Progress In Electromagnetics Research M, Vol. 113, 213–224, 2022

A Sparse Array Design Method Based on Direct-Connection of 4 Uniform Linear Arrays

Liye Zhang*, Weijia Cui, Chunxiao Jian, Bin Ba, and Hao Li

Abstract—In order to obtain the analytical expression of the position of sparse array sensors under the condition of a given total array sensor number, a sparse array design method based on direct-connection of 4 uniform linear arrays (DCUA4) is proposed. By using the only known parameter of the total array sensor number, the sensor number and spacing parameters of four subarrays are obtained by mathematical operation, then the four subarrays are directly connected to realize the design of sparse array. It is proved that the aperture of the sparse array is large, and there are no holes. Because all the sensors are allocated to four subarrays, the number of small spacing sensor pairs in the array is controlled. The performance of the proposed array is simulated based on the spatial smoothing MUSIC (SS-MUSIC) algorithm. The simulation results show that the proposed DCUA4 can produce a large virtual array aperture, realize high-precision direction of arrival (DOA) estimation under underdetermined conditions, and resist the influence of low mutual coupling.

1. INTRODUCTION

Direction of arrival (DOA) estimation is an important research direction in the field of array signal processing, which has been widely used in radar, sonar, wireless communication, and other fields. It is well known that the traditional DOA estimation is based on uniform linear array (ULA). Classical DOA estimation algorithms, such as MUSIC [1] and Estimating Signal Parameter via Rotational Invariance Techniques (ESPRIT) [2], can estimate the DOA of up to $M-1$ sources using M array sensors. Therefore, when the source number is large, the DOA estimation ability of ULA can only be improved by adding array sensors. In addition, ULA requires that the array sensor spacing is less than or equal to the half-wavelength of the incident signal, which will lead to serious mutual coupling [3].

DOA estimation of signals under underdetermined conditions [4], that is, DOA estimation when the source number is more than array sensor number, is a research hotspot in the field of array signal DOA estimation in recent years. An effective method to solve this problem is to design sparse arrays. The method of calculating difference virtual array [5] can increase the array sensor number available for DOA estimation and realize DOA estimation of signals under underdetermined conditions. At the same time, because the array sensor spacing is no longer constrained by the half-wavelength of the incident signal, the influence of mutual coupling between array sensors is weaker than that of ULA. The minimum redundant array (MRA) [6] is a typical sparse array whose difference virtual array is a ULA with no holes. In the case of M sensors, MRA can produce the largest virtual array aperture. However, for any given sensor number, there is no closed expression for the virtual array aperture that can be obtained by MRA. Therefore, although MRA has good performance, the array sensor position parameters can only be obtained by traversing, so it is difficult to design when array sensor number is large.

Received 4 July 2022, Accepted 31 August 2022, Scheduled 23 September 2022
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The existing sparse array design methods can be divided into two categories: the improved sparse array design method based on coprime array (CA) [7] and the improved sparse array design method based on nested array (NA) [8]. The improved sparse arrays based on CA include shifted coprime array (SCA) [9], thinned coprime array (TCA) [10], coprime array with two-hole difference (CATHDC) [11], coprime array with prime number 3 (CAP3) [12], etc., which have the advantages of fewer sensor pairs with small spacing and less influence on the array by mutual coupling. The improved sparse arrays based on NA, such as augmented nested array I-1 (ANA11) [13], are greatly affected by mutual coupling because of the small pacing between subarray sensors. But their difference virtual array has no holes, and the aperture of the virtual array is larger than that of the improved arrays based on CA, so it is suitable for DOA estimation without considering the effect of mutual coupling.

In order to obtain larger array aperture and anti-mutual coupling ability, a new sparse array structure included in the category of the improved sparse array based on NA is proposed in this paper, which is called sparse array based on direct-connection of 4 uniform linear arrays (DCUA4). For a given array sensor number, the number and spacing parameters of four ULA subarrays can be calculated by accurate mathematical expression. By directly connecting the four subarrays, a sparse array with no holes in the difference virtual array can finally be obtained. The proposed DCUA4 has the following advantages: (1) A large difference virtual array aperture can be obtained, then the signal DOA estimation under underdetermined conditions can be realized. (2) The number of sensor pairs with small spacing is small, then the higher DOA estimation performance can be obtained when considering the effect of low mutual coupling.

The remainder of this paper is presented as follows. Section 2 introduces some basic knowledge about the received signal model of sparse array, spatial smoothing MUSIC (SS-MUSIC) algorithm, and mutual coupling. The proposed DCUA4 is shown in detail in Section 3. Section 4 presents the numerical simulations and analysis to verify the performance of the proposed array. Section 5 summarizes the full paper.

The notations of some operation symbols involved in this paper are shown as follows: $(\cdot)^{\mathrm{T}}$, $(\cdot)^*$, and $(\cdot)^{\mathrm{H}}$ denote the transpose, conjugate, and conjugate transpose of the matrix; $\mathrm{E}\{\cdot\}$ is the statistical expected value operator; $\mathrm{diag}\{\cdot\}$ is the diagonal matrix composed of vector elements; $\mathrm{vec}\{\cdot\}$ is the vectorization of the matrix; $\circ$ is the Khatri-Rao product; $\lceil\cdot\rceil$ and $\lfloor\cdot\rfloor$ are the upward rounding and downward rounding operations; and $\mathrm{Card}\{\cdot\}$ is the number of elements in the statistical set.

2. PRELIMINARY

2.1. The Received Signal Model of Sparse Array

The effectiveness of the array design method is mainly evaluated by its DOA estimation performance. In order to analyze the DOA estimation performance of the proposed DCUA4 design method, this subsection first introduces the received signal model of sparse array.

