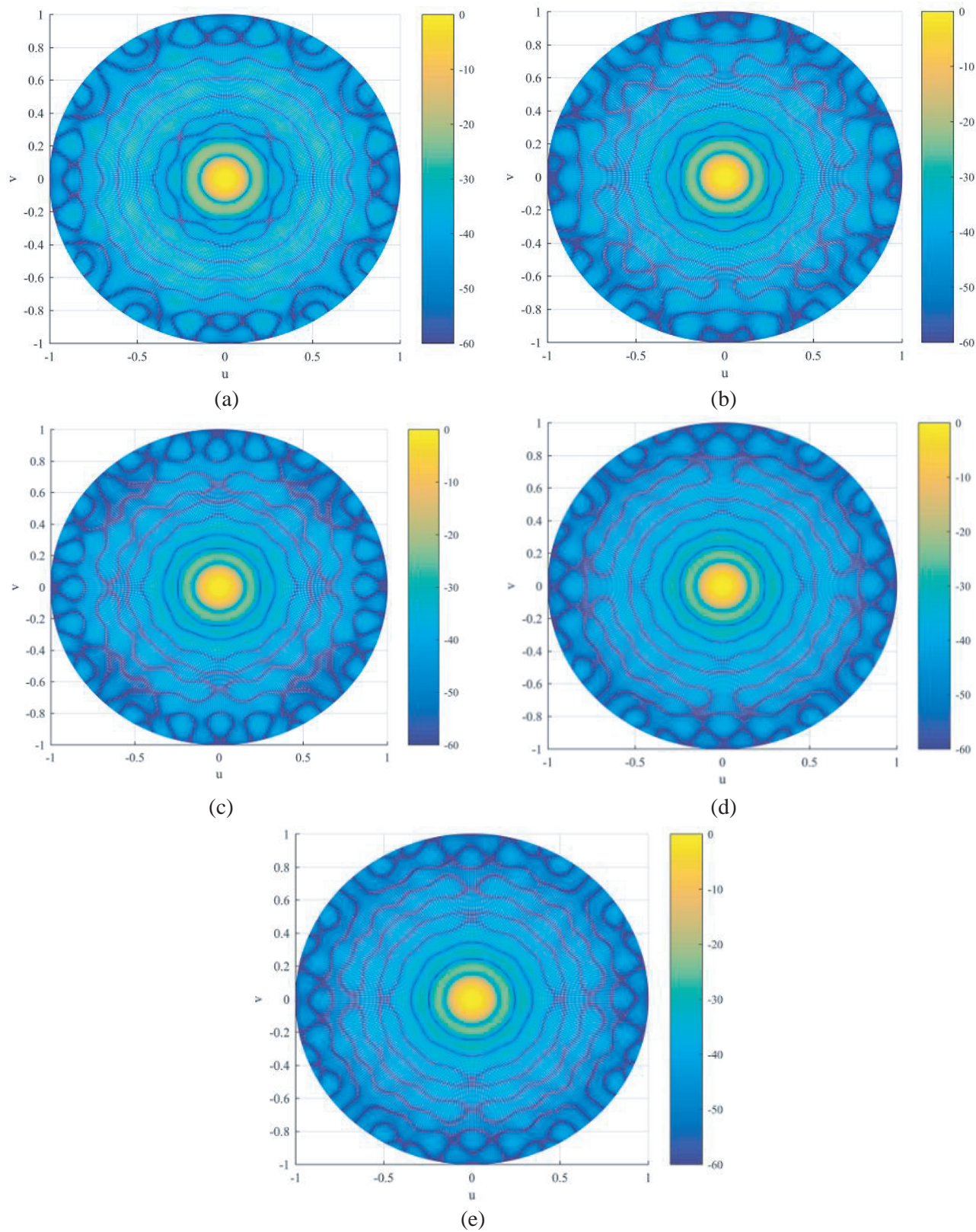
Figure 6. Normalized 3-D power patterns of Simulation A for (a) 2 subarrays, (b) 4 subarrays, (c) 6 subarrays, (d) 8 subarrays, (e) 10 subarrays.

Table 1. Parameters of IWO.

