Progress In Electromagnetics Research B, Vol. 98, 77–86, 2023

Theoretical Study of Electromagnetic Field, Diffracted by Two Slots in a Conducting Screen

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Abstract—A rigorous solution is presented for description of the plane electromagnetic wave diffraction by two parallel slots in a perfectly conducting screen of finite thickness, placed before a dielectric layer, operating as a receiver of radiation in the near zone. The field in this layer is studied for the case of small obstacle dimensions being of the order of the wavelength. It is shown that the best spatial resolution of images from two slots in a dielectric layer is reached together with their optimal focusing, which can be determined by the method proposed earlier for one-slot diffraction.

1. INTRODUCTION

Since the beginning of the new century, there has been a renaissance of interest in the problem of electromagnetic diffraction by a slot in a conducting screen. It is caused by the discovery of the phenomenon of an anomalously high energy concentration of the diffraction field in the near zone of subwavelength apertures [1], i.e., of small apertures with the dimensions of the order of or smaller than the radiation wavelength, and the slot can serve as the simplest model of a two-dimensional aperture. Great interest in such effects is due to the possibility of local exposure of the field on small areas of the irradiated material and their application in spectroscopy [2] and in the technology of optical lithography [3]. Unfortunately, here the theory lags far behind experiment and practical demands. Until now, for explaining the phenomenon of slot diffraction, approximate methods [4, 5] have been used, and they are suitable for optical problems with large sizes of diffraction obstacles in comparison with the wavelength, but they are not applicable for description of fields in the near zone of small slot apertures. With their help, at best, it was possible to estimate the integral intensity of the total field behind the slot, but such theoretical methods are not suitable for a detailed analysis of its spatial structure in the near zone. Such an analysis can be carried out only on the basis of a rigorous solution of the problem of electromagnetic wave diffraction by a slot [6]. Moreover, considering diffraction by subwavelength objects, one needs to take into account the distorting effect of the dielectrics under test themselves, which play the role of electromagnetic radiation receivers, for example, of the material under study in spectroscopy or the photoresist in the lithographic process [7]. There is an analogy between the classical electromagnetic theory and quantum mechanics, where the back action of the measuring device on the objects under study (particles and fields) cannot be neglected [8]. Therefore, for studying the spatial structure of the field in the near zone of a small aperture, it is necessary to apply a rigorous solution of the diffraction problem for a slot in a conducting screen, taking into account the presence of a plane dielectric layer on a substrate behind the screen as the simplest model of an irradiated dielectric. However, the construction of such a solution is complicated, because here we should consider resonance phenomena in the dielectric layer, which causes the excitation of waveguide modes. Nevertheless, such a solution was recently constructed [9], and it served as the bases for the study of the spatial pattern of the field in a thin film at the near diffraction zone of a simple slot structure [7, 10]. It turned out that

Received 5 January 2023, Accepted 3 February 2023, Scheduled 7 February 2023
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one more anomalous aperture effect can manifest itself, namely, the lensless focusing of electromagnetic radiation by a slot in a perfectly conducting screen, when the effective size of the light diffraction spot in the dielectric from the slot is 1.5–2, or even 2.5 times smaller than the size of the slot itself. However, this effect can occur in the near zone and only for certain dimensions of the diffraction scheme, if these dimensions turn out to be of the order of the wavelength of the incident radiation. For estimation of the quality of the diffraction slot image, i.e., a light spot in a dielectric, and for determining the region of existence of this effect, a theoretical method was proposed in [10], which reduces to the calculation of just one dimensionless scalar value, the so-called electric energy focusing parameter in a thin dielectric film. It shows how many times the effective width of the diffraction slot image is less than the width of the slot itself. At small values of this parameter, a strong divergence of the image takes place, but when its value reaches values of the order of 2 or more, the effect of focusing appears. For wide slots, the image quality can further deteriorate due to the nonuniform distribution of energy along the profile, and here, it is necessary to introduce additional values in order to characterize quality of its uniformity. However, for narrow slots of the order of a wavelength, a well-focused image usually has a monotone profile without intensity jumps, and therefore only one focusing parameter is sufficient to estimate the quality of their images.

In addition, for optical lithography, as well as for spectroscopy, another property of diffraction images is of great importance. It is spatial resolution under conditions of close position of several apertures [3]. Obviously, at good focusing of close images, they should not meet and produce noticeable exposure in the area between the slots behind the screen. However, taking into account the non-trivial nature of the interference phenomenon, this conclusion requires demonstrative confirmation. Therefore, in this paper, we consider a rigorous solution of the diffraction problem for two closely spaced slots in a conducting screen and use it to study the conditions for optimal focusing and spatial resolution of the diffraction images of these slots in a thin dielectric layer behind the screen. A similar problem was solved before, say, in the work [11], which clearly shows the shortcomings and contradictions of the traditional optical approach to small apertures. The premise of this paper is to specify the field on a very narrow slot in the form of one fundamental slot mode, and then calculate the field in the rest of the space. Meanwhile, the field on the slot is initially unknown, and in order to determine it adequately, one has to consider the expansion of this field in sinusoidal slot modes, most of which decay in the direction normal to the slot and screen boundaries. In this case, the truth of the total diffraction solution can be validated theoretically only in one way: by testing the accuracy of the fulfillment of the boundary conditions for the fields on both sides of the boundaries of the slots and conducting screens. If the boundary conditions are satisfied with the required accuracy, then the solution of the diffraction problem is correct, but if not, then there are serious reasons to doubt the truth of all the results of theoretical study. The accuracy of fulfillment of the boundary conditions directly depends on the number of modes inside and outside the slot taking into consideration in the solution. For example, for slots with a width of the order of a wavelength, the minimum relative error of fulfilling these conditions on the order of 0.01–0.001 is achieved if one considers 100 sinusoidal slot modes, almost all of which are decaying, and with decrease in the width of the slots, the number of required modes only increases. Thus, limiting ourselves to one or two modes of a small slot of a simple form, it is impossible to obtain a reasonable solution of the diffraction problem that will satisfy the boundary conditions in the near zone, and therefore the application of such a solution in this zone turns out to be illegal.

