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A T-Shaped Polyomino Subarray Design Method for Controlling Sidelobe Level

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Abstract—Partitioning large planar antenna arrays into smaller subarrays reduces the system costs and gives many other advantages. In this article, symmetrical T-shaped tetromino subarrays are suggested to perform the partition process of the large planar arrays. Different structures of T-shaped tetromino subarrays have been obtained by simply rotating its orientation by multiple angles of 90 degrees such that the entire planar array aperture can be filled. Two array architectures based on the different T-shaped tetrominos are constructed. The amplitude weights of the designed subarrays are optimized by means of the genetic algorithm such that the resulting array patterns have low sidelobe level. In the first architecture, all the elements in the original array are divided into several subarrays based on three T-shape structures, while in the second architecture all the elements are combined into eight different T-shapes. To control the sidelobe level in the proposed T-shaped tetromino subarrays, a surface mask boundary function is included in the optimization process to find the optimum weights of the T-shaped subarrays. Simulation results showed that the sidelobes can be reduced to less than -20 dB in the first architecture and less than -25 dB in the second architecture, in addition to a significant reduction in the complexity of the feeding network for each one. Moreover, detailed connections of the feeding network circuitry of the used T-shaped tetromino subarray structures are given for practical implementation.

1. INTRODUCTION

A large planar array consists of hundreds or more radiating elements. Usually the weights in terms of amplitude and phase of these array elements are controlled by the transmitter/receiver (T/R) units, which are cascaded with each element in the array with a signal processing execution. Due to a large number of elements in the array (i.e., an increasing in T/R units), this leads to a decrease in the speed of signal processing implementation, as well as an increase in the costs of establishing the array system [1–4]. Initially, the idea of the subarray technique is used to overcome these problems. However, applying the traditional subarray technique leads to the creation of several problems such as the generation of uncontrolled periodic sidelobe level (SLL) [5], as well as the generation of grating lobes [6, 7]. Therefore, it is necessary to study other subarray forms to solve these problems.

In the published literature, such as in [8], a Butler matrix based on nested subarrays for achieving low SLL was obtained by rotating their matrix. In [9], lower SLL is achieved by using irregular and independent subarrays through the use of quadratic programming. In [10] and [11], a hybrid genetic algorithm and modified particle swarm were used respectively to optimize the weights of the subarrays in order to obtain a low SLL. Also, an excitation matching method was suggested to build a subarray structure and optimizing their weights to get the required radiation pattern [12].

Sometimes, using certain regular shapes such as rectangular or square ones in dividing the array aperture into smaller subarrays is impractical due to the appearance of grating lobes [12]. Although it is easy to construct identical small rectangular or square subarrays, it is impractical as mentioned.

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One solution to this problem is to create nonidentical rectangular or square subarrays by using different orientations with each separate shape [4, 13]. Other methods have been used to reduce the SLL in linear arrays [14, 15].

One of the modern methods used to reduce the SLL as well as to cancel the undesirable grating lobes is the use of new forms of irregular subarrays based on the polyomino shapes. In order to reduce the costs in practice (i.e., reducing the complexity), it is important to use a small number of irregular subarray shapes such as L-octomino and polyhex subarray shapes [16]. Also, in [17], new shapes have been proposed using tetromino and octomino to obtain the desired pattern.

In this paper, an efficient subarray shape based on a symmetrical T-shaped tetromino is proposed to partition the large planar arrays such that their radiation patterns meet some desired specification like low SLL. Such subarray structures can be simply designed in practice with lower cost. The final array consists of a set of subarrays in the form of T-shaped tetromino. To avoid undesirable grating lobes by realizing irregular subarrays, each T-shaped tetromino in the proposed array is rotated by an angle equal to 90, 180, and 270 degrees. The practical feeding network circuitry for each designed T-shaped tetromino subarray is illustrated. A genetic algorithm (GA) was used to find the optimum amplitude weights at the subarray level.

