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Uncertainty Analysis Method of Computational Electromagnetics Based on Clustering Method of Moments

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Abstract—Uncertainty analysis is one of the hot research issues in the field of computational electromagnetics in the past five years. The Method of Moments is a non-embedded uncertainty analysis method with relatively high computational efficiency and has the unique advantage of not being affected by the "curse of dimensionality". However, when the nonlinearity between the simulation input and output is large, the accuracy of the Method of Moments is not ideal, which severely limits its application in the field of computational electromagnetics. In this paper, an improved strategy based on the central clustering algorithm is proposed to improve the expected value prediction results of the Method of Moments, thereby improving the accuracy of the overall uncertainty analysis. At the same time, the co-simulation technology of MATLAB software and COMSOL software is completed, then the accuracy and computational efficiency of the proposed algorithm in this paper are quantitatively verified. In this case, the clustering Method of Moments is effectively popularized in commercial electromagnetic simulation software.

1. INTRODUCTION

Uncertainty analysis is a hot topic in the field of computational electromagnetics in recent five years, in order to accurately describe the random factors caused by vibration [1, 2], manufacturing tolerance [3, 4], and lack of knowledge [5–7] in the actual electromagnetic engineering environment.

Monte Carlo Method (MCM) is the most widely used uncertainty analysis method, and it has attracted the attention of computational electromagnetism for its advantages of high calculation accuracy and easy programming [8–10]. MCM is based on the law of weak large numbers, which reflects the researchers' cognition of the concept of uncertainty, that is, considering all possible situations. Therefore, in theoretical research, MCM simulation results are usually used as standard data to verify the accuracy of other uncertainty analysis methods, rather than test data. In other words, the difference between the simulation results of the MCM and the measured results is the basis for judging whether the uncertainty analysis method is effective, while the difference between the simulation results of other methods and the MCM is the basis for judging the effectiveness of the uncertainty analysis method itself. However, the computational efficiency of the MCM is very low, and it is completely uncompetitive in solving complex electromagnetic simulation problems.

Subsequently, some efficient uncertainty analysis methods, such as Direct Solution Technique (DST) [11], Unscented Transforms (UT) [12, 13], and Perturbation Method (PM) [14], have received increasing attention, but they have not been widely applied due to their poor accuracy.

In 2014, the generalized polynomial chaos theory was gradually improved in the field of computational fluid dynamics and introduced into the field of computational electromagnetics. The theory includes two kinds of uncertainty analysis methods, namely Stochastic Galerkin Method (SGM) [15, 16] and Stochastic Collocation Method (SCM) [17, 18]. The convergence of chaotic polynomial is relatively high, so SGM and SCM have the dual advantages of high calculation accuracy

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and high calculation efficiency. However, when the number of random variables increases, the number of chaotic polynomials required by the SGM and the number of matching points required by the SCM increase exponentially, which seriously reduces the computational efficiency. When the number of random variables is more than 20, the computational efficiency is not even as good as that of the MCM, resulting in long simulation time that cannot be realized, which is the "curse of dimensionality" problem. In 2018, a new sparse grid strategy was introduced to the generalized polynomial chaos theory, but its effect was only to alleviate the "curse of dimensionality" problem. The application scope is increased to 30 random variables, but the essence of exponential growth is not changed [19].

Since 2019, the "curse of dimensionality" of uncertainty analysis method has been a bottleneck problem that needs to be solved urgently in the field of computational electromagnetics. The representative research results are the Stochastic Reduced Order Models (SROM) of Liverpool University [20] and the Method of Moments (MoM) of York University [21]. SROM carries out deterministic electromagnetic simulation at specific sampling points and completes uncertainty analysis based on statistical methods to avoid the occurrence of "curse of dimensionality". However, at this stage, there is no method to effectively determine the number of specific sampling points required for SROM, so its accuracy cannot be guaranteed. MoM expands the expression by approximating the Taylor formula between the simulation input and simulation output, and characterizes uncertainty analysis by using the expected value results and the standard deviation results [21]. However, when the nonlinearity between simulation input and output is large, the accuracy of the algorithm is difficult to guarantee. In [22], an Improved MoM (IMoM) based on Richardson extrapolation method is proposed to improve the accuracy of the MoM in calculating the standard deviation results, but the accuracy of the expected value results has not been solved.