$iter\_max$	P_MAX	S_{max}	S_{min}	n	P	σ_{max}	σ_{min}
50	25	10	1	3	12	0.1	10^{-5}

3.1. Simulation A: Only BCE Is Considered ($\beta_1 = 30, \beta_2 = 0$)

In this example, the number of array elements is 316. So, there are $N = 79$ array elements in the first quadrant. The fitness function is given in Equation (10) to enhance BCE. Fig. 6 shows the radiated power pattern. The radiated power is mainly concentrated in the receiving area shown by the yellow area while a small amount of power is "lost" in other directions. Table 2 shows the optimized boundary and excitation amplitude of each subarray. Table 3 gives the BCE of the array antenna. When the number of the subarrays is 2, 4, 6, 8, 10, the BCEs of the antenna array are 89.59%, 92.15%, 92.78%, 93.04%, 93.11%. Compared with [10], the BCE obtained in this paper can be improved by at least 0.46%. In order to show the effectiveness of the proposed algorithm, the synthesis result is compared with other methods which are given in Table 4. The number of subarrays is chosen as 10. In [6], the whole array antenna is uniformly excited. The BCE of the array antenna is 83.46%. In [8], the excitation amplitude is Gauss distribution, and the BEC is 92.31%. When the excitation amplitude of each array element is optimized, the BCE is 93.23 [9]. From Table 4, it can be seen that the synthesis result is better than the results optimized in [6] and [8]. The BCE obtained in this paper is lower than the result obtained in [9]. In [9], each array element must have an amplifier which can improve the complexity of the array antenna. However, the method proposed in this paper requires only 10 amplifiers which can

Table 2. Boundary and excitation amplitude for each subarray of Simulation A.

| Q	Boundary of each subarray ρ'_q	Subarray excitation amplitude w_q
2	$[0, 3.3364, 5]$	$[1, 0.5172]$
4	$[0, 2.5749, 3.3364, 4.3742, 5]$	$[1, 0.7600, 0.5440, 0.3437]$
6	$[0, 1.9049, 2.5749, 3.3364, 3.9538, 4.4310, 5]$	$[1, 0.8629, 0.7130, 0.5461, 0.4362, 0.2928]$
8	$[0, 1.4587, 2.3727, 3.1830, 3.3364,$ $3.9538, 4.3742, 4.6513, 5]$	$[1, 0.8713, 0.7307, 0.6425,$ $0.5329, 0.4195, 0.3448, 0.2606]$
10	$[0, 1.4587, 2.1516, 2.3727, 3.0218, 3.3364,$ $3.6922, 3.9538, 4.3742, 4.6513, 5]$	$[1, 0.8999, 0.8399, 0.7380, 0.6500, 0.5574,$ $0.5205, 0.4217, 0.3390, 0.2622]$

Table 3. Results of maximum BCE in Simulation A.

Q	Ref. [10] BCE(%)	This method BCE (%)
2	89.13	89.59
4	90.69	92.15
6	92.13	92.78
8	92.49	93.04
10	92.51	93.11

Table 4. The BCE results in Simulation A comparison with other methods.

	Ref. [6]	Ref. [8]	Ref. [9]	Ref. [10]	This method
Maximize BCE	83.46%	92.31%	93.23%	92.51%	93.11%
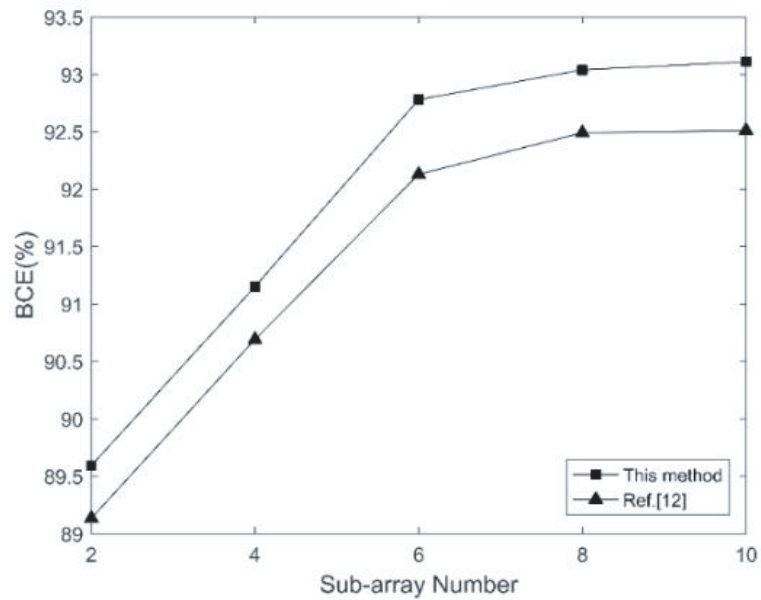
**Figure 7.** Comparison of maximum BCE in Simulation A.

greatly reduce the engineering implementation difficulty and cost of the array antenna [10]. Fig. 7 gives the BECs of different subarrays. It can be seen with the increase of the number of subarrays, the BCE is also increased. The structure of the subarray division after optimization is given in Fig. 8. Table 5 gives the number of elements of each subarray after the subarray division.