2. MATHEMATICAL MODEL FOR T-SHAPED TETROMINO STRUCTURE

The problem of using regular rectangular and square subarray shapes in partitioning the large planar arrays is solved by suggesting a certain T-shaped tetromino structure. To proceed with this irregular subarray shape, a mathematical model of the T-shaped tetromino structure should be given.

In this section, an equi-axial square planar array is considered. This array contains several equal-sized rectangular and square cells (i.e., a set of regular subarrays). Let's consider an array structure consisting of a set of N rows and M columns located on the x and y axes, where the rows and columns are parallel to the x-axis and y-axis, respectively, as shown in Figure 1.

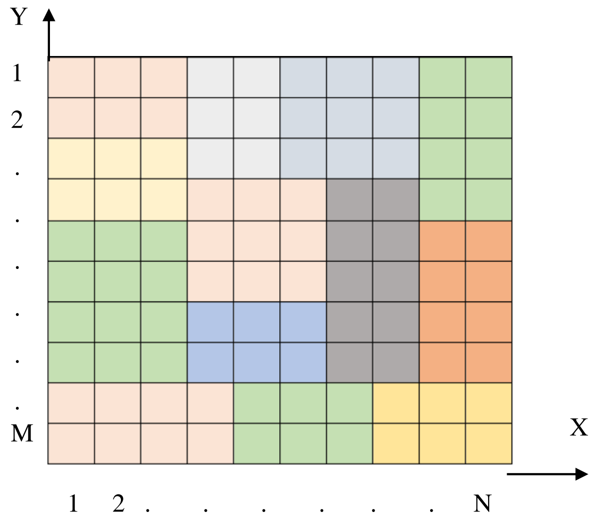


Figure 1. Structure of the square planar array with regular rectangular and square subarrays.

It is worth to mention that the physical distances between the radiating elements are equal and uniform, thus, this parameter was not included in the optimization procedures. Accordingly, the separation distance between any two successive elements on the x and y axes is referred to as dx and dy, respectively.

In order to construct the mathematical model, firstly, the array is represented by an empty matrix structure filled with zeros.

$$A = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix} \tag{1}$$

Then this structure is filled with the required T-shaped tetromino with different orientations and rotations. This polyomino can be rotated inside the structure by multiple angles of 90 degrees. In total there are four orientations of the polyomino shape. The first one is the zero orientation which will be considered as the reference direction; the second one is the 180 directions which is defined by two rows and three columns; and the 90 and 270 directions are defined by three rows and two columns, as follows:

$$T^0_{0\mathrm{deg}} = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} & 0 \end{bmatrix}, \quad T^1_{90\mathrm{deg}} = \begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \\ 0 & \mathbf{1} \end{bmatrix}, \quad T^2_{180\mathrm{deg}} = \begin{bmatrix} 0 & \mathbf{1} & 0 \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \end{bmatrix}, \quad T^3_{270\mathrm{deg}} = \begin{bmatrix} \mathbf{1} & 0 \\ \mathbf{1} & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}$$

By having a matrix structure with a four-way orientation, it is possible to construct and fill the final array with required T-shaped polyomino in positions x and y. It is important to consider that the same direction subarrays cannot be lined up consecutively inside the structure so that no occupied element remains outside the polyomino shapes. Also, the prohibition of crossing the shapes outside the boundaries of the overall array structure is included, i.e., all T-shapes with their four orientations must be arranged inside the main structure. Then, the structure can be written as follows:

$$L_\gamma(x,y) = A_{x,y} + \sum_{i=1}^{q-1} A_{xi,yi} \tag{2}$$

where γ and q are the orientation and number of elements in the tetrominos, respectively. If $L = 0$, it means that placing the tetrominos in the specified place is possible. Let's give an example of placing the first T-shaped tetromino in the zero direction (reference direction) in an empty structure composed of size $10 * 10$ as follows:

$$A = \begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathbf{1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{3}$$

Now we have a matrix structure (array matrix) model and a tetromino orientation (orientation matrix) model with conditions for placing and arranging the polyomino. This criterion allows us to describe the geometric relationships between the elements in the system. In addition, the electromagnetic relationships of the elements must be considered to calculate the desired beam pattern.

