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Microwave Imaging of Small Scatterers by MUSIC Algorithm Using a Novel Source Number Detection Method

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Abstract—Microwave imaging of small scatterers is an inverse scattering problem, and recently, the MUSIC algorithm has been proposed to solve this type of problem. The MUSIC algorithm, by assuming that the number of targets is a priori known, can locate the scatterers from the peaks of the well-known pseudospectrum. The noise and multiple scattering create ambiguity to detect the number of targets. Usually, information-based algorithms such as Akaike information criterion (AIC) and minimum description length (MDL) are employed for source number estimation. However, in the cases of low signal-to-noise ratio (SNR) and close targets, the performance of these methods is seriously degraded. In the present work, we propose a two-step approach to enumerate the scatterers in microwave imaging applications for cases where traditional methods fail. Firstly, the MUSIC algorithm is applied to locate all possible targets by assuming the maximum number of targets, and secondly, we can discriminate between the real and unreal targets by using a novel formula that acts as a spatial filter. The efficiency of the proposed method has been examined through various simulation tests using numerical and experimental datasets, and the results verify that the method can accurately specify the location and number of scatterers in 2D microwave imaging applications.

1. INTRODUCTION

Microwave imaging for the preliminary detection of breast cancer has attracted much interest in the few past decades, mainly because the dielectric properties of malignant breast tissue are significantly different from those of normal tissue at microwave frequencies [1]. Moreover, the non-ionizing, non-compressive nature and relatively low-cost hardware of microwave imaging are attractive aspects compared to other conventional imaging methods such as X-ray mammography and magnetic resonance imaging (MRI). However, it is known that the detection of tumors in the background medium from the measured scattered field is an inverse problem where it is difficult to solve due to its intrinsic non-linearity and ill-posedness [2].

Under certain assumptions on the geometry of the targets and for exact data, it is possible to detect the number and location of small scatterers by using a finite number of measurements in the aforementioned inverse scattering problems. The theoretical results that guarantee the unique identifiability of unknown multiple scatterers in such problems can be found in [3–5]. The MUSIC algorithm is a well-known linear technique that can be employed for the tomography of small inclusions in both full- and limited-view inverse scattering problems. The results in the literature show that the MUSIC method is a promising technique for quasi-real-time qualitative microwave imaging [6–9]. The multiple signal classification (MUSIC) algorithm uses a well-known pseudospectrum to detect the location of targets. The number of targets is prerequisite knowledge for computing the spectrum, and applying the wrong number of targets will create spurious peaks which lead to performance degradation.

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However, the MUSIC algorithm for guaranteeing good imaging performance requires a priori knowledge of the number of scatterers. This problem is addressed as source number estimation in the literature, and several attempts have been made to solve it [10, 11].

Many studies have been conducted to develop algorithms for number detection and location estimation of small scatterers (tumors) in microwave tomographic imaging [10–13]. One group of these algorithms, namely the nonlinear inversion methods, is based on the Newton-type iteration technique that can successfully characterize the number and location of small scatterers within the imaging domain. However, iteration-based methods suffer from inaccurate initial prediction, slow convergence, improper regularization, intensive computation, and the local minimization problem. Hence, for an alternative, linear algorithms have been proposed to overcome the problem of fast, stable, and accurate localization of small targets. Despite that the linear algorithms cannot provide quantitative information about the dielectric anomalies of the targets, it will be possible to obtain the location of targets in a few seconds. Thus, the results of the linear methods can be employed as a good initial guess for nonlinear inversion algorithms to essentially reduce the computational efforts.

The AIC and MDL are two typically used methods that have been applied to source number detection in signal processing for a long time. The utilization of these methods for the enumeration of electromagnetic scatterers in microwave tomography applications has been studied in a few papers. In [10], Pourahmadi et al. illustrated that the scattering from the small scatterers can be, mathematically, modeled as the MDL model, and they applied the MDL to enumerate the targets required for the MUSIC algorithm. In [11], we have proposed a method, named FMDL, which combines the multifrequency data in MDL to improve the MDL performance for the source number estimation. However, the accuracy of the MDL and FMDL for number detection is decreased when the number of snapshots (transmitters) is small, and/or the signal-to-noise ratio (SNR) is low. Furthermore, the MDL criterion is usually effective to estimate a small number of scatterers and often fails when encountering a large number of targets. The second-order statistic of eigenvalues (SORTE) [14], the ratio of adjacent eigenvalues (RAE) [15], and the optimal thresholding method [16] are the other alternative methods, which are based on the eigenvalue gap measure, for source number detection. Compared with the MDL and AIC methods, these methods require low computational costs and simple implementations. However, their performances are extremely degraded for low SNR cases and when few sensors are used. Hence, motivated by these challenges, we propose a two-step efficient method based on the MUSIC algorithm combined with a novel source number detection approach to detect and localize the small scatterers more accurately than the classical methods in microwave imaging applications. The work in this paper can be considered for different scattering scenarios and different applications [17, 18]. In both cases, the proposed method can be used fruitfully, when being properly adapted.

In the first step of the proposed method, the MUSIC is applied to estimate the total location of possible targets by assuming a hypothetical number of targets which can be obtained by thresholding the eigenvalues so that the resulting number of targets is greater than the true target number. After that, we can compute the target reflectivities by employing the resultant target locations, and then using a novel detection method, one can distinguish between the actual and spurious targets. Afterward, in the second step, one can apply the MUSIC method once more with the correct number of targets to accurately estimate the location of actual targets. Through various simulations, we show that the proposed algorithm can improve the performance of the MUSIC algorithm for microwave imaging of targets in 2D inverse problems, and the results validate the superiority of the method over other competing methods.

