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Optimisation of Directed Energy Systems' Positions Subject to Uncertainty in Operations

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Abstract—Directed energy weapons (DEWs) have been identified as valuable assets in future land and joint combat. High-power radio frequency (HPRF) is a form of DEW which can neutralise robotic systems by discharging electromagnetic (EM) radiation over a region to couple system electronics. Its widespread effect enables the simultaneous disruption of groups of electronic systems, such as swarms of unmanned aerial systems (UASs). Since EM radiation is a distance-based effect, the arrangement of defensive HPRF systems with respect to their target is critical to understanding their utility and viability.

Consequently, a mathematical model to assess the effectiveness of HPRF DEW positioned at a given location is formulated. Towards this, a combat scenario specialised to land operations is defined. The assumptions required to formulate the scenario geometrically and mathematically are also outlined. Provided with the position of an effector, it is then possible to quantify the vulnerability of a UAS swarm in terms of a disruption probability. This accounts for uncertainty stemming from UAS and swarm behaviour and assumes that UASs are independent and identically distributed. The model also draws upon the work previously conducted at Defence Science Technology Group (DSTG) which derived an HPRF disruption probability function.

An optimisation of the disruption probability is undertaken in terms of the position of a single narrowband HPRF effector. Under a hypothesised set of HPRF and threat parameters, maximal swarm defeat probabilities are examined in different swarm deployment regions and HPRF beam widths. This led to the discovery of various tradeoffs between aforementioned features. In particular, under a fixed beam width, proximity to the swam provided an increased defeat probability but reduced the beam's coverage of the swarm. Hence, numerous UASs might not be affected by EM radiation throughout the engagement, reflected in a reduction to the swarm defeat probability.

1. INTRODUCTION

Various international programs investigating directed energy weapons (DEWs) have recognised the technologies' potential to support defence systems. Both high energy laser (HEL) and high power radio frequency (HPRF) DEWs provide defence solutions through different means. HEL DEWs deliver thermal hard-kill effects to a single target by concentrating a beam of electromagnetic (EM) radiation on its surface. Alternatively, HPRF DEWs propagate EM effects over a region with the ability to impact multiple targets simultaneously. Disruption occurs through the coupling of electronics, where emitted HPRF radiation is absorbed through antennas or electronic components [1]. HPRF technologies are often classed in terms of their frequency distribution. Intuitively, wideband refers to systems which deliver EM radiation over a broad range of frequencies, while narrowband system's frequencies are more concentrated.

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The capabilities of HPRF DEWs offer an asymmetric countermeasure to groups of non-kinetic threats. Given a rapidly increasing interest in swarming robotics [2], HPRF systems might provide an unprecedented level of utility. Military applications range from Icarus Swarm's radioactivity sensing swarm [3] to DARPA's OFFensive Swarm-Enabled Tactics (OFFSET) program comprising upwards of 250 weaponised unmanned aerial systems (UASs) and unmanned ground systems (UGSs) [4]. Meanwhile, Epirus has demonstrated the effectiveness of their Leonidas high-power microwave (HPM) counter-UAS (C-UAS) system in disrupting unmanned aerial system (UAS) swarms [5].

Should the Australian Defence Force (ADF) express an interest in this technology, then an understanding of their utility and how to maximise it should be obtained. In particular, decision makers need to understand how HPRF effectors should be positioned in a defensive context. This will ultimately determine how HPRF DEWs and complementary technologies are integrated over the immediate future.

Towards this objective, [6] formulated and applied an HPRF disruption probability function to a stochastic performance prediction model. The research presented in the following sections adopts the HPRF disruption probability model to optimise the effectiveness of an HPRF DEW against a swarm of UASs with respect to the effector's position. In doing so, relationships between swarm behaviours, HPRF features and the disruption probability are also distinguished.

The article is structured as follows. Section 2 outlines assumptions around the underlying combat scenario and subsequently constructs the problem from a mathematical perspective. Next, Section 3 derives an expression for the probability of disrupting the swarm. Further development yields an objective function which captures uncertainty around various components of combat. This is applied in Section 4 which examines the optimal effector positions and relationships between HPRF parameters, swarm behaviour, and the disruption probability.

