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# A Systematic Study of Low *SLL* Two-Way Pattern in Shared Aperture Radar Arrays

**Theodoros Samaras[1, *] and John N. Sahalos[1, 2]**

**Abstract**—A systematic study of a low *SLL* (sidelobe level) two-way pattern in shared aperture arrays is presented. Three or two-weight excitations are used for the elements of the transmit and receive arrays depending on the requirements. The receive array has a smaller number of elements by not receiving from some of the edge elements of the transmit array. The condition of appearance of certain minor lobes of the transmit array pattern at certain nulls of the receive one helps to find the ratio between the number of elements of the receiving and transmit arrays. In the case of more than one possible ratio, the optimum ratio is the one that gives the lowest *SLL*. In the three-weight array the total number of the transmit elements is equal to the that of the two higher excitations plus the number of elements of the highest one. In the two-weight excitation, the higher weight elements of the transmit array are chosen to be approximately one half of the total elements. The excitations of both arrays are found by equating the level of the higher two unequal sidelobes of the two-way array factor. The three-weight array design is presented for the first time while the two-weight case gives lower peak *SLL* than those of the literature. Our work contains the important steps of the design and the main aspects of the implementation. The resulting peak *SLL* of the two-way array pattern reaches up to less than −51.2 dB and less than −56.5 dB for the two- and three-weight cases correspondingly.

## 1. INTRODUCTION

Antennas with a narrow main beam and low sidelobes make up the main parts of radar systems. Design techniques of several radar antenna arrays can be found in the literature. In [1], spaceborne antennas for planetary exploration were presented. Radar systems principles with interesting details of antennas were given in [2]. Fenn [3] presented an interesting analysis of adaptive antennas and phased arrays for radar and communication systems. A comprehensive introduction of radar technology with examples of modern systems based on active array antennas are given in [4]. Transmit and receive phased arrays at the same aperture [5–9] turned out to be an excellent structure for radars. Such a structure has great performance with reduced bulkiness and low manufacturing costs. Haupt [10–12] proposed the design of a radar system by using the constraint that certain nulls of the receive pattern are in the directions of the peak of certain sidelobes of the transmit one. The idea has been proven extremely useful and successful. In [13], Haupt's idea was extended and improved by using the condition of equating the levels of the two higher minor lobes of the two-way radiation pattern. Moreover, in [13, 14] the use of two-weight excitation for the elements of the transmit and receive arrays reduced the *SLL* to less than −50 dB.

In [14], several examples were presented for the ratios $\frac{N_r}{N_t}$ and $\frac{M}{N_t}$ for the two-weight arrays. $N_t$, $N_r$, and $M$ are the number of transmit, receive, and higher weight elements of the arrays. In [15], a parametric optimization has been applied to arrays with a three-weight amplitude excitation, and the

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* Corresponding author: Theodoros Samaras (theosama@auth.gr).
[1] Radiocommunications Laboratory, Department of Physics, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece.
[2] Department of Engineering, University of Nicosia, 46 Makedonitissas Avenue, CY-2417 Nicosia, Cyprus.

$SLL$ achieved became less than $-57\,\mathrm{dB}$. It is noticed that arrays with combined even and odd number of elements cannot be implemented [15]. Expressions (2) and (3) of [15] can be used only if both arrays contain either even or odd number of elements.

In [16–18] and the references therein [17], one can find several studies dealing with radar arrays including element mutual coupling [16, 18], element patterns [16], etc. The above give radars with low $SLL$ that helps to reject interference and ground clutter. In most of the usual approaches, an amplitude taper to the transmit and/or receive array is applied. Amplitude tapers need expensive feed networks. Thus, to avoid bulkiness and cost we can use arrays with a smaller number of weight element excitation. This is important because, compared to arrays with amplitude taper, this approach results in similar (slightly worse) performance but requires a simpler practical implementation.

In the current work, we present a three-weight array design for the first time. Also, compared with previous works, it improves and optimizes the two-weight array case. For all arrays, it presents the most important rules for choosing the even or odd number of elements in combination with the calculation of the optimum level of the excitation. The conditions of appearance of minor lobes of the transmit array at the direction of nulls of the receive one give the ratio $\frac{N_r}{N_t}$. In the resulting two-way pattern, the condition is imposed that the two mostly unequal highest sidelobes become equal. This condition gives the optimum level of the weight excitation. The reduction of the $SLL$ for the two-weight arrays becomes better than that of [13] and [14] with $SLL$ up to less than $-51.2\,\mathrm{dB}$. For the three-weight array, the $SLL$ was found to be less than $-56.5\,\mathrm{dB}$. Radar systems with such peak $SLL$ improve their anti-jamming performance by reducing background noise and interference.

## 2. THREE-WEIGHT AMPLITUDE EXCITATION

Let us consider a transmit and a receive linear array of discrete elements with three-weight excitation. The elements are along the $z$-axis with equal inter-element distance $d$ (Fig. 1).

We set $W_1 = 1 + W$, $W_2 = 2$, $W_3 = 3$, and we have the transmit and receive array factors $AF_t(\theta)$ and $AF_r(\theta)$ as follows [19]:

$$\begin{aligned} AF_t(\theta) &= \frac{e^{-j\frac{N_t-1}{2}kd\cos\theta}}{(1+W)N_t+(1-W)M+L}\left\{\frac{(1+W)\sin\left(\frac{N_t}{2}kd\cos\theta\right)+(1-W)\sin\left(\frac{M}{2}kd\cos\theta\right)+\sin\left(\frac{L}{2}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \\ AF_r(\theta) &= \frac{e^{-j\frac{N_r-1}{2}kd\cos\theta}}{(1+W)N_r+(1-W)M+L}\left\{\frac{(1+W)\sin\left(\frac{N_r}{2}kd\cos\theta\right)+(1-W)\sin\left(\frac{M}{2}kd\cos\theta\right)+\sin\left(\frac{L}{2}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \end{aligned} \tag{1}$$

The number of elements with weights $W_1$ is $N_t - M$, i.e., $\frac{N_t-M}{2}$ from each side of the array. The number of elements with weight higher than $W_1$ is $M$, and the number of elements with $W_3 = 3$ is $L$. To simplify the procedure, we assume at first that $W = 0$ and $N_t = M + L$ with $M > L$. The transmit and receive array factors become

$$AF_t(\theta) = 2\frac{e^{-j\frac{N_t-1}{2}kd\cos\theta}}{N_t}\left\{\frac{\sin\left(\frac{N_t}{4}kd\cos\theta\right)\cos\left(\frac{M}{4}kd\cos\theta\right)\cos\left(\frac{L}{4}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \tag{2}$$

$$AF_r(\theta) = \frac{e^{-j\frac{N_r-1}{2}kd\cos\theta}}{N_r+N_t}\left\{\frac{\sin\left(\frac{N_r}{2}kd\cos\theta\right)+2\sin\left(\frac{N_t}{4}kd\cos\theta\right)\cos\left(\frac{M-L}{4}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \tag{3}$$

We assume that $d = \frac{\lambda}{2}$, and from (2) we can see that the positions of nulls of $AF_t$ are at the angles

$$\begin{aligned} \theta_i^t &= \cos^{-1}\left(\frac{4i}{N_t}\right), \quad i = 1, 2, 3, \ldots \\ \theta_l^t &= \cos^{-1}\left[\frac{2(2l-1)}{M}\right], \quad l = 1, 2, 3, \ldots \\ \theta_m^t &= \cos^{-1}\left[\frac{2(2m-1)}{L}\right], \quad m = 1, 2, 3, \ldots \end{aligned} \tag{4}$$

We look at several combinations of $N_t$, $L$, $M$ where two of the above three conditions give the same angle.