2. MUSIC IMAGING ALGORITHM

Consider a two-dimensional microwave imaging system with TM incident containing N_t transmitters and N_r receivers in the form of antenna arrays as shown in Fig. 1. The ith antenna is localized at the position $\mathbf{R}_i^t$ for transmitting and $\mathbf{R}_i^r$ for receiving. Several small scatterers (targets) with unknown locations $\mathbf{X}_m$, $m = 1, ..., M$ located in a background medium are irradiated by the transmitters that are separately excited and produce incident wavefields that propagate into the medium. The targets scatter the wave that, partly, reaches the receivers. Using the Helmholtz equation for a 2D problem in the frequency domain, the received wave at the ith receiver due to the jth transmitter can be formulated

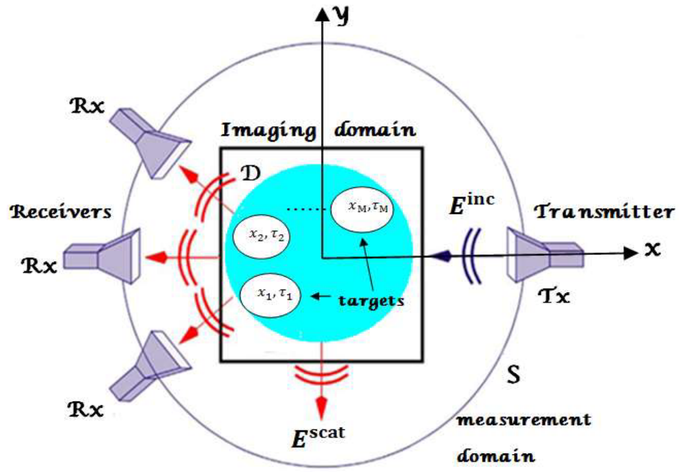


Figure 1. Configuration of the microwave imaging system.

as

$$\psi_j(\mathbf{R}_i^r, \omega) = \psi_j^{inc}(\mathbf{R}_i^r, \omega) + \int G_0(\mathbf{R}_i^r, \mathbf{r}, \omega) O(\mathbf{r}, \omega) \psi_j(\mathbf{r}, \omega) \mathrm{d}\mathbf{r} \tag{1}$$

where $G_0(\mathbf{r}, \mathbf{r}', \omega)$ is the Green's function between positions $\mathbf{r}$ and $\mathbf{r}'$ corresponding to the background medium, and $\psi_j^{inc}(\mathbf{r}, \omega)$ and $\psi_j(\mathbf{r}, \omega)$ are the incidences and total waves, respectively, measured at position $\mathbf{r}$ and produced by the jth transmitter. The scatterers are characterized by the object distribution function $O(\mathbf{r}, \omega)$ which can be defined by

$$O(\mathbf{r}, \omega) = k^2(\mathbf{r}, \omega) - k_0^2(\omega) \tag{2}$$

where $k_0(\omega)$ is the wavenumber of the background medium, and $k(\mathbf{r}, \omega)$ is the wavenumber of the total medium (background plus targets). Then, the scattered wave at position $\mathbf{r}$ can be simply obtained by subtracting the total field $\psi_j(\mathbf{r}, \omega)$ from the incident field $\psi_j^{inc}(\mathbf{r}, \omega)$. After that, under the Born approximation and by assuming small transceivers and small targets [19], we can form the $N_r \times N_t$ scattering matrix $\mathbf{K}$, which is called multi-static response matrix (MSR), as

$$\mathbf{K}(\omega) = \sum_{m=1}^{M} \tau_m(\omega) \mathbf{g}_{0,r}(\mathbf{X}_m, \omega) \mathbf{g}_{0,t}^T(\mathbf{X}_m, \omega) \tag{3}$$

in which $\mathbf{g}_{0,t}(\mathbf{X}_m, \omega)$ and $\mathbf{g}_{0,r}(\mathbf{X}_m, \omega)$, respectively, are the transmitting and receiving background Green's function vectors at the target location $\mathbf{X}_m$ that can be defined as

$$\mathbf{g}_{0,t}(\mathbf{X}_m, \omega) \equiv \left[G_0(\mathbf{X}_m, \mathbf{R}_1^t, \omega), ..., G_0(\mathbf{X}_m, \mathbf{R}_{N_t}^t, \omega)\right]^T \tag{4}$$

$$\mathbf{g}_{0,r}(\mathbf{X}_m, \omega) \equiv \left[G_0(\mathbf{R}_1^r, \mathbf{X}_m, \omega), ..., G_0(\mathbf{R}_{N_r}^r, \mathbf{X}_m, \omega)\right]^T \tag{5}$$

where T stands for the transpose operation. Also, τ_m in Eq. (3) is the mth target reflectivity (or scattering strength) which is related to the dielectric properties of the targets (see [20] for a detailed description), and it can be expressed in terms of the object profile of the mth target as

$$\tau_m(\omega) = \int O_m(\mathbf{r}, \omega) \mathrm{d}\mathbf{r} \tag{6}$$

The M disjoint profile $O_m(\mathbf{r}, \omega)$, $m = 1, ..., M$ with an effective size that is small to the wavelength centered at location $\mathbf{X}_m$ formed the object profile $O(\mathbf{r}, \omega)$. Hence, one can write

$$O(\mathbf{r}, \omega) \equiv \sum_{m=1}^{M} O_m(\mathbf{r} - \mathbf{X}_m, \omega) \approx \sum_{m=1}^{M} \tau_m(\omega) \delta(\mathbf{r} - \mathbf{X}_m) \tag{7}$$