When dealing with the computational electromagnetic simulation problem with a large number of random variables, other methods cannot be realized due to the long simulation time, so it is still necessary to choose one of the SROM and IMoM to complete the simulation.

At the same time, many simulations in computational electromagnetics need to be completed by the finite element method in commercial electromagnetic simulation software, such as FEKO, CST, and COMSOL. How to promote the uncertainty analysis method in commercial electromagnetic simulation software is another research hotspot in recent years. In January 2022, COMSOL company released version 6.0 software, which introduced uncertainty analysis module into commercial electromagnetic simulation software for the first time. However, its module is based on the MCM and SCM as the core, unable to solve the "curse of dimensionality" problem, which will cause trouble to users. The most intuitive problem is that when the user introduces too many uncertain parameters, it will lead to that the construction cannot be solved within the specified time, resulting in a waste of time cost, and more seriously, the user is often not clear about the specific reasons for that the program can not be completed, resulting in further waste of computing resources.

In this paper, an improved strategy based on central clustering algorithm is proposed to improve the accuracy of the expected simulation results of method of moments, so as to completely solve the problem of "curse of dimensionality". At the same time, the proposed method is applied to COMSOL commercial electromagnetic simulation software to achieve efficient uncertainty analysis under high-dimensional random variables, and further expand the application scope of uncertainty analysis method in computational electromagnetics.

2. MOM AND ITS RICHARDSON EXTRAPOLATION METHOD IMPROVEMENT

In uncertainty analysis, the input of electromagnetic simulation is no longer deterministic parameters, and it must be modeled by random variables, as shown in formula (1), where ξ_i represents the random variable, and ξ is the set of random variables, which is the uncertainty input of the whole simulation.

$$\xi = \{\xi_1, \xi_2, \dots, \xi_n\} \tag{1}$$

MoM is based on the Taylor formula expansion theory. Assuming that the simulation input has a unique random variable ξ_1, the relationship between the simulation output and the input is:

$$y(\xi_1) = y(\overline{\xi_1}) + \left.\frac{dy}{d\xi_1}\right|_{\xi_1=\overline{\xi_1}} \times \left(\xi_1 - \overline{\xi_1}\right) + \left.\frac{d^2y}{d\xi_1^2}\right|_{\xi_1=\overline{\xi_1}} \times \frac{\left(\xi_1 - \overline{\xi_1}\right)^2}{2} + \dots, \tag{2}$$

where $\overline{\xi_1}$ represents the expected value of a random variable. $y(\xi_1)$ represents the simulation output related to random variable ξ_1 and ignores the high order expansion. The approximate calculation results of expected value and variance are as follows:

$$E(y) \approx y(\overline{\xi_1}) \tag{3}$$

$$\sigma^2(y) = E(y^2) - E^2(y) \approx \left(\frac{dy}{d\xi_1}\right)^2 \sigma_{\xi_1}^2 \tag{4}$$

When n random variables are considered in formula (1), the approximate calculation results of expected value and standard difference are:

$$E(y) \approx y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right) \tag{5}$$

$$\sigma(y) = \sqrt{\left(\frac{dy}{d\xi_1}\right)^2 \sigma_{\xi_1}^2 + \cdots + \left(\frac{dy}{d\xi_i}\right)^2 \sigma_{\xi_i}^2 + \cdots \left(\frac{dy}{d\xi_n}\right)^2 \sigma_{\xi_n}^2} \tag{6}$$

The output parameter sensitivity $\frac{dy}{d\xi_i}$ in formula (6) can be calculated by differential calculation in the following formula:

$$\frac{dy}{d\xi_i} = \frac{y_{EM}\left(\overline{\xi_1}, \cdots \overline{\xi_i} + \delta_i \cdots, \overline{\xi_n}\right) - y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right)}{\delta_i} \tag{7}$$

Among them, perturbation δ_i is the step size in the difference formula; the value is $\delta_i = \frac{\max(\xi_i) - \min(\xi_i)}{2}$; $\max(\xi_i)$ is the maximum of random variable ξ_1; $\min(\xi_i)$ is the minimum of random variable ξ_1; $y_{EM}()$ is the result of deterministic electromagnetic simulation at a specific point.

