Progress In Electromagnetics Research Letters, Vol. 106, 103–110, 2022

Parameters of the Probability Density Function of Fluctuations of the Apparent Radiation Center from the Helicopter Propeller Hub

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Abstract—The properties of the angular glint of radar reflections from the propeller hub are considered. Analytical expressions were obtained based on a known multipoint geometric model of the propeller hub to calculate the probability density function parameters of angular glint for the planes, azimuth, and elevation angle for a single-blade rotor at an arbitrary rotation angle of the head. The relations obtained for a propeller hub with a single blade are generalized to the case of a propeller hub with an arbitrary number of blades. It is shown that the angular glint of the propeller hub is a random process with periodically changing parameters. The theoretical results are confirmed by mathematical modeling.

1. INTRODUCTION

Modern radar stations (radars) measure and evaluate a large number of parameters of the electromagnetic field scattered by the radar object. The results of their evaluation determine such parameters as the range to the object, the speed of approach, the angular position of the object, the size and shape of the object.

Simulation is widely used at various stages of radar development and during their modernization [1, 2]. The completeness and reliability of modeling are largely determined by the variety and elaboration of simulation models of radar objects [3]. Thus, the task of developing new and refining existing models of radar objects is relevant.

One class of radar object of considerable interest in modeling is an aircraft with rotating blades. This class of objects can include helicopters and unmanned aerial vehicles (e.g., quadcopters and octocopters).

It is known that the echo signal from such objects is formed by reflections from the fuselage, blades, and propeller hub and is unsteady [4]. When modeling reflections from them, geometric models are widely used, which are a set of independent points having a geometric configuration that coincides with the real object, each of which emits signals equivalent to reflections from the replaced object fragment [5]. For example, when modeling helicopter reflections for X-band radar, the fuselage comprises dozens of points, the propeller hub, and each of the blades requires hundreds of points [6]. This significantly limits the applicability of such models for real-time echo generation.

In addition, real radar objects are characterized by the angular glint phenomenon — fluctuations of the phase front of the reflected electromagnetic wave caused by the multipoint structure of the radar object. Angular glint is an important characteristic to determine the angular position and angular dimensions of a reflecting object [7, 8]. Angular glint can lead to target tracking failures, coordinate measurement errors [9]. In addition, this phenomenon can be used for target recognition [10]. As a rule, it is characterized by the probability density function (PDF) and spectral power density [10, 11].

Received 22 June 2022, Accepted 22 August 2022, Scheduled 13 September 2022
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PDF characterizes the angular dimensions of the object, and for each of the angular coordinates it is defined by the expression [10]:

$$W(\xi) = \frac{\mu}{2 \cdot \left(1 + \mu^2 \cdot (\xi - m)^2\right)^{3/2}}, \tag{1}$$

where ξ is the generalized coordinate (azimuth or elevation angle); m is the expectation of angular glint on the generalized coordinate under consideration ξ; μ is the value characterizing the width of the distribution on the considered generalized coordinate ξ.

The spectral properties of angular glint characterize the rate of fluctuations of the apparent center of radiation and are determined by the parameters of joint motion of the object and radar.

The literature considers the issues of determining the PDF of angular glint of point [12, 13] and distributed radar objects [11] and estimating their spectral composition. To analyze the properties of angular glint, geometric models of ground, and air targets, various extended objects were compiled [5, 10, 14]. The sources [1, 15, 16] consider the issues of hardware-in-loop modeling of the angular noise of radar objects. In all cases, it was assumed that the object is a rigid structure and reflects a stationary echo signal. No analysis of the statistical properties of the angular glint of the objects reflecting the non-stationary echo signal was performed.

At the same time, it is noted in the literature that the non-stationarity of reflections from helicopters leads to errors in measuring their coordinates and tracking [6]. This indicates that the formation of angular glint (in fact, the reproduction of the spatial configuration of the radar object and its change over time) in the simulation of such objects is significant.

The traditional approach to analyzing reflections from complex radar objects is to decompose this object into its component parts, analyze the reflections separately for each of these parts, and then generalize the results obtained [17–19]. As mentioned earlier, the helicopter is decomposed into three elements: the fuselage, blades, and propeller hub. The object of the present study is the helicopter propeller hub angular glint parameters. Thus, this work aims to determine the dependence of the probability density parameters of the helicopter propeller hub angular glint on the rotation angle of the propeller hub.