2. SOLUTION OF DIFFRACTION PROBLEM

Let us consider a stationary two-dimensional problem of diffraction of a plane electromagnetic wave by two slots in a perfectly conducting screen of finite thickness. And at a certain distance H behind the screen, a plane dielectric layer is placed on a substrate (Fig. 1). The thickness of the latter is usually very large and has little effect on the nature of the solution for the field in the layer, and therefore the assumption of an unlimited thickness of the substrate will not be erroneous. Let the plane of incidence of the diffracting wave coincide with the coordinate plane xy, which is orthogonal to the z axis parallel to the infinite edges of the slots. In this case, Maxwell's equations allow the separation of the field into two different polarizations H and E (in optics they are usually denoted as TE and TM) [4, 12], each of which is determined by an independent complex scalar function $u(x,y)$ of two coordinates x and y. In

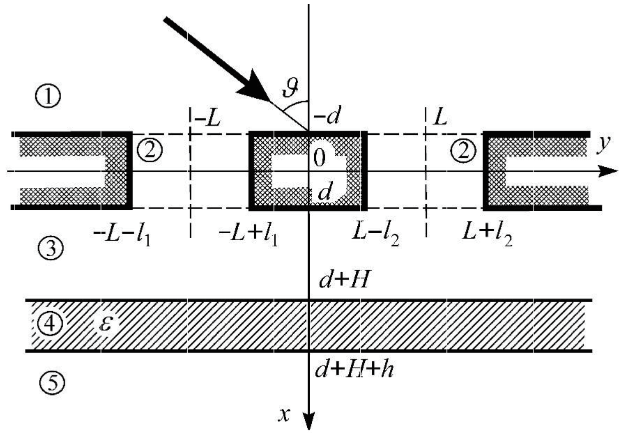


Figure 1. Geometry of the problem for two slots diffraction system.

this case, the spatial components of the electric and magnetic vectors of each polarization are expressed in terms of this function as follows [4, 9]:

$$\left\{ \begin{array}{c} E_z \\ \varepsilon^{-1}(x)H_z \end{array} \right\} = u \, ; \quad \left\{ \begin{array}{c} H_x \\ -E_x \end{array} \right\} = -\frac{i}{k}\frac{\partial u}{\partial y} \, ; \quad \left\{ \begin{array}{c} -H_y \\ E_y \end{array} \right\} = -\frac{i}{k}\frac{\partial u}{\partial x} \, , \tag{1}$$

where $k = \omega/c$ is the wavenumber; ω is the circular frequency of the field; c is the speed of light; $\varepsilon(x)$ is the piecewise constant function and equals to the unite in empty space and the dielectric permittivity ε or ε_s of a dielectric inside that. The upper symbols in the left sides of Eq. (1) refer to H polarization, and the lower symbols correspond to E polarization. $i = (-1)^{1/2}$ is the imaginary unite. The right-hand sides of Eq. (1) contain different values for different polarizations, since for them the values of the functions u at the same points in space differ from each other due to differences in the boundary conditions. In Eq. (1), as well as everywhere below, the exponential factor $\exp(-i\omega t)$, determining the dependence of stationary fields on time t, is omitted.

The boundary conditions for the fields are reduced to the well-known requirements of the continuity of the tangential components of the electric and magnetic vectors at the boundaries between different dielectrics and the vanishing of the tangential components of the electric field on a perfectly conducting surface [4, 11]. According to Eq. (1), at the boundaries of the dielectric layer, we have:

$$(u)_{x=d+H-0} = \varepsilon^{\nu}(u)_{x=d+H+0} \, ; \quad \varepsilon^{\nu}(u)_{x=d+H+h-0} = \varepsilon_s^{\nu}(u)_{x=d+H+h+0}; \tag{2a}$$

$$(\partial u/\partial x)_{x=d+H-0} = (\partial u/\partial x)_{x=d+H+0} \, ; \quad (\partial u/\partial x)_{x=d+H+h-0} = (\partial u/\partial x)_{x=d+H+h+0}, \tag{2b}$$

where ν is zero for H polarization and $\nu = 1$ for E polarization, and the symbol "0" denotes an infinitesimal positive value. Taking into account the presence of two slots, the boundary conditions at the screen boundaries $x = -d$ and $x = d$ are formulated as follows:

$$(1-\nu)(u)_{x=\mp d} + \nu(\partial u/\partial x)_{x=\mp d} = 0 \quad \text{for} \quad y \le -L - l_1; -L + l_1 \le y \le L - l_2; y \ge L + l_2 \tag{3}$$

directly on a perfectly conducting surface, and

$$\left. \begin{array}{l} (u)_{x=\mp d-0} = (u)_{x=\mp d-0} \\ (\partial u/\partial x)_{x=\mp d-0} = (\partial u/\partial x)_{x=\mp d-0} \end{array} \right\} \text{for } -L - l_1 < y < -L + l_1; \;\; L - l_2 < y < L + l_2 \, , \tag{4}$$

where there is no conductor. In addition, similar conditions must be satisfied on the inner walls of the slots:

$$(1-\nu)(u) + \nu(\partial u/\partial y) = 0 \quad \text{for} \quad y = -L \pm l_1 \text{ or } \; y = L \pm l_2, \text{ when } -d \le x \le d. \tag{5}$$

We will solve the problem using the eigenmodes method. Let us divide the entire space of field distribution into 5 various homogeneous regions of simple geometry, indicated by numbers in Fig. 1.