Obviously, when the nonlinearity between input and output of electromagnetic simulation is large, the approximate error in formula (2) will seriously affect the calculation accuracy of MoM. In order to improve the accuracy of standard deviation simulation results, [22] proposes an approximate method of sensitivity difference formula based on Richardson extrapolation, which solves the influence of nonlinearity on calculation accuracy in the calculation process of standard deviation. The principle is as follows:

$$\begin{aligned}\frac{dy}{d\xi_i} &= 2 \times \frac{y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i} + \dfrac{\delta_i}{2}, \cdots, \overline{\xi_n}\right) - y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right)}{\dfrac{\delta_i}{2}} \\ &\quad - \frac{y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i} + \delta_i \cdots, \overline{\xi_n}\right) - y_{EM}\left(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}\right)}{\delta_i}\end{aligned} \tag{8}$$

However, the accuracy of the expected value in formula (5) is still affected by nonlinearity, and the calculation accuracy of the IMoM needs to be improved.

3. CLUSTERING METHOD OF MOMENTS

In order to improve the accuracy of MoM in predicting the expected value, this paper proposes a clustering method of moments based on central clustering algorithm. In the process of center clustering, the optimization algorithm is needed to find the cluster center. Because the constructed optimization problem is relatively simple, the general intelligent optimization algorithm can be realized. In this paper, genetic algorithm [23, 24] is used to optimize. The concrete steps of clustering method of moments are as follows:

Step 1, similar to the MCM, the random variable set $\xi = \{\xi_1, \xi_2, ..., \xi_n\}$ in formula (1) is sampled; the number of sampling points is K_ξ; and the form of a single sampling point is $M_i = \left\{M_{\xi_1}^i, M_{\xi_2}^i, ..., M_{\xi_n}^i\right\}$. Based on the law of weak large number, the sampling point is convergent, that is, a large number of sampling points can fully characterize the uncertainty of simulation input parameters.

Step 2, to describe similarity, define the Euclidean distance between two chromosomes $L(M_i, M_j)$, as follows:

$$L(M_i, M_j) = \sqrt{\sum_{k=1}^{n} \left(M_{\xi_k}^{i} - M_{\xi_k}^{j}\right)^2}. \tag{9}$$

Step 3, several representative sampling points are selected to replace all the sampling points M_i, $i = 1, 2, ..., K_\xi$. The number of representative sampling points is t_ξ. Each representative sampling point N_i, $i = 1, 2, ..., t_\xi$ is an n-dimensional data, which can represent the proportion of the original sampling points, and the percentage is supposed to be P_i, $i = 1, 2, ..., t_\xi$.

Step 4, sorting the original sampling points from 1 to K_ξ, the parameters to be identified of genetic algorithm are the serial numbers of the representative sampling points, and their values are all in the range of [1, K_ξ]. The minimum Euclidean distance $L_{\min}(M_i)$ between each sampling point and t_ξ representative sampling points is calculated, and the calculation formula is as follows:

$$\min\left\{L(M_i, M_{N_1}), L(M_i, M_{N_2}), ..., L(M_i, M_{N_{t_\xi}})\right\}. \tag{10}$$

Step 5, the fitness function value of chromosome M_i can be calculated as $Fin(M_i) = \sum_{i=1}^{K_\xi} L_{\min}(M_i)$.

Step 6, through the conventional selection, crossover and mutation operation of genetic algorithm, the optimal solution representing the number of sampling points is obtained $\left\{N_1^{best}, N_2^{best}, ..., N_{t_\xi}^{best}\right\}$.

Step 7, with the minimum Euclidean distance as the standard, the percentage $\left\{P_1^{best}, P_2^{best}, ..., P_{t_\xi}^{best}\right\}$ of the number of sampling points represented by t_ξ sampling points is counted, and it is obvious that each percentage P_i^{best} is greater than zero, and the sum is 1.

Step 8, deterministic electromagnetic simulation is carried out at each sampling point N_i^{best}, and the output value of simulation results is denoted as $y_{EM}(N_i^{best})$.

Step 9, the expected value of uncertainty analysis results is calculated as follows

$$E(y) = \sum_{i=1}^{t_\xi} \left[y_{EM}(N_i^{best}) \times P_i^{best}\right]. \tag{11}$$

Step 10, redetermine the center point $(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n})$ of random variable model

$$(\overline{\xi_1}, \cdots, \overline{\xi_i}, \cdots, \overline{\xi_n}) = \sum_{i=1}^{t_\xi} \left[P_i^{best} \times N_i^{best}\right]. \tag{12}$$

Step 11, formula (6) and formula (8) are applied to predict the standard deviation results of uncertainty analysis.