**Table 5.** Number of subarray elements in Simulation A.

Q	Subarray elements									
	Sub1	Sub2	Sub3	Sub4	Sub 5	Sub6	Sub7	Sub8	Sub9	Sub10
2	37	42	–	–	–	–	–	–	–	–
4	22	15	25	17	–	–	–	–	–	–
6	13	9	15	15	12	15	–	–	–	–
8	8	11	12	6	15	10	7	10	–	–
10	8	7	4	11	7	6	9	10	7	10

Table 6. Boundary and excitation amplitude for each subarray of Simulation B.

Q	Boundary of each subarray ρ'_q	Subarray excitation amplitude w_q
2	$[0, 3.3364, 5]$	$[1, 0.5827]$
4	$[0, 2.2648, 3.3364, 4.3742, 5]$	$[1, 0.8108, 0.6138, 0.4222]$
6	$[0, 2.2648, 2.5750, 3.3364, 3.9538, 4.5972, 5]$	$[1, 0.8615, 0.7910, 0.6416, 0.5211, 0.3737]$
8	$[0, 1.4587, 2.2648, 2.8514, 3.2606,$ $3.8253, 4.3742, 4.5972, 5]$	$[1, 0.9016, 0.7886, 0.7260, 0.6184,$ $0.5501, 0.4485, 0.3472]$
10	$[0, 1.4587, 2.1516, 2.3727, 3.1830,$ $3.3364, 3.7593, 3.9538, 4.3742, 4.5972, 5]$	$[1, 0.9101, 0.8592, 0.7770, 0.7179,$ $0.6172, 0.5950, 0.5237, 0.4376, 0.3530]$
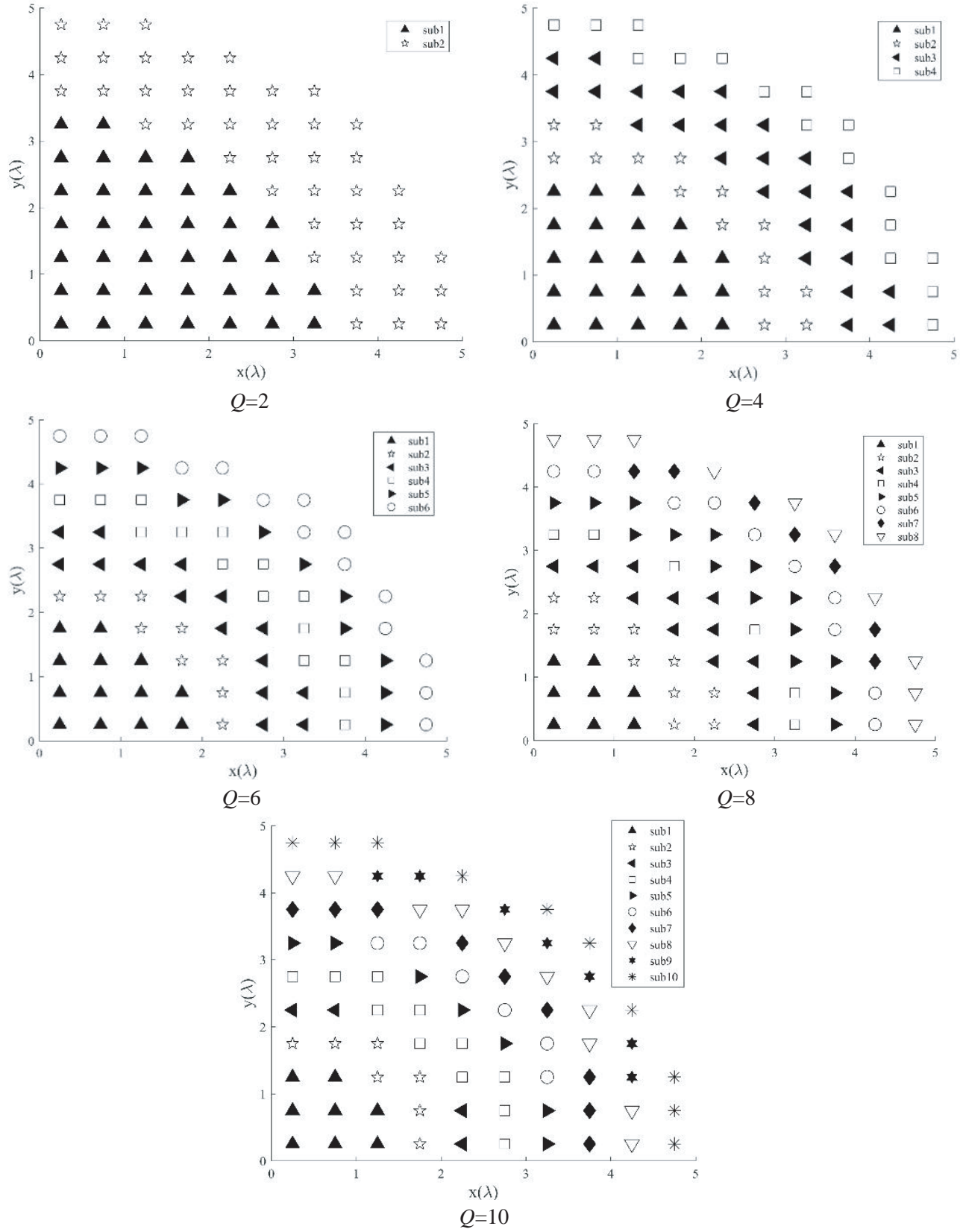
Figure 8. Subarray structure with different number of subarrays in Simulation A.
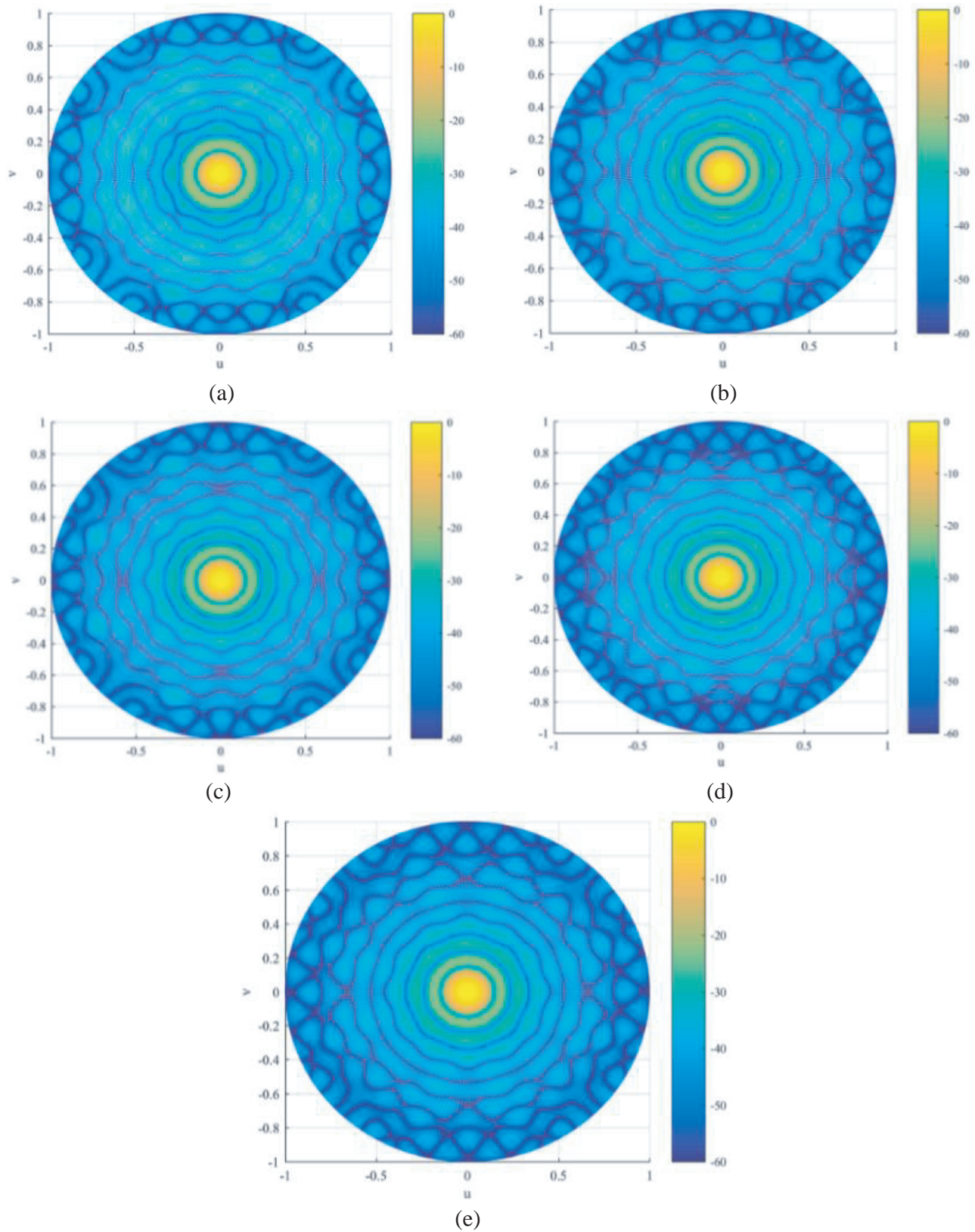
Figure 9. Normalized 3-D power patterns of Simulation B for (a) 2 subarrays, (b) 4 subarrays, (c) 6 subarrays, (d) 8 subarrays, (e) 10 subarrays.