Suppose that K independent far-field narrow-band signals are impinging on a M-sensor sparse array in the direction of $\{\theta_1, \theta_2, \ldots, \theta_K\}$ with power $\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2\}$, respectively, and the array sensor position set of the sparse array is $\mathbb{D} = \{d_1, d_2, \ldots, d_M\}$. Then, the received signal can be expressed as

$$\mathbf{X}(t) = \mathbf{A}\mathbf{S}(t) + \mathbf{N}(t), \tag{1}$$

where the array manifold matrix $\mathbf{A}$ can be expressed as

$$\mathbf{A} = \left[\mathbf{a}_{\mathbb{D}}(\theta_1), \mathbf{a}_{\mathbb{D}}(\theta_2), \ldots, \mathbf{a}_{\mathbb{D}}(\theta_K)\right], \tag{2}$$

$$\mathbf{a}_{\mathbb{D}}(\theta_k) = \left[1, \mathbf{e}^{j\pi d_2 \sin(\theta_k)}, \ldots, \mathbf{e}^{j\pi d_M \sin(\theta_k)}\right]^{\mathrm{T}}. \tag{3}$$

The signal vector $\mathbf{S}(t)$ is

$$\mathbf{S}(t) = \left[S_1(t), S_2(t), \ldots, S_K(t)\right]^{\mathrm{T}}, \tag{4}$$

where $t = 1, 2, \ldots, J$, J represents the snapshots number. The noise vector $\mathbf{N}(t)$ is expressed as

$$\mathbf{N}(t) = \left[N_1(t), N_2(t), \ldots, N_M(t)\right]^{\mathrm{T}}. \tag{5}$$

In the AWGN channel, the components of noise vector satisfy the Gaussian distribution with a mean of 0 and a variance of σ_n^2.

Next, the covariance matrix of the received signal $\mathbf{R}_X$ can be calculated as

$$\mathbf{R}_X = E\left[\mathbf{X}\mathbf{X}^{\mathrm{H}}\right] = \mathbf{A}\mathbf{R}_S\mathbf{A}^{\mathrm{H}} + \sigma_n^2\mathbf{I}_M, \tag{6}$$

where $\mathbf{R}_S = \mathrm{diag}[\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]$. By vectorization of $\mathbf{R}_X$, we get

$$\mathbf{Z} = \mathrm{vec}(\mathbf{R}_X) = (\mathbf{A}^* \circ \mathbf{A})\mathbf{p} + \sigma_n^2\mathbf{e}_n, \tag{7}$$

where $\mathbf{p} = \mathrm{diag}[\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2]^{\mathrm{T}}$, $\mathbf{e}_n = \mathrm{vec}(\mathbf{I}_M)$. Define $\mathbf{B} = \mathbf{A}^* \circ \mathbf{A}$, then

$$\mathbf{B} = [\mathbf{b}(\theta_1), \mathbf{b}(\theta_2), \ldots, \mathbf{b}(\theta_K)], \tag{8}$$

$$\mathbf{b}(\theta_k) = \left[e^{j\pi(d_1-d_1)\sin(\theta_k)}, e^{j\pi(d_2-d_1)\sin(\theta_k)}, \ldots, e^{j\pi(d_m-d_n)\sin(\theta_k)}, \ldots, e^{j\pi(d_M-d_M)\sin(\theta_k)}\right]^{\mathrm{T}}. \tag{9}$$

Based on this, the position difference set of physical array sensors is defined as

$$\mathbb{D}_v = \{d_m - d_n | d_m, d_n \in \mathbb{D}\}. \tag{10}$$

In this case, if the elements in $\mathbb{D}_v$ are regarded as the position of the sensors of a virtual linear array, Eq. (7) can be regarded as a received signal vector on the difference virtual array, and Eq. (10) can be regarded as a mapping from a physical array to a difference virtual array.

Based on the above received signal model of sparse array, most of the DOA estimation algorithms are processed by the virtual received signal vector $\mathbf{Z}$ obtained by mapping of $\mathbf{R}_X$. According to the different ideas of algorithms, they can be roughly divided into two categories: algorithms based on spatial smoothing and algorithms based on compressed sensing. The main purpose of this paper is to prove the performance advantages of the proposed array, but the type of DOA estimation algorithm has no direct effect on the performance of the array. Therefore, the SS-MUSIC algorithm [14] is used to evaluate the performance of the proposed DCUA4 in the Section 4.

2.2. Sparse Array DOA Estimation Algorithm Based on SS-MUSIC

According to Eq. (7), the rank of the signal vector $\mathbf{p}$ obtained by the virtual processing of $\mathbf{R}_X$ is 1, that is, the received signals no longer satisfy the independent condition. Therefore, the traditional DOA estimation algorithm based on eigenvalue decomposition of covariance matrix is invalid. In order to solve the problem of DOA estimation of coherent signals, a common method is spatial smoothing. In this subsection, the SS-MUSIC algorithm commonly used in sparse array DOA estimation is briefly introduced.

Assuming that the virtualized linear array has L continuous sensors which can be divided into N subarrays by sliding. Each subarray has the same structure with the array sensor number L_0, where $N = \lceil L/2 \rceil$ and $L_0 = L - N + 1$. Denote the signal vector of the virtual received signal vector $\mathbf{Z}$ corresponding to the continuous part of the virtual array sensors as $\mathbf{Z}_1$, then the received signal vector corresponding to the n-th subarray is represented as $\mathbf{Z}_{1n}$, and the covariance matrix of the subarray is constructed as

$$\mathbf{R}_n = \mathbf{Z}_{1n}\mathbf{Z}_{1n}^{\mathrm{H}}. \tag{11}$$

The covariance matrices of all N subarrays are added to average, and the spatial smoothing covariance matrix is obtained as

$$\mathbf{R} = \frac{1}{N}\sum_{n=1}^{N}\mathbf{R}_n. \tag{12}$$

The spatial smoothing matrix $\mathbf{R}$ is used to replace the original received signal covariance matrix $\mathbf{R}_X$, so the traditional MUSIC algorithm can be used to realize the DOA estimation of sparse array.