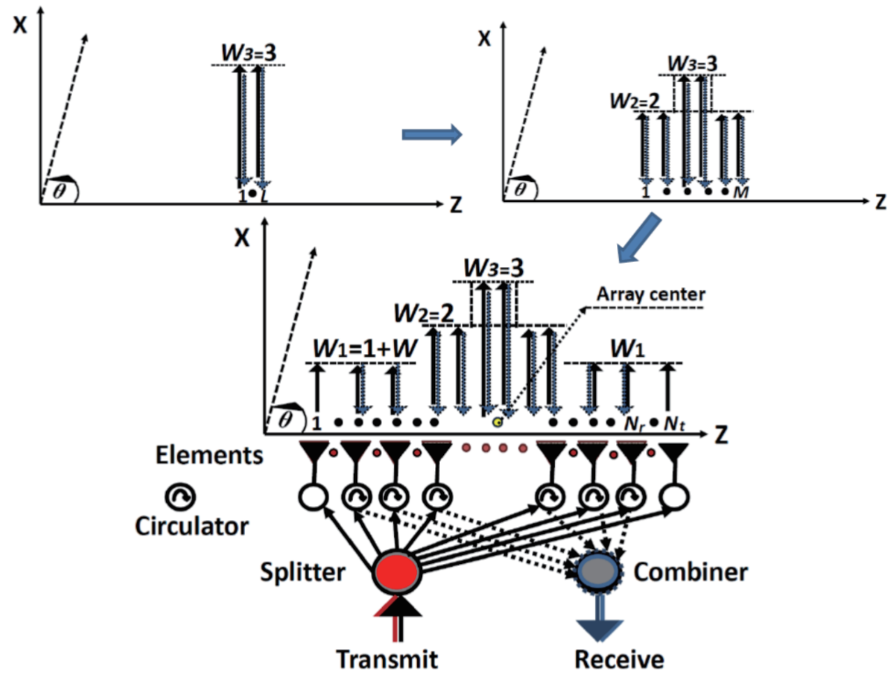


**Figure 1.** Three-weight radar transmit and receive linear arrays.

We start from the first two conditions of (4) and have

$$\begin{aligned} \frac{M}{N_t} &= \frac{2l-1}{2i} > \frac{1}{2} \\ M &= \frac{5}{8}N_t,\ i=4,\ l=3 \Rightarrow L = \frac{3}{8}N_t \\ M &= \frac{7}{12}N_t,\ i=6,\ l=4 \Rightarrow L = \frac{5}{12}N_t \end{aligned} \tag{5}$$

From the first and third conditions of (4) it is

$$\begin{aligned} \frac{L}{N_t} &= \frac{2m-1}{2i} < \frac{1}{2} \\ L &= \frac{3}{8}N_t,\ i=4,\ m=2 \Rightarrow M = \frac{5}{8}N_t \\ L &= \frac{5}{12}N_t,\ i=6,\ m=3 \Rightarrow M = \frac{7}{12}N_t \end{aligned} \tag{6}$$

Finally, from the second and third conditions of (4) we get

$$\begin{aligned} \frac{L}{M} &= \frac{2m-1}{2l-1} \Rightarrow \frac{L}{N_t} = \frac{2m-1}{2m+2l-2} < \frac{1}{2} \\ L &= \frac{3}{8}N_t,\ l=3,\ m=2 \Rightarrow M = \frac{5}{8}N_t \\ L &= \frac{5}{12}N_t,\ l=4,\ m=3 \Rightarrow M = \frac{7}{12}N_t \end{aligned} \tag{7}$$

Expressions (5)–(7) show that all the conditions of (4) are true at the same time for certain relations among $N_t, L, M$. It is noticed that the above relations are not unique. For example, we can have $L = \frac{1}{4}N_t, \frac{7}{16}N_t, \ldots$ with $M = \frac{3}{4}N_t, \frac{9}{16}N_t, \ldots$, respectively. Without excluding other cases, the ones that will be presented here give some interesting results. We assume that the peak of sidelobes is approximately in the middle between ascending nulls [13, 14]. In Tables A1 and A2 of the Appendix we give the position of nulls and sidelobes of the transmit pattern for $L = \frac{3}{8}N_t$ and $L = \frac{5}{12}N_t$. We start the numerical procedure by zeroing (3) for $\cos\theta = \frac{20}{3N_t}$ which is the position of the $3^{rd}$ sidelobe of the transmit array for $L = \frac{3}{8}N_t$. It is found that $\frac{N_r}{N_t} \sim 0.856$. Alternatively, for $\cos\theta = \frac{44}{5N_t}$ which is the position of the $4^{th}$ sidelobe we get $\frac{N_r}{N_t} \sim 0.788$. Also, for the $5^{th}$ sidelobe we have $\cos\theta = \frac{54}{5N_t}$ and $\frac{N_r}{N_t} \sim 0.8$. For an array with $N_t = 80$, $M = 50$, $L = 30$, and $W = 0$, we compare the $SLL$ of the two-way array patterns for $N_r = 0.856 \times 80 \sim 68$, $N_r = 0.788 \times 80 \sim 64$, and $N_r = 0.8 \times 80 = 64$. The receive array with $N_r = 68$ gives the lowest $SLL < -54.7\,\text{dB}$ of the two-way array pattern. This is shown in Fig. 2. For a transmit array with $N_t = 128$, $M = 80$, and $L = 48$, we compare the $SLL$ for $N_r = 0.856 \times 128 \sim 110$, $N_r = 0.788 \times 128 \sim 100$, and $N_r = 0.8 \times 128 \sim 102$. The two-way array factor for $N_r = 100$ has the lowest $SLL < -54.8\,\text{dB}$ and is given in Fig. 3. For $N_r = 110$, it is $SLL < -54.1\,\text{dB}$, and for $N_r = 102$ it is $SLL < -54\,\text{dB}$.

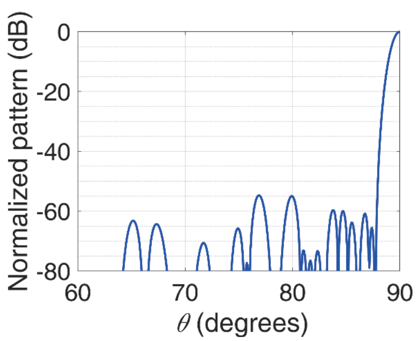


**Figure 2.** A two-way array factor for $N_t = 80$, $M = 50$, $L = 30$ and $N_r = 68$. For $W = 0$, $SLL$ becomes $< -54.7\,\text{dB}$.

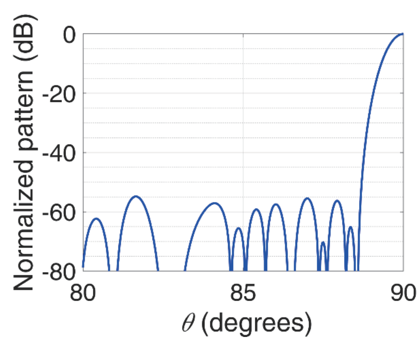


**Figure 3.** A two-way array factor for $N_t = 128$, $M = 80$, $L = 48$ and $N_r = 100$. For $W = 0$, $SLL$ becomes $< -54.8\,\text{dB}$.

