Progress In Electromagnetics Research Letters, Vol. 105, 27–32, 2022

Guarantees of Minimum Performance Levels with Directed Energy Weapons

Graham V. Weinberg*

Abstract—The integration of directed energy weapons (DEWs) into modern military platforms is of considerable interest to those examining the impact of emerging technology on the future fighting force. Hence the performance prediction of DEWs is of importance. The purpose of this study is to develop a simple framework where the minimum number of DEWs deployed in an operational setting can be determined, to achieve a desired level of performance.

1. INTRODUCTION

Directed energy weapons (DEWs) have been identified as an emerging technology which has the potential for specialised applications for the future fighting force [1, 2]. The two such weapons of considerable interest are high energy lasers (HELs) and high power radio frequency (HPRF) DEWs. HEL DEWs apply a narrowly focused beam of energy, with wavelength smaller than 10^{-6} metres, to defeat threats through a thermal effect [3, 4] while HPRF DEWs direct a pulsed beam of microwave energy towards a target, such that surface currents are induced on it to produce system coupling and consequent disruption [5, 6]. A major military application of these DEWs is to defeating airborne threats from a ground based platform [7, 8]. Such threats of interest include missiles and uncrewed aerial vehicles (UAVs), and the latter application of DEWs has received much attention [10–12]. This is because UAVs can be used by ground-based forces to conduct intelligence gathering exercises, and can also be used in swarms to cause potential disruptions [9].

Given the utility of DEWs for defeating airborne threats there has been much interest in the performance prediction of DEWs, and in particular, the case where a degree of collaboration between defensive platforms is utilised. Recent work at Defence Science and Technology Group (DSTG) has not only examined the performance analysis of single DEW effectors [13], but has explored their application to active protection systems (APS). An APS permits a series of armoured fighting vehicles (AFVs) to be equipped with an automatic threat detection and defeat system [14, 15]. Studies at DSTG have investigated the concept of collaboration between AFVs equipped with APS which utilise DEWs [16–19]. The framework in which collaboration for APS has been studied at DSTG has been to assume that a fleet of AFVs consist of a subset of vehicles with a target detection, classification and tracking capability, while a second subset contains DEW capabilities. These two subsets of vehicles are not necessarily disjoint, and the operating protocol is that when the AFV team identifies a suitable target it schedules the most suitable DEW-enabled vehicle to eliminate the threat.

Hence the analysis undertaken at DSTG has not examined the application of multiple DEWs to defeat a single threat. Consequently, this study will introduce a basic methodology in which this can be investigated. Of more importance is the fact that this framework will permit the examination of the question of what is the minimum number of DEWs required to ensure a guaranteed minimum likelihood of threat defeat within an operational setting. This will also allow one to determine the specifications of DEWs within an AFV combat team, to ensure that threats of interest can be defeated successfully with a sufficiently large probability.

Received 1 June 2022, Accepted 24 June 2022, Scheduled 13 July 2022
* Corresponding author: Graham V. Weinberg (graham.weinberg@defence.gov.au).
The author is with the Defence Science and Technology Group, Australia.

2. NUMBER OF NECESSARY EFFECTORS

The purpose of this section is to derive an expression for the probability of disruption of a target when an arbitrary number of DEWs are focused on it. Towards this objective some assumptions will be imposed and justified. In addition to this, a useful expression will be derived to permit the determination of the number of DEW effectors to guarantee a minimum probability of disruption of the target. Throughout the following attention will be restricted to HEL DEWs, and thus DEW will be used for brevity.

Suppose that there are a series of N DEWs deployed on AFVs within the theatre of operations, and that they are focused on a single threat. A DEW's peak power density can be used to measure the applied power incident on the target [19]. It is assumed that these DEWs strike the target's surface area in different positions, and that the target has a vulnerability threshold $\mathcal{U}$, such that if any of the resultant power densities of the DEWs on the target exceed this threshold then the target has been disabled. Hence one can consider the target defeated if at least one DEW's resultant power density exceeds the surface area vulnerability threshold [19].

