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Concentric Magnetic Gear without Separate Modulator Structure — A Theoretical Study Based on Harmonics Interaction

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Abstract—This paper theoretically describes a new concept of passive contactless concentric magnetic gear, which, unlike the existing ones, does not use any separate modulator structure, and instead, a set of strength modulated permanent magnet pole pieces are introduced on the outer permanent magnet rotor structure. Mathematical analysis shows that stable operation in this proposed system is possible with any specific gear ratio, dependent on the number of pole pieces and on the choice of modulation constant of the pole strength variation. The system described is simpler because of the absence of separate modulator structure. The concept is new, leads to less parts count, and hence deserves consideration due to its simplicity. A simple simulation study result is also included at the end, which confirms the presented theory. The main contribution of the paper is the introduction of a new concept for designing magnetic gears using fewer physical components and showing that it is a viable design and able to produce a tangible toque at a particular gear ratio. In addition, the mathematical theory in the paper leads to interesting new results indicated in the design section of the paper, which have not been seen in the literature known to the author.

1. INTRODUCTION

Magnetic gear technology, including those pertaining to concentric magnetic gears, has been well described in the existing literature in reasonable depth by various researchers and documented in the literature [1–11]. The technology enables physical contactless movement of two mechanical elements, i.e., rotors, and the torque is transmitted through the interaction between magnetic fields, which are normally permanent magnets. Basic structure of the system generally consists of an inner rotating member containing permanent magnets and an outer rotating member, also consisting of permanent magnets. In addition, a vital component of the system is a modulator, consisting of a regular ferromagnetic material based structure which provides modulation of the magnetic field created by the permanent magnets in the rotors. The structure is shown in Fig. 1 [12]. In this figure the outer ring and inner ring show the permanent magnets in red and blue colored blocks, with the colors red and blue representing say, the north and south poles, respectively. The modulator is shown by the white and grey rectangular blocks between the inner and outer rotors, with the grey blocks made of some ferromagnetic material of high magnetic permeability. The white blocks could be air gap or some non-magnetic or low permeability material which can allow a mechanically robust structure for the purpose of physical fabrication. The physical details of the magnetic gear can be found in the literature [1–12]. While the inner and outer rotors are movable, in general the modulator structure can remain stationary, but it may be allowed to move in certain special situations. It should be noted that there will be appropriate physical arrangement in this gear system so that the structure is held together while allowing the rotors to rotate.

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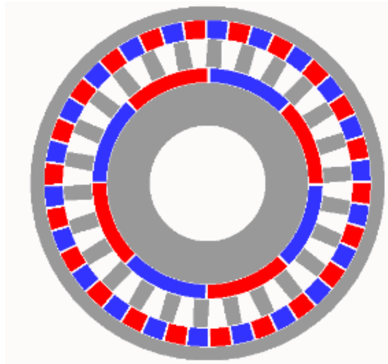


Figure 1. A magnetic gear structure [12].

The basic principle of torque transfer occurs through the magnetic field interactions between the outer and inner permanent magnet structures. The modulator changes the magnetic field between the inner and outer rotors through the grey and white blocks in the modulator. When the magnetic field from the permanent magnets encounters the grey block, it allows good passage of the magnetic field with higher strength, and when the white blocks are encountered, the magnetic field is weakened. From mathematical analysis of the magnetic field, it can be shown that only when the speed ratio between the outer and inner rotors has a specific value related to the number of inner and outer rotor permanent magnet pole pairs, and the number of modulating pole pieces (the grey and white blocks which are not permanent magnets), it is possible to transmit rotation between the outer and inner rotors with some tangible torque [1, 2]. In this system, torque transmission can happen when Equation (1) is satisfied.

$$P_o + P_i = N_m \tag{1}$$

where P_o, P_i, and N_m are the number of pole pairs of the inner rotor, and number of pole (ferromagnetic material pieces) pieces of the modulating structure [1–12]. It has also been shown [1, 8] that the transmission of rotation can happen when there is a specific gear ratio (i.e., speed ratio) G between the inner and outer rotating members, and the relation shown in Equation (2) is satisfied.

$$G = (\omega_o/\omega_i) = -(P_i/P_o) \tag{2}$$

where ω_o and ω_i are the rotational speeds of the outer and inner rotating members of the magnetic gear system. It may be noted here that there are some other variants of Equation (2) as well [8].

Although the above system works, it has three members which make it structurally complex with larger part count. In this paper, a completely new structure is proposed which does away with the modulator structure described above. Instead, the modulation is introduced in a completely different way by introducing magnetic pole pieces of variable strengths in either of the permanent magnet rotor structures, i.e., either the outer or the inner rotor structure. This paper also provides a general theory of the methodology and shows that it is a viable system. The system described is obviously simpler because of the absence of a separate modulation structure. The concept is totally new with lower parts count, hence deserves consideration for application due to its simplicity and robustness. The items noted in this paragraph are very specific contributions of this paper, which also provides a general theory of the magnetic gear system without using a modulating structure. Finally, some simulation results are included, which indicate that the theoretical equations developed in this paper are sustainable. It is to be noted that this is a theoretical paper, and at this stage the general theory has been validated through simple simulations results provided in the paper, and although a physical device has not been fabricated yet, the author believes that with the theory itself being completely new, it merits the attention of researchers. Although the creation of such a physical system may seem like difficult with variable strength permanent magnet pole pieces, it may be a very good candidate for additive manufacturing technology based fabrication. The research on additively manufactured permanent magnets is progressing very impressively at this time at Oak Ridge National Labs (ORNL) in the USA, in addition to other places [13, 14]. The progress of the work noted in those References [13, 14] suggests that creation of such a system noted in this paper by using additive manufacturing methods is not at all far-fetched.