The array factor is characterized by the presence of amplitude and phase excitation through which the radiation pattern is controlled, as follows [14]:

$$AF(\theta, \emptyset) = \sum_{n=1}^{N} \sum_{m=1}^{M} w_{nm} e^{-jk[ndx(u-u_o)+mdy(v-v_o)]} \tag{4}$$

where $w_{nm} = a_{nm} e^{jp_{nm}}$, where a_{nm} and p_{nm} are amplitude and phase excitation, respectively; $k = 2\pi/\lambda$, where k is the wave number, and λ is the wavelength; u_o and v_o represent the main beam steering. Assuming that each element in the main array is linked with RF (attenuator and phase shifter), but in the case of using subarrays, all elements in it are linked with one RF, so the array factor equation must be rewritten. Let's take an array of $N * M$ filled with $N_o * M_o$ rectangular subarrays with amplitude only weighting, i.e., $p_{nm} = 0$ for further simplifying the complexity, then the subarray factor will be written as:

$$AF_{sub}(\theta, \emptyset) = \sum_{n=1}^{N_o} \sum_{m=1}^{M_o} a_{nm} e^{-jk[ndx(u-u_o)+mdy(v-v_o)]} \tag{5}$$

But in the case of T-shaped tetromino subarrays, the problem becomes more complicated because the shapes are irregular and rotated, and differ from the rectangular ones. In fact, this structure is described mathematically by knowing the orientation and coordinates of all the new subarrays, so the subarray factor will be expressed as:

$$AF_{sub}(\theta, \emptyset) = \sum_{i=1}^{q} a_i e^{-jk\left[T_{i,1}^{\gamma} dx(u-u_o)+T_{i,2}^{\gamma} dy(v-v_o)\right]} \tag{6}$$

In the end, the whole array factor containing T-shaped tetromino only is calculated, so it is necessary to know the positions and orientation of each polyomino in the final structure as:

$$AF(\theta, \emptyset) = \sum_{i=1}^{I} AF_{sub}(\theta, \emptyset) e^{-j2\pi f T_i} \tag{7}$$

where I is the number of subarrays in the main array, and T_i is the time delay and can be calculated from the coordinates in the array as [18, 19]:

$$T_i = \frac{1}{c}\left[x_i dx\right](u - u_o) + \frac{1}{c}\left[y_i dy\right](v - v_o) \tag{8}$$

where c is the light velocity. To get the desired radiation pattern by controlling a certain level of the side lobes with the proposed subarrays, an additional condition is added to the optimization process called the surface mask boundary function (SMBF). Numerically, this SMBF can be written as:

$$SMBF = \sum_{x} \sum_{y} \mid AF(\theta, \emptyset) - mask\ limit \mid^2 \tag{9}$$