Let the incident plane wave have the form:

$$u_0(x, y) = \exp\left\{ ik \left[\alpha_0(x + d) + \beta_0 y \right] \right\} \, , \tag{6}$$

where $\alpha_0 = \cos\vartheta$ and $\beta_0 = \sin\vartheta$ are the parameters of wave propagation along the axes x and y, and ϑ is the angle of wave incidence on the surface of a screen. Then, in region 1 in front of the screen (at $x \leq -d$), we can write the following representation for the field as an expansion in the continuous spectrum of plane wave modes:

$$\begin{aligned} u_1(x,y) &= \{\exp[ik\alpha_0(x+d)] - (-1)^\nu \exp[-ik\alpha_0(x+d)]\}\exp(ik\beta_0 y) \\ &\quad +(-1)^\nu \int_{-\infty}^{+\infty} \alpha^{-\nu} B(\beta)\exp[-ik\alpha(x+d) + ik\beta y]\mathrm{d}\beta, \end{aligned} \tag{7}$$

where $B(\beta)$ are the unknown mode amplitudes. Here, the incident wave (6) and the wave reflected from the screen are separated explicitly, and the diffraction field is written as a Fourier integral over plane waves, where the factor $\exp[-ik\alpha(x+d)]$ is added to each plane-wave component. It describes the propagation of this component in the half-space $x < -d$ and ensures satisfaction of the Helmholtz equation [4, 11], if we set

$$\alpha = \sqrt{1-\beta^2}. \tag{8}$$

In order for the field (7) not to increase with distance from the screen, it is necessary to choose a branch of the square root (8) with a non-negative imaginary part.

In the regions behind the screen, the fields are also represented as Fourier integrals with respect to the tangential propagation parameter β. For all three areas, these fields can be written in the form of a general formula:

$$u_m(x,y) = \int_{-\infty}^{+\infty} \frac{f_m(\beta,x)}{D(\beta)} A(\beta)\, \exp\left(ik\beta y\right) \mathrm{d}\beta\ , \tag{9}$$

where m is the index (number) of the region of field propagation ($m = 3$; 4; 5);

$$\begin{aligned} f_3(\beta,x) &= \alpha^{-\nu}\left[D_0(\beta)e^{ik\alpha(x-d)} + R(\beta)e^{ik\alpha(2H+d-x)}\right]; \\ f_4(\beta,x) &= \alpha^{-\nu}T_{34}(\beta)\left[e^{ik\gamma(x-H-d)} + R_{45}(\beta)e^{ik\gamma(2h+H+d-x)}\right]e^{ik\alpha H}; \\ f_5(\beta,x) &= \alpha^{-\nu}T_{34}(\beta)T_{45}(\beta)e^{ik\gamma h}\exp\left[ik\gamma_s(x-h-d)\right]; \\ \gamma &= \sqrt{\varepsilon-\beta^2}; \quad \gamma_s = \sqrt{\varepsilon_s-\beta^2} \end{aligned}$$

are the parameters of normal wave propagation in the dielectric and in the substrate ($\mathrm{Im}\gamma_s \geq 0$), and $A(\beta)$ are the unknown amplitudes of the continuous spectrum modes behind the screen.

Additional amplitude factors in the integrands of (9) are introduced in order to satisfy the conditions (2) at the boundaries of a dielectric layer. If

$$R_{43}(\beta) = \frac{\gamma-\alpha\varepsilon^\nu}{\gamma+\alpha\varepsilon^\nu}; \quad T_{34}(\beta) = \frac{2\alpha}{\gamma+\alpha\varepsilon^\nu}; \quad R_{45}(\beta) = \frac{\gamma\varepsilon_s^\nu-\gamma_s\varepsilon^\nu}{\gamma\varepsilon_s^\nu+\gamma_s\varepsilon^\nu}; \quad T_{45}(\beta) = \frac{2\gamma\varepsilon^\nu}{\gamma\varepsilon_s^\nu+\gamma_s\varepsilon^\nu}; \tag{10}$$

$$D(\beta) = D_0(\beta) + (-1)^\nu R(\beta)e^{2ik\alpha H}; \tag{11}$$

$$D_0(\beta) = 1 - R_{43}(\beta)R_{45}(\beta)e^{2ik\gamma h}; \quad R(\beta) = R_{45}(\beta)e^{2ik\gamma h} - R_{43}(\beta) \tag{12}$$

then these conditions are identically satisfied for each mode of the continuous spectrum, i.e., for each individual component of the integrals (9). Here, the values $R_{ij}(\beta)$ and $T_{ij}(\beta)$ (10) have the meaning of reflection and refraction coefficients of a plane electromagnetic wave with a tangential propagation parameter β on a plane boundary between dielectric media with numbers i and j [4, 9].