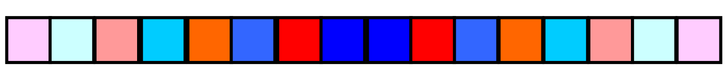
There has been some research done [15–17] using harmonic magnetic gears and nutation magnetic gears where air gap variation is done differently to modulate the magnetic field as the gear rotates. It is true that harmonic gears also do not need a separate modulation structure. But harmonic gears [15] depend on the use of a low speed rotor which is flexible and is deformed by some sliding contact to allow the variation of the air gap. The high speed rotor of this gear has to be profiled accordingly. Reference [15] also alludes to the difficulty of physical fabrication for such gears beyond one variation cycle of the air gap per mechanical rotation. It also needs a rigid stator where the permanent magnets are located, and as noted above, also needs two sets of rotors with high and low speeds, respectively. So, in terms of parts count the design has its complexity. Nutation magnetic gears [16, 17] use a different process, where there is a nutation sleeve at angle with respect to the input shaft. One set of permanent magnets is attached to a rotor connected to this bent shaft. The other set of permanent magnets is attached to a rotor which is connected to a shaft that is straight. This mechanical structure leads to wobbling of the rotor which is connected to the bent shaft with respect to the rotor connected to the straight shaft. This leads to air gap variation between the two sets of rotors and hence the variation of magnetic field strength. There is also a guide pin connected to the housing of the whole gear system to constrain the motion of the rotor connected to the bent axis [16]. So, even though this nutation gear does not have a separate modulator structure, it also has complexity in its design. The proposed method in this paper, to the best of the belief of this author, is a new addition to the methods of creating a magnetic gear without a separate modulation structure. The method, at an initial look, may seem like a difficult process from the fabrication point of view. But as noted in the previous paragraph, with significant advances in the area of additive manufacturing [13, 14] over the past years, where the option of magnetization exists so that magnetic strength of a permanent magnet can be controlled during the additive manufacturing process itself, it probably will not be too difficult to achieve a variable strength permanent magnet rotor set by using such a process. Of course, one can use permanent magnets fabricated separately with variable field strengths using conventional methods to magnetize those and attach those on a rotor later. As regards comparison between different magnetic gears, i.e., harmonic, nutation, and the method described in this paper, all of them are without separate modulation structures, which could be part of a future research undertaking and could involve study of both the performance and the fabrication process.

2. DESCRIPTION OF THE SYSTEM

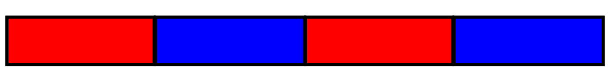
2.1. Structural Representation

For the sake of simplified discussion, let us cut the structure of the magnetic gear's outer and inner rotors shown earlier in Fig. 1 and spread it flat as shown in Fig. 2, covering a full rotation. It should be noted that the following figures (Figs. 2 and 3) are cut up view of the conceptual structure shown in Fig. 1, and these two figures, i.e., Fig. 2 and Fig. 3, are not meant to be an exact representations of Fig. 1 in terms of pole count, etc.

In Fig. 2, let us assume that color red represents north, and color blue represents south poles. The upper row represents the outer rotor with larger diameter, and the lower row represents the inner rotor



(a)



(b)

Figure 2. Cut up view of the magnetic gear structure. (a) Outer rotor structure. (b) Inner rotor structure.

with smaller diameter. The modulator structure has been removed in this system shown in Fig. 2. However, the strengths of the magnetic pole pairs as they progress from left to right increase in field strength, and for the sake of this discussion it is assumed that the field strength achieves maximum when it is at the middle of the structure, then again the strength starts decreasing till it reaches the right edge. After that, the magnetic strength profile is repeated since the magnetic structure's circular shape is repeated. For convenience of understanding, in Fig. 2, the color of the poles has been shown to start with lighter colors at the edges and becomes darker as it goes towards the middle to represent higher strength of the magnetic pole pairs. For the inner rotor, unlike the outer rotor, the magnetic field strengths of the poles do not change.

The structure shown in Fig. 2 is repeated in Fig. 3, where the magnetic field strength magnitude is modulated. It should be noted that only the magnitude, not the polarity, is indicated in Fig. 3 for keeping the diagram simple. This simple modulation can be further enhanced, and an amplitude

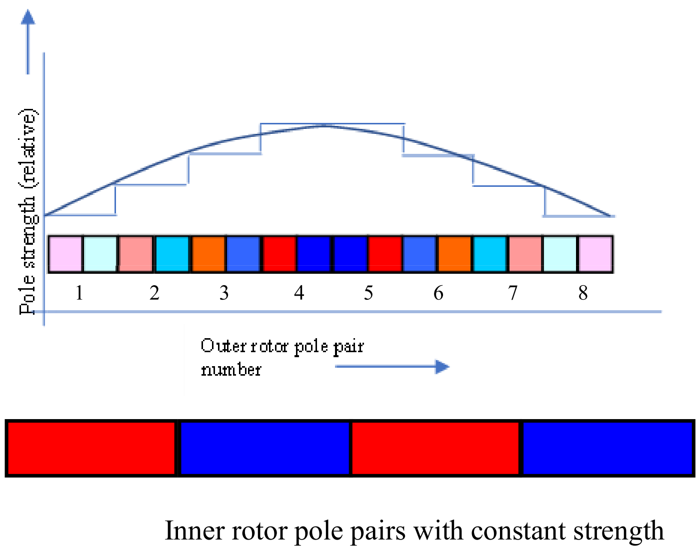


Figure 3. Cut up view of the magnetic gear structure showing pole strength (relative) of the outer rotor pole pairs.