3.2. Simulation B: Both BCE and CSL Are Considered ($\beta_1 = 30, \beta_2 = 45$)

In this example, BEC and CSL are both considered where β_1=30, β_2=45 in Equation (10). The size of the array aperture, array element number, spacing of adjacent array elements, fitness function, and IWO parameters are the same as those of Simulation A. Table 6 shows the optimized boundary and excitation amplitude of each subarray. Fig. 9 gives the power pattern of the best synthesis result in different subarrays. Table 7 gives the BCE and CSL of the array antenna. When the number of subarrays is 2, 4, 6, 8, 10, the BCEs of the antenna array are 89.34%, 91.74%, 92.30%, 92.50%, 92.60%, and the CSLs are -12.98 dB, -12.34 dB, -12.08 dB, -11.99 dB, -11.97 dB. Compared with [10], the BCE obtained in this paper can be improved by at most 1.05%, and the CSL can be reduced by at most 0.41 dB. Table 8 gives the synthesis results compared with other methods. The number of the
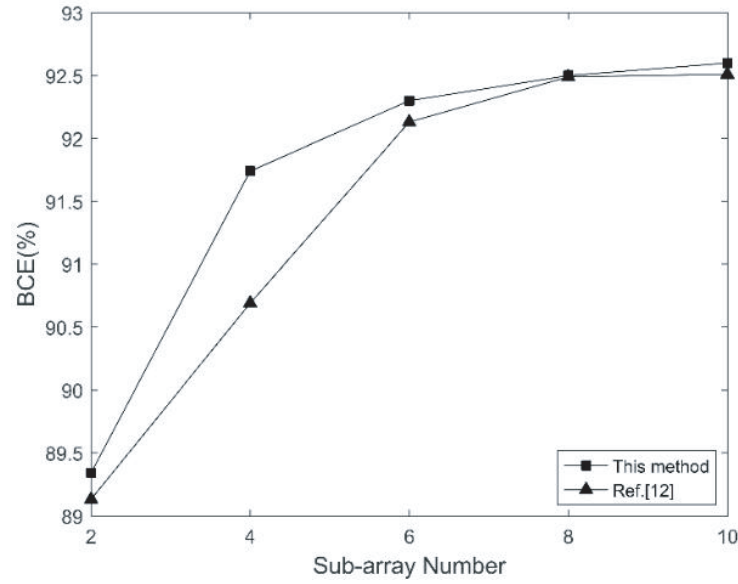