2.3. Mutual Coupling

In practical applications, there is mutual influence between any two sensors in the array antenna, which is called mutual coupling effect. This subsection introduces the array received signal model considering mutual coupling and the method to evaluate the array mutual coupling.

[15] shows that the mutual coupling coefficient c_d between the two sensors is inversely proportional to the spacing d. The mutual coupling matrix of linear array can be modeled as a B-band Toeplitz matrix $\mathbf{C}$. When the sensor spacing is greater than B, the mutual coupling between array sensors can be ignored. Then, the matrix $\mathbf{C}$ is expressed as

$$[\mathbf{C}]_{mn} = \begin{cases} 0 & |d_m - d_n| > B \\ c_{|d_m - d_n|} & |d_m - d_n| \le B \end{cases} \tag{13}$$

where $[\mathbf{C}]_{mn}$ represents the n-th element of the m-th row of $\mathbf{C}$. $d_m, d_n \in \mathbb{D}$ are normalized sparse array sensor positions, and $c_0 = 1 > |c_1| > |c_2| > \ldots > |c_B| > 0$.

When the mutual coupling between array sensors is considered, the received signal model of sparse array will be rewritten from (1) to

$$\tilde{\mathbf{X}}(t) = \mathbf{CAS}(t) + \mathbf{N}(t), \tag{14}$$

Since the mutual coupling coefficient depends on the spacing between adjacent sensors, the weight function [16] can be used to characterize the mutual coupling of physical arrays. To this end, a set $\mathbb{M}(s)$ consisting of all sensor pairs with spacing equals to s is first introduced, whose expression is

$$\mathbb{M}(s) = \{(n_1, n_2) | n_1 - n_2 = s, n_1, n_2 \in \mathbb{D}\}. \tag{15}$$

Then, for the physical array whose array sensor location set is $\mathbb{D}$, its weight function is defined as

$$w(s) = \text{Card}\{\mathbb{M}(s)\}. \tag{16}$$

Therefore, the physical meaning of $w(s)$ is the number of sensor pairs with spacing equals to s.

3. THE PROPOSED SPARSE ARRAY DESIGN METHOD

In this section, the proposed sparse array based on the direct-connection of 4 uniform linear arrays is given, which is called DCUA4, and its structure is shown in Figure 1.

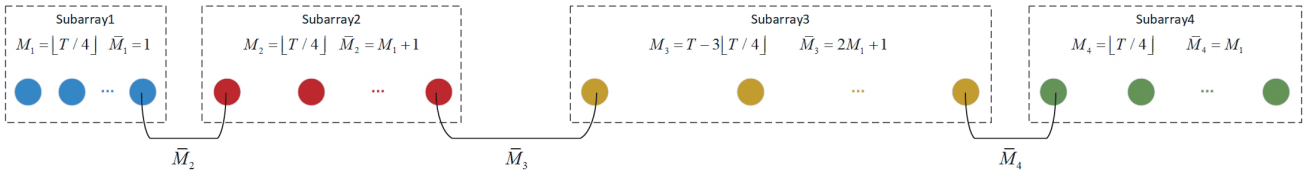

Figure 1. Sparse array based on direct-connection of 4 uniform linear arrays.

The proposed DCUA4 is obtained by directly connecting four uniform linear arrays with different sensor spacings. When the total array sensor number is $M (M \ge 4)$, the sensor position parameters of the proposed array are given as

$$\mathbb{D}_{\text{DCUA4}} = \mathbb{L}_1 \cup \mathbb{L}_2 \cup \mathbb{L}_3 \cup \mathbb{L}_4, \tag{17}$$

where,

$$\begin{cases} \mathbb{L}_1 = \{m_1 \bar{M}_1 | 0 \le m_1 \le M_1 - 1\} \\ \mathbb{L}_2 = \{m_2 \bar{M}_2 + (M_1 - 1)\bar{M}_1 | 1 \le m_2 \le M_2\} \\ \mathbb{L}_3 = \{m_3 \bar{M}_3 + M_2 \bar{M}_2 + (M_1 - 1)\bar{M}_1 | 1 \le m_3 \le M_3\} \\ \mathbb{L}_4 = \{m_4 \bar{M}_4 + M_3 \bar{M}_3 + M_2 \bar{M}_2 + (M_1 - 1)\bar{M}_1 | 1 \le m_4 \le M_4\} \end{cases}. \tag{18}$$

In Eq. (18), M_i and $\bar{M}_i$ represent the number and spacing of the i-th ($i \in [1, 2, 3, 4]$) subarray, respectively, which have values of

$$\begin{cases} M_1 = \lfloor M/4 \rfloor, \bar{M}_1 = 1 \\ M_2 = M_1, \bar{M}_2 = M_1 + 1 \\ M_3 = M - 3M_1, \bar{M}_3 = 2M_1 + 1 \\ M_4 = M_1, \bar{M}_4 = M_1 \end{cases}. \tag{19}$$

Proposition for the DCUA4 with the total sensor number of $M(M \leq 4)$, the array sensor position range of continuous difference virtual array can be obtained from $-(2M-1)M_1 + 4M_1^2 - M + 1$ to $(2M-1)M_1 - 4M_1^2 + M - 1$, where $M_1 = \lfloor M/4 \rfloor$.