The assumption concerning the difference of the point of impact of the series of DEWs on the target facilitates the mathematical model, and eliminates the need to consider the complex interaction of DEW beams on the target.

Suppose that DEW j has a peak power density $I_j(t)$ on the target at time t. Its units are in Watts per metre square (as for the threshold $\mathcal{U}$). Hence if $\phi_j(t)$ is the probability that DEW j defeats the target by time t then $\phi_j(t) = \mathbb{P}(I_j(t) > \mathcal{U})$. It is also assumed that the DEWs illuminate the target independently, which is an assumption consistent with the surface strike point assumption adopted previously. Then the probability of the target being defeated is exactly the probability that at least one of the DEWs has a power density on the target that exceeds the threshold $\mathcal{U}$. Consequently,

$$\begin{aligned} & \mathbb{P}(\text{Target disrupted by time } t) \\ = \; & \mathbb{P}(\text{At least one DEW's power density on target exceeds } \mathcal{U} \text{ by time } t) \\ = \; & 1 - \mathbb{P}(\text{None of the DEW's power densities on target exceeds } \mathcal{U} \text{ by time } t) \\ = \; & 1 - \prod_{j=1}^{N} \mathbb{P}(I_j(t) < \mathcal{U}) = 1 - \prod_{j=1}^{N}(1 - \phi_j(t)), \end{aligned} \tag{1}$$

where the second last equality in (1) follows by the independence assumption. Therefore (1) enables one to calculate the probability of target disruption when a series of DEWs are applying energy to it. If relevant DEW performance characteristics are introduced then one can evaluate (1) in practice.

Rather than examine (1) it is instead of significant military utility to examine the following question. Suppose that in a given operational setting one is interested in determining the *minimum* number of DEWs to be deployed so that a guaranteed minimum likelihood of target defeat is assured. To put this into a tangible context, one may be interested in the minimum number of AFVs, each equipped with a DEW, to be assured that a threat of a particular type will be eliminated with a given likelihood of defeat. This question can be investigated in a mathematical setting by examining what is the minimum N in (1) such that the final probability in the latter is at least a fixed level $\epsilon \in (0, 1)$ uniformly over time t.

Examining (1) the requirement above can be specified by searching for the smallest N such that

$$1 - \prod_{j=1}^{N}(1 - \phi_j(t)) \geq \epsilon. \tag{2}$$

Through a rearrangement of (2), and by noting that the logarithm function is increasing and that when an inequality is normalised by a negative number the inequality direction is reversed, one arrives at

$$\sum_{j=1}^{N} \frac{\log(1 - \phi_j(t))}{\log(1 - \epsilon)} \geq 1. \tag{3}$$

Consequently, based upon (3), it is required to determine the solution to

$$\min_N \left\{ \sum_{j=1}^{N} \frac{\log(1-\phi_j(t))}{\log(1-\epsilon)} \geq 1 \right\}. \tag{4}$$

Given a series of DEW parameters and an appropriate power density function it is then possible to determine $\phi_j(t)$ and produce plots of (3), and then search for the smallest N so that (3) exceeds unity over all permissible values of t. In addition this can be done to investigate the appropriate DEW power level required to achieve the desired probability of defeat. The next section will illustrate this through specific examples.

3. CASE STUDIES

To illustrate how (4) can provide insights into the tradeoff between the number of deployed DEW effectors and the maintenance of a desired level of performance, this section considers particular examples. A simple physical scene is assumed for the combat space to simplify the numerical analysis. The threat is assumed to be such that a series of platforms with DEWs are on an intercept course for it. Relative to this threat it is thus assumed that each platform moves toward it with a fixed velocity, and that it will reach the threat after a given time. Although the effectors are on platforms on an intercept course toward the target, it is assumed that their DEW beams will not cause collateral damage to team members. Hence for modelling simplicity the threat is assumed stationary and on the same horizontal plane as the platforms deploying the DEWs.