In region 2 (inside the screen), one should take into account the presence of modes of two slots at once, inside each of which the field is represented as a discrete Fourier series:

$$\begin{aligned} u_2(x,y) &= \sum_{n=1}^{+\infty}\{\Phi_{(1s)n}(x)\cos[k\xi_{(1\mathrm{s})\mathrm{n}}(y+L)] + i\Phi_{(1\mathrm{a})\mathrm{n}}(x)\sin[k\xi_{(1\mathrm{a})\mathrm{n}}(y+L)]\}\,\theta(l_1^2-(y+L)^2) \\ &\quad + \sum_{n=1}^{+\infty}\{\Phi_{(2s)n}(x)\cos[k\xi_{(2\mathrm{s})\mathrm{n}}(y-L)] + i\Phi_{(2\mathrm{a})\mathrm{n}}(x)\sin[k\xi_{(2\mathrm{a})\mathrm{n}}(y-L)]\}\,\theta(l_2^2-(y-L)^2), \end{aligned} \tag{13}$$

where

$$\Phi_N(x) = \sigma_N^{-\nu}\{a_N \exp[ik\sigma_N(d+x)] + (-1)^\nu b_N \exp[ik\sigma_N(d-x)]\},$$

N is a multiindex $N = (1s)n$; $(1a)n$; $(2s)n$; $(2a)n$, in which indices 1 and 2 denote the belonging of the parameters to one of the two slots; θ is the Heaviside step function [4], which bounds the region of field existence to the interior of two slots $-L-l_1 \le y \le -L+l_1$ and $L-l_2 \le y \le L+l_2$ with the half-widths l_1 and l_2. The propagation parameters of symmetric (index s) and antisymmetric (index a) modes of both slots along the y axis are determined similarly to the case of one slot [6, 9] as follows:

$$\xi_{(\mathrm{1s,2s)n}} = \frac{\pi}{kl_{1,2}}\left(n - \frac{1+\nu}{2}\right); \quad \xi_{(\mathrm{1a,2a)n}} = \frac{\pi}{kl_{1,2}}\left(n - \frac{\nu}{2}\right); \quad \sigma_{(K)n} = \sqrt{1-\xi^2_{(K)n}}, \tag{14}$$

where K is a symbol for the doubled index: $K = 1s$; $2s$; $1a$; $2a$. The first two relations (14) ensure the fulfillment of the boundary conditions (5) on the conducting walls of the slots $y = -L \pm l_1$ and $y = L \pm l_2$, and the last Eq. (14) is the condition for the Helmholtz equation to be satisfied for modes.

Substituting the expressions for the fields (7), (9), and (13) into the boundary Eqs. (3) and (4), we obtain a system of linear equations for the mode amplitudes outside and inside the slots. Using the conditions of modes orthogonality in various regions [9], from this system one can express the amplitudes of the modes on both sides of the screen in terms of the amplitudes of the slot modes:

$$B(\beta) = \frac{kl_1}{2\pi}e^{ik\beta L}\sum_{m=1}^{+\infty}\left[\Phi_{(1s)m}Q_m^{(1s)}(\beta)+\Phi_{(1a)m}Q_m^{(1a)}(\beta)\right]+\frac{kl_2}{2\pi}e^{-ik\beta L}\sum_{m=1}^{+\infty}\left[\Phi_{(2s)m}Q_m^{(2s)}(\beta)+\Phi_{(2a)m}Q_m^{(2a)}(\beta)\right]; \tag{15a}$$

$$A(\beta) = \frac{kl_1}{2\pi}e^{ik\beta L}\sum_{m=1}^{+\infty}\left[\Psi_{(1s)m}Q_m^{(1s)}(\beta)+\Psi_{(1a)m}Q_m^{(1a)}(\beta)\right]+\frac{kl_2}{2\pi}e^{-ik\beta L}\sum_{m=1}^{+\infty}\left[\Psi_{(2s)m}Q_m^{(2s)}(\beta)+\Psi_{(2a)m}Q_m^{(2a)}(\beta)\right], \tag{15b}$$

where the overlap integrals of the modes from various regions $Q_m^{(1s,1a)}$ and $Q_m^{(2s,2a)}$ for each of the two slots are determined in the same way as in the case of an only slot [9]:

$$Q_n^{(K\mathrm{s,Ka})}(\beta) = \frac{2}{l}\int_0^l \left\{\begin{array}{l}\cos\left(k\xi_{(K\mathrm{s)n}}\right)\cos\left(k\beta y\right)\\ \sin\left(k\xi_{(K\mathrm{a)n}}\right)\sin\left(k\beta y\right)\end{array}\right\}\mathrm{dy} = \frac{2\beta^v\xi^{1-\mathrm{v}}_{(\mathrm{Ks,Ka)n}}}{\beta+\xi_{(\mathrm{Ks,Ka)n}}}\cdot\frac{\sin\left[kl(\beta-\xi_{(K\mathrm{s,Ka)n}})\right]}{kl(\beta-\xi_{(K\mathrm{s,Ka)n}})},$$

$K = 1$ or $K = 2$. This provides the opportunity to simplify our diffraction problem and reduce it to solving the transformed system of boundary equations (4) with respect to the amplitudes of slot modes:

$$\sum_{m=1}^{+\infty}\left[\Phi_{(1s)m}V_{nm}^{(1s)}+\Phi_{(2s)m}\bar{V}_{nm}^{(1s2s)}-i\Phi_{(2a)m}\bar{V}_{nm}^{(1s2a)}\right]+\zeta_{(s)n}\bar{\Phi}_{(1s)n} = 2\alpha_0^{1-\nu}Q_n^{(1s)}(\beta_0)\exp(-ik\beta_0 L) \tag{16a}$$

$$\sum_{m=1}^{+\infty}\left[\Phi_{(1a)m}V_{nm}^{(1a)}-i\Phi_{(2s)m}\bar{V}_{nm}^{(1a2s)}+\Phi_{(2a)m}\bar{V}_{nm}^{(1a2a)}\right]+\zeta_{(a)n}\bar{\Phi}_{(1a)n} = 2\alpha_0^{1-\nu}Q_n^{(1a)}(\beta_0)\exp(-ik\beta_0 L) \tag{16b}$$