Proof The array sensor location set of the difference virtual array is expressed as

$$\begin{aligned} \mathbb{V}_{\mathrm{DCUA4}} &= \mathrm{diff}(\mathbb{D}_{\mathrm{DCUA4}}, \mathbb{D}_{\mathrm{DCUA4}}) \\ &= \mathrm{diff}([\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_4], [\mathbb{L}_1, \mathbb{L}_2, \mathbb{L}_3, \mathbb{L}_4]). \\ &= \cup_{i=1}^{4} \cup_{i=1}^{4} \mathrm{diff}(\mathbb{L}_i, \mathbb{L}_j) \end{aligned} \tag{20}$$

When $i = j$, $\mathrm{diff}(\mathbb{L}_i, \mathbb{L}_j)$ represents the deviation set of sensor position of the i-th subarray, in which the elements are symmetric with respect to 0. We take all the elements in the set that are greater than or equal to 0 to form $\mathbb{V}_{ii}$. When $i \neq j$, $\mathrm{diff}(\mathbb{L}_i, \mathbb{L}_j)$ represents the cross difference set of sensor position of the i-th and the j-th subarrays which we recorded as $\mathbb{V}_{ij}$. Since $\mathrm{diff}(\mathbb{L}_i, \mathbb{L}_j) = -\mathrm{diff}(\mathbb{L}_j, \mathbb{L}_i)$, we only need to discuss the cross difference set of sensor position of the subarrays when $i < j$.

(1) Analysis of the subarray sensor position deviation set.

The sensor position deviation set of subarray 1 is expressed as

$$\mathbb{V}_{11} = [0, M_1 - 1]\bar{M}_1 = [0, M_1 - 1]. \tag{21}$$

The sensor position deviation set of subarray 2 is expressed as

$$\mathbb{V}_{22} = [0, M_2 - 1]\bar{M}_2 = [0, M_1 - 1](M_1 + 1). \tag{22}$$

The sensor position deviation set of subarray 3 is expressed as

$$\mathbb{V}_{33} = [1 - M_3, M_3 - 1]\bar{M}_3 = [0, M - 3M_1 - 1](2M_1 + 1). \tag{23}$$

The sensor position deviation set of subarray 4 is expressed as

$$\mathbb{V}_{44} = [1 - M_4, M_4 - 1]\bar{M}_4 = [0, M_1 - 1]M_1. \tag{24}$$

(2) Analysis of the subarray sensor position cross difference set.

The sensor position cross difference set of subarray 1 and subarray 2 is expressed as

$$\begin{aligned} \mathbb{V}_{12} &= [1, M_2]\bar{M}_2 + (M_1 - 1)\bar{M}_1 - [0, M_1 - 1]\bar{M}_1 \\ &= [0, M_1 - 1] + [1, M_1](M_1 + 1) \end{aligned}. \tag{25}$$

The sensor position cross difference set of subarray 1 and subarray 3 is expressed as

$$\begin{aligned} \mathbb{V}_{13} &= [1, M_3]\bar{M}_3 + M_2\bar{M}_2 + (M_1 - 1)\bar{M}_1 - [0, M_1 - 1]\bar{M}_1 \\ &= [0, M_1 - 1] + [1, M - 3M_1](2M_1 + 1) + M_1(M_1 + 1) \end{aligned}. \tag{26}$$

The sensor position cross difference set of subarray 1 and subarray 4 is expressed as

$$\begin{aligned} \mathbb{V}_{14} &= [1, M_4]\bar{M}_4 + M_3\bar{M}_3 + M_2\bar{M}_2 + (M_1 - 1)\bar{M}_1 - [0, M_1 - 1]\bar{M}_1 \\ &= [0, M_1 - 1] + [1, M_1]M_1 + (M - 3M_1)(2M_1 + 1) + M_1(M_1 + 1) \end{aligned}. \tag{27}$$

The sensor position cross difference set of subarray 2 and subarray 3 is expressed as

$$\begin{aligned} \mathbb{V}_{23} &= [1, M_3]\bar{M}_3 + M_2\bar{M}_2 + (M_1 - 1)\bar{M}_1 + [1, M_2]\bar{M}_2 - (M_1 - 1)\bar{M}_1 \\ &= [1, M - 3M_1](2M_1 + 1) + [0, M_1 - 1](M_1 + 1) \end{aligned}. \tag{28}$$

The sensor position cross difference set of subarray 2 and subarray 4 is expressed as

$$\begin{aligned} \mathbb{V}_{24} &= [1, M_4]\bar{M}_4 + M_3\bar{M}_3 + M_2\bar{M}_2 + (M_1 - 1)\bar{M}_1 - [1, M_2]\bar{M}_2 - (M_1 - 1)\bar{M}_1 \\ &= [1, M_1]M_1 + [0, M_1 - 1](M_1 + 1) + (M - 3M_1)(2M_1 + 1) \end{aligned}. \tag{29}$$

The sensor position cross difference set of subarray 3 and subarray 4 is expressed as

$$\begin{aligned} \mathbb{V}_{34} &= [1, M_4]\bar{M}_4 + M_3\bar{M}_3 + M_2\bar{M}_2 + (M_1 - 1)\bar{M}_1 - [1, M_3]\bar{M}_3 - M_2\bar{M}_2 - (M_1 - 1)\bar{M}_1 \\ &= [1, M_1]M_1 + [0, M - 3M_1 - 1](2M_1 + 1) \end{aligned}. \tag{30}$$

By finding the union of the above subarray sensor position deviation sets and the subarray sensor position cross difference sets, the elements of the virtual array sensor position set $\mathbb{V}_{\mathrm{DCUA4}}$ greater than or equal to 0 can form a set, that is,

$$\begin{aligned}\mathbb{V}^{+}_{\mathrm{DCUA4}} &= \mathbb{V}_{11} \cup \mathbb{V}_{22} \cup \mathbb{V}_{33} \cup \mathbb{V}_{44} \cup \mathbb{V}_{12} \cup \mathbb{V}_{13} \cup \mathbb{V}_{14} \cup \mathbb{V}_{23} \cup \mathbb{V}_{24} \cup \mathbb{V}_{34} \\ &= [0, M_1 - 1 + M_1 M_1 + (M - 3M_1)(2M_1 + 1) + M_1(M_1 + 1)]. \\ &= [0, (2M - 1)M_1 - 4M_1^2 + M - 1]\end{aligned} \tag{31}$$

Then, the elements in $\mathbb{V}_{\mathrm{DCUA4}}$ which are less than or equal to 0 can be expressed as $\mathbb{V}^{-}_{\mathrm{DCUA4}} = -\mathbb{V}^{+}_{\mathrm{DCUA4}} = [-(2M - 1)M_1 + 4M_1^2 - M + 1, 0]$.