2. PROBLEM FORMULATION

In this Section, the physical scenario for which the mathematical model will be based upon is described. Suppose that a swarm of UASs are simultaneously deployed by Red Force some distance from a Blue Force target. The purpose of the swarm is to disable the Blue target using their weaponised payload. Hence, the passive target is defended by a single HPRF DEW which is stationed within the battle space. Suppose that the engagement ranges are sufficiently small such that a supporting detection system will invariably detect the swarm upon deployment. This is facilitated by an open environment with unimpeded sight lines. It is assumed that the UASs are dispersed over a small region which the narrowband HPRF effector tracks over time. Suppose that the effector is stationary but is able to slew at a rate allowing the DEW's beam to remain centralised on the swarm throughout the engagement. Depending on the width of the HPRF beam, the entire swarm may not be affected by EM radiation at all times. If at least one UAS is not disrupted before reaching the Blue target, then the Red Force operation is deemed successful. For mathematical simplicity, it is also assumed that a two-dimensional perspective of the combat scenario is sufficiently accurate to provide valuable insights.

Under these assumptions, the combat scenario can be depicted geometrically as illustrated in Figure 1. The combat area is restricted to a circular area of radius r_e. The Red Force swarm is composed of n_s individual drones which travel towards B_{target} positioned at the origin. The swarm's position is defined in terms of its centre denoted in polar form by $(r_e - v_s t, \theta_s)$ with respect to B_{target}. Here, v_s is the velocity of all UASs in the swarm; θ_s is the swarm deployment angle; and t simply denotes the current time. Hence, at time 0, the swarm centre lies on the edge of the combat area. UASs are distributed uniformly over a circular area with radius r_s, around the swarm centre. Let the position of an individual UAS be denoted by (d_u, θ_u) with respect to the swarm centre, where $d_u \in [0, r_s]$. Hence, a UAS can reach B_{target} at time $(r_e - r_s)/v_s$ which denotes the engagement end time, T. The Blue Force HPRF DEW, B_1, protecting B_{target} propagates effects within a beam spanning an angle of ω. Its position is given by (d_1, θ_1) with respect to the origin.

Figure 1 provides an example of how the combat scenario might evolve. However, the position at which the swarm is deployed and how its drones are arranged are unknown. As a result, these sources of uncertainty are captured within the disruption probability function. This is demonstrated in the following section which derives the objective function.