For the condition that $L = \frac{5}{12}N_t$, zeroing (3) for $\cos\theta = \frac{64}{7N_t}$ which is the position of the $4^{th}$ sidelobe, we have $\frac{N_r}{N_t} \sim 0.825$. Alternatively, for $\cos\theta = \frac{20}{N_t}$ which is the position of the $5^{th}$ sidelobe, we have $\frac{N_r}{N_t} = 0.8$, while for the $6^{th}$ sidelobe, $\cos\theta = \frac{102}{7N_t}$ and $\frac{N_r}{N_t} \sim 0.796$. Using an array with $N_t = 120$, $M = 70$, $L = 50$, and $W = 0$, we compare the $SLL$ of the two-way array patterns for $N_r = 0.825 \times 120 \sim 100$ or 98, $N_r = 0.8 \times 120 \sim 96$, and $N_r = 0.796 \times 120 \sim 96$. The lowest resulting $SLL$ which is $< -54.3\,\text{dB}$ was found for $N_r = 98$. Fig. 4 shows the corresponding two-way factor. For $N_r = 96$, it is $SLL < -53.1\,\text{dB}$, and for $N_r = 100$, it is $SLL < -52\,\text{dB}$.

For arrays with an odd number of elements, the design can take place by subtracting (or adding)

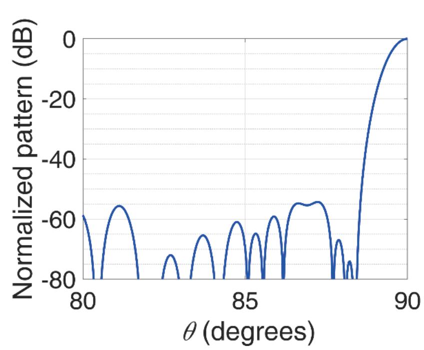


**Figure 4.** A two-way array factor for $N_t = 120$, $M = 70$, $L = 50$ and $N_r = 98$. For $W = 0$, SLL becomes $< -54.3\,\text{dB}$.

one element from (to) the corresponding even arrays. Thus, we have

$$N_{to} = N_{te} - 1, \quad M = Odd\left\{\frac{5}{8}N_{te}\right\} \quad \text{and} \quad L = Odd\left\{\frac{3}{8}N_{te}\right\}$$

or

$$N_{to} = N_{te} - 1, \quad M = Odd\left\{\frac{7}{12}N_{te}\right\} \quad \text{and} \quad L = Odd\left\{\frac{5}{12}N_{te}\right\}$$

The nulls of the transmit array for $W = 0$ cannot be derived analytically. We suppose that the $\cos\theta$ of nulls have a small difference from the ones of even number of elements. Thus, using Equations (4) we can write them by adding to the numerator small quantities $|x_i|$, $|x_l|$, $|x_m| \ll 1$. The angles of nulls can then be written as:

$$\begin{aligned}
\theta_i^t &= \cos^{-1}\left(\frac{4i + x_i}{N_t}\right), \quad i = 1, 2, 3, \ldots \\
\theta_l^t &= \cos^{-1}\left[\frac{2\,(2l-1) + x_l}{M}\right], \quad l = 1, 2, 3\ldots \\
\theta_m^t &= \cos^{-1}\left[\frac{2\,(2m-1) + x_m}{L}\right], \quad m = 1, 2, 3\ldots \\
x_i &= a_i,\ x_l = a_l,\ x_m = a_m \text{ for } L = \frac{3}{8}N_{te} \\
x_i &= b_i,\ x_l = b_l,\ x_m = b_m \text{ for } L = \frac{5}{12}N_{te}
\end{aligned} \tag{8}$$

The positions of nulls are given in Table A3 of the Appendix.

The transmit array factor is written as

$$AF_t(\theta) = \frac{e^{-j\frac{N_{te}-2}{2}kd\cos\theta}}{N_{te} - 1 + M + L}\left\{\frac{\sin\left(\frac{N_{te}-1}{2}kd\cos\theta\right) + \sin\left(\frac{M}{2}kd\cos\theta\right) + \sin\left(\frac{L}{2}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \tag{9}$$

By zeroing the numerator of (9), we can find approximately the values $a_i$ and $b_i$. For the first five nulls of the transmit array, $a_i$ and $b_i$ are given in Table A4 of the Appendix. We keep in mind that the position of the sidelobes is approximately in the middle between ascending nulls. Thus, for example, $\cos\theta$ of the $3^{rd}$ sidelobe of the transmit array is at

$$\cos\theta^3 = \frac{1}{2}\left(\frac{16 + a_3}{3N_{te}} + \frac{8 + a_4}{N_{te}}\right) = \frac{1}{3N_{te}}\left(20 + \frac{170}{N_{te}}\right) \tag{10}$$

If the above sidelobe is at a null of the receive array factor, then by zeroing (3) and using (10) for $W = 0$ we get

$$\sin\left[\frac{N_{ro}}{N_{te}}\frac{\pi}{3}\left(10 + \frac{86}{9N_{te}}\right)\right] + \sin\left[\frac{5}{24}\pi\left(10 + \frac{86}{9N_{te}}\right)\right] + + \sin\left[\frac{\pi}{8}\left(10 + \frac{86}{9N_{te}}\right)\right] = 0 \tag{11}$$

The ratio $\frac{N_{ro}}{N_{te}}$ can be found for a given $N_{te}$. Let us have a transmit array with $\left(\frac{M}{N_{te}}, \frac{L}{N_{te}}\right) = \left(\frac{5}{8}, \frac{3}{8}\right)$. For $N_{te} = 40$ we have that $N_{to} = 40 - 1 = 39$, $M = Odd[(5/8)40] = 25$, $L = Odd[(3/8)40] = 15$. Solving (11) we find that $\frac{N_{ro}}{N_{te}} \sim 0.844$ which gives $N_{ro} = 33$. Fig. 5 shows the two-way array factor which has an $SLL < -54.6$ dB.

For the case $\left(\frac{M}{N_{te}}, \frac{L}{N_{te}}\right) = \left(\frac{7}{12}, \frac{5}{12}\right)$, we take the $\cos\theta$ for the $4^{th}$ sidelobe of a transmit array. It is

$$\cos\theta^4 = \frac{1}{2}\left(\frac{8 + b_4}{N_{te}} + \frac{72 + b_5}{7N_{te}}\right) = \frac{1}{7N_{te}}\left(64 + \frac{7}{3N_{te}} + \frac{54\cos\left(\frac{\pi}{7}\right)}{7N_{te}\left[1 + \cos\left(\frac{\pi}{7}\right)\right]}\right) \tag{12}$$

If the sidelobe of the transmit array is at a null of the receive array factor, then by following the previous procedure we can find the ratio $\frac{N_{ro}}{N_{te}}$.

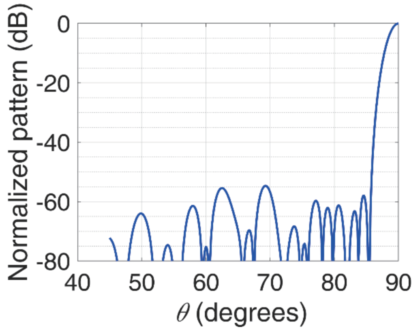


**Figure 5.** The two-way array factor for $N_t = 39$, $M = 25$, $L = 15$ and $N_r = 33$. For $W = 0$, $SLL$ becomes $< -54.6\,\text{dB}$.

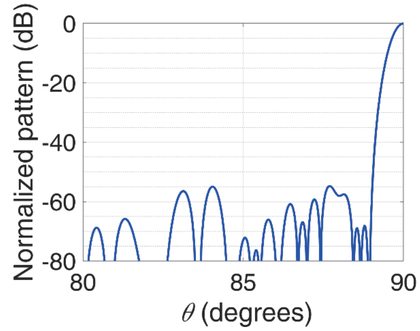


**Figure 6.** A two-way array factor for $N_t = 177$, $M = 105$, $L = 73$, $N_r = 147$, $SLL$ becomes $< -54.7\,\text{dB}$.