Finally, the sensor position set of the difference virtual array is obtained as $\mathbb{V}_{\mathrm{DCUA4}} = [-(2M - 1)M_1 + 4M_1^2 - M + 1, (2M - 1)M_1 - 4M_1^2 + M - 1]$. ■

When the total sensor number M is 17, the actual sensor distribution and virtual sensor distribution of the proposed DCUA4 are shown in Figure 2.

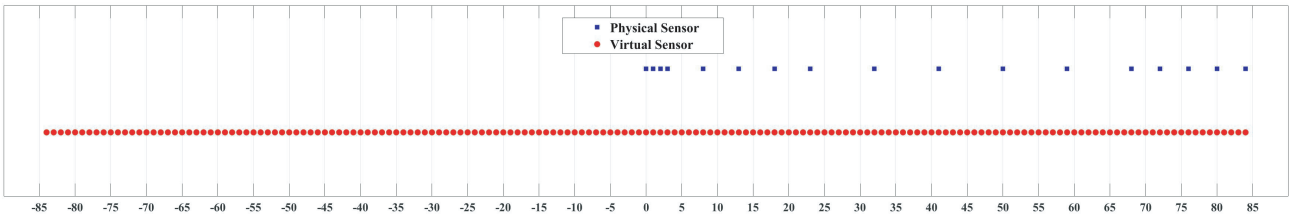

Figure 2. The actual sensor distribution and virtual sensor distribution of the proposed DCUA4 when $M = 17$.

4. NUMERICAL SIMULATIONS

In order to verify the performance of the proposed sparse array design method, comparative numerical simulations are designed in this section to compare the performance of the proposed DCUA4 with classical sparse arrays such as CA, NA, and other sparse arrays proposed in recent years such as SCA, TCA, CAP3, CATHDC, ANA11, in continuous virtual array aperture, weight function, and DOA estimation performance.

4.1. Comparison of Continuous Virtual Array Aperture

The aperture size of continuous virtual array is the most direct parameter that affects the performance of sparse array DOA estimation. The larger the aperture of the continuous virtual array is, the more sources can be estimated by the array, and the higher the accuracy of the DOA estimation result is. Figure 3 shows the variation of the aperture size of continuous virtual arrays of different arrays when the number of physical sensors varies from 1 to 25.

The simulation results show that, compared with other arrays, the aperture size of the continuous difference virtual array of the proposed DCUA4 is close to that of the ANA11, and better than other arrays. CA, TCA, SCA, CATHDC, and CAP3 are sparse arrays based on CA. One of the major defects of this kind of sparse arrays is that there are a large number of holes in their difference virtual arrays, so the continuous virtual array apertures are poor compared with the improved sparse arrays based on NA.

4.2. Comparison of Weight Function

It is mentioned in Subsection 2.3 that the mutual coupling between array sensors will affect the DOA estimation accuracy of sparse arrays. The weight function can directly show the mutual coupling of the array, and especially the values of $w(1)$, $w(2)$, and $w(3)$ reflect the mutual coupling most obviously.

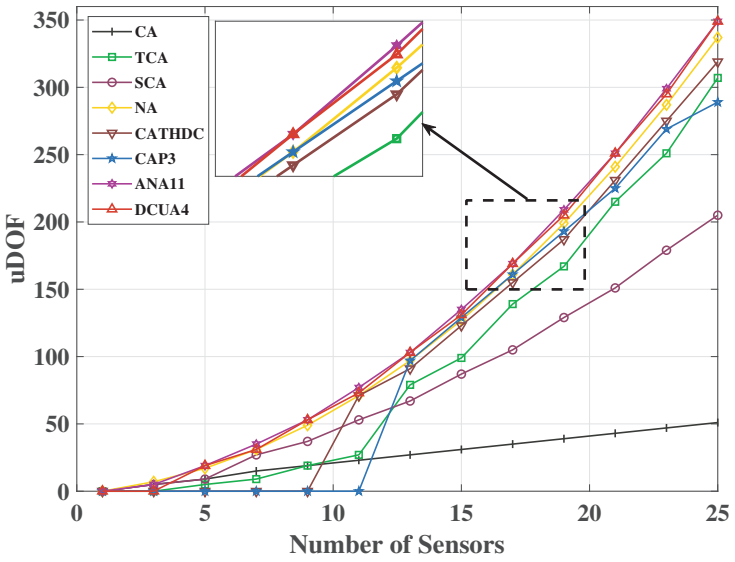

Figure 3. The number of continuous virtual array sensors of different sparse arrays varies with the actual number of array sensors.

Table 1 shows the array sensor configuration of the proposed DCUA4 and other arrays when the array sensor number M is 17.

Table 1. The array sensor configuration of the proposed DCUA4 and the other arrays when $M = 17$.

Array	Configuration of Array Sensor
CA	$0, 7, 11, 14, 21, 22, 28, 33, 35, 42, 44, 49, 55, 56, 63, 66, 70$
TCA	$0, 7, 9, 14, 18, 21, 27, 28, 35, 42, 49, 56, 72, 81, 90, 99, 108, 117$
SCA	$0, 7, 14, 21, 28, 35, 42, 46, 49, 56, 57, 63, 68, 70, 79, 90, 101$
CATHDC	$0, 3, 6, 9, 12, 13, 15, 18, 21, 32, 40, 48, 56, 64, 75, 77, 79$
CAP3	$0, 3, 6, 9, 12, 13, 15, 18, 21, 35, 43, 51, 59, 67, 78, 80, 82$
NA	$0, 1, 2, 3, 4, 5, 6, 7, 8, 17, 26, 35, 44, 53, 62, 71, 80$
ANA11	$1, 2, 3, 4, 10, 20, 30, 40, 50, 60, 70, 80, 81, 82, 83, 84, 85$
DCUA4	$0, 1, 2, 3, 8, 13, 18, 23, 32, 41, 50, 59, 68, 72, 76, 80, 84$

According to Eq. (16), Figure 4 shows the weight function $w(s), |s| \leq 4$ of different arrays when the sensor number M is 17.