For simple representation, the variable ω that refers to the frequency domain is deleted from the following equations. The data matrix $\mathbf{K}$, defined in Equation (3), is a key quantity that is applied to form an image of the targets (reconstruction of the object profile). So, imaging algorithms such as MUSIC aim to estimate the location of targets ($\mathbf{X}_m$) from the measured scattered field formed by the matrix $\mathbf{K}$. The theory of the MUSIC algorithm is based on the eigenvalue decomposition of the time-reversal matrix $\mathbf{T} = \mathbf{K}^H\mathbf{K}$ as

$$\mathbf{K}^H\mathbf{K}\mathbf{u}_j = \sigma_j^2\mathbf{u}_j, \quad \mathbf{K}\mathbf{K}^H\mathbf{v}_j = \sigma_j^2\mathbf{v}_j \tag{8}$$

where H denotes the Hermitian; $\mathbf{u}_j$ is the jth eigenvector of matrix $\mathbf{T}$; $\mathbf{v}_j$ is the jth eigenvector of matrix $\mathbf{T}^H$; and σ_j^2 is the jth singular value of the matrix $\mathbf{T}$. The matrix $\mathbf{T}$ is known as the time-reversal matrix [20], and it is shown that those eigenvectors associated with the nonzero eigenvalues span the signal subspace, which is related to the target locations. The eigenvectors with $\sigma_j^2 = 0$ span the noise subspace. The eigenvectors of matrix $\mathbf{K}$ are the singular vectors of matrix $\mathbf{T}$, and its singular values σ_j are the square root of the eigenvalues of $\mathbf{T}$ (i.e., σ_j^2). So instead of using the singular value decomposition (SVD) of the matrix $\mathbf{T}$, we can use the SVD of matrix $\mathbf{K}$ [20] for the time-reversal approach. The MUSIC algorithm is based on the orthogonality of the signal and noise subspaces. So, the MUSIC pseudo-spectrum at each point $\mathbf{X}$ can be obtained as follows

$$P_{r,t}(\mathbf{X}) = \frac{1}{\sum\limits_{\sigma_j=0} \left|\mathbf{u}_j^H \mathbf{g}_{0,r}(\mathbf{X})\right|^2 + \sum\limits_{\sigma_j=0} \left|\mathbf{v}_j^H \mathbf{g}_{0,t}^*(\mathbf{X})\right|^2} \tag{9}$$

where $*$ denotes the complex conjugation. When $\mathbf{X}$ coincides with the unknown scatterer locations (i.e., $\mathbf{X} = \mathbf{X}_m$), the pseudospectrum $P_{r,t}(\mathbf{X})$ will create peaks. As can be seen from Eq. (9), the number of zero eigenvalues (noise subspace) or the number of non-zero eigenvalues (signal subspace) must be *a priori* known so that the MUSIC algorithm can accurately determine the location of scatterers. However, if the number of scatterers is overestimated or underestimated, then the MUSIC algorithm may give spurious peaks. So the accurate detection of targets number is the major challenge of the MUSIC method for target localization.

2.1. RAE Method

One simple method for scatterers enumeration is based on the ratio of adjacent eigenvalues (RAE) [15] of the matrix $\mathbf{K}$ that has simple implementation and low execution time. By applying SVD on matrix $\mathbf{K}$ and sorting the resulting eigenvalues in descending order, one can obtain the eigenvalues $\sigma_1 \geq \cdots \geq \sigma_M \geq \sigma_{M+1} \geq \cdots \geq \sigma_{N_t} \geq 0$, which contain M larger eigenvalues $\sigma_1 \geq \cdots \geq \sigma_M$ associated with the scatterers, and the remaining $N_t - M$ eigenvalues related to the noise are, theoretically, equal, i.e., $\sigma_{M+1} = \cdots = \sigma_{N_t} = \gamma^2$, where γ^2 is the variance of the noise. Then, we can compute the ratio between the adjacent singular values as

$$\mathrm{RAE}(j) = \frac{\sigma_j(\omega)}{\sigma_{j+1}(\omega)} \tag{10}$$

where $j = 1, ..., N_t - 1$. The method selects j as the targets number for which the criterion in Eq. (10) is maximized. The method acts well in usual utilization, but for microwave imaging applications, the performance of this method is considerably degraded in the cases of low SNR values and the use of a small number of sensors. Another alternative method based on the information-theoretic criteria for number estimation is the MDL method. The authors in [10], presented the method that employs the MDL algorithm to detect the number of targets for the MUSIC algorithm in microwave imaging applications. A brief description of this method is given in the next section.

2.2. MDL Method

The MDL method [21] is an information-based method that can be used to examine the eigenvalues of a sample covariance matrix to specify how many of the smallest eigenvalues of the covariance matrix are equal. This algorithm consists of minimizing a criterion over the number of detectable sources. In microwave imaging applications, assuming that the scatterers are considered as sources and the different illumination of transmitters assumed as multiple snapshots, the following criterion can be employed for MDL to characterize the number of targets as

$$\mathrm{MDL}(d) = N_t \ln\left(\left(\frac{1}{N_t-d}\sum_{j=d+1}^{N_t}\sigma_j\right)^{N_t-d} \Bigg/ \prod_{j=d+1}^{N_t}\sigma_j\right) + \frac{1}{2}d(2N_t-d)\ln(N_t), \quad d=0,\ldots,N_t-1 \quad (11)$$

The estimate of the number of scatterers is the value of d which minimizes the above criterion.

In [10], the MDL method is applied to the estimation of the target number in microwave imaging applications using both synthetic and experimental data, and the results verified the accurate enumeration of targets with low complexity. However, the performance of MDL degrades when the SNR value is low, and the targets are closely located. Moreover, the MDL cannot perform well when the number of transmitters (N_t) is comparable with the number of scatterers. Also, the limited number of transmitters (snapshots) leads to a considerable gradient for the noise eigenvalues, and it causes MDL to overestimate the number of targets even at high SNRs as discussed in [10]. In this paper, we propose a novel detection method that outperforms the classical detection methods such as RAE and MDL in microwave imaging applications, and using a preprocessing approach, we modify the method to produce consistent estimates in both low and high SNRs.