Suppose that DEW effector $j \in \{1, 2, \ldots, N\}$ travels on its platform at ν_j metres per second and that it would take T_j seconds to reach the target. Hence the distance to the target at time t is $R_j(t) = \max\{0, \nu_j(T_j - t)\}$. The DEWs are assumed to operate in the same setting as applied in [13, 19], whose models for propagation have been based upon that in [20]. The key operational parameters are that all DEWs operate at a wavelength of $\lambda = 1.045 \times 10^{-6}$ metres, with a power level P_j Watts and duty cycle C_j. The DEW dwell time on the target is modelled by a truncated exponential distribution with parameter μ_τ seconds and that the target has an average illumination area parameter μ_σ square metres. As in [13] it will be assumed that $\mu_\tau = 1/3$ seconds and $\mu_\sigma = ((1.045^2)/9) \times 10^{-8}\mu_r$, where μ_r is the radar cross section (RCS) of the target in square metres.

Based upon [13], the probability of DEW disruption of the target can be shown to be

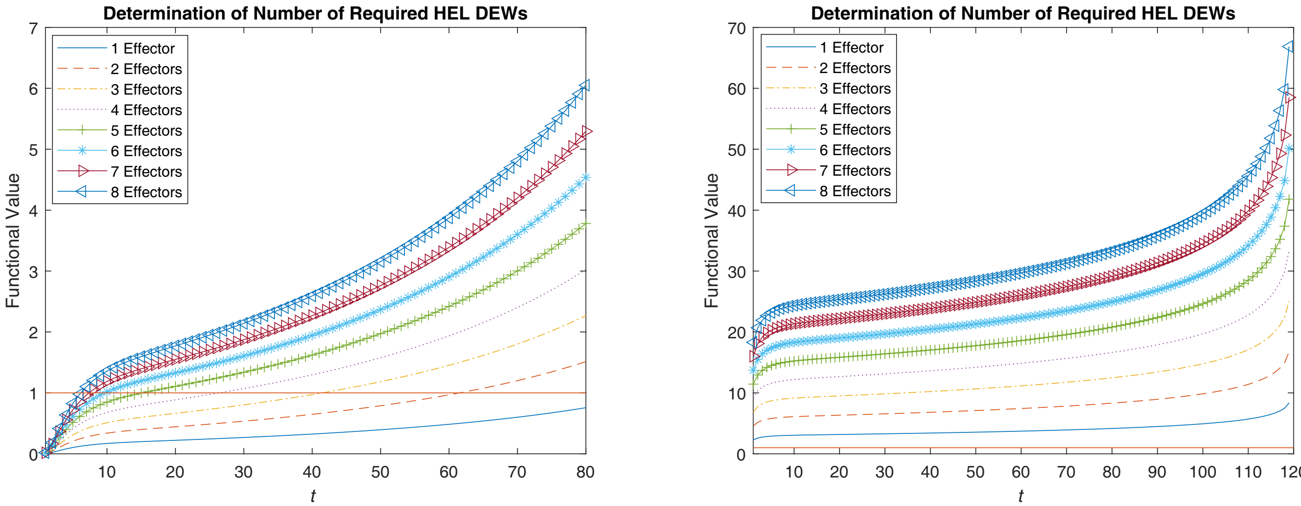
$$\phi_j(t) = \frac{\mu_\tau}{1 - e^{-\mu_\tau t}} \int_0^t e^{-\mu_\tau s} e^{-\mu_\sigma \frac{\mathcal{U}}{\kappa P_j C_j} \frac{\nu_j^2 (T_j - t)(T_j - t + s)}{s}} ds. \tag{5}$$

where $\kappa = 1.8330 \times 10^{-5}$.

In the examples to follow it will be assumed that $C_j = 1$ for all j (full duty cycle), $\mu_r = 10$ (target has small RCS, such as a UAV), $\nu_j = 20$ (slowly moving DEW platforms) and $T_j = 120$. The interest will be in the impact on N as P_j, $\mathcal{U}$ and ϵ are varied. The power level of each DEW will be considered equal (so that $P_j = P$ for all j), so that the analysis is investigating the situation where each military asset has an equivalent DEW. In each example the cases where $N \in \{1, 2, \ldots, 8\}$ are only shown for brevity.