$$\sum_{m=1}^{+\infty}\left[\Phi_{(1s)m}\bar{V}_{nm}^{(2s1s)}+i\Phi_{(1a)m}\bar{V}_{nm}^{(2s1a)}+\Phi_{(2s)m}V_{nm}^{(2s)}\right]+\zeta_{(s)n}\bar{\Phi}_{(2s)n} = 2\alpha_0^{1-\nu}Q_n^{(2s)}(\beta_0)\exp(ik\beta_0 L) \tag{16c}$$

$$\sum_{m=1}^{+\infty}\left[i\Phi_{(1s)m}\bar{V}_{nm}^{(2a1s)}+\Phi_{(1a)m}\bar{V}_{nm}^{(2a1a)}+\Phi_{(2a)m}V_{nm}^{(2a)}\right]+\zeta_{(a)n}\bar{\Phi}_{(2a)n} = 2\alpha_0^{1-\nu}Q_n^{(2a)}(\beta_0)\exp(ik\beta_0 L) \tag{16d}$$

$$\sum_{m=1}^{+\infty}\left[\Psi_{(1s)m}W_{nm}^{(1s)}+\Psi_{(2s)m}\bar{W}_{nm}^{(1s2s)}-i\Psi_{(2a)m}\bar{W}_{nm}^{(1s2a)}\right]-\zeta_{(s)n}\bar{\Psi}_{(1s)n} = 0 \tag{16e}$$

$$\sum_{m=1}^{+\infty}\left[\Psi_{(1a)m}W_{nm}^{(1a)}-i\Psi_{(2s)m}\bar{W}_{nm}^{(1a2s)}+\Psi_{(2a)m}\bar{W}_{nm}^{(1a2a)}\right]-\zeta_{(a)n}\bar{\Psi}_{(1a)n} = 0 \tag{16f}$$

$$\sum_{m=1}^{+\infty}\left[\Psi_{(1s)m}\bar{W}_{nm}^{(2s1s)}+i\Psi_{(1a)m}\bar{W}_{nm}^{(2s1a)}+\Psi_{(2s)m}W_{nm}^{(2s)}\right]-\zeta_{(s)n}\bar{\Psi}_{(2s)n} = 0 \tag{16g}$$

$$\sum_{m=1}^{+\infty}\left[i\Psi_{(1s)m}\bar{W}_{nm}^{(2a1s)}+\Psi_{(1a)m}\bar{W}_{nm}^{(2a1a)}+\Psi_{(2a)m}W_{nm}^{(2a)}\right]-\zeta_{(a)n}\bar{\Psi}_{(2a)n} = 0 \tag{16h}$$

where $\zeta_{(s)n} = 1 + \nu\delta_{1n}$; $\zeta_{(a)n} = 1$, δ_{1n} is the Kroneker's symbol ($\delta_{1n} = 1$ at $n = 1$ and $\delta_{1n} = 0$ at $n \neq 1$). The amplitudes of slot modes $a_{(1s,1a)m}$, $a_{(2s,2a)m}$ and $b_{(1s,1a)m}$, $b_{(2s,2a)m}$ enter Eqs. (15) and (16) through linear combinations

$$\Phi_M = a_M + b_M \exp(2ik\sigma_M d)\,; \quad \Psi_M = a_M \exp(2ik\sigma_M d) + b_M\,;$$
$$\bar{\Phi}_N = \sigma_N^{1-2\nu}\,[a_N - b_N \exp(2ik\sigma_N d)]\,; \quad \bar{\Psi}_N = \sigma_N^{1-2\nu}[a_N \exp(2ik\sigma_N d) - b_N]\,,$$

where M and N are multiindexes: $M = (1s)m$; $(1a)m$; $(2s)m$; $(2a)m$; $N = (1s)n$; $(1a)n$; $(2s)n$; $(2a)n$, and the amplitude-independent coefficients of system (16) have the form

$$\left\{\begin{matrix} V_{nm}^{(1s),(1a)} \\ W_{nm}^{(1s),(1a)} \end{matrix}\right\} = \frac{kl_1}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} Q_n^{(1s),(1a)}(\beta)\, Q_m^{(1s),(1a)}(\beta)\,\mathrm{d}\beta\,; \tag{17a}$$

$$\left\{\begin{matrix} V_{nm}^{(2s),(2a)} \\ W_{nm}^{(2s),(2a)} \end{matrix}\right\} = \frac{kl_2}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} Q_n^{(2s),(2a)}(\beta)\, Q_m^{(2s),(2a)}(\beta)\,\mathrm{d}\beta\,; \tag{17b}$$

$$\left\{\begin{matrix} \bar{V}_{nm}^{(1s2s),(1a2a)} \\ \bar{W}_{nm}^{(1s2s),(1a2a)} \end{matrix}\right\} = \frac{kl_2}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} \cos(2k\beta L)\, Q_n^{(1s),(1a)}(\beta)\, Q_m^{(2s),(2a)}(\beta)\,\mathrm{d}\beta\,; \tag{18a}$$

$$\left\{\begin{matrix} \bar{V}_{nm}^{(1s2a),(1a2s)} \\ \bar{W}_{nm}^{(1s2a),(1a2s)} \end{matrix}\right\} = \frac{kl_2}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} \sin(2k\beta L)\, Q_n^{(1s),(1a)}(\beta)\, Q_m^{(2a),(2s)}(\beta)\,\mathrm{d}\beta\,; \tag{18b}$$

$$\left\{\begin{matrix} \bar{V}_{nm}^{(2s1s),(2a1a)} \\ \bar{W}_{nm}^{(2s1s),(2a1a)} \end{matrix}\right\} = \frac{kl_1}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} \cos(2k\beta L)\, Q_n^{(2s),(2a)}(\beta)\, Q_m^{(1s),(1a)}(\beta)\,\mathrm{d}\beta\,; \tag{18c}$$