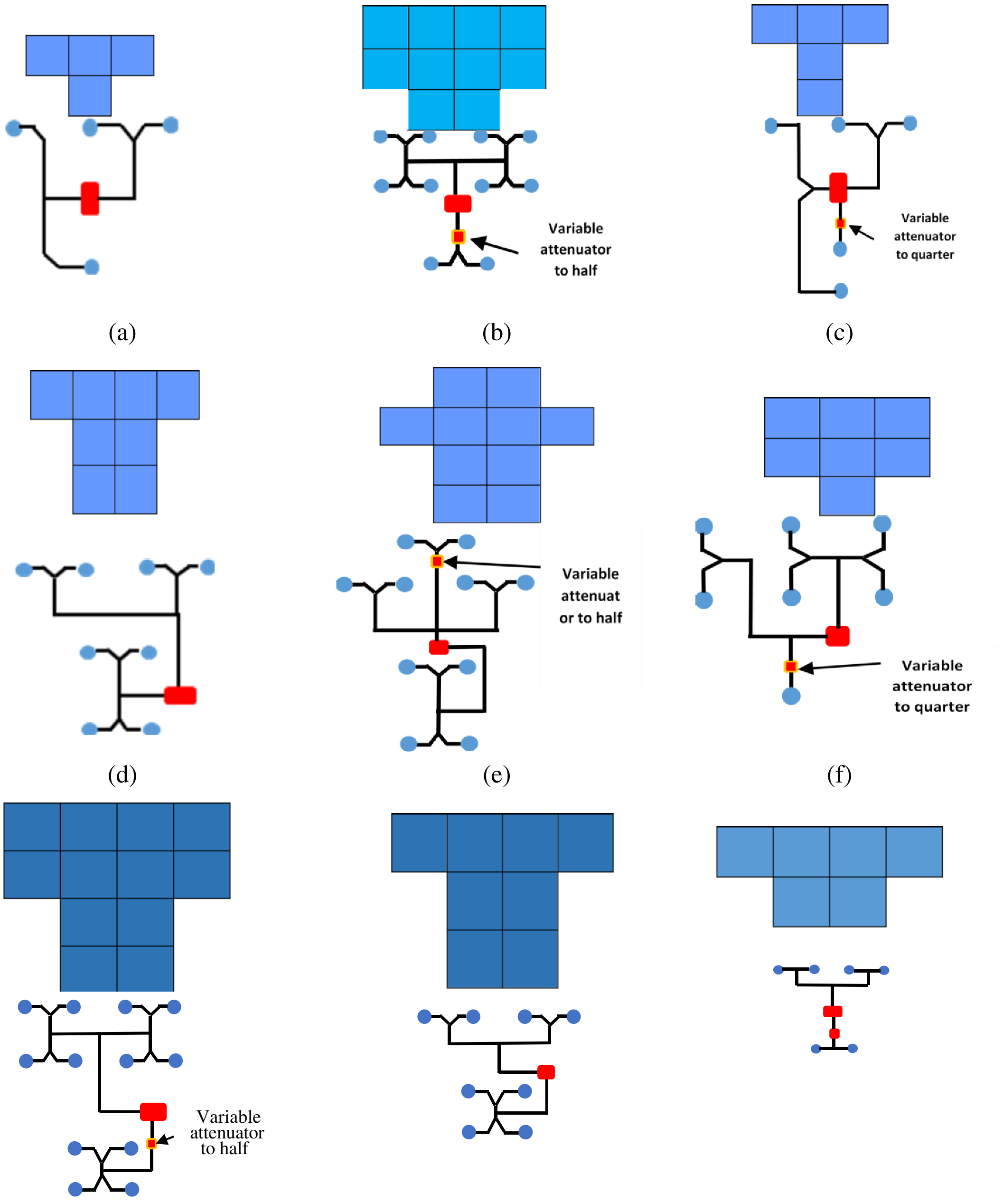
2.1. Different Shapes of T-Shaped Tetrominos Structure with Power Dividers

Usually, it is expected that designing or proposing irregular subarrays will add cost and complexity to the manufacturing process. However, selecting or suggesting identical polyomino with rotating at certain levels leads to a reduction in complexity and cost. In addition, selecting just one polyomino shape makes it easier to design lossless power dividers. Figure 2 shows a set of different T-shaped tetrominos with a power divider for each one, and each T-shape can be rotated at multiple 90 degrees. It is noticed from this figure that T-shape can be controlled to fill the large planar array. In addition, the power dividers of all T shapes in any direction are identical and uniform in distribution, but due to the rotation property, the excitation points may lead to the creation of an irregular pattern. In all the proposed power dividers, the power is evenly distributed to each element. Because of the difference in the number of elements in the proposed T-shapes, attenuators were used in the structure of the power divider in some of the proposed forms in order to unify the power distribution in the subarray.

2.2. Simulation Results

To improve the effectiveness of the proposed method, two examples were examined. In the two examples, the genetic algorithm was used as an optimization process with the following properties: population size of 20; selection is roulette; crossover is single point; mutation rate is 0.16. Also, in order to reduce the complexity as little as possible, the amplitude only excitation control (i.e., the phase excitation is set to zero) was used with taking the central symmetry property in the distribution of the array elements. The total number of elements in the array was chosen according to the proposed examples. The performance of the SLL reduction is demonstrated in each considered example.