2.3. Optimal Thresholding Method

The method proposed in [16], which is based on the thresholding of the singular values of the data covariance matrix, can be used for target enumeration in microwave imaging applications. The method is inspired by the recovery of low-rank matrices from noisy data using hard singular value thresholding, where empirical singular values below a threshold λ are set to zero. In the case of the $n \times n$ matrix in the white noise of level σ, the optimal value for the hard threshold is $2.309\sqrt{n}\sigma$ when σ is known, and when σ is unknown, it equals $2.858^* y_{med}$, where y_{med} is the median of eigenvalues of matrix $\mathbf{K}$. The performance of the optimal thresholding method is superior to the MDL method under low SNR values and superior to the AIC method under high SNR values.

3. PROPOSED DETECTION METHOD

The block diagram of the proposed method is given in Fig. 2. First, the matrix $\mathbf{K}$, formed from the measured scattered field sensed at receivers due to the presence of targets (scatterers) for the total number of active transmitters, is employed in the MUSIC algorithm to estimate the location of targets by assuming the hypothetical number of targets (N) which is larger than the actual number of targets (M). For example, for an array of N_t elements, the location and number of $N_t - 1$ targets can be estimated at most, thus letting $N = N_t - 1$. Then these obtained locations were applied to calculate

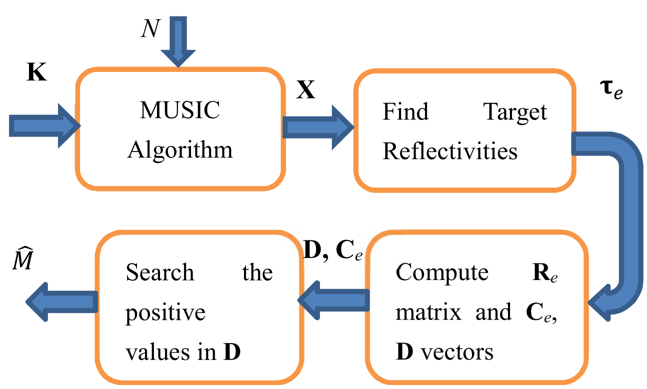


Figure 2. Block diagram of the proposed detection method.

the associated target reflectivities (τ_i, $i = 1, ..., N$). The resulting target locations contain the M actual target locations and the $N - M$ spurious target locations. In the final stage, we must distinguish the M largest values of $|\tau_i \tau_i'|$ associated with the actual targets using a detection law. Now, the detailed description of the method is explained in the following. Firstly, we ignore the influence of noise and assume that N is the maximum number of possible targets, then we can rewrite Eq. (3) in the matrix form as [22]

$$\bar{\mathbf{K}} = \mathbf{G}(\mathbf{X})\boldsymbol{\tau} \tag{12}$$

in which $\bar{\mathbf{K}}$ is the $N_r N_t \times 1$ long column vector or vectorized form of matrix $\mathbf{K}$; $\boldsymbol{\tau} = [\tau_1, \tau_2, ..., \tau_N]^T$ is the $N \times 1$ column vector containing all possible target reflectivities; and $\mathbf{G}(\mathbf{X})$ is the matrix with dimension $N_r N_t \times N$ that can be expressed as

$$\mathbf{G}(\mathbf{X}) = \begin{bmatrix} G_0(\mathbf{R}_1^r, \mathbf{X}_1)G_0(\mathbf{X}_1, \mathbf{R}_1^t) & \cdots & G_0(\mathbf{R}_1^r, \mathbf{X}_N)G_0(\mathbf{X}_N, \mathbf{R}_1^t) \\ G_0(\mathbf{R}_1^r, \mathbf{X}_1)G_0(\mathbf{X}_1, \mathbf{R}_2^t) & \cdots & G_0(\mathbf{R}_1^r, \mathbf{X}_N)G_0(\mathbf{X}_N, \mathbf{R}_2^t) \\ \vdots & \vdots & \vdots \\ G_0(\mathbf{R}_{N_r}^r, \mathbf{X}_1)G_0(\mathbf{X}_1, \mathbf{R}_{N_t}^t) & \cdots & G_0(\mathbf{R}_{N_r}^r, \mathbf{X}_N)G_0(\mathbf{X}_N, \mathbf{R}_{N_t}^t) \end{bmatrix} \tag{13}$$

where $\mathbf{X} \equiv [\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_M, \mathbf{X}_{M+1}, ..., \mathbf{X}_N]$ contains all possible target locations including M actual targets and $N - M$ spurious targets that can be obtained by the MUSIC algorithm. The matrix $\mathbf{G}(\mathbf{X})$ can be rewritten as $\mathbf{G}(\mathbf{X}) = [\bar{G}(\mathbf{X}_1), \bar{G}(\mathbf{X}_2), ..., \bar{G}(\mathbf{X}_N)]$ in which $\bar{G}(\mathbf{X}_m) = \mathrm{vec}[\mathbf{g}_{0,r}(\mathbf{X}_m)\mathbf{g}_{0,t}^T(\mathbf{X}_m)]$ is the $N_r N_t \times 1$ vector (vec($\cdot$) represents the vectorized form of a matrix). Due to the absence of noise, the estimation of target reflectivities $\boldsymbol{\tau}_e$ from Eq. (11) is obtained by

$$\boldsymbol{\tau}_e = [\mathbf{G}^H(\mathbf{X})\mathbf{G}(\mathbf{X})]^{-1}\mathbf{G}^H(\mathbf{X})\bar{\mathbf{K}} \tag{14}$$

where $\boldsymbol{\tau}_e = [\tau_1, ..., \tau_M, \tau_{M+1}, ..., \tau_N]^T$ is the estimated target reflectivities for the total number of possible targets. In the noise-free case, τ_m could be zero when $m \geq M + 1$, and the target number can be obtained by the number of nonzero target reflectivities. In this case, Eq. (14) can be simplified as $\boldsymbol{\tau}_e = [\boldsymbol{\tau}; \mathbf{Z}_{N-M\times 1}]$ in which $\boldsymbol{\tau} = [\tau_1, ..., \tau_M]^T$ is the real target reflectivities, and $\mathbf{Z}_{N-M\times 1}$ is a $(N-M)\times 1$ column vector with zero elements.