Compared with the IMoM, the clustering method of moments adds t_ξ times of deterministic electromagnetic simulation calculations, and the impact of this increase in simulation time can be almost ignored. In other words, the clustering method of moments not only improves the accuracy of uncertainty analysis, but also retains the unique advantages of the MoM to solve the “curse of dimensionality” problem.

It is worth noting that the optimization problem only needs to find the number of sampling points, so the problem to be optimized is relatively simple. When the genetic algebra is sufficient, the genetic algorithm can inevitably solve the optimal solution without considering the convergence problem or suboptimal solution of the genetic algorithm itself.

4. PARALLEL CABLE CROSSTALK SIMULATION EXAMPLE

In order to verify the accuracy of the clustering method of moments proposed in this paper, the simulation example of parallel cable crosstalk proposed in [18] is used to complete the uncertainty analysis, and the model is shown in Figure 1. The figure shows two cables with a length of 0.5 meters

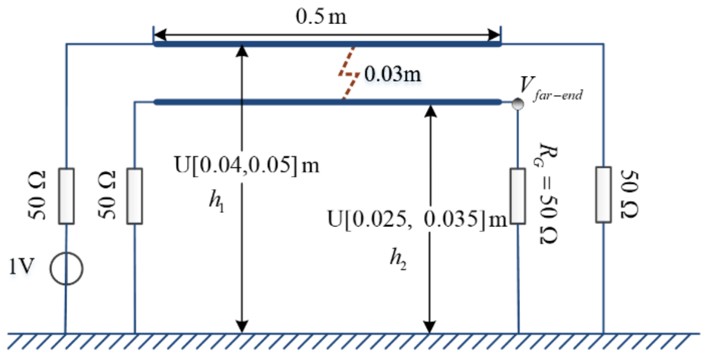


Figure 1. Model diagram of parallel cable crosstalk simulation example [18].

and a parallel distance of 0.03 meters. These two cables have geometric parameter uncertainty at the height from the ground. The random variable model is as follows:

$$\begin{cases} h_1(\xi_1) = 0.045 + 0.005 \times \xi_1 \text{ [m]} \\ h_2(\xi_2) = 0.03 + 0.005 \times \xi_2 \text{ [m]} \end{cases} \tag{13}$$

ξ_1 and ξ_2 are random variables satisfying uniform distribution in $[-1, 1]$. The excitation source is an AC power supply with amplitude of 1 V, and its frequency range is from 1 MHz to 100 MHz. The resistance of the model is 50 Ω, and the simulation output is the crosstalk voltage value of the disturbed load end. Other parameters of the model are consistent with [18].

According to the clustering method of moments, the random variable set $\{\xi_1, \xi_2\}$ is sampled. The number of sampling points is 6400, which represents K_ξ in Section 3. The number of selected representative sampling points is 8, which is t_ξ in Section 3. Obviously, $L(M_i, M_j)$ is the Euclidean distance of two-dimensional data. The genetic algorithm is carried out, and the results are shown in Table 1.

Table 1. Representative sampling points and their percentages for the clustering method of moments.

	Representative sampling points	**Corresponding percentage**
1	$N_1^{best} = \{-0.6338, 0.5835\}$	$P_1^{best} = 0.1327$
2	$N_2^{best} = \{-0.6862, -0.1864\}$	$P_2^{best} = 0.1248$
3	$N_3^{best} = \{0.7271, -0.2957\}$	$P_3^{best} = 0.1106$
4	$N_4^{best} = \{-0.4472, -0.6906\}$	$P_4^{best} = 0.1397$
5	$N_5^{best} = \{-0.0385, 0.8603\}$	$P_5^{best} = 0.0933$
6	$N_6^{best} = \{0.4329, -0.7173\}$	$P_6^{best} = 0.1244$
7	$N_7^{best} = \{0.8643, 0.7700\}$	$P_7^{best} = 0.1045$
8	$N_8^{best} = \{0.1176, 0.1859\}$	$P_8^{best} = 0.1700$

According to formula (6), formula (8), and formula (11), the uncertainty analysis results can be obtained, as shown in Figure 2(a) and Figure 2(b). Figure 2(a) is the expected value of crosstalk voltage, while Figure 2(b) is the standard deviation. The deterministic electromagnetic simulation was completed on the original 6400 sampling points, and the obtained the MCM uncertainty analysis results were used as standard data to verify the effectiveness of the proposed method.