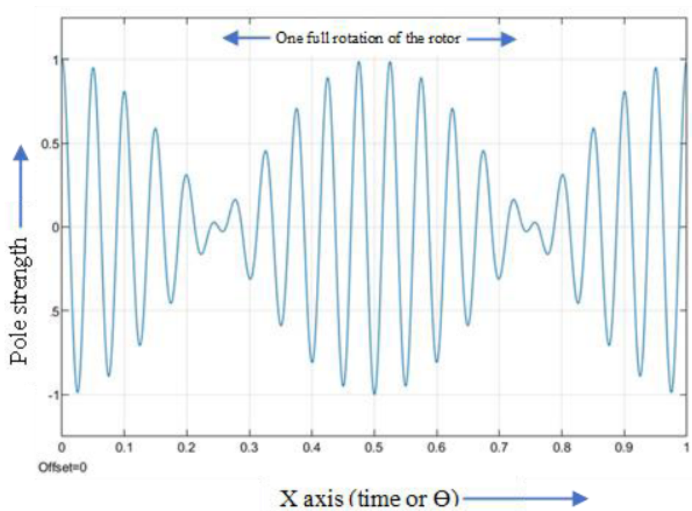


Figure 4. Amplitude modulation of the magnetic field, either as a function of space angle Θ, or as a function of time t, assuming the rotor is rotating.

modulation of the type shown in Fig. 4 can be created. Fig. 4 shows the amplitude of modulation of a time function and the nature of the waveform. Fig. 3 will pictorially look more and more close to Fig. 4 if there were more magnetic poles introduced in the system, and the diagram of Fig. 3 was compressed horizontally. In this amplitude modulated magnetic structure, the nature of the waveform will be a function of both space (i.e., angular position Θ) and time t when the amplitude modulated magnetic structure rotates. If the rotor is in motion, then its magnetic field is a function of time at a particular space angular position Θ, and similarly if a particular moment of time is considered, then it will be a function of the position Θ. In this figure, obviously, a full cycle can represent one full rotation of the rotor structure, after which the cycle repeats. In Fig. 4, it can be assumed that the rotor is rotating, and hence the magnetic field strength seen from a stator frame of reference will be a function of both variables space angle Θ and time t. An observation which should be noted here is that Fig. 4 represents an ideal version of a modulated system. In a real physical construct of such a system since the number of permanent magnet pole pieces is finite, such an ideal version of modulation shown in Fig. 4 may not be realizable, which may lead to some additional harmonics, etc. However, the mathematical theory presented in the next section remains valid because any additional harmonics will not contribute to an average torque, when being subjected to the coupling of the interactive harmonics presented through the equations in the next section.

3. MATHEMATICAL THEORY

3.1. Mathematical Analysis

Based on the description in the previous section and Fig. 4, the mathematical analysis of the proposed magnetic gear system can be formulated as follows in the next paragraphs. It may be noted here that in order to find the gear ratio of the regular magnetic gear with the modulation structure in the existing literature, the analysis is normally based on trying to create the same number of pole pairs in one of the space harmonics involving the rotating members [1] in order to produce toque. This paper is taking a completely general mathematical approach and shows that it is really not necessary to make such assumptions about bringing equal pole numbers in the picture at all while analyzing such structures, and shows that the gear ratio follows automatically through the mathematical analysis such that a tangible torque can be produced.

Based on Fig. 4, the flux density due to the permanent magnets in the outer rotor, with magnetic strengths modulated, can be expressed as follows in order to capture the graph in Fig. 4:

$$\lambda_o(t,\theta) = A\cos\{P_o * (\omega_o t - \theta)\} * \cos\{nP_o * (\omega_o t - \theta)\} \tag{3}$$

In Equation (3), the subscript "o" indicates outer rotor, λ_o the flux, A the maximum amplitude of the flux, P_o the number of pole pairs, ω_o the angular velocity, and θ the mechanical spatial angle with respect to a stationary frame of reference. All these quantities are for the outer rotor, and n is defined as a modulation factor so that the graph in Fig. 4 can be depicted mathematically. It should be noted that the quantity n is a real number and not necessarily an integer.

Similarly, for the inner rotor, where the magnetic flux strengths are not modulated, the flux can be expressed as:

$$\lambda_i(t,\theta) = B\cos\{P_i * (\omega_i t - \theta - \alpha)\} \tag{4}$$

In Equation (4), the subscript "i" indicates inner rotor, λ_i the flux, B the maximum amplitude of the flux, P_i the number of pole pairs, ω_i the angular velocity, θ the same spatial angle as before, and α (a constant) is the spatial angle indicating the angular separation between the outer and inner rotor magnetic fields at $t = 0$, or any other time for that matter. It may be worthwhile to mention here that the outer rotor is chosen for the purpose of modulating the pole piece strengths simply because there are more pole pieces in the outer rotor, and hence it is more convenient to modulate the pole strength there from the perspective of physical fabrication if so needed. In principle, one could choose the inner rotor as well for modulating the pole strength, and the theory presented will be similar and equally valid.