2. FORMULATIONS AND MODEL

2.1. Multipoint Propeller Hub Model

A propeller hub is a geometrically complex object. Several sources suggest that instead of completely reproducing the propeller hub configuration for modeling, a geometric model composed of a set of reflectors distributed in space with a given probability density in the coordinates of radius ($f_r(r)$), azimuth ($f_\theta(\theta)$), and height ($f_z(z)$) can be used [6, 20, 21]:

$$\begin{cases} f_r(r) = \dfrac{2}{r_{hub}} \cdot e^{-2 \cdot r / r_{hub}}; \\ f_\theta(\theta) = A \cdot Unif\left[\theta_{blade} - \dfrac{\pi}{N}; \theta_{blade} + \dfrac{\pi}{N}\right] + B \cdot Norm\left[\theta_{blade} + \beta; \Delta\theta\right]; \\ f_z(z) = Unif\left[-\dfrac{h_{hub}}{2}; \dfrac{h_{hub}}{2}\right], \end{cases} \tag{2}$$

where r_{hub} and h_{hub} — the radius and height of the propeller hub; θ_{blade} — the angular position of the blade at the initial moment; β — the angle of blade rotation relative to the initial moment; $\Delta\theta$ — the angular width of about 1 degree; N — the number of propeller blades; $Unif[a, b]$ — the uniform distribution in the range from a to b; $Norm[m, \sigma]$ — normal distribution with mathematical expectation m and variance σ^2; A and B — weight ratios, which determine the distribution of reflectors in the propeller hub.

Parameters A and B can be changed to simulate the propeller hubs of different aircraft [6]. It can be seen from (2) that an increase in the parameter B will lead to a concentration of the propeller hub reflectors in the blade mounting area.

The origin of the cylindrical coordinate system is aligned with the geometric center of the sleeve. The oZ axis is aligned with the propeller hub axis. It is required to use at least a hundred points per propeller blade mounting area to model the propeller hub.

Each propeller hub point reflects a signal with a Doppler shift proportional to the speed of convergence of the point and the radar. The amplitude of the reflected signal for all points of the geometric model of the sleeve is assumed to be the same. The initial phases are random and distributed equivocally between 0 and 360 degrees. It is noted in [6, 20, 21] that the temporal and spectral realization of the echo signal from such a geometric model corresponds to the characteristics of a real propeller hub. In this work, we will apply this model to calculate the parameters of the PDF of the angular noise of reflections from the propeller hub.

2.2. Ratios for Calculating the Parameters of the Propeller Hub PDF

Knowing the multipoint propeller hub model, it is possible to calculate the parameters of the angular glint PDF for the angular coordinates azimuth and elevation angle. To do this, let us transfer to normalized coordinates. On the oZ axis, normalize to half of the propeller hub height ($h_{hub}/2$). On the oR axis, normalize to the radius of the propeller hub (r_{hub}). Thus, in the planes, the azimuth and elevation angle of the reflector coordinates will take values in the range of $[-1; 1]$.

As shown from (2) for the plane elevation angle, there is a uniform distribution of reflectors in space. The parameters of the angular glint PDF for a radar object with such a reflectivity distribution in space are defined in [10] and are equal to: $m_\theta = 0$, $\mu_\theta = \sqrt{3}$.

Let us calculate the parameters of the propeller hub angular glint PDF for the azimuth plane. The relations that allow calculating the required values are given in [10]:

$$m = \frac{\int\limits_{\xi} \xi \cdot F_R(\xi)\, d\xi}{\int\limits_{\xi} F_R(\xi)\, d\xi}; \tag{3}$$

$$\mu = \sqrt{\frac{\int\limits_{\xi} F_R(\xi)\, d\xi}{\int\limits_{\xi} (\xi - m)^2 \cdot F_R(\xi)\, d\xi}} \tag{4}$$

where $F(\xi)$ is the function determining the density function of reflection intensity along the considered generalized coordinate ξ (azimuth or elevation angle).

For the horizontal plane, the function $F_R(\theta)$ is determined by the sighting direction of the propeller hub and the number of propeller blades.

Consider the change of function $F_R(\theta)$ by the example of a propeller with one blade. When sighting the propeller hub along the normal to the propeller blade (Fig. 1(a)), the reflector distribution function of the geometric model is defined by expression 1 of system (2).