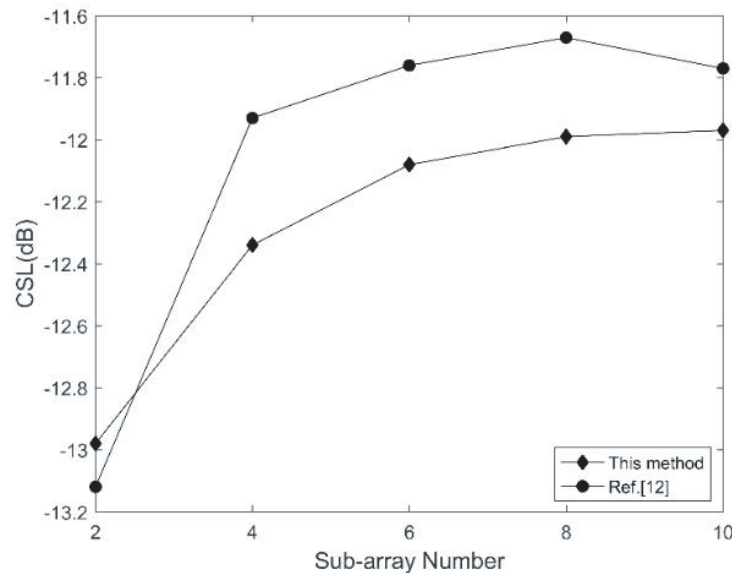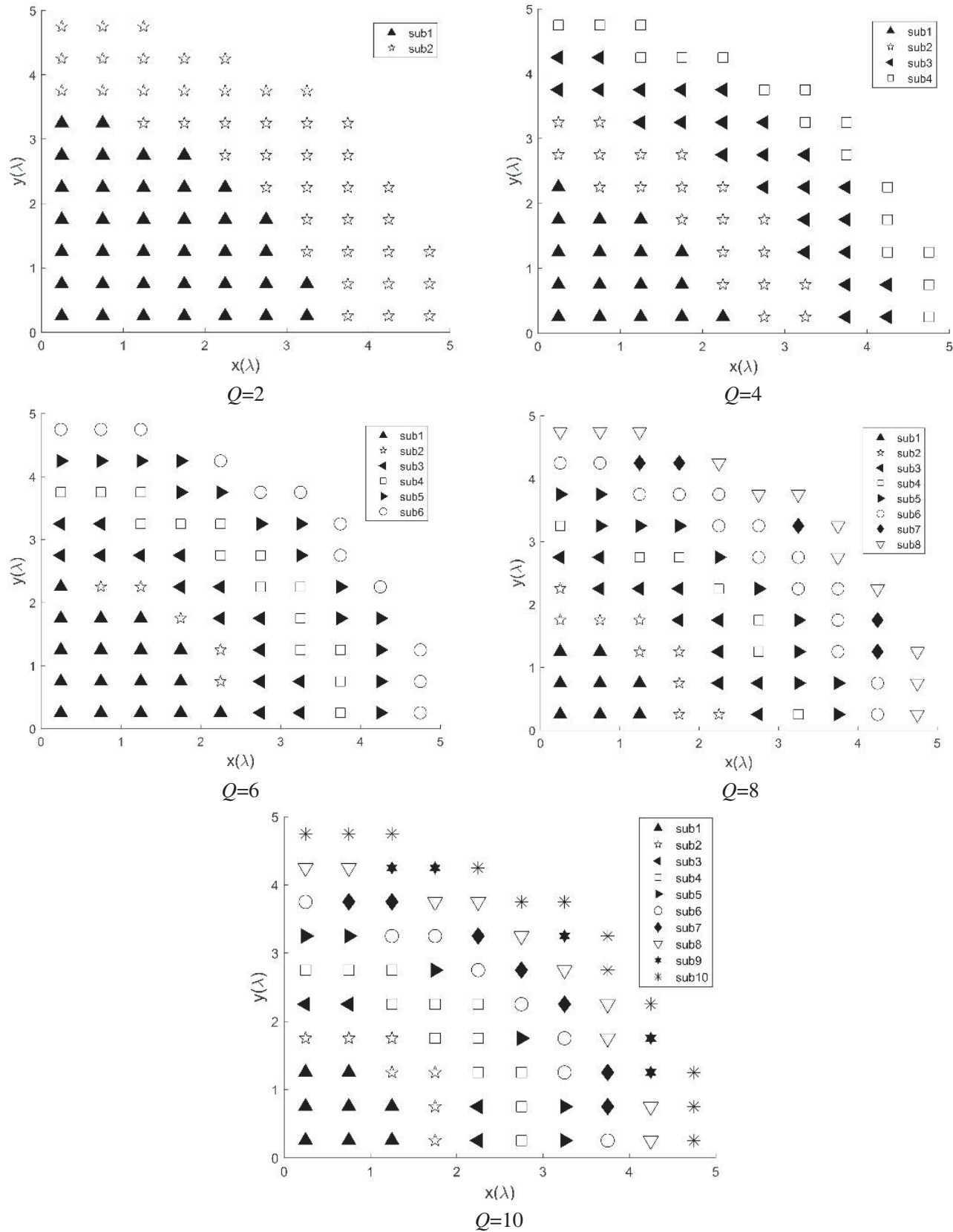**Figure 10.** Comparison of maximum BCE in Simulation B.