According to the simulation results, it can be concluded that the CA, TCA, and SCA have lower weight function values, because this kind of arrays uses subarrays with larger sensor spacing leading to fewer sensor pairs with small spacing. On the other hand, for each array improved based on NA, the mutual coupling between the array sensors has a great influence because of the existence of the subarray with sensor spacing of 1. Because the proposed DCUA4 allocates all array sensors to four subarrays, the length of the subarray whose sensor spacing equals to 1 is reduced to some extent, and the influence of mutual coupling is weakened.

4.3. Comparison of the Performance of DOA Estimation

Using the array to estimate the DOA of the received signals is the most intuitive way to evaluate the sparse array. In this section, the DOA estimation results of the proposed DCUA4 and other arrays are given according to the SS-MUSIC algorithm described in Subsection 2.2.

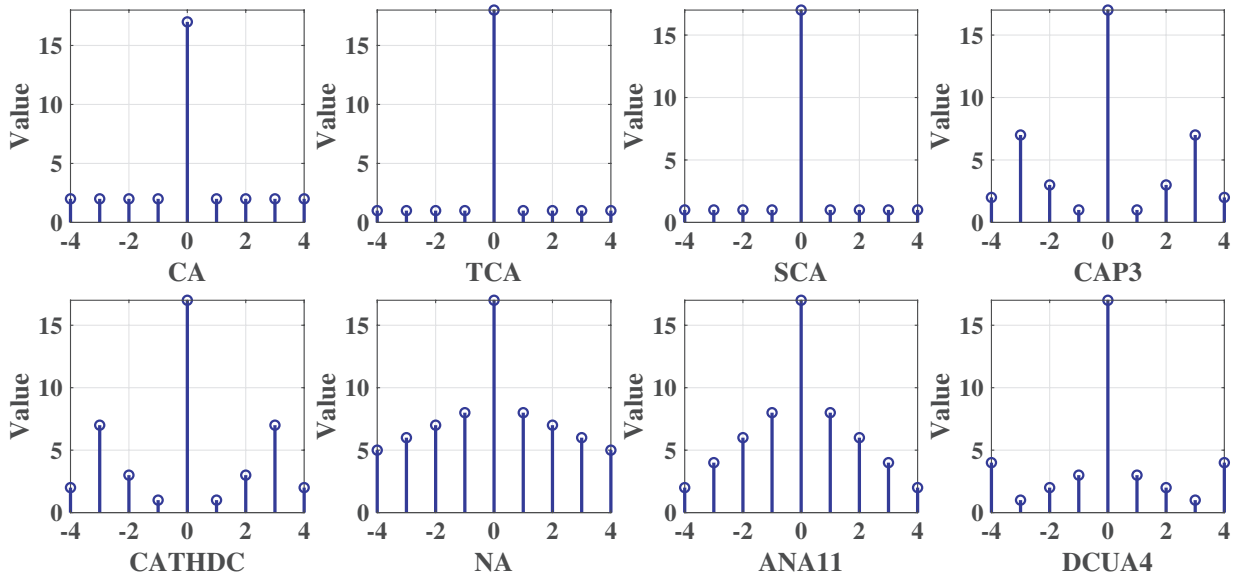

Figure 4. The weight function $w(s), |s| \leq 4$ of different arrays when $M = 17$.

4.3.1. Underdetermined Estimation

The ability of underdetermined estimation is a major advantage of sparse arrays over ULA. Figure 5 shows the DOA estimation results of the proposed DCUA4 under underdetermined conditions.

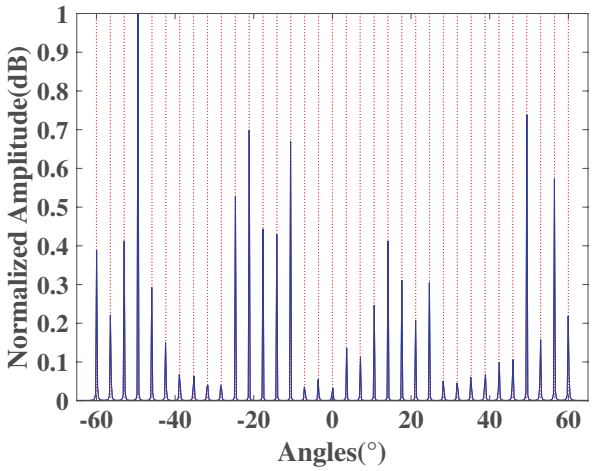

Figure 5. The DOA estimation results of the proposed DCUA4 under underdetermination conditions when $M = 17$.

Simulation parameters: sensor number $M = 17$, source number $K = 51$, and impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= 5$ dB, snapshots $J = 5000$.

The simulation results show that the proposed DCUA4 can estimate the DOAs of 51 sources under the condition of 17 physical sensors, indicating that the proposed DCUA4 has the ability of underestimation.

4.3.2. Root Mean Square Error

In this section, we give the root mean square error (RMSE) of DOA estimation for different arrays without considering the influence of mutual coupling and considering the effect of mutual coupling. The

definition of RMSE is expressed as

$$\text{RMSE} = \sqrt{\frac{1}{KL}\sum_{k=1}^{K}\sum_{l=1}^{L}[(\hat{\theta}_{k,l} - \theta_k)^2]}, \tag{32}$$

where K represents the number of signal sources, L the number of Monte Carlo simulations, $\hat{\theta}_{k,l}$ the estimated angle of the k-th signal source in the l-th Monte Carlo simulation, and θ_k the real angle of the k-th signal source.