The function $F_\alpha(r)$, which determines the intensity distribution density of the reflection in the azimuthal plane, as seen from expression 2 of the system (2), consists of two components (terms). Evenly distributed reflectors determine the first term at all angles around the propeller hub axis. It is independent of the propeller hub angle. The reflectors are distributed exponentially, symmetrically about the propeller hub axis in terms of the radial component. The second term determines the concentration of reflecting points near the blade. As the propeller hub rotates, the configuration of the reflecting points in space changes. When the sighting direction is normal to the line of blade location, the angular size is maximum. Sighting the propeller hub parallel to the blade (Fig. 1(b)) leads to the concentration of the propeller hub reflectors in the center of the two-point model — the angular size is minimal. Thus, the density function of the reflectivity distribution along the radial coordinate at an

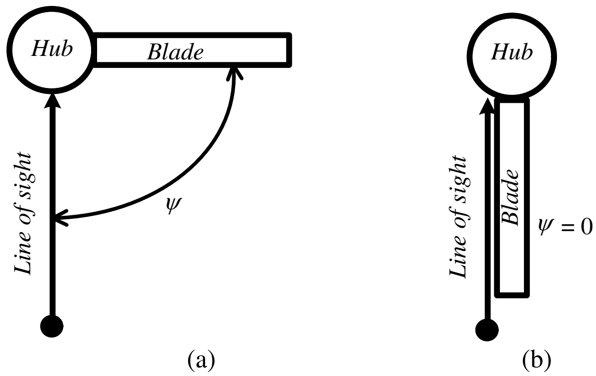

Figure 1. Sighting options for the propeller hub and propeller blades, (a) sighting along the normal to the blade; (b) sighting along the blade.

arbitrary rotation angle of the propeller hub is defined by the expression:

$$F_\alpha(r) = \frac{2 \cdot B}{L} \cdot \exp\left(\frac{-2 \cdot |r|}{L}\right) + \begin{cases} \frac{2 \cdot A}{L} \cdot \exp\left(\frac{-2 \cdot r}{L \cdot \sin(\psi)}\right), & r \geq 0 \\ 0, & r < 0 \end{cases}, \tag{5}$$

where ψ is the angle between the blade and the sighting direction (see Fig. 1).

Calculation of the mathematical expectation of the angular glint in the azimuth plane by (3), taking into account the normalization of the geometric model to the radius of the propeller hub, gives:

$$m_\alpha = \frac{\int\limits_{-\infty}^{\infty} r \cdot F_\alpha(r)\,dr}{\int\limits_{-\infty}^{\infty} F_\alpha(r)\,dr} = \frac{\int\limits_{-\infty}^{\infty} 2 \cdot r \cdot B \cdot \exp(-2 \cdot |r|)\,dr + \int\limits_{0}^{\infty} 2 \cdot r \cdot A \cdot \exp\left(\frac{-2 \cdot r}{\sin(\psi)}\right) dr}{\int\limits_{-\infty}^{\infty} 2 \cdot B \cdot \exp(-2 \cdot |r|)\,dr + \int\limits_{0}^{\infty} 2 \cdot A \cdot \exp\left(\frac{-2 \cdot r}{\sin(\psi)}\right) dr}$$
$$= \frac{A \cdot \sin^2(\psi)}{2 \cdot A \cdot \sin(\psi) + 2 \cdot B} \tag{6}$$

The second integral in the numerator (6) has primes $[0; \infty]$. These limits correspond to $\psi \in [0; \pi]$ (the blade is positioned to the right relative to the observation point). At $\psi \in (-\pi; 0)$ (the blade is to the left of the observation point), the integration limits will change to $[-\infty; 0]$. In this case, when (6) is calculated, a minus sign will appear in front of the variable A. In general, the expression for determining the mathematical expectation of the angular glint of a single-blade propeller hub can be written as:

$$m_\alpha = \frac{\mathrm{Sign}(\psi) \cdot A \cdot \sin^2(\psi)}{2 \cdot \mathrm{Sign}(\psi) \cdot A \cdot \sin(\psi) + 2 \cdot B}, \tag{7}$$

where: $\mathrm{Sign}(\psi) = \begin{cases} 1, & \psi \in [0; \pi]; \\ -1, & \psi \in (-\pi; 0). \end{cases}$