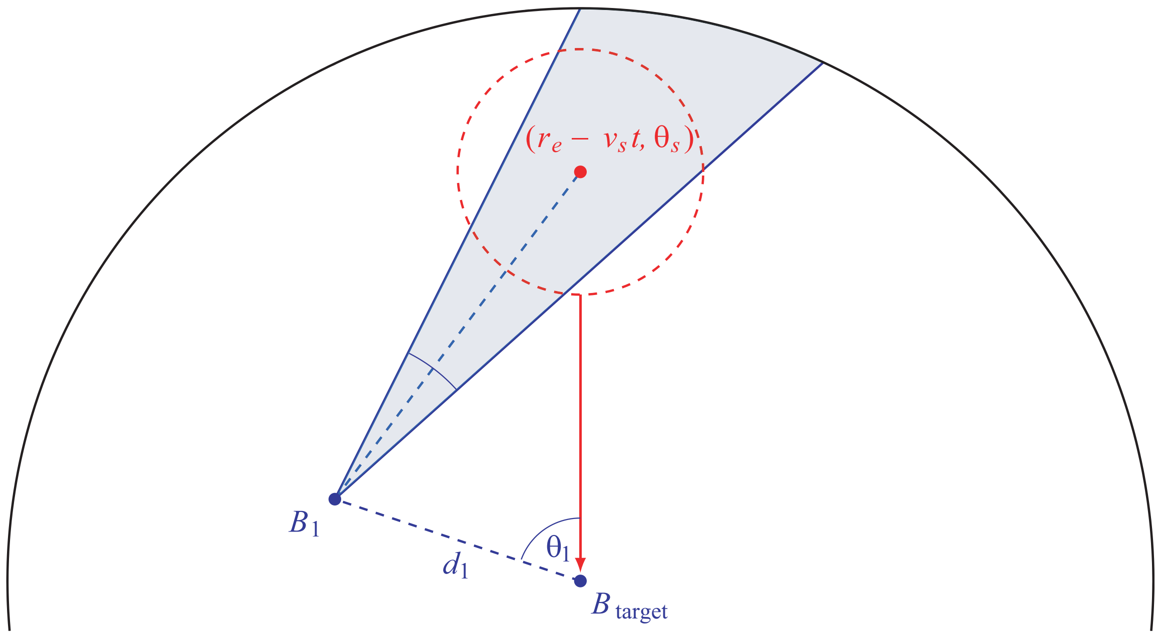

Figure 1. Geometry of the HPRF defence scenario where directed energy effects are confined to a beam which covers an angle of ω. The position of B_1 is described in polar coordinates by (d_1, θ_1) while the swarm centre is given by $(r_e - v_s t, \theta_s)$.

3. OBJECTIVE FUNCTION

The focus of the optimisation problem is the objective function which expresses the probability of defeating the swarm before reaching B_{target} given the position of the HPRF effector. In terms of parameters, the polar coordinates describing the effector's position, (d_1, θ_1), are inputs to the function. An expression describing the probability of an HPRF DEW disrupting a threat over a given time period was recently developed by [6]. Here, the power density imposed on the threat by the effector throughout the engagement is given by,

$$I(T) = \int_0^T \frac{P_B A_B A_R}{\lambda^2 D^2(s)}\, ds, \tag{1}$$

where P_B is the HPRF power level (Watts) functioning at a wavelength of λ (m). Variable A_B is the HPRF antenna's effective aperture area (m^2), A_R the effective aperture area of the UAS antenna (m^2), and $D(s)$ the distance separating the UAS and effector at time s (m). It is assumed that A_R can be modelled through an exponential distribution with parameter μ_R, motivated by a Swerling I target model. This construction assumes that the power discharged by the DEW has a cumulative impact on the UAS, evident in the integral with respect to time. This is applicable to a narrowband HPRF DEW since electromagnetic waves are continuously irradiated at approximately the same frequency. Hence, it is implicitly assumed that the wavelength at which the DEW operates is damaging to the system. By relating the power density that a threat absorbs to a threshold level, τ, measured in W/m^2, the following probability equation is obtained.

$$\mathbb{P}\left(I(T) > \tau\right) = \exp\left(\frac{\mu_R \tau \lambda^2}{P_B A_B}\left[\int_0^T D^{-2}(s)\, ds\right]^{-1}\right). \tag{2}$$

The distance separating an individual UAS and the HPRF effector is dependent on a number of geometric variables which describe the positions of both forces. Hence, (2) is reinterpreted as,

$$\mathbb{P}\left(I(T) > \tau | d_1, \theta_1, d_u, \theta_u, \theta_s\right) = \exp\left(\frac{\mu_R \tau \lambda^2}{P_B A_B}\left[\int_0^T D^{-2}(s | d_1, \theta_1, d_u, \theta_u, \theta_s)\, ds\right]^{-1}\right). \tag{3}$$

The integral from 0 to T assumes that the Red threat is illuminated across the entire engagement. However, this may not hold for all positions within the swarm as illustrated in Figure 1. Subsequently, the following indicator function is introduced,

$$\mathbf{1}(t|d_1, \theta_1, d_u, \theta_u, \theta_s) = \begin{cases} 1, & \text{if the threat is illuminated at time } t \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

Its boolean status is determined by converting the beam boundaries to parametric functions and checking if the UAS lies between them. This is evaluated at each given UAS position within the swarm and allows (3) to be reformulated as,

$$\mathbb{P}\left(I(T) > \tau | d_1, \ldots, \theta_s\right) = \exp\left(\frac{\mu_R \tau \lambda^2}{P_B A_B}\left[\int_0^T \mathbf{1}(s|d_1, \ldots, \theta_s) D^{-2}(s|d_1, \ldots, \theta_s)\, ds\right]^{-1}\right). \tag{5}$$

It was previously asserted that individual UAS would be distributed uniformly over a circular area around the swarm centre, as illustrated in Figure 1. Given a radius of r_s, the joint probability density of the UAS position is given by,

$$p(d_u, \theta_u) = \frac{d_u}{\pi r_s^2}. \tag{6}$$

By applying the law of total probability to (5) and integrating with respect to d_u and θ_u, the defeat probability of an individual threat within the swarm is acquired.