(1) without considering the influence of mutual coupling effect

Figure 6 and Figure 7 show the variation of RMSE with Signal to Noise Ratio (SNR) and snapshots when the proposed DCUA4 and other arrays use SS-MUSIC algorithm to estimate DOA without considering the influence of mutual coupling, respectively.

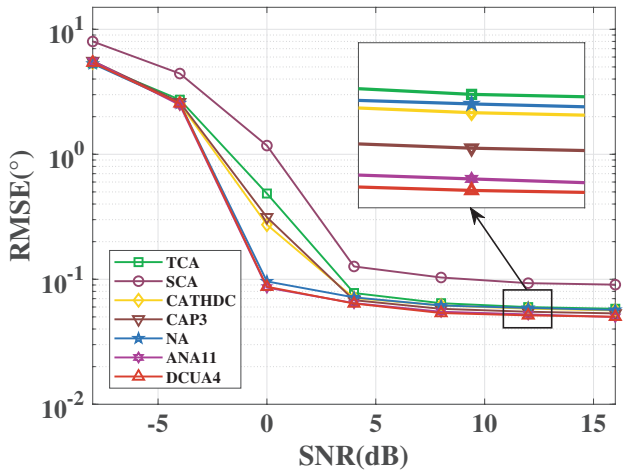

Figure 6. Variation of RMSE with SNR of DOA estimation results for different sparse arrays without considering the influence of mutual coupling effect.

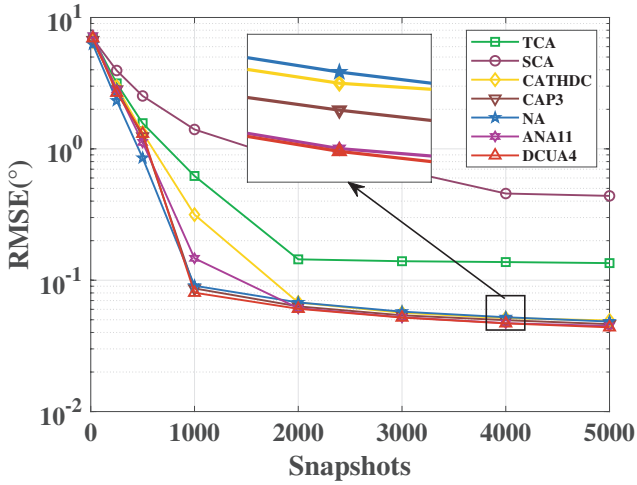

Figure 7. Variation of RMSE with snapshots of DOA estimation results for different sparse arrays without considering the influence of mutual coupling effect.

Simulation parameters: sensor number $M = 17$, source number $K = 35$, and impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= [-8, -4, 0, 4, 8, 16]$ dB, snapshots $J = 2000$, and the number of Monte Carlo simulations is 200.

Simulation parameters: sensor number $M = 17$, source number $K = 35$, impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= 5$ dB, snapshots $J = [20, 50, 100, 200, 500, 1000, 2000, 5000]$, and the number of Monte Carlo simulations is 200.

The simulation results show that the RMSE of the proposed DCUA4 is close to the ANA11 when the SNR and snapshots change, but better than the other arrays, which means that the DOA estimation result of sparse array is only related to the aperture of difference virtual array without considering the effect of mutual coupling. Because the aperture of the proposed DCUA4 and ANA11 is larger than that of the other arrays, their DOA estimation performance is the best.

(2) considering the influence of mutual coupling effect

Figure 8 and Figure 9 show the variation of RMSE with the mutual coupling coefficient c when the proposed DCUA4 and the other arrays use the SS-MUSIC algorithm to estimate the DOA.

Simulation parameters: $B = 3$, mutual coupling coefficient c is uniformly distributed at the interval of 0.02 in the range of $[0.01, 0.15]$, sensor number $M = 17$, source number $K = 12$, impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= 0$ dB, snapshots $J = 1000$, and the number of Monte Carlo simulations is 500.

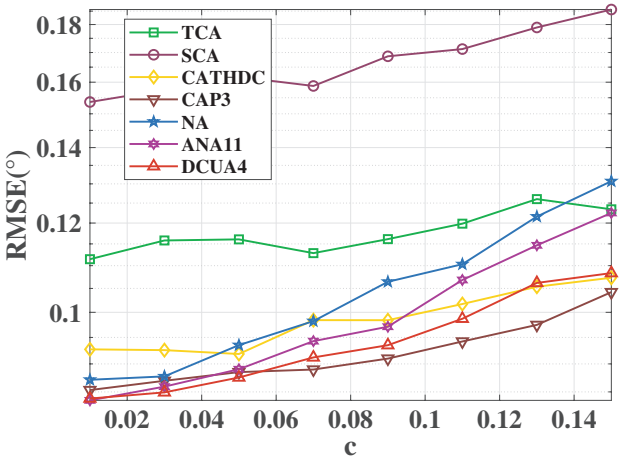

Figure 8. RMSE of DOA estimation results for different sparse arrays with the mutual coupling coefficient $0.01 \leq c \leq 0.15$.

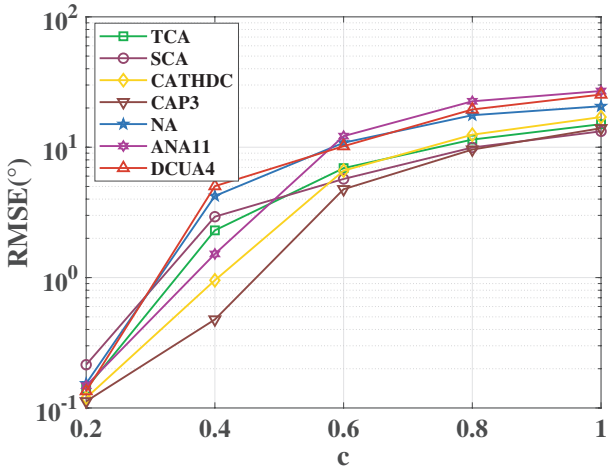

Figure 9. RMSE of DOA estimation results for different sparse arrays with the mutual coupling coefficient $0.2 \leq c \leq 1$.