In the first example, a planar array structure with $10 * 10$ elements is considered. This structure is tiled with three T-shaped tetromino structures. Figure 3 shows the 2D and 3D distributions of the



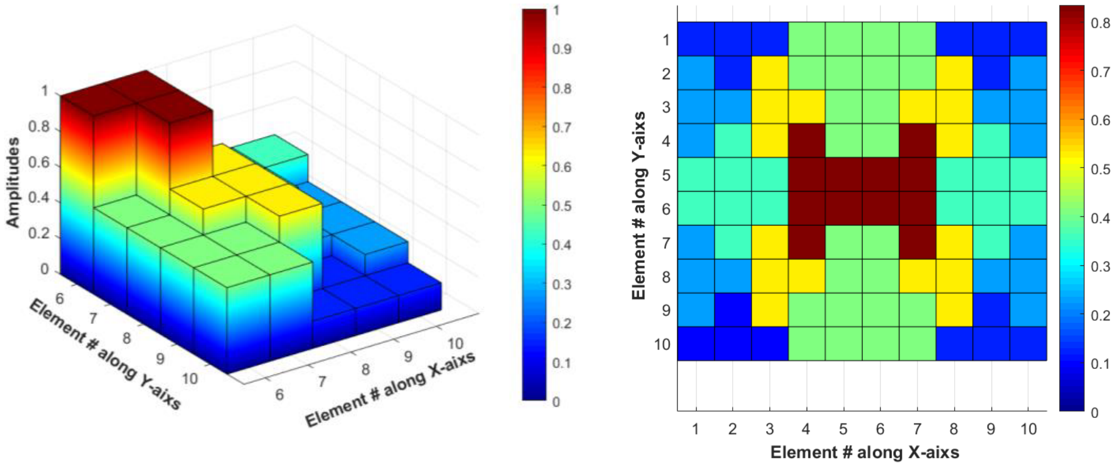
(a) (b) (c)

(d) (e) (f)

(g) (h) (i)

Figure 2. Different structures of T-shaped tetrominos. (a) Four elements T-shaped tetromino with corresponding power divider. (b) Ten elements T-shaped tetromino with corresponding power divider. (c) Five elements T-shaped tetromino with corresponding power divider. (d) Eight elements T-shaped tetromino with corresponding power divider. (e) Ten elements T-shaped tetromino with corresponding power divider. (f) Seven elements T-shaped tetromino with corresponding power divider. (g) Twelve elements T-shaped tetromino with corresponding power divider. (h) Eight elements T-shaped tetromino with corresponding power divider. (i) Six elements T-shaped tetromino with corresponding power divider.

optimized amplitudes only assigned to the first quarter of the array with the sharing of the center elements, as these weights were derived using the genetic algorithm and based on the symmetry of the origin. Figure 3(b) shows the top view of fully symmetrical weights distribution. Also, Figure 4 shows



(a) (b)

Figure 3. The 2D and 3D distribution of the optimized amplitudes only weighting. (a) 3D distribution of first quarter assigned to each subarray. (b) Top view of 3D distribution of fully planar array with subarrays.

the 2D and 3D radiation patterns. Here, optimized weights were applied to the main array to reduce the SLL to less than -20 dB using tetrominos only and according to the proposed SMBF. Table 1 shows the number of T-shaped tetrominos used.