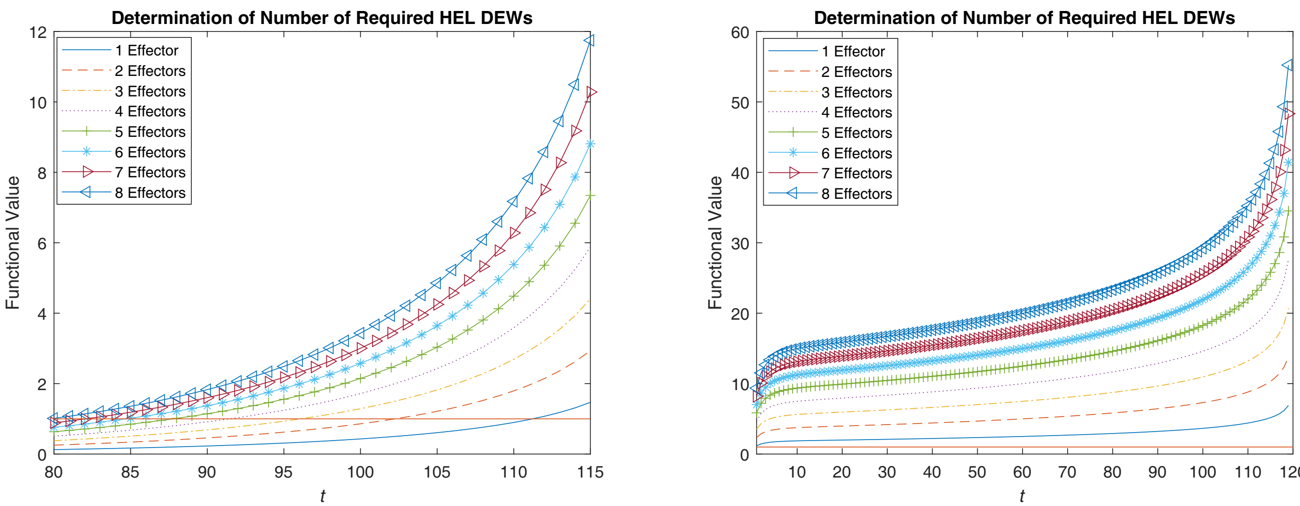
For the first case suppose that $\epsilon = 0.8$ and $\mathcal{U} = 1$. The two plots in Figure 1 graph (3) for two different power levels, and for N varying from one to eight. The left subplot is for the case where $P_j = 10^3$ for all j, and the right subplot corresponds to $P_j = 10^6$ for all j. Also included in these plots is a horizontal line at unity for reference. The left plot demonstrates that for a power level of 10^3 Watts there will not be a sufficient number of effectors to achieve the disruption probability of $\epsilon = 0.8$, unless the number of effectors is significantly larger than eight. However, the right subplot, which is for the case where each effector has power level of 10^6 Watts, shows that a minimum of three will achieve the desired minimum disruption probability. Further increasing the power level of DEWs causes this minimum number to reduce.

The next set of examples can be found in Figure 2. Here the same minimum disruption probability is maintained but the vulnerability threshold has been increased to $\mathcal{U} = 10$. The left subplot is for a power level of 10^3 Watts for each effector. As can be observed it would require significantly more than



(a) (b)

Figure 1. Two illustrations of (3), plotted as a function of t, to enable determination of (4), with different DEW parameterisations and threshold characteristics.