Feature Selective Validation (FSV) method is used to determine the effectiveness of the simulation results. The FSV value provided by this method can quantitatively evaluate the difference between the simulation data and standard data, and the FSV value corresponds to the qualitative evaluation standard. More details about the FSV method can be referred to [25–27]. With the results of the MCM as the standard data, the FSV values of different uncertainty analysts are shown in Table 2. For the

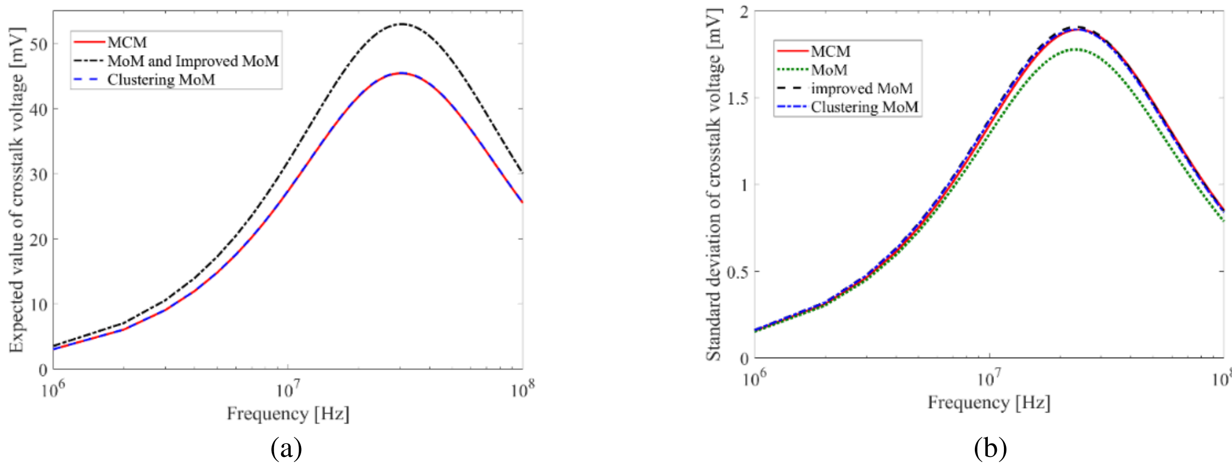


Figure 2. Uncertainty analysis of crosstalk voltage. (a) Expected value result. (b) Standard deviation results.

Table 2. The FSV values of uncertainty analysis results.

	MoM	**IMoM**	**Clustering MoM**
Expected results	0.2328	0.2328	0.0017
Standard deviation results	0.1061	0.0295	0.0327

expected value results, the results of the MoM and theIMoM are calculated by formula (5). The FSV value is 0.2328, which is only the evaluation grade of 'Good', while the FSV value of Clustering MoM is 0.0017, which is the evaluation grade of 'Excellent'. The effectiveness of the proposed Clustering MoM in improving the calculation accuracy of the expected value is verified. For the standard deviation results, the FSV value of IMoM and Clustering MoM has little difference, which belongs to the 'Excellent' qualitative evaluation grade, slightly better than the 'Very Good' qualitative evaluation grade of the MoM.

To sum up, with the quantitative evaluation results of the FSV method, the accuracy of the clustering method of moments proposed in this paper can be verified in the simulation example of parallel cable crosstalk, especially in the calculation accuracy of expected value.

5. PROMOTION OF CLUSTERING METHOD OF MOMENTS IN COMSOL SOFTWARE

In order to apply clustering method of moments to COMSOL commercial electromagnetic simulation software, it is necessary to call COMSOL software automatically. At this time, COMSOL software needs to be regarded as a black box model; the deterministic simulation input parameters are given; and the uncertainty analysis post-processing is completed through the clustering method of moments. In this paper, the co-simulation platform of COMSOL software and MATLAB software is built to solve this problem, and the implementation process is shown in Figure 3.