Let us take a transmit array with $N_{t0} = 178 - 1 = 177$, $M = Integ \cdot (7/12)178 = (103 \text{ or } 105)$, $L = Integ \cdot (5/12)178 = (73 \text{ or } 75)$. For $N_{te} = 178$, it is found that $\frac{N_{ro}}{N_{te}} \sim 0.828$ which gives $N_{ro} = 147$. From all the combinations of $M$ and $L$, the values $M = 105$ and $L = 73$ give the lowest $SLL < -54.7\,\text{dB}$. The two-way array factor for this case is shown in Fig. 6.

From a set of numerical results, it was found that the ratio $\frac{N_{ro}}{N_{te}}$ has a small difference from the corresponding one $\frac{N_{re}}{N_{te}}$. Thus, to simplify the procedure we can suppose that $\frac{N_{ro}}{N_{te}}$ is approximately the same as the corresponding one for even array.

To improve $SLL$, the only one choice is to have $W \neq 0$. In the examples of Figs. 5 and 6 we see that the two higher sidelobes have almost the same level. Thus, equating the level of each one of them to the one of the next higher sidelobe, we expect to find $W$ that gives a lower $SLL$. In Fig. 5 equating the sidelobes at 84.6° and 69.25°, we found that $W = 0.15$. With this amplitude the two-way array factor shows an improved $SLL$ which from $-54.6\,\text{dB}$ becomes less than $-56.0\,\text{dB}$. This is given in Fig. 7.

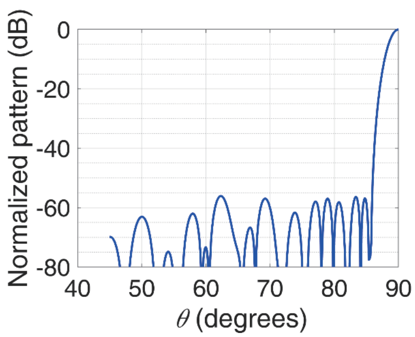


**Figure 7.** The two-way array factor for $N_t = 39$, $M = 25$, $L = 15$ and $N_r = 33$. For $W_1 = 1 + W = 1.15$, $SLL$ becomes $< -56.0\,\text{dB}$.

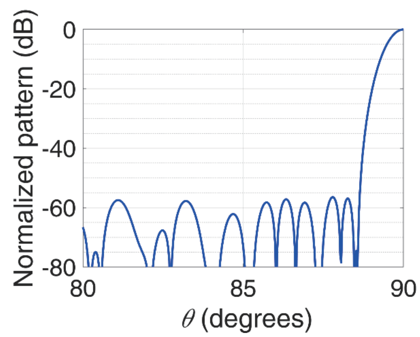


**Figure 8.** The two-way array factor for $N_t = 117$, $M = 75$, $L = 45$ and $N_r = 99$. For $1 + W = 1.15$, $SLL$ becomes $< -56.5\,\text{dB}$.

An array with $N_t = 118 - 1 = 117$ can have $L = (73 \text{ or } 75)$, $L = (43 \text{ or } 45)$, and $N_r = 99$. For $W = 0$, the min $SLL < -55.4\,\text{dB}$ was found for $M = 75$ and $L = 45$. Using $W = 0.15$, the $SLL$ is improved and has a value $< -56.5\,\text{dB}$. This is given in Fig. 8.

The performance analysis of three-weight amplitude excitation shows two-way array factors with $SLL$ less than $-56.5\,\text{dB}$.

The three-weight case for $W = 0$ gave two-way patterns which were improved for $W = 0.15$. This happens for both even and odd numbers of elements. Apart from the two examples given above, this value of $W = 01.5$ was found to provide the best result for many other cases tested. In the examples of the case of the two-weight arrays, the values of $W$ are $< 0$ and differ by an average of $\sim 14\%$. In this case, the relation between the numbers of elements has a greater effect. A general rule for all cases is to equate the two sidelobes of higher levels for $W = 0$ and find the best value of $W$.

It is noticed that the amplitude excitation was assumed to be $W_1 = 1 + W$, $W_2 = 2$, $W_3 = 3$. If $W_2$ and $W_3$ were changed, the results would be affected under many conditions. Our proposed solution is simple, and we believe that it offers relatively useful and satisfactory results.

## 3. REDUCTION OF THE NUMBER OF WEIGHTS

Now we assume that in a three-weight array we have $L = 0$. Using the condition $N_t = M + L$, we have that $N_t = M$. In this case, the arrays from three-weight become uniform, and the ratio $\frac{N_r}{N_t}$ was analytically given in the past [11–13].

For $L = 0$ without the condition $N_t = M$, the linear arrays will have two-weight excitation. These are shown in Fig. 9. Our goal is not to repeat [13] and [14]. Our effort is to extend them and somehow to optimize the two-weight arrays.

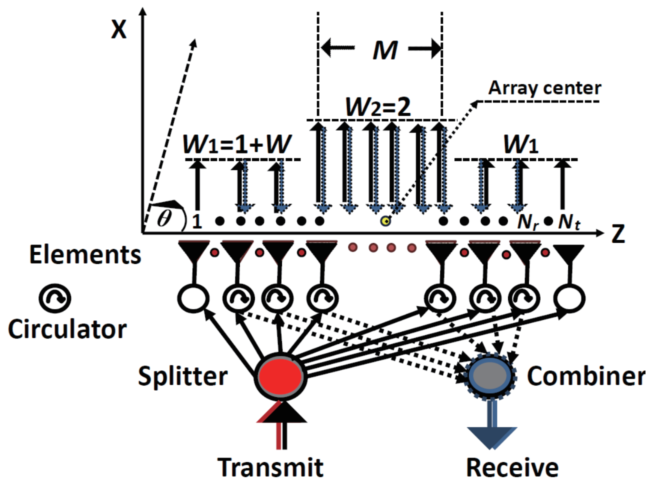


**Figure 9.** Two-weight radar transmit and receive linear arrays.

To have an analytical solution for the position of nulls and sidelobes of the transmit array, we suppose that $M = \frac{N_t}{2}$. If we set $Z = \frac{1-W}{1+W}$, the array factors given in (1) become

$$\begin{aligned} AF_t(\theta) &= \frac{e^{-j\frac{N_t-1}{2}kd\cos\theta}}{N_t\left(1+\frac{Z}{2}\right)}\left\{\frac{\sin\left(\frac{N_t}{4}kd\cos\theta\right)\left[2\cos\left(\frac{N_t}{4}kd\cos\theta\right)+Z\right]}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \\ AF_r(\theta) &= 2\frac{e^{j\frac{N_r-1}{2}kd\cos\theta}}{N_r+\frac{ZN_t}{2}}\left\{\frac{\sin\left(\frac{N_r}{2}kd\cos\theta\right)+Z\sin\left(\frac{N_t}{4}kd\cos\theta\right)}{\sin\left(\frac{1}{2}kd\cos\theta\right)}\right\} \end{aligned} \tag{13}$$

The directions of nulls of the transmit array are found by zeroing the numerator of $AF_t$ in (13). We set $\cos^{-1}\left(\frac{Z}{2}\right) = y$ and get

$$\sin\left(\frac{N_t}{4}kd\cos\theta\right)\left[\cos\left(\frac{N_t}{4}kd\cos\theta\right)+\cos y\right] = 0 \tag{14}$$

For $d = \frac{\lambda}{2}$, the nulls are at the angles

$$\begin{aligned} \theta_i^t &= \cos^{-1}\left(\frac{4i}{N_t}\right), \quad i = 1, 2, 3, \ldots \\ \theta_l^t &= \cos^{-1}\left[\frac{4(2l-1)}{N_t} \pm \frac{4}{N_t\pi}y\right], \quad l = 1, 2, 3\ldots \end{aligned} \tag{15}$$