$$\mathbb{P}\left(I(T) > \tau | d_1, \theta_1, \theta_s\right) = \frac{1}{\pi r_s^2}\int_{-\pi}^{\pi}\int_0^{r_s} \mathbb{P}\left(I(T) > \tau | d_1, \theta_1, d_u, \theta_u, \theta_s\right) d_u\, dd_u\, d\theta_u. \tag{7}$$

If all UASs are independently and identically distributed over the swarm, then the probability that all n_s threats are defeated by the HPRF DEW is given by,

$$\mathbb{P}\left(I_n(T) > \tau | d_1, \theta_1, \theta_s\right) = \mathbb{P}\left(I(T) > \tau | d_1, \theta_1, \theta_s\right)^{n_s} \tag{8}$$

$$= \left[\frac{1}{\pi r_s^2}\int_{-\pi}^{\pi}\int_0^{r_s} \mathbb{P}\left(I(T) > \tau | d_1, \theta_1, d_u, \theta_u, \theta_s\right) d_u\, dd_u\, d\theta_u\right]^{n_s}. \tag{9}$$

Blue Force is uncertain of the angle at which Red Force deploys the swarm. Hence, assume that deployment can occur uniformly over a subset of the engagement boundary centred on the previously assumed trajectory, $\theta_s = 0$. Let the boundary extend 2ϕ radians such that $\theta_s \in [-\phi, \phi]$. Hence, the probability of defeating a swarm of n_u threats deployed in the region $[-\phi, \phi]$ is,

$$\mathbb{P}\left(I_n(T) > \tau | d_1, \theta_1\right) = \frac{1}{2\phi}\int_{-\phi}^{\phi}\left[\frac{1}{\pi r_s^2}\int_{-\pi}^{\pi}\int_0^{r_s} \mathbb{P}\left(I(T) > \tau | d_1, \theta_1, d_u, \theta_u, \theta_s\right) d_u\, dd_u\, d\theta_u\right]^{n_u} d\theta_s \tag{10}$$

Alternatively, a minimisation of the probability at least one threat that remains undefeated can be achieved by subtracting the above from 1. Hence,

$$\mathbb{P}\ (\text{at least 1 threat remains})$$

$$= 1 - \mathbb{P}\left(I_n(T) > \tau | d_1, \theta_1\right) \tag{11}$$

$$= 1 - \frac{1}{2\phi}\int_{-\phi}^{\phi}\left[\frac{1}{\pi r_s^2}\int_{-\pi}^{\pi}\int_0^{r_s} \mathbb{P}\left(I(T) > \tau | d_1, \theta_1, d_u, \theta_u, \theta_s\right) d_u\, dd_u\, d\theta_u\right]^{n_u} d\theta_s \tag{12}$$

In order to solve (12), numerical integration must be employed. Analytical means are not feasible due to the intricate geometry involved in calculating $D(s)$, which inherently depends on d_u, θ_u, and θ_s.

4. APPLICATION

Consider the scenario where Red Force deploys a UAS swarm at $r_e = 400\,\text{m}$ due north of B_{target}, such that $\theta_s = 0$. Suppose that the swarm consists of 6 drones which are randomly distributed over a circular area of radius $r_s = 25\,\text{m}$. They approach at a velocity of $v_s = 20\,\text{m/s}$ reaching their desired location at $T = 18.75\,\text{s}$. Supplied with this information, Blue Force can position an HPRF DEW to obtain the greatest probability of defeating all UASs within the swarm. It is assumed that the DEW is powered by $P_B = 100\,\mathrm{kW}$ and operates at a wavelength of $\lambda = 0.23\,\mathrm{m}$. The directive antenna's effective aperture is $A_B = 1\,\mathrm{m}^2$ while that of each UAS is described by an exponential distribution with mean $\mu_R = 0.1\,\mathrm{m}^2$. The disruption threshold of each equates to $\tau = 10\,\mathrm{W/m}^2$. These parameters obtained from open source literature are substituted into (12) to yield the probability that at least one drone is not defeated by time T.