Under the white Gaussian noise (WGN) assumption, we can obtain

$$\boldsymbol{\tau}_e\boldsymbol{\tau}_e^H = \begin{bmatrix} \boldsymbol{\tau}^H & \mathbf{Z}_{M\times(N-M)} \\ \mathbf{Z}_{(N-M)\times M} & \mathbf{Z}_{(N-M)\times(N-M)} \end{bmatrix} + \sigma^2[\mathbf{G}^H(\mathbf{X})\mathbf{G}(\mathbf{X})]^{-1} \tag{15}$$

where σ^2 is the variance of noise, and $\mathbf{Z}_{a\times b}$ is a matrix with size $a \times b$ having zero elements. After that, we can define the matrix $\mathbf{R}_e$ as follows

$$\mathbf{R}_e = \boldsymbol{\tau}_e\boldsymbol{\tau}_e^H - \sigma_{\min}^2[\mathbf{G}^H(\mathbf{X})\mathbf{G}(\mathbf{X})]^{-1} \tag{16}$$

where $\sigma_{\min}^2$ is the minimum singular value of matrix $\mathbf{K}$. Hence, we can define

$$\mathbf{C}_e = \mathrm{diag}(\mathbf{R}_e) = [c_1, c_2, ..., c_M, c_{M+1}, ..., c_N] \tag{17}$$

where the first M elements of $\mathbf{C}_e$ represent the power of actual scatterers, i.e., $[c_1, ..., c_M] = \left[\left|\tau_1^2\right|, ..., \left|\tau_M^2\right|\right]$, and the remaining elements of $\mathbf{C}_e$ correspond to the power of noise or spurious targets. Next, we define the vector $\mathbf{D} = [d_1, ..., d_M, d_{M+1}, ..., d_N]$ in which d_i is calculated as

$$d_i = c_i - \frac{1}{N}\sum_{i=1}^{N} c_i \tag{18}$$

Since the first M elements of $\mathbf{C}_e$ are much larger than the rest ones, the first M elements of $\mathbf{D}$ are positive whereas the remaining $N - M$ elements are negative. Therefore, we can estimate the number of actual targets by searching the total number of positive elements of $\mathbf{D}$.

The total steps of the proposed method for microwave imaging by MUSIC algorithm using the proposed scatterers enumeration are given as follows:

Step 1. Apply the MUSIC algorithm with an assumed number of targets (N), firstly, to obtain the total possible target locations $\mathbf{X} \equiv [\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_M, \mathbf{X}_{M+1}, ..., \mathbf{X}_N]$. The hypothetical target number is assumed to be larger than the true one (i.e., $N > M$).

Step 2. Putting the resulting locations from step 1 in Eq. (13), we compute the target reflectivities associated with the actual and spurious targets.

Step 3. Calculate the coefficients $\mathbf{C}_e = [c_1, ..., c_M, c_{M+1}, ..., c_N]$ using Eq. (16) and Eq. (17).

Step 4. Compute the vector $\mathbf{D} = [d_1, ..., d_M, d_{M+1}, ..., d_N]$ using Eq. (18).

Step 5. Search the total number of positive elements of vector $\mathbf{D}$ as the estimated actual number of scatterers.

Step 6. Apply the MUSIC algorithm with the actual number of targets, and secondly, accurately localize the targets.

The choice of N will not seriously affect the performance of the proposed detection method. For example, it can be assumed to be the number of transceivers minus one, which is the maximum number of detectable targets by the array. In our tests, we assume that the maximum number of possible targets, i.e., N, is equal to or more than twice the number of actual targets. It must be noted that the proposed method is efficient for only point-like scatterers, where the size of each scatterer is much smaller than the wavelength, so in the case of extended targets where the profile of the unknown scatterers must be identified, the use of a thresholding strategy to determine the appropriate value of N is essential for imaging. The idea in [23] is an effective method for determining the threshold value to efficiently specify the locations and/or shapes of a collection of small inclusions.

As a final point, the target reflectivity for the proposed method is achieved under the Born approximation, but the proposed method empirically performs reasonably well in the cases of the presence of multiple scattering between the targets. To investigate the exact case of multiple scattering, we can use the iterative formula in [21] for computing the scattering strengths. The next steps of the algorithm are the same as before.

4. SIMULATION RESULTS

In the following experiments, the probability of detecting the correct number of targets is the performance metric that is assessed for various SNR values. In all simulations, we take 500 times Monte-Carlo runs, and the probability of detection is defined by $P = P_k/P_t$ where P_t denotes the number of Monte-Carlo simulations, and P_k means the number of the correct detection. In the simulations, the geometric arrangement of the sensors is similar to the real Fresnel data [24], but the geometric shapes of the receivers are different for the two cases. Also, the scattering data matrix $\mathbf{K}$ is generated by (11), and additive white Gaussian noise $\mathbf{W}$ is added to this simulated data ($\hat{\mathbf{K}} = \mathbf{K} + \mathbf{W}$). The signal-to-noise value (SNR) in dB is defined as

$$\text{SNR} = 20 \log_{10} \frac{\|\mathbf{K}\|}{\|\mathbf{W}\|} \tag{19}$$

where $\|.\|$ denotes the Euclidean norm value.