Consider expression (4) for the propeller hub of a single-blade propeller. Taking into account the normalization of coordinates of points in the geometric model, substitute $F_\alpha(r)$ in (4):

$$\mu_\alpha = \sqrt{\frac{\int\limits_{-\infty}^{\infty} F_\alpha(r)\,dr}{\int\limits_{-\infty}^{\infty} (r - m_\alpha)^2 \cdot F_\alpha(r)\,dr}}$$

$$= \sqrt{\frac{\int\limits_{-\infty}^{\infty} 2 \cdot B \cdot \exp\left(-2 \cdot |r|\right) dr + \int\limits_{0}^{\infty} 2 \cdot A \cdot \exp\left(\frac{-2 \cdot r}{\sin\left(\psi\right)}\right) dr}{\int\limits_{-\infty}^{\infty} 2 \cdot \left(r - m_\alpha\right)^2 \cdot B \cdot \exp\left(-2 \cdot |r|\right) dr + \int\limits_{0}^{\infty} 2 \cdot A \cdot \left(r - m_\alpha\right)^2 \cdot \exp\left(\frac{-2 \cdot r}{\sin\left(\psi\right)}\right) dr}} \tag{8}$$

Write the expressions for each of the integrals:

$$\int\limits_{-\infty}^{\infty} 2 \cdot B \cdot \exp\left(-2 \cdot |r|\right) dr \;=\; 2 \cdot B; \tag{9}$$

$$\int\limits_{0}^{\infty} 2 \cdot A \cdot \exp\left(\frac{-2 \cdot r}{\sin\left(\psi\right)}\right) dr \;=\; 2 \cdot A \cdot \sin\left(\psi\right); \tag{10}$$

$$\int\limits_{-\infty}^{\infty} 2 \cdot \left(r - m_\alpha\right)^2 \cdot B \cdot \exp\left(-2 \cdot |r|\right) dr \;=\; B + 2 \cdot B \cdot m_\alpha^2; \tag{11}$$

$$\int\limits_{0}^{\infty} 2 \cdot A \cdot \left(r - m_\alpha\right)^2 \cdot \exp\left(\frac{-2 \cdot r}{\sin\left(\psi\right)}\right) dr \;=\; 0.5 \cdot A \cdot \sin^3\left(\psi\right) - m_\alpha \cdot A \cdot \sin^2\left(\psi\right) + m_\alpha^2 \cdot A \cdot \sin\left(\psi\right) \tag{12}$$

Thus, the expression defining the parameter $\mu_\alpha(\psi)$:

$$\mu_\alpha\left(\psi\right) = \sqrt{\frac{2 \cdot B + 2 \cdot A \cdot \sin\left(\psi\right)}{B + 2 \cdot B \cdot m_\alpha^2 + 0.5 \cdot A \cdot \sin^3\left(\psi\right) - m_\alpha \cdot A \cdot \sin^2\left(\psi\right) + m_\alpha^2 \cdot A \cdot \sin\left(\psi\right)}}. \tag{13}$$

Relations (7) and (13), determining the parameters of the angular glint PDF in the azimuth plane for the propeller hub with one blade, are not difficult to generalize to the case of a propeller with an arbitrary number of blades. For this purpose, write the expression determining the intensity distribution of reflection intensity of such a propeller hub, denoting ψ_i as a sighting angle of i-th blade:

$$F_\alpha\left(r\right) = \frac{2 \cdot B}{L} \cdot \exp\left(\frac{-2 \cdot |r|}{L}\right) + \sum_{i=1}^{N} \begin{cases} \dfrac{2 \cdot A}{L} \cdot \exp\left(\dfrac{-2 \cdot r}{L \cdot \sin\left(\psi_i\right)}\right), & r \geq 0; \\ 0, & r < 0. \end{cases} \tag{14}$$