$$\left\{\begin{matrix} \bar{V}_{nm}^{(2s1a),(2a1s)} \\ \bar{W}_{nm}^{(2s1a),(2a1s)} \end{matrix}\right\} = \frac{kl_1}{\pi}\int_{-\infty}^{+\infty} C(\beta)\,\alpha^{1-2\nu} \sin(2k\beta L)\, Q_n^{(2s),(2a)}(\beta)\, Q_m^{(1a),(1s)}(\beta)\,\mathrm{d}\beta\,, \tag{18d}$$

where $C(\beta) = 1$ for the upper coefficients, denoted by the letter V and $C(\beta) = \bar{D}(\beta)/D(\beta)$ for the lower coefficients W, $\bar{D}(\beta) = D_0(\beta) - (-1)^\nu R(\beta)\exp(2ik\alpha H)$; D_0 and R are functions (12).

The additive regularization of matrix integrals (17), (18) and field integrals (9) near the complex zeros of the function $D(\beta)$ (11) is carried out in exactly the same way as in the case of an only slot in the screen [9]. The spatial structure of the diffraction field by two slots is also calculated using Eqs. (1), (7), (9), (13) by analogy with this simpler case.

3. SPATIAL PATTERN OF THE DIFFRACTION FIELD

The diffraction field of two slots is not a superposition of the diffraction fields from each slot separately, because these fields are determined independently of each other, and when being superimposed, each of them will violate conditions (4) at the boundaries of the other slot. In order to satisfy these conditions simultaneously for both slots, the terms with coefficients (18) appear in Eqs. (16), which describe the mutual matching of fields inside the slots during diffraction. However, calculations show that in the absence of such terms, when all coefficients (18) are equal to zero, the diffraction field will differ little from the field described by the rigorous solution with nonzero coefficients (18). The mutual influence of diffraction fields is more or less noticeable (with a relative difference of less than 5%) only in the space between the slots and at very small distances from the screen (up to 0.1λ–0.2λ). In other regions of the space behind the screen, this effect can be neglected, and the resulting diffraction field can be considered as a simple superposition of independent diffraction fields from two slots.

Let us consider the spatial structure of the slot images, i.e., light diffraction spots, in a thin dielectric film ($d + H \leq x \leq d + H + h$), which is located behind the conducting screen on a thick substrate (Fig. 1) and operates as an optical radiation detector, an object under spectroscopic study, or an exposed photoresist in lithography. Calculations show that the diffraction field varies slightly over the depth of the dielectric, so in this direction it can be averaged. As a result, one can consider the relative electric energy density in that as a function of only one tangential coordinate y [10]:

$$W(y) = \varepsilon \frac{1}{W_0 h} \int_{d+H}^{d+H+h} |E(x,y)|^2 \mathrm{dx} \tag{19}$$

where W_0 is the electric energy density of the incident plane wave; $\varepsilon = n^2$ is the dielectric constant of the film; n is its refractive index; $E(x,y)$ is the electric field value inside the film, calculated by Eqs. (1) and (9). We shall consider the conditions of the best resolution of two images from different slots and the conditions for their optimal focusing in the smallest area on the film, which are of the greatest interest for local optical spectroscopy and lithography. As it turned out, for a theoretical estimation of these conditions, it is sufficient to use one scalar parameter, namely the index of the relative decrease in the diffraction slot image, or, in short, the diffraction focusing parameter F [10], which is determined according to the formula

$$F = (l_1 + l_2)\, W_3 / (W_1 W_2)\ , \tag{20}$$

where

$$W_m = \int_{-\infty}^{+\infty} W^m(y) \mathrm{dy}\ , \quad m = 1;\ 2;\ 3$$

is the integral over the dielectric layer surface of the electric energy density of the field (19) to power m. However, here, in contrast to the case of an only slot, for the focusing parameter (20) one obtains the value averaged over two slots.

The quality of the diffraction image depends on a large number of geometric parameters: the half-thickness of a perfectly conducting screen d and the thickness of the dielectric film h, the distance between the screen and the film H, the half-widths of the slots l_1 and l_2, and also on the distance between them $\Delta = 2L - l_1 - l_2$. Therefore, let us consider a simplified case of diffraction of a normally incident plane electromagnetic wave on two identical slots of equal half-width $l_1 = l_2 \equiv l$, when the dielectric film and substrate are transparent with constant refractive indices $n = 1.60$ and $n_s = 1.46$, respectively. With such initial data, the diffraction problem turns out to be completely symmetrical in the tangential coordinate y (Fig. 1), and consequently the focusing parameter F will be the same for both slots under consideration. Fig. 2 shows the results of calculations of this parameter for radiation of two various polarizations. It can be seen that for a system of two slots, as well as for one, the H-polarized radiation can be focused much more efficiently than the E-polarized one, and the larger values of the focusing parameter can be achieved for it.