Based on regular trigonometry considerations, Equation (3) can be expanded as follows:

$$\begin{aligned}\lambda_o(t,\theta) &= \frac{A}{2}\left\{\cos\left[\{P_o*(\omega_o t-\theta)\}+\{nP_o*(\omega_o t-\theta)\}\right]+\cos[\{P_o*(\omega_o t-\theta)\}-\{nP_o*(\omega_o t-\theta)\}]\right\}\\ &= \frac{A}{2}\left[\cos\{[(1+n)P_o*(\omega_o t)]-[(1+n)P_o*\theta]\}+\cos\{[(1-n)P_o*(\omega_o t)]-[(1--n)P_o*\theta]\}\right]\\ &= \frac{A}{2}\left[\{\lambda_{o1}(t,\theta)\}+\{\lambda_{o2}(t,\theta)\}\right] \qquad (5)\end{aligned}$$

where,

$$\lambda_{o1}(t,\theta)=\cos\left\{[(1+n)P_o*(\omega_o t)]-[(1+n)P_o*\theta]\right\} \qquad (6)$$

and,

$$\lambda_{o2}(t,\theta)=\cos\left\{[(1-n)P_o*(\omega_o t)]-[(1-n)P_o*\theta]\right\} \qquad (7)$$

In order to produce torque, the two magnetic fields $\lambda_o(t,\theta)$ and $\lambda_i(t,\theta)$ will need to react with each other. In particular, to evaluate the torque, one needs to multiply these two quantities and $\sin(\alpha)$, based on the regular torque equation between two magnetic fields detailed in various electric machine theory textbooks [18]. Since $\lambda_o(t,\theta)$ has two separate terms indicated in Equations (6) and (7), we can do this multiplication with one term at a time and see the effect.

Hence, we can write as follows:

$$\begin{aligned}T_1 &= \frac{A}{2}[\lambda_{o1}(t,\theta)*\lambda_i(t,\theta)] = 1^{\text{st}}\text{ component of torque} \qquad (8)\\ &= \frac{AB}{2}\left[\cos\left\{[(1+n)P_o*(\omega_o t)]-[(1+n)P_o*\theta]\right\}*\cos\{P_i*(\omega_i t-\theta-\alpha)\}\right] \qquad (9)\end{aligned}$$

which is,

$$\begin{aligned}&\frac{AB}{4}\left\{\cos[P_i*(\omega_i t)+(1+n)P_o*(\omega_o t)-(1+n)P_o\theta-P_i\theta-P_i\alpha]\right\}\\ &+\frac{AB}{4}\left\{\cos[P_i*(\omega_i t)-(1+n)P_o*(\omega_o t)+(1+n)P_o\theta-P_i\theta-P_i\alpha]\right\} \qquad (10)\end{aligned}$$

The term defined by the first part of Equation (10) is denoted by T_{11} as follows:

$$T_{11}=\frac{AB}{4}\left\{\cos\left[P_i*(\omega_i t)+(1+n)P_o*(\omega_o t)-(1+n)P_o\theta-P_i\theta-P_i\alpha\right]\right\} \qquad (11)$$

which is a function of time and space angle. This means that at a particular angle θ, it is a function of time only, and similarly at a particular time t, it is a function of θ only. As we can see from above term in Equation (11), the cosine function will lead to a time-average torque equal to zero at any angle θ due to the time t variation. Hence, in Equation (11), to make the torque time invariant (so that the time-average does not become zero in general), we need to equate the term within the cosine term which contains time t, equal to zero, so that the overall cosine term becomes time invariant. This requires that,

$$P_i*(\omega_i t)+(1+n)P_o*(\omega_o t)=0 \qquad (12)$$

implying that,

$$\frac{\omega_i}{\omega_o}=-(1+n)\frac{P_o}{P_i}=G. \qquad (13)$$

So, the above Equation (13) indicates the speed ratio, or correspondingly the gear ratio of the device required to have a non-zero time-average torque. It should be noted that once this gear ratio is chosen, based on the first part of Equation (10) requirement, and then this is used in the second part of Equation (10), the time varying terms in the second part of (10) will create a zero average, i.e., there will be no contribution to time-average torque from the second part of Equation (10). Hence, we can, for our purpose, ignore the second part of Equation (10) from further consideration in our analysis. The same applies to the second part in Equation (5), i.e., $\lambda_{o2}(t,\theta)$, and we can ignore that as well. But the end conclusion regarding the gear ratio and torque will be the same, regardless of which component of torque is chosen for harmonic analysis, since this is a physical device with only one solution related to the gear ratio and torque. The same conclusion would have resulted if we chose any other components of $\lambda_o(t,\theta)$ for our analysis, and the value of gear ratio and average torque would have been the same.