Positions of the first nulls and sidelobes are given in Table A5 of the Appendix. We choose to have nulls of the pattern of the receive array at the positions $\theta_1$ and $\theta_3$ of the $1^{st}$ and $3^{rd}$ sidelobes of the transmit pattern. Thus, by using (13) for the $AF_r(\theta)$ we get

$$\begin{aligned} \sin\left(\frac{N_r}{2}\pi\cos\theta_1\right) + Z\sin\left(\frac{N_t}{4}\pi\cos\theta_1\right) = 0 \\ \sin\left(\frac{N_r}{2}\pi\cos\theta_3\right) + Z\sin\left(\frac{N_t}{4}\pi\cos\theta_3\right) = 0 \end{aligned} \tag{16}$$

$\cos\theta_1 = \frac{4}{N_t} - \frac{2}{N_t\pi}y$ and $\cos\theta_3 = \frac{6}{N_t} + \frac{2}{N_t\pi}y$ are found from the ascending nulls of $AF_t(\theta)$. Equation (16) can be written as

$$\begin{aligned} \sin\left[\frac{N_r}{N_t}(2\pi - y)\right] + 2\cos y\cos\frac{y}{2} = 0 \\ \sin\left[\frac{N_r}{N_t}(3\pi + y)\right] - 2\cos y\cos\frac{y}{2} = 0 \end{aligned} \tag{17}$$

It is

$$\frac{1}{2} < \frac{N_r}{N_t} < 1 \tag{18}$$

Adding the two equations of (17) we get

$$\sin\left[\frac{N_r}{N_t}(2\pi - y)\right] + \sin\left[\frac{N_r}{N_t}(3\pi + y)\right] = 0 \tag{19}$$

Thus

$$2\sin\left[\frac{N_r}{N_t}\left(\frac{5\pi}{2}\right)\right]\cos\left[\frac{N_r}{N_t}\left(\frac{\pi}{2} + y\right)\right] = 0 \tag{20}$$

From (20) we get

$$\left[\frac{N_r}{N_t}\left(\frac{5\pi}{2}\right)\right] = k\pi \to k = 2, \frac{N_r}{N_t} = 0.8 \tag{21}$$

$$\left[\frac{N_r}{N_t}\left(\frac{\pi}{2} + y\right)\right] = \frac{2l-1}{2}\pi, l = 1, 2, \ldots \tag{22}$$

Using any equation of (17) we have from (21) and (22) the following expressions

$$2\cos y\cos\frac{y}{2} - \sin\left[0.8 \times \left(\frac{\pi}{2} + y\right)\right] = 0 \tag{23}$$

$$2\cos y\cos\frac{y}{2} - (-1)^l\cos\left[\frac{N_r}{N_t}\left(\frac{5\pi}{2}\right)\right] = 0 \tag{24}$$

Solution of (23) gives

$$y = \frac{\pi}{3} \to W = 0 \tag{25}$$

In (24) the ratio $\frac{N_r}{N_t}$ is a fraction of two integers. Thus, we can have one of the following ratios

$$\frac{N_r}{N_t} = \frac{N_t - 2}{N_t}, \frac{N_t - 4}{N_t}, \frac{N_t - 6}{N_t}, \ldots, \frac{N_t - \left(\frac{N_t}{2} - 2\right)}{N_t} \tag{26}$$

We modify (24) to the following cubic equation

$$4\left(\cos\frac{y}{2}\right)^3 - 2\left(\cos\frac{y}{2}\right) - (-1)^l\cos\left[\frac{N_r}{N_t}\left(\frac{5\pi}{2}\right)\right] = 0 \tag{27}$$

By taking an example for a transmit array with $N_t = 40$ and $M = 20$ we can solve the cubic equation and have the values for $Z = 2\cos y$ (Table 1).

**Table 1.** $Z$ versus the ratio of $\frac{N_r}{N_t}$.

| $\frac{N_r}{N_t}$ | $\frac{38}{40}, \frac{26}{40}, \frac{22}{40}$ | $\frac{36}{40}, \frac{28}{40}$ | $\frac{34}{40}, \frac{30}{40}$ | $\frac{32}{40}$ |
|---|---|---|---|---|
| $Z$ | 0.485 | 0.857 | 1.057 | 1.13 |

Testing all the possible values of $\frac{N_r}{N_t}$ we can find the one with the lowest $SLL$ of the two-way array factor. This gives for the first time *the optimum ratio of* $\frac{N_r}{N_t}$ which in our case is $\frac{N_r}{N_t} = \frac{32}{40} = 0.8$. It is important to say that for $Z = 1$ the two-way array factor has an $SLL < -49.6\,\text{dB}$ while for $Z = 1.13$ it has an improved $SLL < -50.46\,\text{dB}$. For either $Z = 1$ or $Z = 1.13$, the two-way array factor has unequal sidelobe levels. Thus, to improve $SLL$, we apply the same method as for the three-weight arrays. We keep the ratio $\frac{N_r}{N_t}$ constant and equate the level of the two higher sidelobes. The array factor for either $Z = 1$ or $Z = 1.13$ has the above sidelobes approximately at the same angles. These sidelobes are shown in Table 2. Thus, it is proposed next, for practical reasons, to use the case of $Z = 1$.

**Table 2.** Positions of the two higher sidelobes.

| Angle | $\theta_1$ | $\theta_2$ |
|---|---|---|
| Degrees | $83^\circ.5$ | $69^\circ$ |
| $SLL$ for $Z = 1.0$ | $-49.6\,\text{dB}$ | $-51.0\,\text{dB}$ |
| $SLL$ for $Z = 1.35$ | $-51.7\,\text{dB}$ | $-50.46\,\text{dB}$ |

$AF(\theta)$ at the angles 83.5° and 69.0° is:

$$AF(\theta_1) = \frac{\sin\left(\frac{N_t}{4} z_1\right)}{\left[\sin\left(\frac{1}{2} z_1\right)\right]^2} \left[2\cos\left(\frac{N_t}{4} z_1\right) + Z\right] \left[\sin\left(\frac{N_r}{2} z_1\right) + Z \sin\left(\frac{N_t}{4} z_1\right)\right] \tag{28}$$
$$z_1 = \pi\cos\theta_1 = \pi\cos(83^\circ.5)$$

and

$$AF(\theta_2) = \frac{\sin\left(\frac{N_t}{4} z_2\right)}{\left[\sin\left(\frac{1}{2} z_2\right)\right]^2} \left[2\cos\left(\frac{N_t}{4} z_2\right) + Z\right] \left[\sin\left(\frac{N_r}{2} z_2\right) + Z \sin\left(\frac{N_t}{4} z_2\right)\right] \tag{29}$$
$$z_2 = \pi\cos\theta_2 = \pi\cos(69^\circ.0)$$

In our first effort we equate the second parts of (28) and (29) and get a quadratic equation with respect to $Z$. Solving the equation analytically we have the final value of $Z$ which gives $Z = 1.09$. In Fig. 10, the two-way array factor is presented. It has an $SLL < -50.6\,\text{dB}$ which is lower than those given before. In [13] and [14], the two higher sidelobe levels of the two-way array factors are not equal. This means that they do not give the lowest $SLL$. In [14], an array with $N_t = 80$, $N_r = 64$, and $Z = 1.13$ gave $SLL < -51.05\,\text{dB}$. With $Z = 1.09$ a lower value of $SLL < -51.18\,\text{dB}$ was found.

For a radar with $N_t = 128$, $M = 64$ it is $\frac{N_r}{N_t} = \frac{102}{128}$ which is not exactly 0.8, and the value of $Z$ is found to be $\sim 1.11$. The two-way array factor is shown in Fig. 11, and the $SLL$ becomes $< -51.10\,\text{dB}$.