Firstly, there are two sources of deterministic electromagnetic simulation input parameters. One is the optimization result $\{N_1, N_2, ..., N_{t_\xi}\}$ of genetic algorithm, and the other is the input parameter with perturbation δ_i in formula (8). Secondly, the sub-function of MATLAB is generated by COMSOL software, and the input parameters are modified. At the same time, TXT file is used for data transmission. When the MATLAB software reads the TXT file, the pointer programming method can be used to eliminate the useless information rows generated by COMSOL software. For example:

(1) ——fid = fopen (strcat('EM_simulation_result', '.txt'));

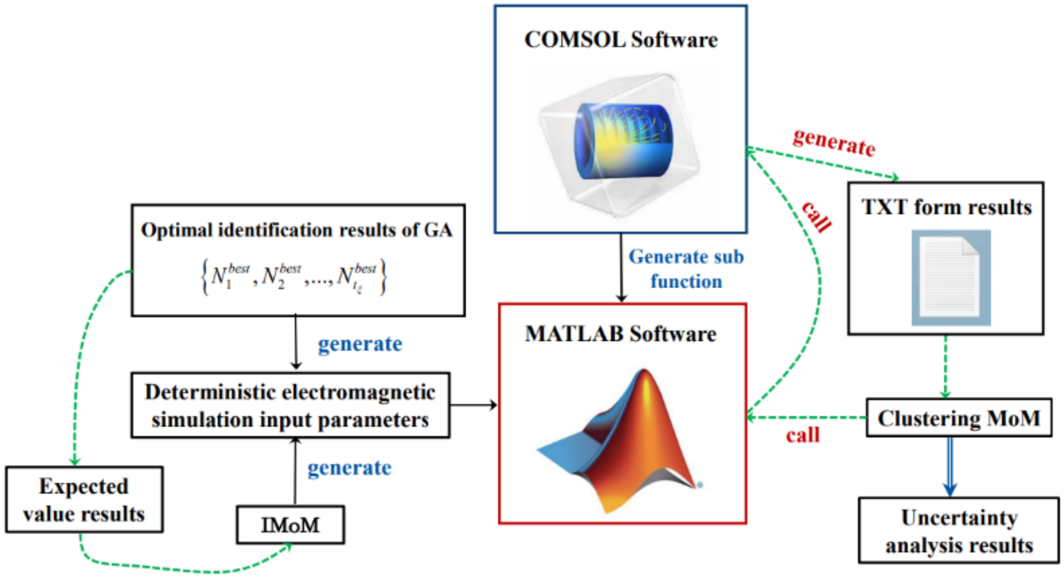


Figure 3. Construction of a co-simulation platform between COMSOL software and MATLAB software.

(2) ——data = textscan (fid, '%f', 'HeaderLines', 8);

Finally, the calculation results of the expected value are obtained by formula (11), and the calculation results of the standard deviation are obtained by formula (6) and formula (8).

Next, the "Antenna Crosstalk Simulation on Aircraft Fuselage" case in COMSOL official website (Figure 4) is used to verify the effectiveness of the proposed joint simulation scheme [28]. The detailed model of the case and the download link can be referenced. In order to reflect the advantages of clustering method of moments in solving high dimensional random variable model, a transmitting antenna (Antenna 3) is added to the original model, and its parameters are completely consistent with Antenna 1, only different in position.

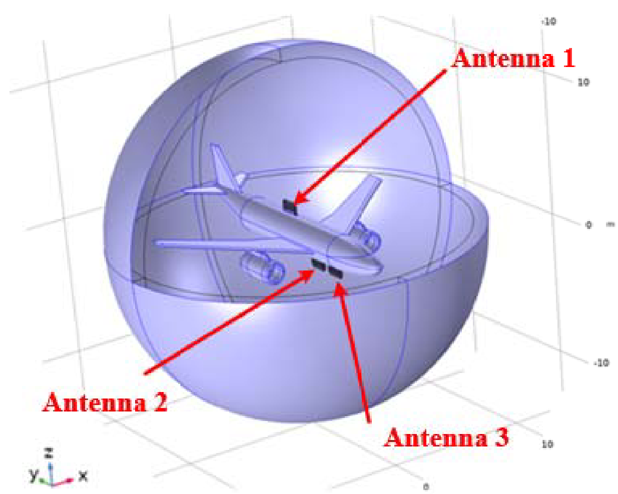


Figure 4. Screenshot of the case of aircraft antenna crosstalk in COMSOL software.