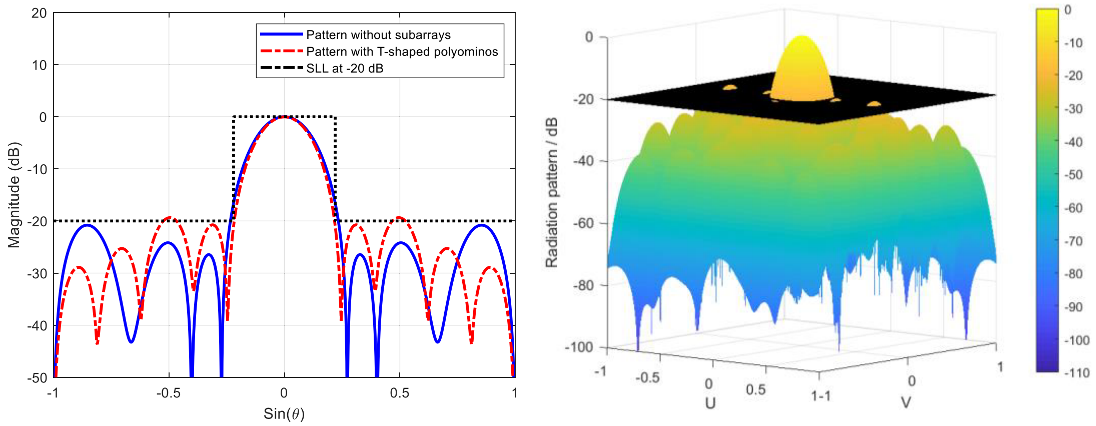
Table 1. Details of T-shaped tetromino structures of the first example.

Parameter	Value	Power divider
Total number of T-shaped tetromino	20	Three different networks only
Number of 4 elements T-shaped tetromino	16	One network structure with rotation
Number of 12 elements T-shaped tetromino	2	One network structure with rotation
Number of 6 elements T-shaped tetromino	2	One network structure with rotation
Number of symmetrical 4 elements T-shaped tetromino	4	One network structure with rotation
Number of symmetrical 12 elements T-shaped tetromino	1	One network structure with rotation
Number of symmetrical 6 elements T-shaped tetromino	1	One network structure with rotation
Total number of power divider	6	
Desired SLL	< -20 dB	-

The reduction in number of tetrominos leads to a simplification of the network from a practical point of view, as well as a reduction in the mutual coupling between the T-shaped tetrominos, in addition to a reduction in the cost.

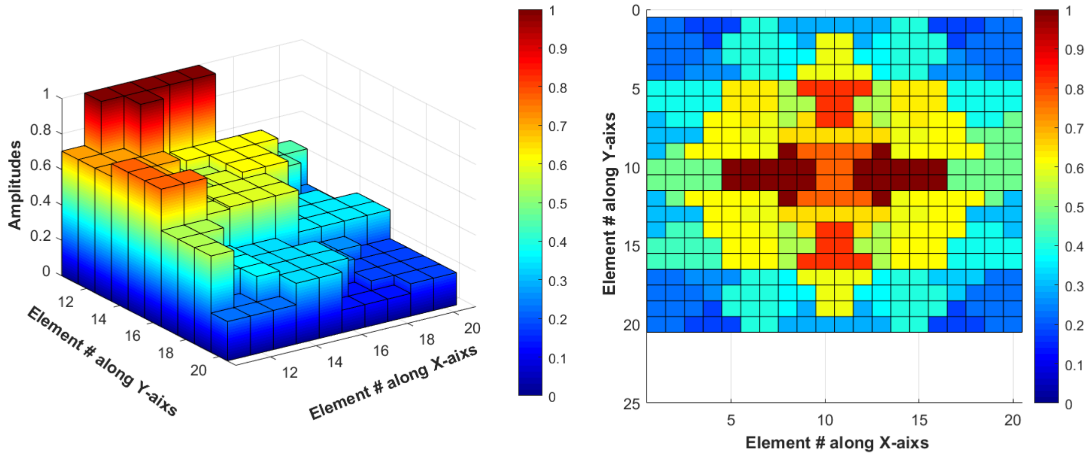
In the second example, a planar array structure of $20 * 20$ elements is examined. This structure is tiled with eight T-shaped tetromino structures. Figure 5 shows the 2D and 3D distributions of the optimized amplitudes-only weights that appear in the first quarter of the array with the shared center elements, while Figure 6 shows the corresponding 2D and 3D radiation patterns.

In this example, the complexity of the feeding network was reduced by using eight different structures of T-shaped tetromino with their rotations. A genetic algorithm is applied to implement this case. After 6 independent runs, the obtained SLL was lower than -25 dB, which gives an improvement



(a) (b)

Figure 4. The 2D and 3D radiation pattern of the first example. (a) 2D radiation pattern. (b) 3D radiation pattern.