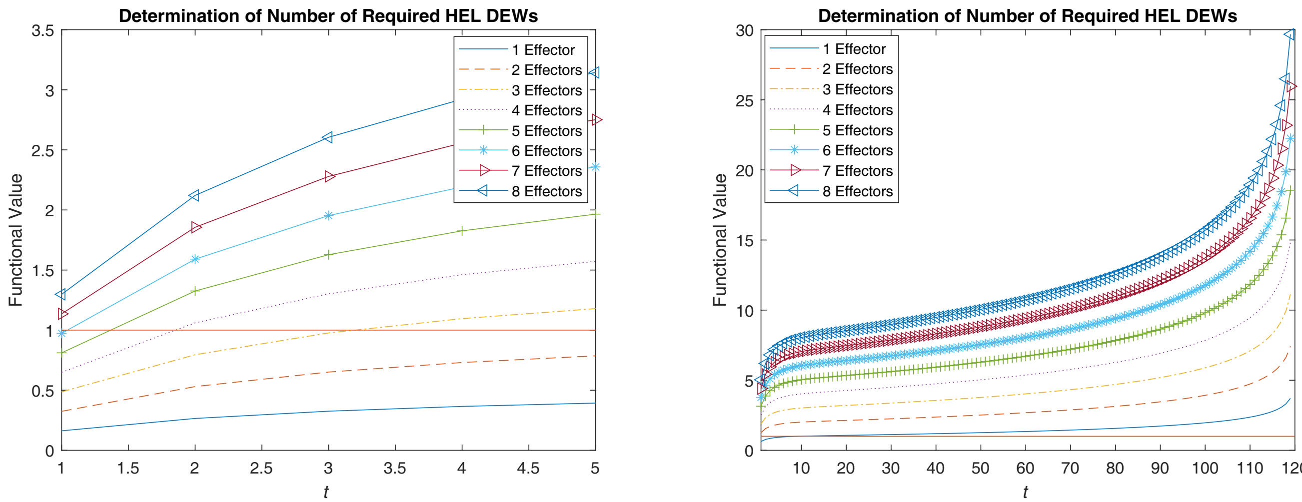
(a) (b)

Figure 2. Another set of illustrations of (3), plotted as a function of t, to enable determination of (4).

eight effectors to guarantee the desired probability of disruption. However, when the power level is increased to 10^6 Watts, a minimum of one effector will be sufficient, which is demonstrated in the right subplot.

The final case study can be observed in Figure 3, and the left subplot supposes that $\mathcal{U} = 100$, corresponding to a target with a large disruption threshold. In this situation the minimum probability of disruption has been set to $\epsilon = 0.95$, and the power level of each DEW has been set to 10^6 Watts. The figure shows that a minimum of six effectors are sufficient in this situation. If the power is reduced it was found that the number of effectors must exceed eight, while if the power is increased, the minimum N reduces as expected.

The right subplot in Figure 3 increases the target disruption threshold to $\mathcal{U} = 10,000$, while maintaining $\epsilon = 0.95$. This is the situation where the target is sufficiently shielded against thermal effects. The power level of each DEW was increased to 10^8 Watts to achieve the results in this figure.



(a) (b)

Figure 3. Two further examples of (3) to determine the minimum N in (4).

Based upon it one can conclude that a minimum of two effectors will be sufficient to guarantee the desired performance under the assumed conditions. If the power is reduced then it will require significantly more than eight DEW effectors to achieve the desired performance.

4. CONCLUSIONS

The main purpose of this study was to provide a quantification of the probability of disruption of a single threat when a series of DEWs are applying energy to it. Based upon this it was then possible to introduce a methodology to determine the minimum number of DEWs required to ensure a minimum likelihood of target disruption. The case studies demonstrated that for low to moderate disruption thresholds ($\mathcal{U} \leq 10$) a uniform power level of 10^6 Watts would imply only a few platforms would require DEWs, while if the power level was reduced below this setting it would result in requiring a large number of DEWs. This situation was also observed when the disruption threshold was increased to beyond 100, and the key observation was that the DEW power level would have to increase beyond 10^8 Watts to result in the choice of $N = 2$ being sufficient to ensure a disruption probability of at least 0.95.