Figure 11. Comparison of CSL in Simulation B.

Figure 12. Subarray structure with different number of subarrays in Simulation B.

subarrays is chosen as 10. In [6], the whole array antenna is uniformly excited. The BCE of the array antenna is 83.46%. In [8], the excitation amplitude is Gaussian distributed with a BEC of 92.31% and CSL of -12.18 dB. The CSL obtained by the proposed method is 0.21 dB higher. However, the BCE can be improved by 0.29%. When the excitation amplitude of each array element is optimized, the BCE is 93.23%, and the CSL is -11.72 dB [9]. It is observed that the BCE obtained by the proposed method is 0.63% lower. Nevertheless, the CSL can be suppressed by 0.25 dB. From Table 8, it can be seen that the synthesis results are better or partially better than the other results. Fig. 10 and Fig. 11 give the BEC and CSL of different subarrays. It can be seen that the BCE increases, and the CSL decreases as the number of subarrays increases. Fig. 12 gives the structure of the subarray partitioning after optimization. The element number of each subarray is given in Table 9.

Table 7. BCE and CSL results of Simulation B.

Q	Ref. [10] BCE (%)	This method BCE (%)	Ref. [10] CSL (dB)	This method CSL (dB)
2	89.13	89.34	-13.12	-12.98
4	90.69	91.74	-11.93	-12.34
6	92.13	92.30	-11.76	-12.08
8	92.49	92.50	-11.67	-11.99
10	92.51	92.60	-11.77	-11.97

Table 8. The BCE and CSL result in Simulation B comparison with other methods.

	Ref. [6]	Ref. [8]	Ref. [9]	Ref. [10]	This method
Maximize BCE	83.46%	92.31%	93.23%	92.51%	92.60%
CSL	–	-12.18 dB	-11.72 dB	-11.77 dB	-11.97 dB

Table 9. Number of subarray elements in Simulation B.

Q	subarray elements									
	Sub1	Sub2	Sub3	Sub4	Sub 5	Sub6	Sub7	Sub8	Sub9	Sub10
2	37	42	–	–	–	–	–	–	–	–
4	17	20	25	17	–	–	–	–	–	–
6	17	5	15	15	15	12	–	–	–	–
8	8	9	11	7	12	15	5	12	–	–
10	8	7	4	12	6	8	7	10	5	12

4. CONCLUSION

In this paper, a new synthesis method of planar array antenna for WPT systems is introduced. The whole array aperture is divided into several subarrays according to the distance between the array element and the center of the array antenna which can reduce the complexity of the feed network and the cost of the array antenna. IWO algorithm is used to optimize the subarray division and the excitation amplitude of each subarray. The subarray division method introduced in this paper can avoid the overlap of the adjacent subarrays effectively. Also, as only the subarray boundary and the excitation amplitude of each subarray are optimized. the number of optimization parameters can be reduced greatly which can reduce the complexity of the algorithm. The synthesis results show the effectiveness of the proposed method. Compared with other synthesis methods, the method introduced in this paper can obtain better or partially better results.

ACKNOWLEDGMENT

REFERENCES

1. Mirbozorgi, S. A., H. Bahrami, M. Sawan, et al., "A smart multicoil inductively coupled array for wireless power transmission," *IEEE Transactions on Industrial Electronics*, Vol. 61, No. 11, 6061–6070, 2014.

2. Massa, A., G. Oliveri, F. Viani, et al., "Array designs for long-distance wireless power transmission — State-of-the-art and innovative solutions," *Proceedings of the IEEE*, Vol. 101, No. 6, 1464–1481, 2013.