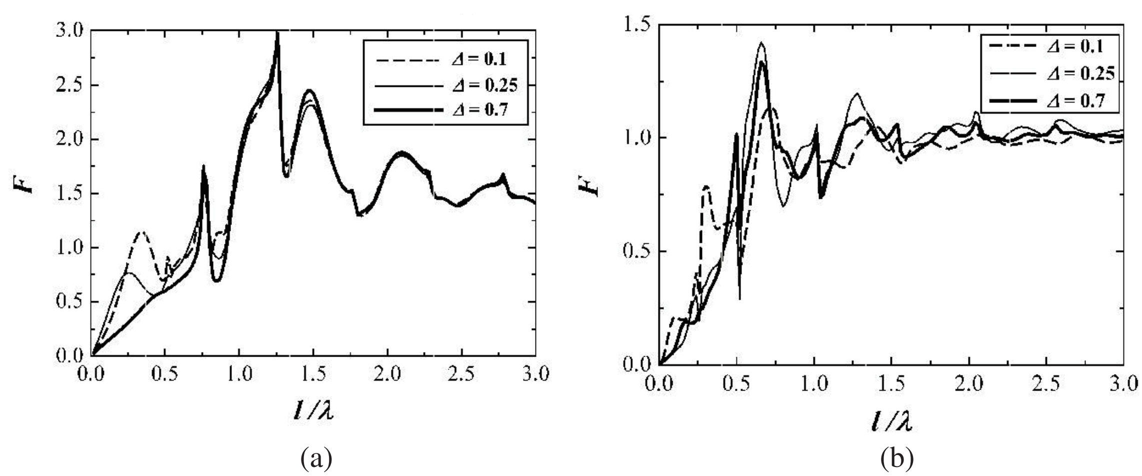
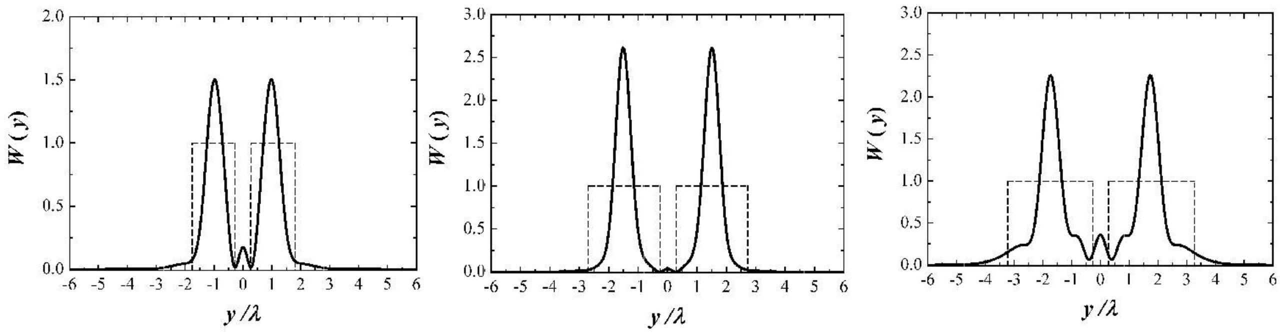
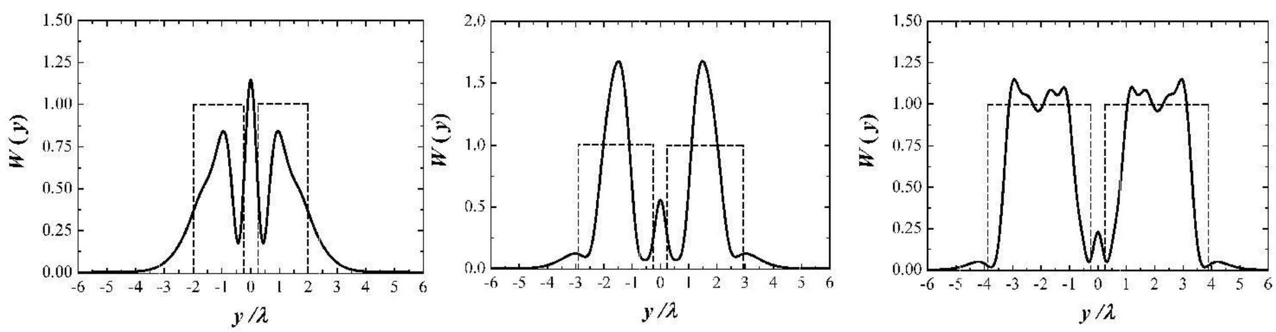


Figure 2. The dependence of the focusing parameter F of two identical slots on their half-width $l_1 = l_2 \equiv l$ at the diffraction of a plane wave of (a) H polarization and (b) E polarization for three various values of the distance between the edges of the slots $\Delta = 2(L-l)$, when the screen half-thickness $d = 0.75\lambda$, the dielectric film thickness $h = 1.2\lambda$, and the distance between screen and film $H = 0.6\lambda$.



(a) (b) (c)

Figure 3. Distribution of the intensity of diffraction radiation of H polarization from two identical slots with the half-width $l_1 = l_2 \equiv l =$ (a) 0.76λ; (b) 1.26λ; (c) 1.48λ in the dielectric film with the thickness $h = 1.2\lambda$, which is located at the distance $H = 0.6\lambda$ from a conducting screen with slots, at the same thickness of the screen $2d = 1.5\lambda$ and the same distance between the edges of the slots $\Delta = 2(L - l) = 0.5\lambda$. The dotted line shows the projections of two slot apertures onto the film.



(a) (b) (c)

Figure 4. The same as for Fig. 3, but for the slot half-width $l_1 = l_2 \equiv l =$ (a) 0.86λ; (b) 1.32λ; (c) 1.8λ.

Figure 3 shows spatial distributions of the electric energy of the H-polarized diffraction field in a dielectric film for three various maxima of this parameter, and Fig. 4 shows it for three various minima. A comparison of these two figures confirms the fact that the focusing parameter F, calculated by formula (20), makes it possible to characterize the focusing properties of a diffraction system with any number of slots, as well as to estimate the diffraction resolution of such a system. Indeed, it can be seen from Fig. 4 that, at relatively small values of the focusing parameter, a noticeable concentration of energy is observed in the area between the slots in the form of additional field maxima. Naturally, their presence nullifies the spatial resolution of the slot images. On the contrary, at large values of the focusing parameter (Fig. 3), when $F > 2$, almost all the field energy is rather uniformly concentrated in small areas directly behind the slots, and the magnitude of the interference field between them is minimal. However, such a pattern is realized only for the H-polarized radiation, while for E polarization, even at the maximum values of the focusing parameter ($F < 1.5$), one observes significant distortions of the uniform profile of the diffraction images of the slots and, at the same time, a noticeable illumination of the area between the slots (Fig. 5). The comparison of Figs. 3 and 4, as well as Fig. 2, clearly demonstrates one more feature of small slot diffraction: a very strong dependence of the focusing properties of slot apertures on their width $2l$, when a change in the half-width of slots by only 0.1λ–0.2λ can lead to a sharp change in the quality of the diffraction image.