Now, let us evaluate the total torque (averaged over time and then integrated over the whole circumference over a space angle 2π). If we use Equation (10), then we get the total time-average torque from the first part of Equation (10) integrated over the whole 2π radians as follows:

Total torque $= T_{1av}$ (time-averaged) is:

$$\begin{aligned} T_{1av} &= \frac{AB}{4}\int_0^{2\pi}\{\cos[-(1+n)P_o\theta - P_i\theta - P_i\alpha]\}\,d\theta \\ &= \frac{AB}{4}\int_0^{2\pi}\{\cos\left[-\left(1+n\right)P_o\theta - P_i\theta\right]\}\{\cos[-P_i\alpha]\} - \{\sin\left[-\left(1+n\right)P_o\theta - P_i\theta\right]\}\{\sin[-P_i\alpha]\}d\theta \\ &= \{\cos[-P_i\alpha]\}\frac{AB}{4}\int_0^{2\pi}\{\cos\left[-\left(1+n\right)P_o\theta - P_i\theta\right]\}\,d\theta \\ &\quad -\{\sin[-P_i\alpha]\}\frac{AB}{4}\int_0^{2\pi}\{\sin\left[-\left(1+n\right)P_o\theta - P_i\theta\right]\}\,d\theta \\ &= \frac{\{\cos[-P_i\alpha]\}AB*\sin\{\left[-\left(1+n\right)P_o - P_i\right]2\pi\}}{4[-(1+n)P_o - P_i]} + \frac{\{\sin[-P_i\alpha]\}AB*\cos\{\left[-\left(1+n\right)P_o - P_i\right]2\pi\}}{4[-(1+n)P_o - P_i]} \\ &\quad -\frac{\{\sin[-P_i\alpha]\}AB}{4[-(1+n)P_o - P_i]} \end{aligned} \tag{14}$$

once the integral limits from 0 to 2π are placed in the above equations.

At this point we can multiply Equation (14) with $-\sin(P_i\alpha)$, so that the torque angle can be taken into account. Doing so leads to the total torque, which is,

$$\begin{aligned} \{-\sin(P_i\alpha)\}\{T_{1av}\} &= \frac{\{\cos[-P_i\alpha]\}\{-\sin[P_i\alpha]\}AB*\sin\{\left[-\left(1+n\right)P_o - P_i\right]2\pi\}}{4[-(1+n)P_o - P_i]} \\ &\quad +\frac{\{-\sin[P_i\alpha]\}\{\sin[-P_i\alpha]\}AB*\cos\{\left[-\left(1+n\right)P_o - P_i\right]2\pi\}}{4[-(1+n)P_o - P_i]} \\ &\quad -\frac{\{-\sin[P_i\alpha]\}\{\sin[-P_i\alpha]\}AB}{4[-(1+n)P_o - P_i]} \end{aligned} \tag{15}$$

It should be noted here that this multiplication by $-\sin(P_i\alpha)$ is necessary and is not a duplication. This is because α in Equation (4) was due to the separation of the inner rotor magnetic field with respect to the outer rotor. But the multiplication by $-\sin(P_i\alpha)$ is now needed to account for torque between the two magnetic fields separated by an angle. Equation (15) shows that if the torque angle $\alpha = 0$, then there is no torque, which is consistent with the torque equation for magnetic systems. If $P_i\alpha = \pi/2$, then the first term in Equation (15) disappears due to the cosine term in front. In this case $\sin\, P_i\alpha = 1$, and Equation (15) can be simplified as follows:

$$T_{1av} = \frac{AB*\cos\left\{\left[-\left(1+n\right)P_o - P_i\right]2\pi\right\}}{4[-(1+n)P_o - P_i]} - \frac{AB}{4[-(1+n)P_o - P_i]} \tag{16}$$

In order to have a nonzero torque from the above equation, the quantity represented by $\cos\{[(1+n)P_o + P_i]2\pi\}$ cannot be equal to 1, implying that $(1+n)P_o + P_i$ cannot be an integer. A simple way to achieve this is to choose the cosine function magnitude above equal to 0, implying that the term inside the cosine within the braces is $\pi/2$. A couple of examples can be chosen. For example, if

$$[(1+n)P_o + P_i] = (\pm)\frac{1}{4}, \frac{3}{4}, \frac{5}{4}, \frac{7}{4}, ..., \frac{m}{4}, ..., \text{etc.} \tag{17}$$

where m is an odd number, which will result in:

$$T_{1av} = -\frac{AB}{4[-(1+n)P_o - P_i]} = -\frac{AB}{m} \tag{18}$$

Similarly, if,

$$[(1+n)P_o + P_i] = (\pm)\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, ..., \frac{m}{2}, ..., \text{etc.} \tag{19}$$

then

$$T_{1av} = -\frac{2AB}{4[-(1+n)P_o - P_i]} = -\frac{AB}{m}. \tag{20}$$

So, either way it leads to the same result. From Equation (15) it is easy to see that with integer values of permanent magnet pole pairs, P_o, P_i, and integer value of n, the average torque will always be zero. Hence, one cannot actually achieve a finite torque in this system, but one can get a very close non-integer value of n, which can lead to a close enough desirable gear ratio G, as per requirement, while having integer values of pole pair number P_o and P_i. This issue of a non-integer G is discussed in the subsection below.

3.2. Design Process of the Magnetic Gear

In order to design a magnetic gear, which, for our purpose, implies making the choice of the pole pair numbers, P_o and P_i, the modulation number n, and to achieve a specific gear ratio G, one needs to fulfill Equations (13) to (15) so that a tangible torque can be obtained to support any load torque. This involves satisfying certain constrains, based on a desired gear ratio and the space available to accommodate the gears. This also involves an optimization process since the numbers P_o and P_i are integers representing the pole pair numbers of the outer and inner rotors. While trying to satisfy Equation (15) one will find that it is impossible to get the exact integral gear ratio, but it is possible to get as close to an integer value as possible but not exactly equal to it, based on the chosen integer values of pole numbers and the resulting value of m in Equation (17) or (18). This situation can be compared to an analogous situation to something like the fact that one can get as close as possible to a particular real number by using rational numbers (quotient of integers), but never exactly reach it. However from engineering perspective, we can get a magnetic gear design very close to our desired gear ratio, etc. by choosing a reasonable number of pole pair numbers. From a practical viewpoint, it is actually relatively easy if one uses an excel type of spreadsheet and uses Equations (15) to (16) and come up with a design, based on the requirements.