3. Zhu, L., Z. Chen, Y. Guo, et al., "A novel multi-coil magnetically-coupled resonance array for wireless power transfer system," *2016 IEEE Wireless Power Transfer Conference (WPTC)*, 2016.

4. Shi, X. and J. R. Smith, "Reconfigurable and adaptive coupled relay resonator platform for a moving receiver," *2019 International Workshop on Antenna Technology (iWAT)*, 2019.

5. Morita, M., T. Sasatani, R. Takahashi, et al., "Surface routing for wireless power transfer using 2-D relay resonator arrays," *IEEE Access*, Vol. 9, 133102–133110, 2021.

6. Baki, A., K. Hashimoto, N. Shinohara, et al., "Isosceles-trapezoidal-distribution edge tapered array antenna with unequal element spacing for solar power satellite," *IEICE Transactions on Communications*, Vol. E91-B, No. 2, 527–535, 2008.

7. Blank, S. J. and M. F. Hutt, "Transmitter illumination taper as a design parameter for wireless power transmission systems," *2012 IEEE/MTT-S International Microwave Symposium Digest*, Montreal, QC, Canada, 2012.

8. Takeshita, S., "Power transfer efficiency between focused circular antennas with Gaussian illumination in Fresnel region," *IEEE Transactions on Antennas and Propagation*, Vol. 16, No. 3, 305–309, 1968.

9. Oliveri, G., L. Poli, and A. Massa, "Maximum efficiency beam synthesis of radiating planar arrays for wireless power transmission," *IEEE Transactions on Antennas and Propagation*, Vol. 61, No. 5, 2490–2499, 2013.

10. Li, X., B. Duan, L. Song, et al., "Study of stepped amplitude distribution taper for microwave power transmission for SSPS," *IEEE Transactions on Antennas and Propagation*, Vol. 65, No. 10, 5396–5405, 2017.

11. Manica, L., P. Rocca, M. Pastorino, et al., "Boresight slope optimization of subarrayed linear arrays through the contiguous partition method," *IEEE Antennas and Wireless Propagation Letters*, Vol. 8, No. 4, 253–257, 2009.

12. Rocca, P., L. Manica, R. Azaro, et al., "A hybrid approach for the synthesis of sub-arrayed monopulse linear arrays," *IEEE Transactions on Antennas andPropagation*, Vol. 57, No. 1, 280–283, 2009.

13. Keizer, W., "Affine transformation for synthesis of low sidelobe patterns in planar array antennas with a triangular element grid using the IFT method," *IET Microwaves, Antennas & Propagation*, Vol. 14, No. 8, 830–834, 2020.

14. Keizer, W. P. M. N., "Synthesis of thinned planar circular and square arrays using density tapering," *IEEE Transactions on Antennas and Propagation*, Vol. 62, No. 4, 1555–1563, 2014.

15. Mohammed, J. R., "A method for thinning useless elements in the planar antenna arrays," *Progress In Electromagnetics Research Letters*, Vol. 97, 105–113, 2021.

16. Liu, G. X., X. Qin, and Q. H. Zhang, "Linear array synthesis for wireless power transmission based on brain storm optimization algorithm," *International Journal of Antennas and Propagation*, Vol. 2021, No. 7, 1–8, 2021.

17. Zhou, H. W., X. X. Yang, and S. Rahim, "Synthesis of the sparse uniform-amplitude concentric ring transmitting array for optimal microwave power transmission," *International Journal of Antennas & Propagation*, Vol. 2018, 1–8, 2018.

18. Li, J., J. Pan, and X. Li, "A novel synthesis method of sparse nonuniform-amplitude concentric ring arrays for microwave power transmission," *Progress In Electromagnetics Research C*, Vol. 107, 1–15, 2020.

19. Sallam, T. and A. M. Attiya, "Low sidelobe cosecant-squared pattern synthesis for large planar array using genetic algorithm," *Progress In Electromagnetics Research M*, Vol. 93, 23–34, 2020.

20. Mandal, S. K., G. K. Mahanti, and R. Ghatak, "Differential evolution algorithm for optimizing the conflicting parameters in time-modulated linear array antennas," *Progress In Electromagnetics Research B*, Vol. 51, 101–118, 2013.

21. Chen, Y., S. Yang, and Z. Nie, "The application of a modified differential evolution strategy to some array pattern synthesis problems," *IEEE Transactions on Antennas and Propagation*, Vol. 56, No. 7, 1919–1927, 2008.