In the first simulation, the array consisting of 36 antennas, which act as transceivers (transmitters and receivers) and are equally spaced on a circle with a radius of 720 mm, is used. Equation (3) is applied to produce the synthetic data. The simulation is performed in two cases: (a) the case with two-point targets with scattering strengths $\tau_1 = 1$ and $\tau_2 = 1.5$ located at positions $(0.25\lambda,\ 0.25\lambda)$ and $(-0.25\lambda,\ -0.25\lambda)$ in the x-y plane, (b) the case with four-point targets with scattering strengths $\tau_1 = 1$, $\tau_2 = 1.2$, $\tau_3 = 1.5$, and $\tau_4 = 1$ located at positions $(0.5\lambda,\ 0.5\lambda)$, $(-0.5\lambda,\ -0.5\lambda)$, $(0.5\lambda,\ -0.5\lambda)$, and $(-0.5\lambda,\ 0.5\lambda)$ in the x-y plane. The operating frequency is set to 2 GHz in two cases. The simulation results for different SNR levels are shown in Figs. 3(a) and (b). As can be seen, the proposed method provides a higher detection probability than the other methods, especially in cases of low SNR and for a large number of targets.

Another simulation is performed with the same array used in the first simulation, assuming small scattering strengths for the targets, to prove that the proposed method is not dependent on the scattering strength values of the targets and can separate real targets from unreal targets under appropriate SNR values. In this simulation, four targets with scattering strength $\tau_1 = 0.1$, $\tau_2 = 0.2$, $\tau_3 = 0.3$, and $\tau_4 = 0.4$ located at positions $(-1\lambda,\ -2\lambda)$, $(2\lambda,\ -1\lambda)$, $(-1\lambda,\ 2\lambda)$, and $(2\lambda,\ 1\lambda)$. The imaging frequency chosen is 2 GHz. The SNR value is set to 10 dB. The results of this experiment are given in Table 1. It can be seen that by analyzing the d_m coefficients, the number of actual targets, which is equal to the number of positive values of d_m, can be well determined.

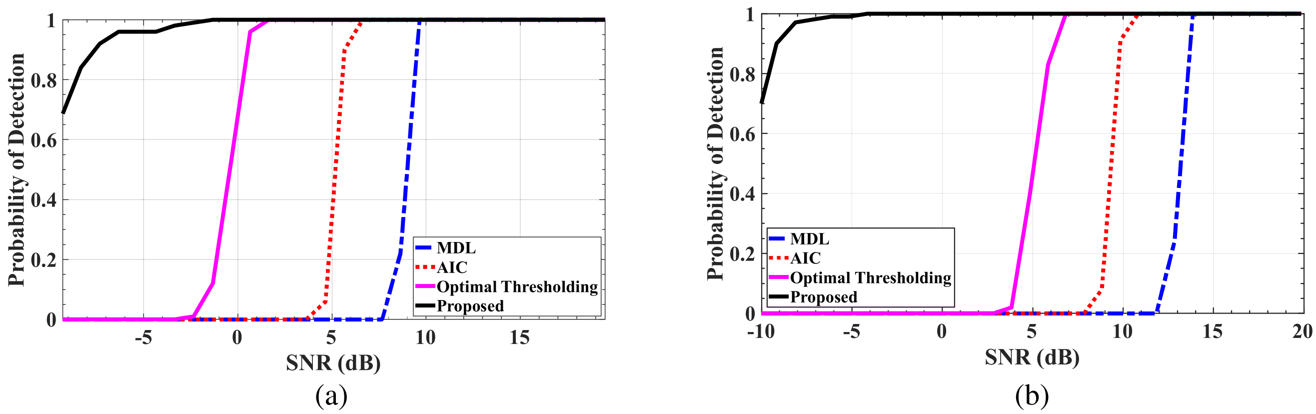


Figure 3. Probability of correct detection in terms of SNR for MDL, RAE, optimal thresholding, and the proposed method by changing the number of targets, (a) 2 targets case and (b) 4 targets case.

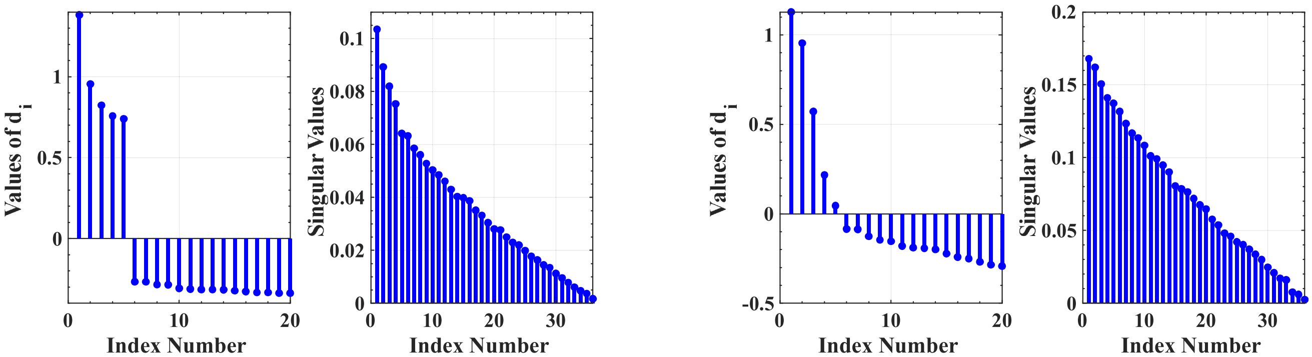
Table 1. The estimated position of real and unreal targets and their scattering strengths.

$\boldsymbol{X}_m$ (mm)	τ_m	$\hat{\boldsymbol{X}}_m$ (mm)	$\hat{\tau}_m$	d_m
(300, −150)	0.1	(300, −150)	0.11	0.03
(−150, 300)	0.2	(−150, 300)	0.21	0.03
(−150, −300)	0.3	(−150, −300)	0.29	0.08
(300, 150)	0.4	(300, 150)	0.34	0.11
		(−150, 262)	0.007	−0.007
		(300, 112)	0.007	−0.007
		(−150, 337)	0.006	−0.007
		(−150, −262)	0.006	−0.007

In the next simulation, 5 nearby targets with the same scattering strengths are irradiated by the array used in the first simulation. This test is performed using very noisy data. The singular values of the **K** matrix for two noise levels are shown in Figs. 4(b) and (d). As can be seen, the number of real targets cannot be distinguished from the number of non-zero eigenvalues of the **K** matrix, while the proposed method was able to detect the number of real targets by counting the number of positive values of the coefficients d_m, as seen in Figs. 4(a) and (c).