An example case can be as follows. Let us suppose that we want a gear ratio of -6. We need to choose the pole pair numbers P_o and P_i and calculate the modulation number n, so that the above gear ratio can be achieved as close as possible, while satisfying Equation (16) through (19) to meet the torque constraints. To do this, for the purpose of illustration, we arbitrarily chose the pole pair numbers $P_o = 2$ and $P_i = 12$ so that they fits the desired magnitude of the gear ratio of 6. An excel spreadsheet was then run using Equations (16) to (19), which led to the gear ratios of -5.875 or -6.125, and the corresponding modulation numbers (n) were -0.020833 and $+0.020833$, with $m = 57$ in Equation (17). It should be noted that the desired integer gear ratio -6.0 lies between numbers $-5.875, -6.125$. The resulting torque is found to be equal to -1.105263 when the gear ratio is -6.125 (as noted earlier based on the equations, the excel sheet also indicates that at exactly -6.0 gear ratio, the torque is zero). To give another example, by choosing pole numbers of 4 and 24 for the inner and outer rotors to make the ratio 6 again, it was found that the gear ratio $G = -6.06$ could be achieved with a modulation number $n = 0.010417$, with $m = 113$ in Equation (17), and G now lies between the number $-5.9375, -6.0625$, with a torque equal to -0.557522. So, the observation from the above is that the gear ratio can be made closer to the pole number ratio of -6 by increasing the pole numbers which will, however, lead to lesser torque supporting capability. This gap between these two possible acceptable gear ratios instead of the number -6 can be closed more and more by an optimum combination of the pole pair values. But if the pole pair values become too large, there can be both space and weight problem and also fabrication issues.

One particular note which may be useful to mention here relates to the fact that magnetic gear design, just like regular mechanical gear design, has two aspects in it. The first one is related to the gear ratio, and the second one relates to the torque that the gear can support. This paper is focused on the first aspect, i.e., the gear ratio, and that the new magnetic gear design concept introduced in this paper is a viable system. The second aspect, which relates to how much maximum torque this gear can support, is related to the strength of the magnetic poles, and this matter is not the focus of this paper. The analogous situation in a regular gear design will be the strength of the materials used to make the gear.

The example design process indicated above in this section, and stated through the write up description above, can be organized in a step by step process through the flowchart indicated in Figure 5. This flowchart shows the generic overview of the steps involved.