Initially, assume that the HPRF beam width is unconstrained and spans the entire combat area such that (4) returns 1 under all circumstances. As a result, the objective function (12) is convex. This is evident in Figure 2, where the defeat probability function to be maximised (10) is evaluated across the region of possible HPRF effector positions. A local optimisation algorithm is sufficient and avoids more computationally intensive global approaches. The Nelder-Mead approach is adopted which iteratively generates a sequence of simplices to approximate an extremum [7]. Under this algorithm, there is also no requirement to evaluate the derivative. No boundaries or constraints are implemented, but a starting point of $(d_1 = 100, \theta_1 = 0)$ is selected.

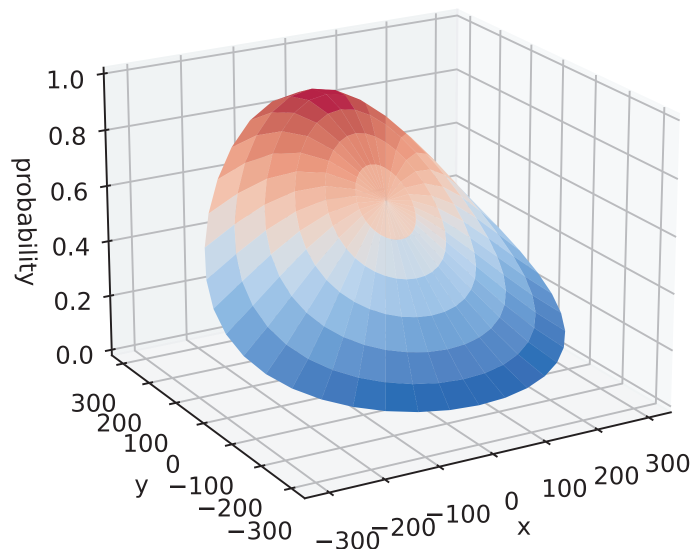

Figure 2. Objective function which illustrates the probability of defeating the swarm deployed from $(x = 0, y = 400)$ when the HPRF effector is located at the plotted position.

The algorithm identifies a maximum swarm defeat probability of 0.272 which is obtained at an HPRF effector position of $(d_1 = 213.5, \theta_1 = -0.016)$, as documented in Table 1. This global maximum shows strong similarities to the swarm position at the half-way point of the engagement where $t = T/2$. This can be demonstrated in a simple one-dimensional analytical construction. By minimising the squared distance separating the threat and effector, the basis of the power density integral (1), the same optimum is observed.

Now, suppose that the exact swarm deployment location is unknown to Blue Force when positioning the HPRF system. However, Blue expects the swarm to be deployed in $[-\phi = -\pi/4, \phi = \pi/4]$. Assume

Table 1. Optimal HPRF locations and corresponding defeat probabilities for different swarm deployment intervals under the assumption the beam spans across the entire swarm.

Deployment interval width (2ϕ)	HPRF distance (d_1)	HPRF angle (θ_1)	Defeat probability
0	213.5	−0.016	0.272
$\pi/2$	193.4	−0.021	0.136
π	189.5	−0.041	0.070

that the Red Force and HPRF effector operates under the same parameters as the previous case. Additionally, no boundaries or constraints are imposed on the optimisation. The Nelder-Mead algorithm identifies a maximum defeat probability of 0.136 when B_1 is positioned at $(193.4, -0.021)$. Evidently, increased uncertainty around the swarm deployment location has shifted the optimal position closer to B_{target}. This is reinforced by the case where $2\phi = \pi$ which yields a maximum defeat probability of 0.070 provided that the HPRF is located at $(189.5, -0.041)$.

These results imply that optimal performance is not necessarily achieved by positioning the HPRF effector such that it is effective under all swarm deployment locations. This is demonstrated when the swarm is deployed from the boundaries, i.e., $\phi = \pi/2$. From the optimal position, UASs never lie within 150 m of the HPRF effector and thus deliver a 0.002 probability of disrupting all 6 threats. However, the chances of disruption are significantly higher and more influential at points central to the deployment interval.

Suppose that the HPRF beam is finite and centred on the swarm's origin. As a consequence, the objective function is non-convex. Hence, a global optimisation technique is required. Despite being most effective in solving problems with discrete search spaces, simulated annealing offers a global metaheuristic within the continuous domain [8]. This problem specifically benefits from an approximate but high quality solution due to small inconsistencies in numerical integration required to solve 12.