Hence if an AFV team is to be assured of being able to defeat a specific type of target then the approach in this letter can provide rules of thumb in terms of the necessary DEW power levels and the number of platforms with such DEWs to ensure a desired likelihood of threat defeat.

REFERENCES

1. Geis, II, J. P., "Directed energy weapons on the battlefield: A new vision for 2025," *Occasional Paper 32*, Center for Strategy and Technology, Air War College, Alabama, USA, 2003.
2. Nielsen, P. E., "Effects of directed energy weapons," National Defence University, Washington, 1994.
3. Anderberg, B. and M. L. Wolbarsht, "High energy laser (HEL) weapons," *Laser Weapons*, Springer, Boston, 1992.
4. Lavan, M., "High energy laser systems for short range defense," *Acta Physica Polonica Series A*, Vol. 115, No. 6, 2009.
5. Bayram, Y., J. L. Volakis, S. K. Myoung, S. J. Doo, and P. Robin, "High power EMI on RF amplifier and digital modulation schemes," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 50, 849–860, 2008.

6. Radasky, W. A., "The threat of intentional interference (IEMI) to wired and wireless systems," *17th International Zurich Symposium on Electromagnetic Compatibility*, 2006.
7. Hafften, M. and R. Stratton, "High energy laser weapon integration with ground vehicles," *NATO Report presented to RTO AVT Symposium*, RTO-MP-AVT-108, 2004.
8. Mantzouris, G., "Thermomechanical analysis of ground-based directed energy weapons on satellites and intercontinential ballistic missiles," Master's Thesis, Naval Postgraduate School, Monterey, 2006.
9. Hu, F., D. X. Ou, and X.-L. Huang, *UAV Swarm Networks: Models, Protocols and Systems*, CRC Press, 2021.
10. Graswald, M., R. Gutser, F. Grabner, B. Meyer, C. Winter, and A. Oelerich, "Defeating UAVs through novel HPEM effectors," *31st International Symposium on Ballistics*, India, 2019.
11. Yun, Q., B. Song, and Y. Pei, "Modeling the impact of high energy laser weapon on the mission effectiveness of unmanned combat aerial vehicles," *IEEE Access*, Vol. 8, 32246–32257, 2020.
12. Lyu, C. and R. Zhan, "Global analysis of active defense technologies for unmanned aerial vehicle," *IEEE Aerospace and Electronic Systems Magazine*, Vol. 37, No. 1, 6–31, 2022.
13. Weinberg, G. V., "Performance prediction of directed energy weapons," *Progress In Electromagnetics Research M*, Vol. 108, 79–88, 2022.
14. Feng, B., X. Liang, W. G. Du, C. L. Qiu, and R. S. Hou, "A stochastic vulnerability analysis method for armored vehicles with active protection systems," *Journal of Physics: Conference Series*, 1507 082051, 2020.
15. Yang, L. and J. Xu, "Analysis on the development of active protection system for tanks and armored vehicles," *Journal of Physics: Conference Series*, 1855 012034, 2021.
16. Weinberg, G. V., "Quantification of combat team survivability with high power RF directed energy weapons," *Progress In Electromagnetics Research M*, Vol. 102, 1–11, 2021.
17. Weinberg, G. V., "A queueing theoretic approach for performance prediction of collaborative active protection systems," *Proceedings of the 24th International Congress on Modelling and Simulation (MODSIM)*, 904–910, Sydney, 2021.
18. Weinberg, G. V., "Determining performance limits for directed energy weapons in collaborative active protection systems," *Proceedings of the 24th International Congress on Modelling and Simulation (MODSIM)*, 911–917, Sydney, 2021.
19. Weinberg, G. V. and M. M. Kracman, "Armoured fighting vehicle team performance prediction against missile attacks with directed energy weapons," ArXiv Preprint, arXiv:2106.14381v1, 2021.
20. Sprangle, P., J. Penano, and B. Hafizi, "Optimum wavelength and power for efficient laser propagation in various atmospheric environments," *Naval Research Laboratory Report*, NRL/MR/6790-05-8907, 2005.