In another simulation, the efficiency of the proposed algorithm to estimate the number of scatterers has been compared with the methods given in our published paper [11]. To perform this experiment, the Monte-Carlo technique was used 500 times, and the results were averaged. In this simulation, three close targets with locations $(0.25\lambda, 0.25\lambda)$, $(-0.25\lambda, -0.25\lambda)$, and $(0.25\lambda, -0.25\lambda)$ in the x-y plane are irradiated by the array containing 24 antennas, which is used as transmitters and receivers placed at a circle with a radius of 240 millimeters. This imaging configuration is similar to the Manitoba microwave tomography system [25]. The results are shown in Fig. 5. The results clearly show the superiority of the proposed algorithm over the other algorithms, particularly for low SNR values.

To better compare the performance of the proposed method with the other competing methods in the cases of a large number of targets and for close targets, another simulation is performed in which 36 antennas as transmitters and receivers are located in a circle with a radius of 720 mm and irradiate 9 scatterers placed in a particular form as shown in Fig. 6(a). Each scatterer is at a distance d from the adjacent scatterer, where d varies from 0.25λ to 2λ. The probability of correct detection of targets in terms of the SNR using the proposed method is obtained over 300 Monte-Carlo runs, and this experiment was performed by changing the distance between the targets (i.e., d) from 0.25λ to 2λ. The frequency of operation is set to 2 GHz. The results of this simulation are shown in Fig. 6(b). As can



(a) (b) (c) (d)

Figure 4. The values of d_i and the singular values for the case of 5 targets with $\tau_m = 1$, $m = 1, ..., 5$ located at $(0,\ 0)$, $(0.25\lambda,\ -0.25\lambda)$, $(-0.25\lambda,\ 0.25\lambda)$, $(-0.25\lambda,\ -0.25\lambda)$ and $(0.25\lambda,\ 0.25\lambda)$, (a), (b) SNR $= -5\,\text{dB}$ and (c), (d) SNR $= -10\,\text{dB}$.

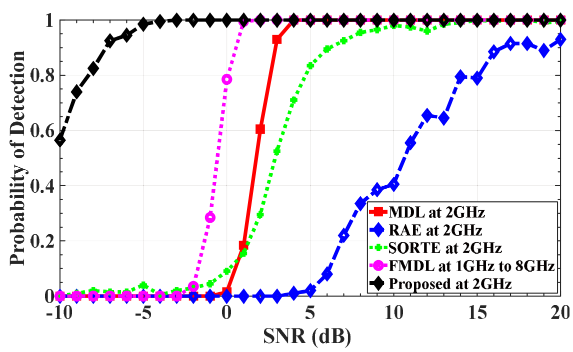
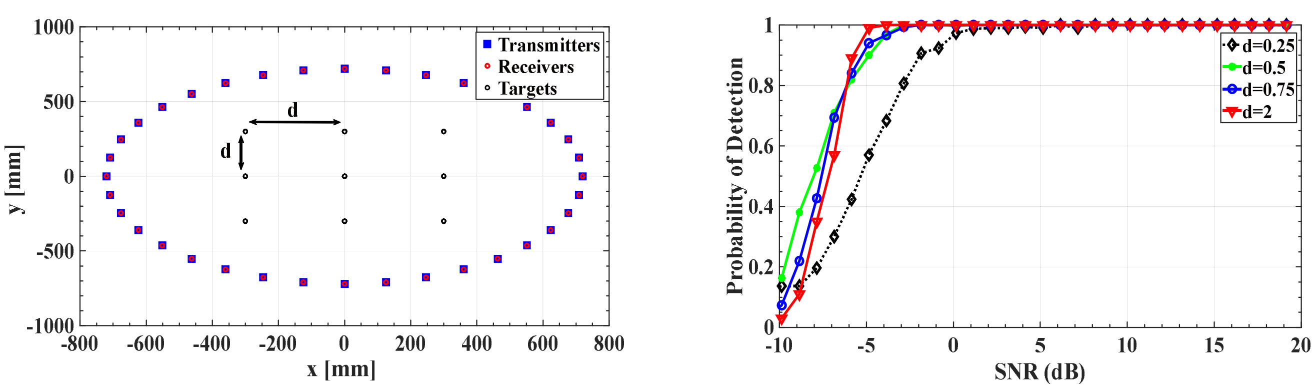


Figure 5. Comparison of the proposed method with our FMDL method proposed in [11].



(a) (b)

Figure 6. Probability of correct detection using the proposed method in terms of SNR by changing the distance between the scatterers (in terms of wavelength), (a) targets and array configurations, (b) probability of correct detection for $d = 0.25\lambda$ to $d = 2\lambda$.

be seen, when the distance between the targets changes from 2λ to 0.5λ, the probability of correctly detecting the targets changes only slightly, and by increasing the SNR value, the proposed method can detect the number of targets with 100% accuracy. However, when the distance between the targets is less than half of the wavelength (for example $d = \lambda/4$), the probability of correct detection decreases

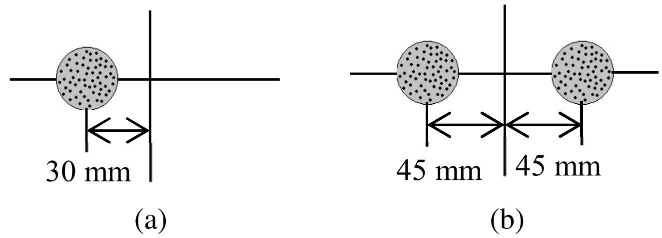


Figure 7. Dielectric targets with permittivity $\varepsilon_r = 3 \pm 0.3$ and radius $r = 15\,\text{mm}$, (a) one dielectric target and (b) two dielectric targets.