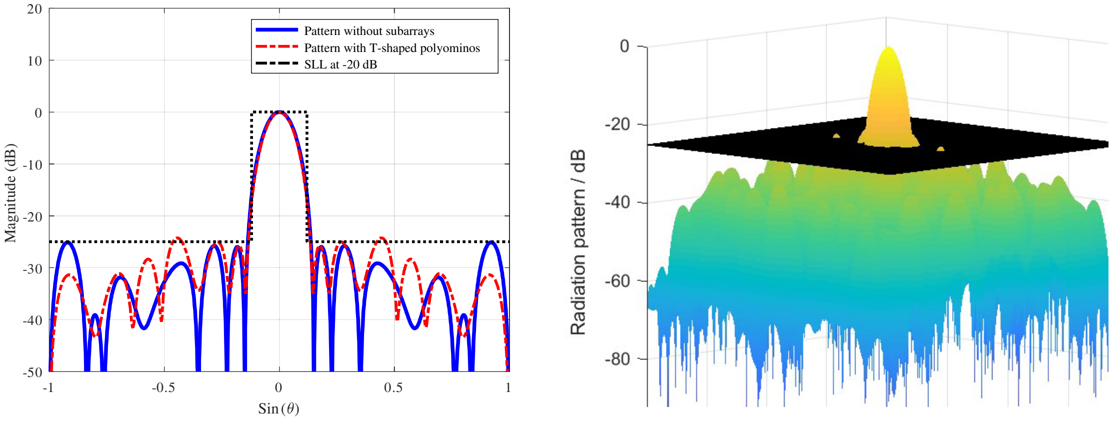


(a) (b)

Figure 5. The 2D and 3D distribution of the optimized amplitudes only weighting of the second example. (a) 3D distribution of first quarter assigned to each subarray. (b) Top view of 3D distribution of fully planar array with subarrays.

Table 2. Details of T-shaped tetromino structures of the second example.

Parameter	Value	Power divider
Total number of T-shaped tetromino	62	Eight different networks
Number of 12 elements T-shaped tetromino	2	One network structure with rotation
Number of 10 elements T-shaped tetromino	14	Two network structures with rotation
Number of 8 elements T-shaped tetromino	8	One network structure with rotation
Number of elements T-shaped tetromino	4	One network structure with rotation
Number of 6 elements T-shaped tetromino	2	One network structure with rotation
Number of 5 elements T-shaped tetromino	4	One network structure with rotation
Number of 4 elements T-shaped tetromino	26	One network structure with rotation
Total number of power divider	8	-
Desired SLL	$< -25\,\mathrm{dB}$	-



(a) (b)

Figure 6. The 2D and 3D radiation pattern of the second example. (a) 2D radiation pattern. (b) 3D radiation pattern.

by 5 dB lower than the results of the first example in which only three structures of T-shaped tetrominos were used. Table 2 shows the details of the number of power dividers structures that are used to implement this example. It can be seen from this table that only 62 structures of the T-shaped tetrominos were tilled. These structures are symmetrically and rotationally distributed.

It is worth noting that the directivity and half power beam width in both cases are not affected. This is an extra advantage of using the proposed polyomino subarray structures.

3. CONCLUSION

The proposed array partitioning method requires one type of T-shaped tetromino subarray, and it represents an easy and efficient way to divide the large square aperture array into smaller subarrays. With each polyomino, different structures can be generated by properly rotating the T-shaped tetromino. The radiation pattern of the designed arrays with T-shaped tetromino may have low sidelobes which are of great interest in practice. It is clear from the present investigation that the desired radiation patterns can be obtained through the use of the symmetric and different structures of T-shaped tetromino with an effective surface boundary mask. The consequences of using such T-shaped subarrays were a significant reduction in the SLL, as well as the complexity and cost of the feeding network. Also, the T-shaped tetromino subarrays reduce the effects of the quantization amplitudes due to the tiling of its different orientations.

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