Based on the above examples, the steps of choosing radar arrays with even number of elements are the following:

(i) *Choose the number of transmit elements* $N_{te}$ *=4k and elements M=2k with higher amplitude.*

(ii) *Find the ratio of* $\frac{N_{re}}{N_{te}}$ *which must be* $\sim 0.8$.

(iii) *Find the two-way array factor for* $W_1$ *=1.0.*

(iv) *Find the final* $W_1$ *by equating the two higher sidelobes of the previous two-way array factor.*

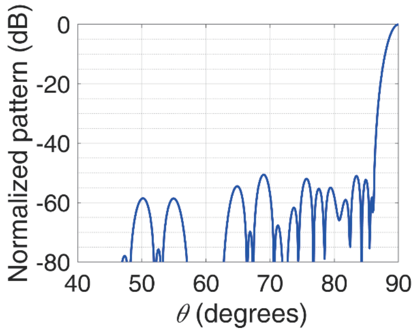


**Figure 10.** The two-way array factor for $N_t = 40$, $M = 20$, $N_r = 32$ and $Z = 1.09$ and $W_1 = 0.957$.

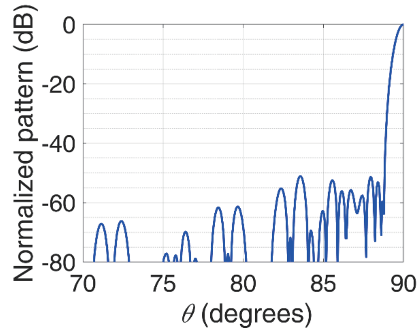


**Figure 11.** The two-way array factor for $N_t = 128$, $M = 64$, $N_r = 102$ and $Z = 1.11$ & $W_1 = 0.948$. *SLL* becomes $< -51.10$ dB.

For arrays with odd number of elements, the design can take place by subtracting (or adding) one element of the corresponding even arrays. Following the procedure of the three-weight case, we have

$$N_{to} = N_{te} - 1, \quad M_o = Odd\left\{\frac{N_{te}}{2}, \frac{N_{te}-2}{2}\right\} \quad \text{and} \quad N_{re} = 0.8N_{te} \sim 2l - 1 \tag{30}$$

The array factors become

$$\begin{aligned} AF_t(\theta) &= 2\frac{e^{-j\frac{N_{to}}{2}\pi\cos\theta}}{(N_{to} + ZM_o)}\left\{\frac{\sin\left(\frac{N_{to}}{2}\pi\cos\theta\right) + Z\sin\left(\frac{M_o}{2}\pi\cos\theta\right)}{\sin\left(\frac{1}{2}\pi\cos\theta\right)}\right\} \\ AF_r(\theta) &= 2\frac{e^{-j\frac{N_{ro}}{2}\pi\cos\theta}}{(N_{ro} + ZM_o)}\left\{\frac{\sin\left(\frac{N_{ro}}{2}\pi\cos\theta\right) + Z\sin\left(\frac{M_o}{2}\pi\cos\theta\right)}{\sin\left(\frac{1}{2}\pi\cos\theta\right)}\right\} \end{aligned} \tag{31}$$

We start our study for $Z = 1$. Zeroing the function

$$F_t(\theta) = \sin\left(\frac{N_{to}}{2}\pi\cos\theta\right) + \sin\left(\frac{M_o}{2}\pi\cos\theta\right) = 2\sin\left(\frac{N_{to}+M_o}{4}\pi\cos\theta\right) \times \cos\left(\frac{N_{to}-M_o}{4}\pi\cos\theta\right) \tag{32}$$

we get the nulls of the transmit array in the following angles

$$\begin{aligned} \theta_i^t &= \cos^{-1}\left[\frac{4i}{N_{to}+M_o}\right], i = 1, 2, 3, \ldots \\ \theta_l^t &= \cos^{-1}\left[\frac{2(2m-1)}{N_{to}-M_o}\right], m = 1, 2, 3\ldots \end{aligned} \tag{33}$$

For the two different cases of (30) we take

(i) $N_{to} = N_{te} - 1$ and $M_o = \mathbf{\frac{N_{te}}{2}} = Odd\ number$

$$\begin{aligned} \theta_i^t &= \cos^{-1}\left[\frac{8i}{(3N_{te}-2)}\right], \quad i = 1, 2, 3, \ldots \\ \theta_l^t &= \cos^{-1}\left[\frac{4(2m-1)}{(N_{te}-2)}\right], \quad m = 1, 2, 3\ldots \end{aligned} \tag{34}$$

(i) $N_{to} = N_{te} - 1$ and $M_o = \mathbf{\frac{N_{te}-2}{2}} = Odd\ number$

$$\begin{aligned} \theta_i^t &= \cos^{-1}\left[\frac{8i}{(3N_{te}-4)}\right], \quad i = 1, 2, 3, \ldots \\ \theta_l^t &= \cos^{-1}\left[\frac{4(2m-1)}{N_{te}}\right], \quad m = 1, 2, 3\ldots \end{aligned} \tag{35}$$

In the directions $\theta_1$ and $\theta_2$ of the two higher sidelobes, we equate the two-way array factor and analytically solve the quadratic equation for $Z$.

Let us take an example for a radar with $N_t = 2 \times 92 - 1 = 183$, $M = 92 - 1 = 91$, and $N_r = 0.8 \times 183 \sim 147$. We equate the higher sidelobes, which are at the angles 88.6° and 85.5°. The solution of the quadratic equation gives $Z \sim 1.14$ and $W_1 = 0.9346$. In Fig. 12, we see the two-way array factor with $SLL < -51.0\,\mathrm{dB}$.

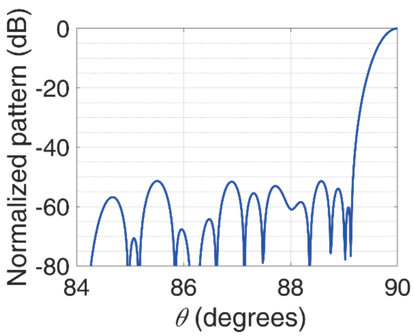


**Figure 12.** The two-way array factor for $N_t = 183$, $M = 91$, $N_r = 147$ and $W_1 = 0.9346$. $SLL$ becomes $< -51.0\,\mathrm{dB}$.

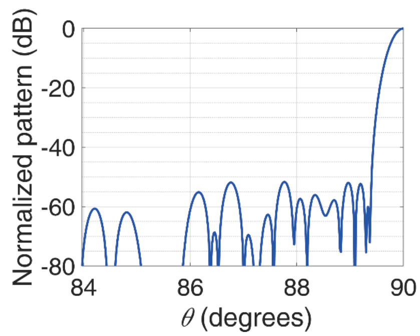


**Figure 13.** The two-way array factor for $N_t = 253$, $M = 127$, $N_r = 203$ and $W_1 = 0.9346$. $SLL$ becomes $< 51.2\,\mathrm{dB}$.

For $N_t = 254 - 1 = 253$, $M = 254/2 = 127$, $Nr = 203$, and $W_1 = 0.9346$, the achieved $SLL$ is less than $-51.2\,\mathrm{dB}$ (see Fig. 13). The results of the examples of [14] were compared with our results, and in all cases our $SLLs$ were lower.

The presented examples have shown that the two-way array factors can achieve $SLL$ up to $< -51.2\,\mathrm{dB}$. This value is lower than the ones given in the literature. Our contribution for the design of the two-weight arrays is finding the optimum ratio $\frac{N_r}{N_t}$ and $W$ by clearly justifying the steps of the procedure. It should be mentioned here that except for [13–15] the authors did not find results of other relevant works to compare.