In this case, eight random variables are introduced, and the specific uncertainty is as follows:

$$\begin{cases}\varepsilon_r(\xi)=4.3\times(1+0.1\times\xi_3),\ \mu_r(\xi)=1+0.02\times\xi_4,\ P_{2,x}(\xi)=0.05\times\xi_5\,[\mathrm{m}],\ P_{2,y}(\xi)=-2.5+0.2\times\xi_6\,[\mathrm{m}],\\ P_{2,z}(\xi)=0.05\times\xi_7\,[\mathrm{m}],\ P_{3,x}(\xi)=0.05\times\xi_8\,[\mathrm{m}],\ P_{3,y}(\xi)=-4.5+0.2\times\xi_9\,[\mathrm{m}],\ P_{3,z}(\xi)=0.05\times\xi_{10}\,[\mathrm{m}].\end{cases} \tag{14}$$

where random variable ξ_3 satisfies the following probability density function relationship:

$$pdf(\xi_3)=\begin{cases}\frac{1}{2}\sin\left(\frac{3\pi}{2}\xi_3\right)+\left(1-\frac{1}{3\pi}\right), & 0\le\xi_3\le 1\\ 0, & \xi_3 \text{ is other values}\end{cases} \tag{15}$$

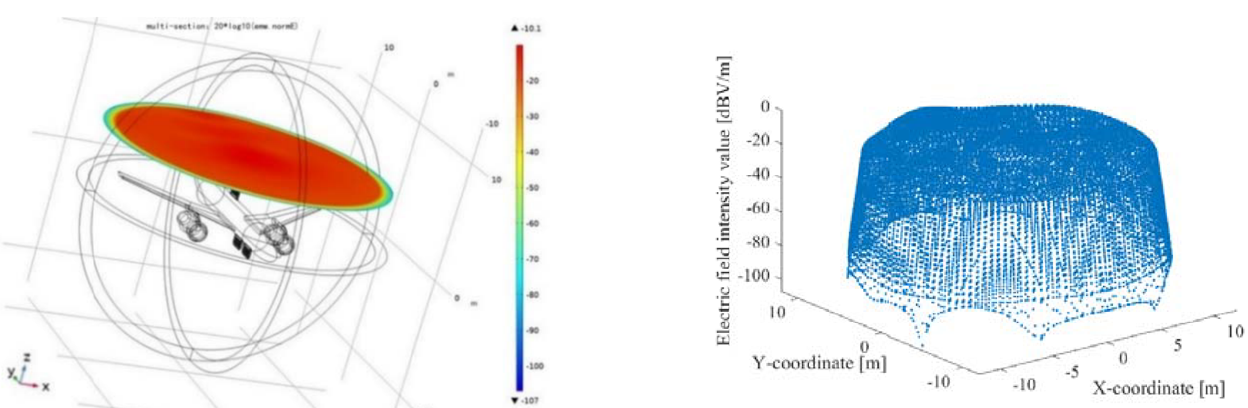
The random variables $\xi_i\,(i=4,\ldots,10)$ are uniformly distributed random variables of interval $[-1, 1]$. $\varepsilon_r(\xi)$ represents the relative dielectric constant in the antenna material, and $\mu_r(\xi)$ represents the relative permeability in the antenna material. $P_{2,x}(\xi)$, $P_{2,y}(\xi)$, and $P_{2,z}(\xi)$ represent the offsets on each coordinate axis during the 'mirror' operation of Antenna 2 in geometric construction. Similarly, $P_{3,x}(\xi)$, $P_{3,y}(\xi)$, and $P_{3,z}(\xi)$ represent the offset of Antenna 3.

The electric field intensity at $z=5\,\mathrm{m}$ is selected as the simulation output, as shown in Figure 5(a). In order to show the results more clearly, it is expressed in the form of decibel.

$$E_{\text{final}}=20\times\log_{10}(E_{\text{norm}})\,[\mathrm{dBV/m}] \tag{16}$$

E_{final} is the final electric field strength.

When the parameters satisfy $\varepsilon_r=4.3$, $\mu_r=1$, $P_{2,x}=0\,\mathrm{m}$, $P_{2,y}=-2.5\,\mathrm{m}$, $P_{2,z}=0\,\mathrm{m}$, $P_{3,x}=0\,\mathrm{m}$, $P_{3,y}=-4.5\,\mathrm{m}$, and $P_{3,z}=0\,\mathrm{m}$, the simulation results of single deterministic electric field intensity are shown in Figure 5(b). It can be seen that the variation range of electric field intensity is close to $100\,\mathrm{dBV/m}$. Therefore, when considering the uncertainty input parameters, the probability density distributions of the maximum, minimum, and average values are calculated and analyzed as the uncertainty analysis results.