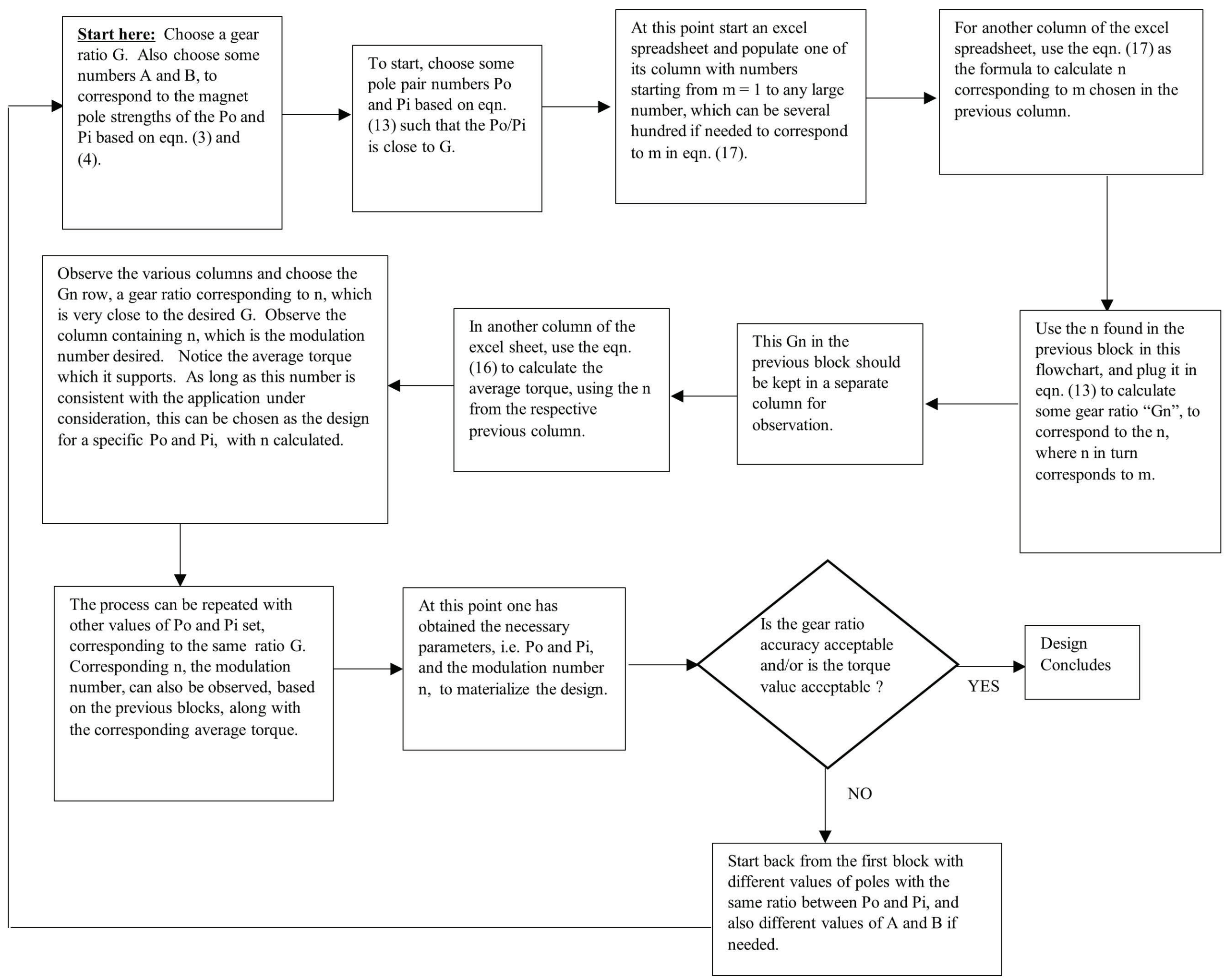


Figure 5. Flow chart showing the overview of the design process indicated in this section.

Let us now examine the consequence of choosing the modulation number n and pole pair values for some special cases. For example, if n is chosen such that $[(1+n)P_o + P_i] = 0$, it leads, when being inserted in Equation (13), to the following.

$$\frac{\omega_i}{\omega_o} = -(1+n)\frac{P_o}{P_i} = 1 \tag{21}$$

which is the trivial situation when the gear ratio is 1, i.e., the whole gear set, i.e., both the rotors, is bound together, moving together with respect to some external frame of reference, which is theoretically consistent with the equations developed earlier in this section and is realizable. Another situation, which is worth mentioning, is that it is possible to have positive gear ratio as well in this system with certain specific values of m in Equation (17) or (19). This was observed during the spread sheet calculation run over a wide range of m.

As noted earlier, in the mathematical analysis presented in this section, only the first component in Equations (5) to (10) was used because only one component was used to make it time invariant over the whole physical range of angle θ. The end results will be the same if any other component was selected, using the same principle. Once a particular component is chosen to obtain a non-zero time-average torque, it leads to other components to produce zero time-average, and hence those do not need to be considered.

3.3. Some Results from Simulation of the Equations Used in this Section

Although spreadsheet based results were used for getting some numerical results, the author also did some Matlab-Simulink based simulation, in order to validate some of the equations above. In particular, a speed was chosen for the outer rotor, a torque selected for the inner rotor, and the simulation was run to see at which speed the torque can be supported. This process was emulated by running a feedback loop based on the torque error, which was converging to zero, leading to a particular inner rotor speed.

A simplified block diagram of the simulation setup done in Matlab-Simulink is shown below in Fig. 6. It should be noted that that the above block diagram is to correlate the equations to show the correctness of the equations and is not meant as a physical dynamic simulation of the magnetic gear described in this paper.

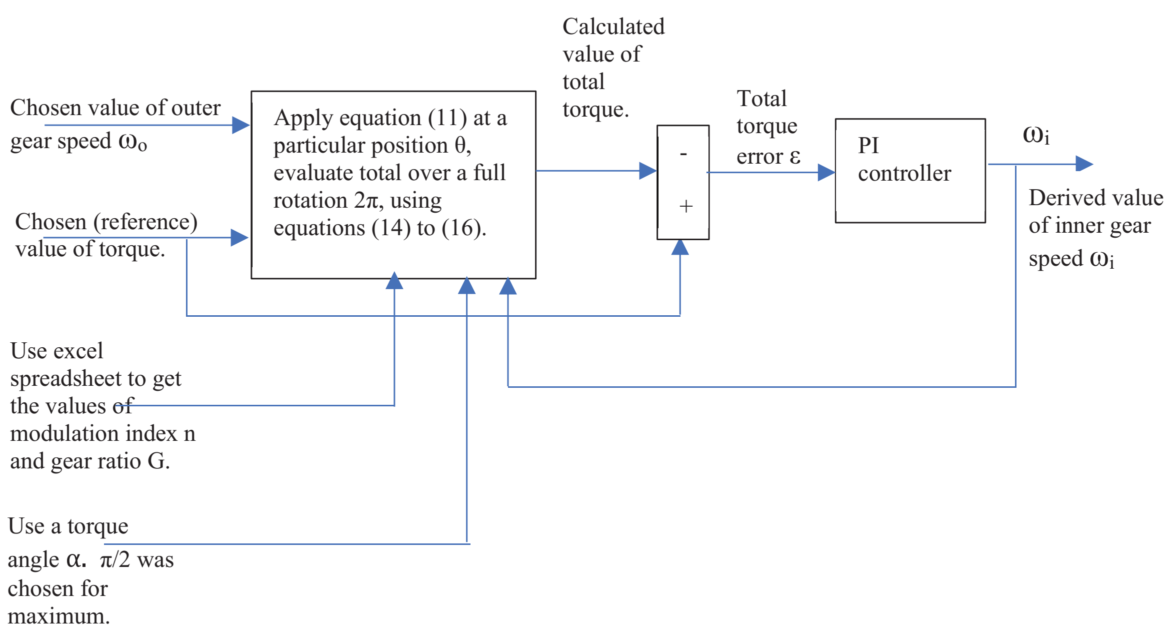


Figure 6. Block diagram showing a possible method emulating of and validating some of the equations used in this paper.