## 4. DISCUSSION

It is known that for radar arrays the more the different weight excitations are, the less the maximum $SLL$ of the two-way patterns is. More weights offer more degrees of freedom in the array design. The limit is to have different weights for each element which will give transmit and/or receive arrays with theoretically the lowest $SLL$ level. The resulting performance compared to that of a simpler practical implementation will require the choice of the excitation.

To have the exact two-way antenna pattern, we must consider the element pattern as well as the mutual coupling between the elements. Mutual coupling results in a loss of the radar signals. It depends on the shape, spacing, and the number of antenna elements. Mutual coupling affects the antenna pattern for larger scan angles [10, 11]. The choice of the elements depends on the scanning requirements and the frequency. In our case, mutual coupling and element patterns were ignored because, if the array is well calibrated, they have little impact on the antenna nulls and sidelobes that are near the main lobe [10, 11]. Thus, our work contains important main aspects of the implementation.

It is noticed [11, 13] that planar arrays can be designed with the same concept of linear arrays. In a planar array [11], the edge elements will be turned off for the receive array. Planar arrays provide more variables and offer higher directivity than linear ones. The same procedure as above can create equally sufficient $SLL$ of two-way array patterns.

All calculations and presentations of the patterns were made by using the ORAMA computer tool [20].

## 5. CONCLUSION

A systematic study for lowering the peak $SLL$ of a radar two-way array factor has been presented. Transmit and receive arrays with two- and three-weight excitations were used. The three-weight excitation is a taper distribution that offers better $SLL$ performance.

The two-weight case combines more the advantages of taper distribution and the simplicity of the feed network of uniform arrays [13]. The amplitude of the element excitation in the middle of the transmit and receive arrays is the same. Certain steps were used for the number of elements and excitations. The elements in the middle for the two-weight case are approximately one half the total of the ones of the transmit array. The ratio $\frac{N_r}{N_t}$ was found from the case of $W_1 = 1$ and by using a pair of conditions of appearance of two minor lobes of the transmit array pattern at the position of certain nulls of the receive one. The corresponding weight in the middle was derived from equating two higher sidelobes which appear in the two-way array factor.

In the case of three-weight excitation, the number of transmit array elements is equal to the total of those that have weight $W_2 = 2$ and $W_3 = 3$. Two cases of $M$ and $L$ were used. One with $M = \frac{5}{8}N_t$, $L = \frac{3}{8}N_t$ and the other with $M = \frac{7}{12}N_t$, $L = \frac{5}{12}N_t$. The ratio $\frac{N_r}{N_t}$ was found from the case of $W_1 = 1$ and the conditions of appearance of minor lobes of the transmit array pattern at the position of certain nulls of the receive one. The excitation $W_1$ was found by equating two higher sidelobes which appear in the two-way array factor. For the case of odd number of elements, the array design is made by subtracting (or adding) one element of the corresponding even numbered array. For a one-weight radar array with $N_t = 1.4N_r$, the peak $SLL$ is $-31.48$ dB [13]. For the two-weight array it is $-51.2$ dB, while for the three-weight array it becomes $-56.5$ dB, as shown in this work. It is obvious that for the two-weight array the detection threshold will be $-19.72$ dB (93.75 times) less than the one-weight array, and for the three-weight array the threshold will be $-25.02$ dB (317.7 times) less. Such values of threshold allow a sufficient performance for the radar systems.

## APPENDIX A.

**Table A1.** Positions of nulls and sidelobes of the three-weight transmit array pattern for $L = \frac{3}{8}N_t$.

| Null no. | $\cos\theta$ | Null no. | $\cos\theta$ | SL no. | $\cos\theta$ | SL no. | $\cos\theta$ |
|---|---|---|---|---|---|---|---|
| 1 | $\frac{16}{5N_t}$ | 6 | $\frac{12}{N_t}$ | 1 | $\frac{18}{5N_t}$ | 6 | $\frac{14}{N_t}$ |
| 2 | $\frac{4}{N_t}$ | 7 | $\frac{16}{N_t}$ | 2 | $\frac{14}{3N_t}$ | 7 | $\frac{81}{5N_t}$ |
| 3 | $\frac{16}{3N_t}$ | 8 | $\frac{16}{N_t}$ | 3 | $\frac{20}{3N_t}$ | 8 | $\frac{81}{5N_t}$ |
| 4 | $\frac{8}{N_t}$ | 9 | $\frac{16}{N_t}$ | 4 | $\frac{44}{5N_t}$ | 9 | $\frac{81}{5N_t}$ |
| 5 | $\frac{48}{5N_t}$ | 10 | $\frac{112}{5N_t}$ | 5 | $\frac{54}{5N_t}$ | 10 | $\frac{81}{5N_t}$ |

**Table A2.** Positions of nulls and sidelobes of the three-weight transmit array pattern for $L = \frac{5}{12}N_t$.

| Null no. | $\cos\theta$ | Null no. | $\cos\theta$ | SL no. | $\cos\theta$ | SL no. | $\cos\theta$ |
|---|---|---|---|---|---|---|---|
| 1 | $\frac{24}{7N_t}$ | 6 | $\frac{12}{N_t}$ | 1 | $\frac{26}{7N_t}$ | 6 | $\frac{102}{7N_t}$ |
| 2 | $\frac{4}{N_t}$ | 7 | $\frac{120}{7N_t}$ | 2 | $\frac{22}{5N_t}$ | 7 | $\frac{76}{5N_t}$ |
| 3 | $\frac{24}{5N_t}$ | 8 | $\frac{72}{5N_t}$ | 3 | $\frac{32}{5N_t}$ | 8 | $\frac{552}{35N_t}$ |
| 4 | $\frac{8}{N_t}$ | 9 | $\frac{16}{N_t}$ | 4 | $\frac{64}{7N_t}$ | 9 | $\frac{20}{N_t}$ |
| 5 | $\frac{72}{7N_t}$ | 10 | $\frac{24}{N_t}$ | 5 | $\frac{78}{7N_t}$ | 10 | $\frac{21}{N_t}$ |

**Table A3.** Positions of nulls of odd number of elements of the three-weight transmit array pattern. $L = \frac{3}{8}N_{te}$, $L = \frac{5}{12}N_{te}$.

| Null no. | $\cos\theta$ | Null no. | $\cos\theta$ | SL no. | $\cos\theta$ | SL no. | $\cos\theta$ |
|---|---|---|---|---|---|---|---|
| 1 | $\frac{16+a_1}{75}$ | 6 | $\frac{12+a_6}{N_{te}}$ | 1 | $\frac{24+b_1}{7N_{te}}$ | 6 | $\frac{12+b_6}{N_{te}}$ |
| 2 | $\frac{4+a_2}{N_{te}}$ | 7 | $\frac{16+a_7}{N_{te}}$ | 2 | $\frac{4+b_2}{N_{te}}$ | 7 | $\frac{120+b_7}{7N_{te}}$ |
| 3 | $\frac{16+a_3}{3N_{te}}$ | 8 | $\frac{16+a_8}{N_{te}}$ | 3 | $\frac{24+b_3}{5N_{te}}$ | 8 | $\frac{72+b_8}{5N_{te}}$ |
| 4 | $\frac{8+b_4}{N_{te}}$ | 9 | $\frac{16+a_9}{N_{te}}$ | 4 | $\frac{8+b_4}{N_{te}}$ | 9 | $\frac{16+_9}{N_{te}}$ |
| 5 | $\frac{48+a_5}{75}$ | 10 | $\frac{112+a_{10}}{5N_{te}}$ | 5 | $\frac{72+b_5}{7N_{te}}$ | 10 | $\frac{24+b_{10}}{N_{te}}$ |