Taking this into account, the parameters of the angular glint in the azimuth plane for the hub propeller hub a multi-blade propeller are defined by the expressions:

$$m_\alpha = \frac{\sum\limits_{i=1}^{N} \left(\mathrm{Sign}\left(\psi_i\right) \cdot A \cdot \sin^2\left(\psi_i\right)\right)}{\sum\limits_{i=1}^{N} \left(2 \cdot \mathrm{Sign}\left(\psi_i\right) \cdot A \cdot \sin\left(\psi_i\right)\right) + 2 \cdot B}, \tag{15}$$

$$\mu_\alpha(\psi) = \sqrt{\frac{2 \cdot B + \mathrm{Sign}(\psi_i) \cdot \sum\limits_{i=1}^{N} 2 \cdot A \cdot \sin(\psi_i)}{B + 2 \cdot B \cdot m_\alpha^2 + \mathrm{Sign}(\psi_i) \cdot \sum\limits_{i=1}^{N} \left(0.5 \cdot A \cdot \sin^3(\psi_i) - m_\alpha \cdot A \cdot \sin^2(\psi_i) + m_\alpha^2 \cdot A \cdot \sin(\psi_i)\right)}} \tag{16}$$

3. RESULTS AND DISCUSSION

3.1. Validation of the Obtained Ratios

Using expressions (15) and (16), calculate the parameters of the directional glint in the azimuth plane depending on the sighting angle. The calculation is made for propeller hubs with 1–5 blades. For all propeller hubs, it is assumed that the blades are equidistant in angle with pitch $\frac{2\cdot\pi}{N}$. The calculation is performed for two propeller hubs with different A and B parameters values. The calculation results are shown in Fig. 2.