Some of the relevant results from the simulation of the equations are shown in Fig. 7. In this example case, the number of inner rotor pole pairs was selected to be 2, and outer rotor pole pair number was chosen as 12. The modulation number n was calculated by using a spreadsheet and Equation (17) for different values of m. In particular, the value of m used from the spread sheet was 57. The number n then was found equal to 0.020833, and the gear ratio G was -6.125 (actually equal to -6.124998). Average torque was specified as 1.10263. The torque error was approaching zero relatively fast as noticed in the simulation. The torque angle $P_i\alpha$ was chosen to be $\pi/2$ to get a maximum value. Outer rotor speed was chosen to be equal to 10 revolutions/sec. (i.e., 10*2π rad/sec or about 63 rad/sec). The output of the simulation was therefore theoretically supposed to be equal to 10*2π rad/sec times the gear ratio $G(-6.125)$ or about -388 rad/sec. Over the length of time the simulation was run, and the predicted speed was slowly converging to the theoretical one. The convergence to the predicted value

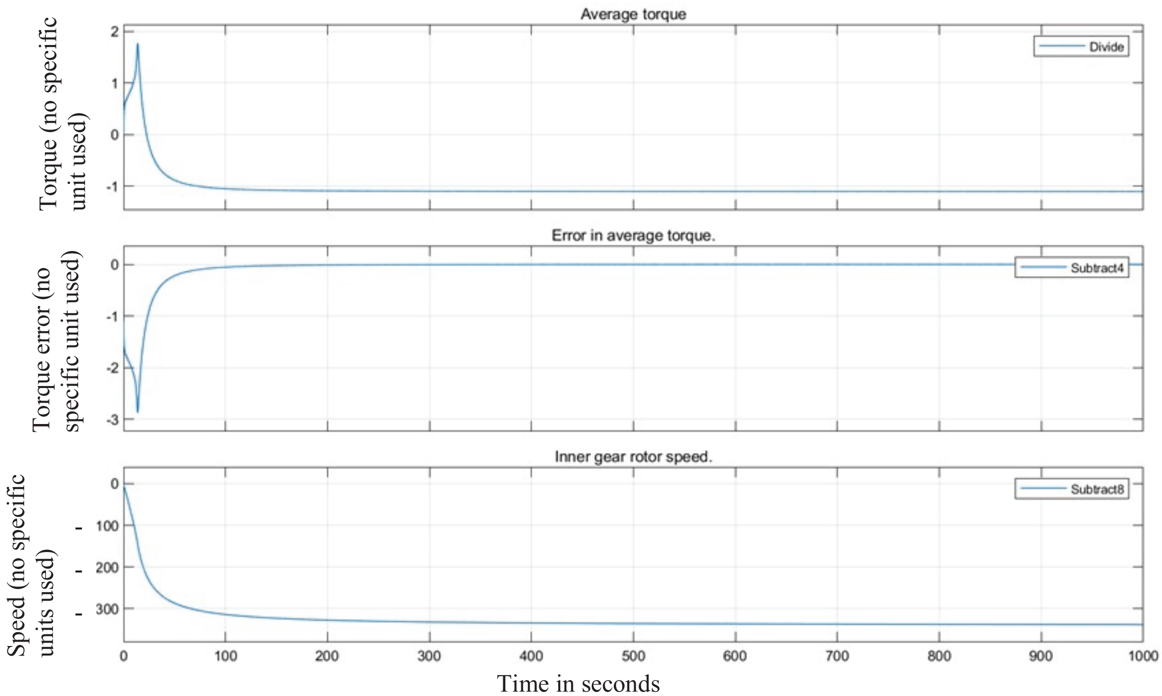


Figure 7. Results from the simulation indicating various quantities used in the equations.

was somewhat slow, and the discrepancy between the speed predicted by the theoretical gear ratio G in Equation (13) and the speed found during the simulation can be attributed to the numerical method, truncation of the numerical values, simulation time step, and the control system feedback used. The author ran the simulation for a very long simulation time and found that the results eventually tended to approach convergence to the correct values. As noted earlier, the simulation results here only represent the behavior of the equations and their numerical validity, and are not dynamic simulation results from a physical simulation of the gear. It is expected that in a real physical magnetic gear the speed will attain the correct value relatively faster. But this will need further research later.

4. SUMMARY, CONCLUSIONS, AND FUTURE RESEARCH

The main contributions of this paper are two specific items. First is that a completely new type of concentric magnetic gear has been proposed, which does not need an extra magnetic material structure for modulation. This, however, comes at the cost of fabricating an outer magnetic structure with variable strength pole pieces. Secondly, the paper has developed a very detailed mathematical analysis of such gears and validated the equations through numerical simulations of the equations. The scope of this paper, as represented through the title of the paper, is limited to the development of a theory based on harmonic interaction, and various papers, while discussing conventional magnetic gears, also use the harmonic interaction principle for analysis. As noted earlier, further research is needed to develop additional simulations to increase the accuracy, and this can include detailed magnetic system analysis prior to physical fabrication. Although the author has not yet undertaken the physical fabrication of the device, it is planned under additional future research, and as noted in the introduction section where [13, 14] were mentioned, such endeavors could include additive manufacturing processes as well. It is believed that the work discussed in this paper will lead to a simpler magnetic gear structure, which can be more robust than the existing three-structure magnetic gears involving the outer and inner rotors, and also a separate modulation structure.

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