$\frac{a_i}{N_t} \ll 1$, $i = 1, 2, \ldots, 10$ and $\frac{b_i}{N_t} \ll 1$, $i = 1, 2, \ldots, 10$

**Table A4.** $a_i$ and $b_i$ of odd number of elements of the three-weight transmit array.

| Null no. | $a_i$ | $b_i$ |
|---|---|---|
| 1 | $\frac{256\cos\left(\frac{12\pi}{10}\right)}{10N_{te}\left[1+\cos\left(\frac{12\pi}{10}\right)\right]}$ | $\frac{288\cos\left(\frac{5\pi}{7}\right)}{7N_{te}\left[1+\cos\left(\frac{5\pi}{7}\right)\right]}$ |
| 2 | $\frac{4}{N_{te}\left[1-\cos\left(\frac{\pi}{4}\right)\right]}$ | $\frac{4}{N_{te}\left[1+\cos\left(\frac{5\pi}{6}\right)\right]}$ |
| 3 | $\frac{124}{9N_{te}}$ | $-\frac{288}{N_{te}}\frac{\cos\left(\frac{2\pi}{5}\right)}{\left[1-\cos\left(\frac{2\pi}{5}\right)\right]}$ |
| 4 | $\frac{8}{N_{te}}$ | $\frac{16}{3N_{te}}$ |
| 5 | $\frac{384\cos\left(\frac{9\pi}{5}\right)}{5N_{te}\left[1+\cos\left(\frac{9\pi}{5}\right)\right]}$ | $\frac{864\cos\left(\frac{\pi}{7}\right)}{7N_{te}\left[1+\cos\left(\frac{\pi}{7}\right)\right]}$ |

**Table A5.** Positions of nulls and sidelobes (SL) of the two-weight transmit array pattern.

| Null no. | $\cos\theta$ | Null no. | $\cos\theta$ | SL no. | $\cos\theta$ | SL no. | $\cos\theta$ |
|---|---|---|---|---|---|---|---|
| 1 | $\frac{4}{N_t} - \frac{4}{N_t\pi}y$ | 6 | $\frac{12}{N_t}$ | 1 | $\frac{4}{N_t} - \frac{2}{N_t\pi}y$ | 6 | $\frac{12}{N_t} + \frac{2}{N_t\pi}y$ |
| 2 | $\frac{4}{N_t}$ | 7 | $\frac{12}{N_t} + \frac{4}{N_t\pi}y$ | 2 | $\frac{4}{N_t} + \frac{2}{N_t\pi}y$ | 7 | $\frac{14}{N_t} + \frac{2}{N_t\pi}y$ |
| 3 | $\frac{4}{N_t} + \frac{4}{N_t\pi}y$ | 8 | $\frac{16}{N_t}$ | 3 | $\frac{6}{N_t} + \frac{2}{N_t\pi}y$ | 8 | $\frac{18}{N_t} - \frac{2}{N_t\pi}y$ |
| 4 | $\frac{8}{N_t}$ | 9 | $\frac{20}{N_t} - \frac{4}{N_t\pi}y$ | 4 | $\frac{10}{N_t} - \frac{2}{N_t\pi}y$ | 9 | $\frac{20}{N_t} - \frac{2}{N_t\pi}y$ |
| 5 | $\frac{12}{N_t} - \frac{4}{N_t\pi}y$ | 10 | $\frac{20}{N_t}$ | 5 | $\frac{12}{N_t} - \frac{2}{N_t\pi}y$ | 10 | $\frac{20}{N_t} + \frac{2}{N_t\pi}y$ |

## REFERENCES

1. Imbriale, W. A., *Spaceborne Antennas for Planetary Exploration*, California, USA, Jet Propulsion Lab., California Institute of Technology, 2006.
2. Raemer, H. R., *Radar Systems Principles*, Boca Raton, FL, CRC Press, 1996.
3. Fenn, A. J., *Adaptive Antennas and Phased Arrays for Radar and Communications*, Boston, USA, Artech House, 2007.
4. Wirth, W.-D., *Radar Techniques Using Antenna Arrays*, 2nd Edition, London, U.K., IET, 2013.

5. Haupt, R. L., "Thinned arrays using genetic algorithms," *IEEE Trans. Antennas Propag.*, Vol. AP-42, No. 7, 993–999, Jul. 1994.
6. Trampuz, C., M. Simeoni, I. E. Lager, and L. P. Ligthart, "Complementarity based design of antenna systems for FMCW radar," *Proc. 5th European Radar Conference*, 216–219, Amsterdam, 2008.
7. Lager, E., C. Trampuz, M. Simeoni, and L. P. Ligthart, "Interleaved array antennas for FMCW radar applications," *IEEE Trans. Antennas Propag.*, Vol. 57, No. 8, 2486–2490, Aug. 2009.
8. Rocca, P. and M. Donelli, 'Low sidelobe ADS-based arrays for FMCW radar," *Proc. IEEE Antennas Propag. Soc. Int. Symp. URSI Nat. Radio Sci.*, 2004–2007, Roznik, Slovenia, 2011.
9. Ferre, N., P. F. Combes, and T. Dusseux, "Transmit-receive optimized patterns for space radar active antennas, " *Proc. IEEE Antennas Propag. Soc. Int. Symp. URSI Nat. Radio Sci. Meeting*, 1240–1243, Seattle, WA, USA, Vol. 2, Jun. 1994.
10. Haupt, R. L., "Optimizing the sidelobe level of a two-way antenna array pattern by thinning the receive aperture" *International Conference of Radar 2018*, Brisbane, Australia, Aug. 2018.
11. Haupt, R. L., "Lowering the sidelobe level of a two-way array factor with uniform transmit and receive arrays," *IEEE Trans. Antennas Propag.*, Vol. 67, No. 6, 4253–4256, Jun. 2019.
12. Haupt, R. L. and P. Nayeri, "Uniform arrays with low sidelobe two-way antenna patterns," *Proc. EuCAP*, 9–13, London, U.K., Apr. 2018.
13. Sahalos, J. N., "Design of shared aperture radar arrays with minimum sidelobe level of the two-way array factor," *IEEE Trans. Antennas Propag.*, Vol. 68, 5415–5420, 2020.
14. Sahalos, J. N., "Lowering the sidelobe level of a two-way pattern in shared aperture radar arrays," *International Journal of Antennas* & *Propagation*, 10, 2021.
15. Rajender, R., K. R. Subhashini, and B. P. Kumar, "Two-way array factor supported by thinning strategy for an improved radar performance," *National Conference on Communications (NCC)*, Kanpur, India, DOI: 10.1109/NCC52529.2021.9530057, Jul. 27–30, 2021.
16. Fenn, A. J., "Theoretical and experimental study of monopole phased array antennas," *IEEE Trans. Antennas Propag.*, Vol. 34, No. 10, 1118–1126, 1985.
17. Mailloux, R. J., *Phased Array Handbook*, 3rd Edition, Artech House, Dedham, Mass., 2017.
18. Sneha, H. L., H. Singh, and R. M. Jha, "Mutual coupling effects for radar cross section (RCS) of a series-fed dipole antenna array," *Hindawi Publishing Corporation, International Journal of Antennas and Propagation*, Vol. 2012, Article ID 601532, 20 pages, doi:10.1155/2012/601532.
19. Balanis, C. A., *Antenna Theory Analysis and Design*, 4th Edition, John Wiley & Sons Inc., New Jersey, USA, 2016.
20. Sahalos, J. N., *Orthogonal Methods for Array Synthesis: Theory and the ORAMA Computer Tool*, John Wiley & Sons Inc., New Jersey, USA, 2006.