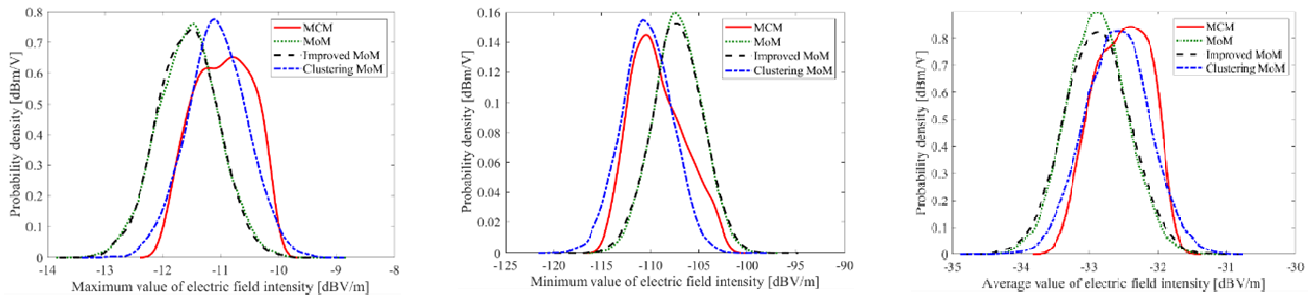


(a) (b)

Figure 5. Electric field strength results at $z=5\,\mathrm{m}$ in software. (a) Screenshot. (b) Values.

In this case, $L(M_i, M_j)$ is the Euclidean distance of 8-dimensional data. Other settings of the algorithm are completely consistent with those in Section 4, which also shows the unique advantages of non-embedded uncertainty analysis methods. Figure 6(a) is the prediction result of probability density distribution of maximum electric field intensity, while Figure 6(b) and Figure 6(c) are the prediction results of minimum and average respectively.

According to the principle of probability theory, the common area between the two probability density curves can be used as an important basis for evaluating their similarity. The closer the value is to 1, the higher the similarity is. With the MCM results as standard data, the common area values of other uncertainty analysis methods are shown in Table 3. For the clustering method of moments proposed in this paper, the public area values are all higher than 0.8, and even higher than 0.9 at the



(a) (b) (c)

Figure 6. Probability density results. (a) Maximum electric field strength. (b) Minimum value of electric field strength. (c) Mean value of electric field strength.

Table 3. Calculation of common area of probability density curves between uncertainty analysis results and the MCM results.

	MoM	**IMoM**	**Clustering MoM**
maximum value	0.6288	0.6316	0.9090
minimum value	0.7386	0.7623	0.8510
mean value	0.6960	0.6808	0.8112

maximum value, which fully illustrates that the accuracy of its uncertainty analysis is high, far better than that of the MoM and IMoM.

Table 4 shows the comparison of the computational efficiency, and the MCM needs 12800 deterministic electromagnetic simulations for 15.7 days. The clustering method of moments only needs sub-deterministic electromagnetic simulation; the simulation time is only 0.96 hours; and the calculation time is less than 0.3% of the MCM, which proves that the clustering method of moments has great advantages in dealing with high-dimensional random variable problems.

Table 4. Comparison of computational efficiency of uncertainty analysis methods.

	MCM	**MoM**	**IMoM**	**Clustering MoM**
Number of deterministic simulations	12800	9	17	25
Total simulation time	15.7 days	0.27 hours	0.5 hours	0.96 hours

6. CONCLUSION

Aiming at the field of computational electromagnetics, this paper proposes a non-embedded uncertainty analysis method called clustering method of moments, which improves the prediction accuracy of the expected value of the traditional method of moments, and then improves the accuracy of the overall uncertainty analysis. Based on the clustering method of moments, the crosstalk simulation of parallel cables with random geometric parameters is realized. With the quantitative evaluation criteria of the FSV method, the effectiveness of the proposed algorithm is verified. The joint simulation platform of MATLAB software and COMSOL software is constructed to realize the promotion of clustering method of moments in commercial electromagnetic simulation software, and its unique advantages in dealing with high dimensional random variable models are verified. The combination of clustering method of moments and COMSOL software is a further promotion of the application of uncertainty analysis method, which solves the problem of "curse of dimensionality" of uncertain input parameters in commercial electromagnetic simulation software and realizes the leading technology to a certain extent.

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