Optimisation is performed under the same parameter values expressed prior. Initially, let the beam cover an angle of $\omega = \pi/6$ radians, equivalent to 30 degrees. When positioned at coordinates $(25.2, -0.45)$, an optimal defeat probability of 0.007 is achieved for a swarm consisting of 6 threats in a deployment interval of size $\pi/2$. If the deployment interval is extended to a size of π, then the optimal HPRF location becomes $(0.15, -2.42)$ which returns a defeat probability of 0.004. For a fixed swarm deployment location, $\theta_s = 0$, the optimal position is $(42.3, -0.07)$ and yields a swarm defeat probability of 0.021. These results are illustrated in Table 2. Evidently, as uncertainty increases, the optimal location of the HPRF effector withdraws to less aggressive positions closer to B_{target}. This behaviour is consistent with the results where directed energy effects were unconfined (Table 1), but now optimums lie in closer proximity to the origin.

Table 2. Optimal HPRF locations and corresponding defeat probabilities for different swarm deployment intervals when HPRF effects are restricted to a beam spanning $\omega = \pi/6$.

Deployment interval width (2ϕ)	Beam width (ω)	HPRF distance (d_1)	HPRF angle (θ_1)	Defeat probability
0	$\pi/6$	42.3	−0.07	0.021
$\pi/2$	$\pi/6$	25.2	−0.45	0.007
π	$\pi/6$	0.15	−2.42	0.004

To understand the impact of different beam widths on the optimal HPRF location, consider ω values of $\pi/8$, $\pi/6$, and $\pi/4$ radians, equivalent to 22.5, 30, and 45 degrees, respectively. Table 3 provides the optimal location and swarm defeat probability for these beam widths under the assumption that the swarm is deployed within an area spanning $\pi/2$. Evidently, increases to beam width cause the optimal

Table 3. Optimal HPRF locations and corresponding defeat probabilities for different HPRF beam widths, ω, while the swarm is assumed to be deployed in the interval $[-\pi/4, \pi/4]$ uniformly.

Deployment interval width (2ϕ)	Beam width (ω)	HPRF distance (d_1)	HPRF angle (θ_1)	Defeat probability
$\pi/2$	$\pi/8$	25.9	−0.32	0.006
$\pi/2$	$\pi/6$	25.2	−0.45	0.007
$\pi/2$	$\pi/4$	24.7	−0.59	0.009
$\pi/2$	2π	193.4	−0.021	0.136

HPRF location to shift marginally closer to B_{target} and simultaneously move further away from $\theta_s = 0$.

The finite beam width appears to introduce a tradeoff around the HPRF effector's proximity to the swarm and the probability of swarm disruption. At close distances, the defeat probability is heightened, but the effector beam occupies a smaller area within the swarm circle. When positioned further away, greater swarm coverage is acquired in return for smaller defeat probabilities. As the beam width increases, its optimal position lies closer to B_{target}, although only marginally.

5. CONCLUSIONS

This study has developed a mathematical framework to calculate the probability of defeating a UAS swarm with a narrowband HPRF DEW subject to various sources of uncertainty. The probability of defeating the entire swarm was subsequently optimised with respect to the position of an HPRF effector within the combat area. Optimal positions and their respective defeat probabilities were obtained under various permutations, which examined the impact of different swarm deployment sites and HPRF beam widths. It became evident that an HPRF effector provides significantly more defensive utility when it is positioned directly along the swarm's trajectory. A tradeoff between the HPRF position and beam width was also observed, such that a beam spanning a smaller area should be more advanced and centralised within the deployment interval.

Current models are restricted to the two-dimensional perspective which is sufficiently accurate for investigating relationships among beam width, optimal location, and deployment intervals. If models were subsequently used to inform the ADF of HPRF DEW utility and their optimal deployment, then this addition will improve its value to decision makers. Additionally, with increased computational power, an exact optimisation algorithm could be implemented, instead of simulated annealing's approximate solution. Covering the combat area with more than one HPRF effector may also be of interest. Analytical efforts towards this goal have yielded functional models when the sources of uncertainty are limited. Various high energy laser (HEL) optimisation models have also been investigated. These share similarities to the HPRF disruption probability, but approach air and missile defence in a different manner.

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