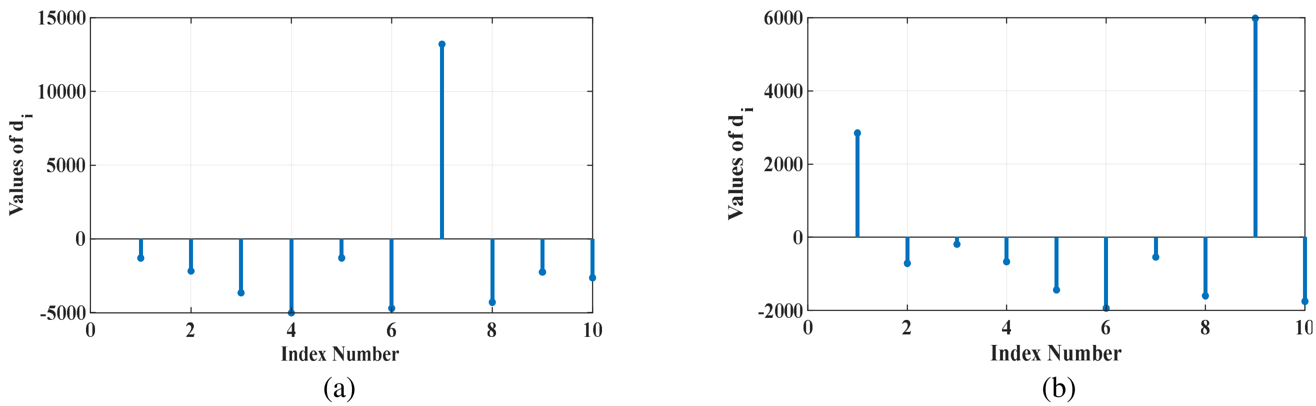


Figure 8. The values of d_i in the case of using real data with external noise by SNR $= 8\,\text{dB}$ and $N = 10$, (a) one target case and (b) two targets case.

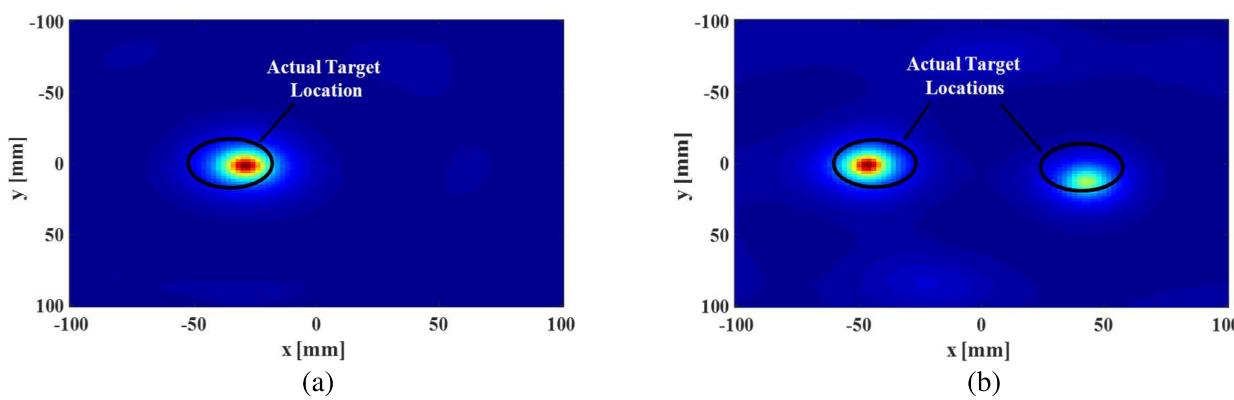


Figure 9. Images resulting from applying the MUSIC algorithm to real data by adding external noise with SNR $= 8\,\text{dB}$, (a) one target case, (b) two targets case.

more strongly by decreasing the SNR value, but as can be seen, the algorithm is still efficient to detect the correct targets at a distance of $\lambda/4$ even in the cases of SNR lower than $5\,\text{dB}$ whereas all other algorithms fail.

In the last simulation, two real data sets obtained from the Fresnel imaging system [24] have been used to evaluate the MUSIC algorithm combined with the proposed technique to estimate the location and number of targets. The targets used in this data are shown in Fig. 7. The Fresnel real data sets were applied in various papers for checking inverse scattering techniques against the experimental data [26, 27]. These experimental data sets have a low noise level, and for testing the algorithm against more noise levels, we add some external noise to this data. Fig. 8 shows the values of d_i defined in

Eq. (18) calculated from the proposed method by employing these two real data sets. As can be seen, from the number of positive values of d_i, we can estimate the number of actual targets. Fig. 9 shows that the targets and their locations are accurately obtained with the help of the proposed algorithm when they are applied to the noisy Fresnel data sets by adding some external noise to this real data.

5. CONCLUSION

The MUSIC method can determine the location of the targets using the peaks of the known pseudospectrum. To form this spectrum, the number of targets must be predetermined. In the absence of noise, the number of targets can be specified from the non-zero eigenvalues of the $\mathbf{K}$ matrix. The presence of noise changes the value of eigenvalues in such a way that it is no longer possible to determine the number of targets. In this article, a method based on a novel detection strategy with low calculation time is proposed to estimate the number and position of 2D scatterers. This method has developed the capability of the MUSIC method to locate small targets in high noise. This method has also been tested with practical microwave imaging data, and promising results have been obtained. The application of the proposed method in cases where the targets have very different reflectivities, as discussed in [28], will be considered in our future work.

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