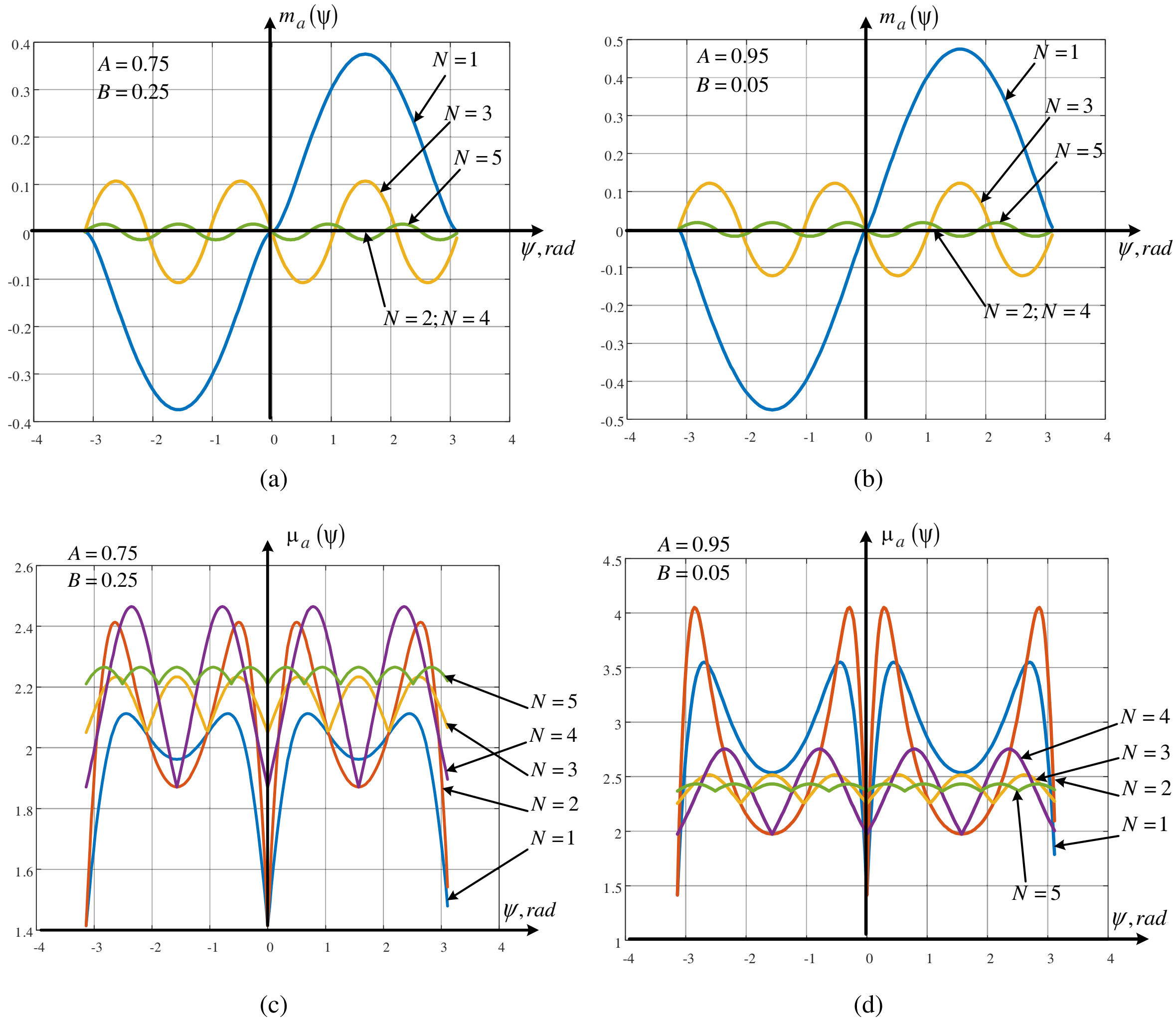

Figure 2. Dependence of angular glint PDF parameters in the azimuth plane of the propeller hub on the sighting angle. (a), (b) mathematical expectation; (c), (d) parameter $\mu_\alpha(\psi)$.

The calculation results are confirmed by mathematical modeling. A multipoint geometric model of the propeller hub was generated. The points of the model are distributed in space according to (2). The propeller hub model used 100 reflection points for each blade mounting area. During the simulation, the reflected signals from each of the points of the multipoint model were generated for a particular propeller hub rotation angle. Their amplitudes were assumed to be the same. The initial phases were uniformly distributed in the interval $[0; 360°]$. The Doppler frequency was determined based on the velocity of approach of the propeller hub point and the observation point. In the plane of azimuth and elevation angle, a monopulse direction-finding of a set of reflected signals was performed using the direction-finding relation [22, 23]:

$$\xi(t) = \mathrm{Re}\left(\frac{F_\Delta(\xi_i) \cdot \dot{s}_i(t)}{F_\Sigma(\xi_i) \cdot \dot{s}_i(t)}\right), \tag{17}$$

where $\dot{s}_i(t)$ — the complex envelope of the signal reflected from the i-th point; F_Δ — the difference radiation pattern; F_Σ — the total radiation pattern; ξ_i — the generalized (azimuth or elevation angle) coordinate of the i-th point.

A histogram was plotted based on the resulting bearing relationship. Histograms were averaged over 20 implementations. The averaged histogram was used to determine the parameters of the angular glint PDF (m and μ) for a given propeller hub rotation angle. As a modeling result for the azimuth plane, dependences similar to those shown in Fig. 2 were obtained. The root mean square deviation of the dependencies of the PDF angular glint parameters obtained as a result of the calculation according to the obtained relations from the modeling results does not exceed 3%. In the plane of the elevation angle, as expected, the PDF parameters of angular glint do not depend on the propeller hub rotation angle and coincide with the theoretical value ($m_\theta = 0$, $\mu_\theta = \sqrt{3}$).

The following conclusions can be drawn from the results of the calculations. In the plane of the elevation angle, the apparent center of radiation fluctuations is stationary. In the azimuth plane, the PDF angular glint parameters are periodic functions of the propeller hub rotation angle. The repetition period is equal to $\frac{2\cdot\pi}{N}$. The stronger the non-stationarity is, the greater the contribution to the reflection is made by the propeller hub fragments to which the blades are attached (manifested by an increase in the A parameter). With this in mind, one can expect a wide variation in the angular glint PDF parameters from the propeller blades, with an even number of propeller blades $m_\alpha \equiv 0$ at any rotation angle of the propeller hub. This can be explained by the symmetrical location of the reflection points relative to the propeller hub axis at any rotation angle.

The parameter μ_α depends on the propeller hub rotation angle at any number of propeller blades. As the number of blades increases, the range of variation of the PDF parameters narrows. Thus, at five blades, the range of variation of angular glint parameters of the PDF does not exceed 5% of the average value.

4. CONCLUSION

It is shown that in the azimuth plane, the fluctuations of the apparent center of reflection of the propeller hub represent a non-stationary random process. The relations making it possible to determine the angular glint PDF parameters at a given rotation angle of the propeller hub are obtained. Theoretical calculations were confirmed by mathematical modeling using a multipoint propeller hub model.

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