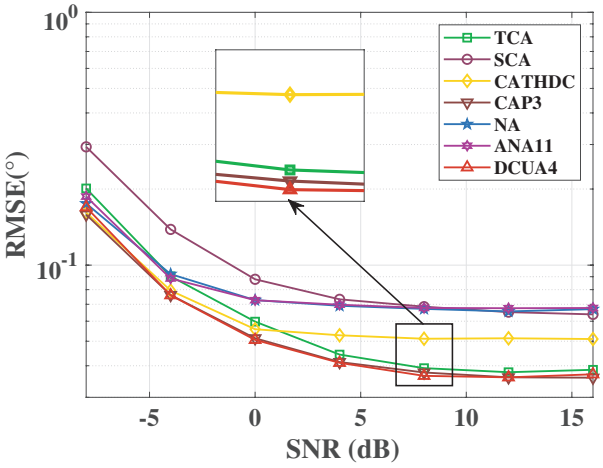

Figure 10. Variation of RMSE with SNR of DOA estimation results for different sparse arrays considering the influence of mutual coupling effect.

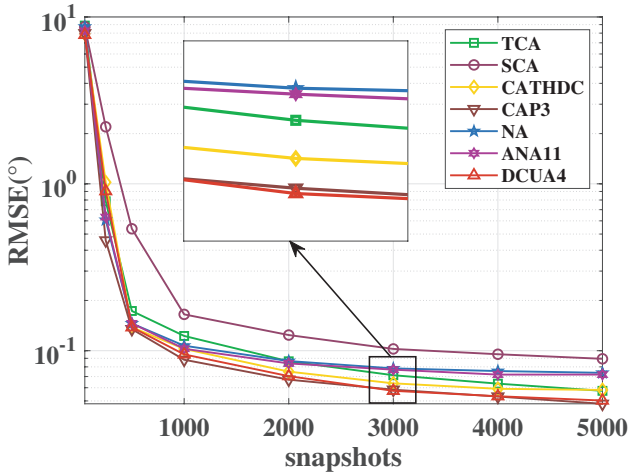

Figure 11. Variation of RMSE with snapshots of DOA estimation results for different sparse arrays considering the influence of mutual coupling effect.

Simulation parameters: $B = 3$, mutual coupling coefficient c is uniformly distributed at the interval of 0.2 in the range of $[0.2, 1]$, sensor number $M = 17$, source number $K = 12$, impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= 0\,\text{dB}$, snapshots $J = 1000$, and the number of Monte Carlo simulations is 500.

The simulation results show that when the mutual coupling coefficient is small ($0.01 \leq c \leq 0.15$), the RMSE of the proposed DCUA4 is close to CATHDC and CAP3 and superior to the other arrays. But when the mutual coupling coefficient is large ($0.2 \leq c \leq 1$), the RMSE of the proposed DCUA4 increases rapidly. The result means that the proposed DCUA4 can achieve better DOA estimation performance when considering the influence of low mutual coupling, but cannot resist the strong influence of mutual coupling.

Figure 10 and Figure 11 show the variation of RMSE with SNR and snapshots when the proposed DCUA4 and other arrays use SS-MUSIC algorithm to estimate DOA considering the influence of mutual coupling, respectively.

Simulation parameters: $B = 3$, $c = 0.1$, sensor number $M = 17$, source number $K = 12$, impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= [-8, -4, 0, 4, 8, 16]$ dB, snapshots $J = 5000$, and the number of Monte Carlo simulations is 200.

Simulation parameters: $B = 3$, $c = 0.1$, sensor number $M = 17$, source number $K = 12$, impinging angles θ are uniformly distributed in the range of $[-60°, 60°]$, SNR $= 0$ dB, snapshots $J = [20, 250, 500, 1000, 2000, 3000, 4000, 5000]$, and the number of Monte Carlo simulations is 200.

According to the simulation results, considering the low mutual coupling effect, the RMSE of the proposed DCUA4 with the change of SNR and snapshots is similar to that of CAP3 and superior to the other arrays, which means that the proposed DCUA4 has the ability to resist the influence of low mutual coupling.

The above simulation results can also qualitatively draw the following conclusions: When the influence of mutual coupling between sensors is considered, the performance of DOA estimation using sparse array is constrained both by array aperture and mutual coupling coefficient. In practical application, the influence of these two aspects should be considered comprehensively.

5. CONCLUSIONS

Based on the direct-connection of 4 uniform linear arrays, a new sparse array, called DCUA4, design method is proposed in this paper. The array can generate a large aperture difference virtual array without holes and achieve a continuous virtual array aperture close to that of the ANA11. By assigning array sensors to four subarrays, the number of small spacing sensor pairs is reduced, then the influence of mutual coupling between array sensors is weakened to some extent.

The simulation results show that the proposed DCUA4 has the ability of underestimation and prove the performance advantage of using it to estimate DOA without considering the influence of mutual coupling and considering the influence of low mutual coupling. However, when the influence of high mutual coupling is considered, the DOA estimation performance of the array deteriorates rapidly. In addition, there is still a subarray with sensor spacing of 1 in the proposed DCUA4, whose length will increase with the increase of the total sensor number, so that the array will be more and more seriously affected by mutual coupling. Therefore, there is still a lot of possibility for improvement for this array.

ACKNOWLEDGMENT

This work was supported in part by Grant No. 62171468 from National Natural Science Foundation of China.

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