It should be borne in mind that our completely rigorous theory has a limited field of application [10], since for short-wave electromagnetic radiation, starting from the near infrared region, real metal screens cannot be considered as perfectly conducting [4, 12]. For visible light, one must employ more suitable

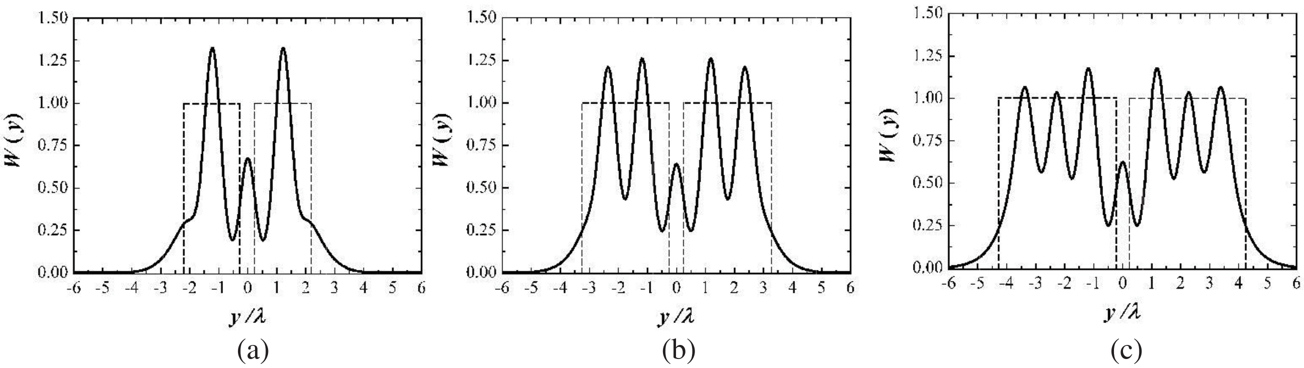


Figure 5. Distribution of the intensity of diffraction radiation of E polarization from two identical slots with the half-width $l_1 = l_2 \equiv l =$ (a) 0.96λ; (b) 1.5λ; (c) 2.0λ in the dielectric film with the thickness $h = 1.2\lambda$, which is located at the distance $H = 0.6\lambda$ from a conducting screen with slots, at the same thickness of the screen $2d = 1.5\lambda$ and the same distance between the edges of the slots $\Delta = 2(L - l) = 0.5\lambda$.

diffraction model with the screen having finite conductivity, but we do not know any works, where such models would be considered.

Unfortunately, we are also unaware of works that could serve as experimental confirmation of the existence of the effect of lensless aperture focusing. At one time, Nye and Liang carried out a number of experiments on a thorough study of the amplitude-phase pattern of the microwave radiation fields of diffraction by one and two slots (see, for example, [13]), but these studies were performed for slots with a width smaller than the wavelength.

4. CONCLUSION

Thus, we have shown that the scalar focusing parameter F (20), introduced in [10] in order to characterize the diffraction image of a single slot, can be successfully used to select the optimal conditions for estimating images from several small slots, despite the explicit dependence of the quality of such images on set of parameters of the diffraction system. Of course, in practice, the choice of many of these geometric parameters will be extremely limited, if not completely excluded. For example, in optical lithography systems, the choice of slot width $2l$ (aperture size), the distance between them Δ, and the thickness of the dielectric film h is dictated to some extent by technological conditions and third-party requirements, but the thickness of the conducting screen $2d$ and the distance H between it and the sensitive film can be varied within certain limits. That is, the examples shown above do not exhaust all the variety of applications of the proposed double-slot diffraction model and are purely demonstrative. Nevertheless, already on the basis of these examples, we can formulate some general conclusions.

First of all, the total diffraction field of two slots can be calculated with high accuracy as the interference field of two independent diffraction fields from each of the two slots separately. Further, to obtain maximum focusing and the highest resolution of diffraction images of slots, it is better to use the H-polarized exposure radiation, whose electric vector is parallel to their boundaries. Here, such a diffraction system is optimal, in which all geometric parameters turn out to be of the order of the radiation wavelength. In particular, the slot width $2l$ should be chosen on the order of 2.2λ–3.2λ. For more narrow slots, the divergence of diffraction radiation sharply increases with all the ensuing consequences, and for wider slots, wave-like interference distortions of the image profile gradually increase. Finally, despite the large number of parameters, which affect the quality of diffraction images, the optimal values of these parameters can be selected using just one scalar value, the focusing parameter F (20). The greater its value is in magnitude, the more effective focusing of images is and the better their mutual resolution is. Our analysis confirms that the maxima of this parameter, depending on the characteristics of the diffraction system of two slots, coincide or almost coincide with the maxima of the focusing parameter for an only slot.

ACKNOWLEDGMENT

The work is supported by the State Program of Scientific Research of the Republic of Belarus "Convergence–2025" (project number 3.02.7) within the framework of the subprogram 11.3 "Interdisciplinary investigations and new emerging technologies".

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