

Swarm UAV Defeat Modelling through Lifetime Distribution Analysis

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Abstract—The problem of defeating a swarm of unmanned aerial vehicles (UAVs) is of considerable importance to the modern warfighter. In recent studies high power radio frequency (HPRF) directed energy weapons (DEWs) have been shown to be suitable for this purpose. Hence there is a need to develop mathematical modelling frameworks to quantify HPRF DEW performance, especially when they are operating in a wideband or ultrawideband mode. Consequently, this paper introduces a novel mathematical model, based upon a new interpretation of UAV vulnerabilities to HPRF DEW, which permits performance assessment to be undertaken. The key to this is to view each UAV through its vulnerabilities to HPRF DEW energy at given frequencies and analyse its impact on the lifetime of each of the UAVs. This results in the definition of an appropriate stochastic process to count the number of UAVs still active in the swarm over a given time interval. Consequently, this permits the determination of minimum HPRF DEW power levels at given frequencies in order to guarantee likelihood of defeat of the swarm before it reaches the HPRF DEW source. Hence the results in this paper will provide a novel framework for determining the specifications of an HPRF DEW's required power distribution over target vulnerabilities to ensure a desired level of system performance.

1. INTRODUCTION

The problem of defeating swarms of unmanned aerial vehicles (UAVs) has emerged as an important topic due to the risks they pose to the modern combat team in an asymmetrically-driven warfighting environment [1–3]. High power radio frequency (HPRF) directed energy weapons (DEWs) have been identified as a suitable countermeasure, since they can provide regional defence against such threats [4]. Such systems couple electromagnetic energy into their targets, and thus can cause a number of system failures [5]. As such, there has been much interest in quantifying the effects of HPRF DEWs when being applied to counter UAV applications [6–8]. Stochastically-driven models for HPRF DEW performance prediction in this context have also been introduced in recent years [9, 10]. Most studies have focused on the consequence of an HPRF DEW operating at a given frequency with a prescribed power level, against a UAV with a single vulnerability threshold at this frequency [11]. The focus in this paper is to instead view a UAV as a series of electromagnetic vulnerabilities and attempt to determine the minimal power requirements in an HPRF DEW source in order to ensure a desired likelihood of threat disruption. This approach will then allow the practical engineer to configure an HPRF DEW so that it distributes its power over these likely vulnerabilities.

Consider the case of an HPRF DEW source operating at a single frequency/wavelength and with a given power level. Specifically, assume that it operates at a wavelength of λ metres (m) with a power level of P_T Watts (W) through a directive antenna with effective aperture area A_T square metres (m²) and that the target has an antenna with an effective aperture area of A_R m². The latter is used to model the surface area on the target likely to be receptive to coupling at the given wavelength. Then

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at a distance of $R(t)$ m at time t seconds (s) from the HPRF DEW location the power density on the threat is given by

$$I(t) = \frac{P_T A_T A_R}{\lambda^2 R^2(t)} = \frac{\kappa}{R^2(t)}, \quad (1)$$

which is measured in Watts per square metre (W/m^2) [10]. The second expression in (1) involving κ has been introduced to separate the time-varying component from the constant element of the power density function.

When the power density on a target exceeds a certain threshold τ it is likely that the target will experience a disruption through coupling of electromagnetic energy. A given HPRF DEW source may be designed to operate over a frequency spectrum, as in a wide-band or ultra-wideband mode, so that it will distribute its power over its characteristic operating frequencies through its underlying waveform. This contrasts to the case where an HPRF DEW source operates in a narrowband mode, dispensing a maximal amount of power in a small bandwidth around its centre frequency.

From the point of view of performance assessment, what would be useful is if one could determine the distribution of HPRF DEW effector power across a target's vulnerabilities, in order to ensure defeat of threats with a prescribed likelihood of success. Hence the purpose of this paper is to introduce a framework in which this may be achieved and illustrate how it may be applied in a specific example. To achieve this it will be useful to produce a mathematical model counting the number of UAVs in a swarm still active over time. The evolution of this model will be shown to be a function of individual UAV lifetimes. Hence it will also be necessary to produce a statistical model for the lifetimes of UAVs as a function of their vulnerabilities. This model development will be the focus of the next section.

2. MODEL FORMULATION

The purpose of this section is to formulate a mathematical model to assess the number of surviving UAVs in the swarm by a given time. Relative to the HPRF DEW system the threat is viewed as a series of vulnerabilities, and the number of surviving threats over time is modelled as a pure death stochastic process. It will be assumed that there is a fixed number of threats N and that the swarm is visible to the HPRF DEW at time $t = 0$. Additionally, it will be supposed that members of the swarm, although spatially distributed, travel at the same speed toward the HPRF DEW; otherwise, the swarm problem may be examined as a single UAV defeat study, as in the approach of [12].

2.1. Jump Process

Suppose that $\phi(t)$ is the number of threats still alive or functional at time t . Hence this process begins in state N and can be reduced to zero if the engagement proceeds indefinitely. The number of threats defeated by time t is therefore given by $N - \phi(t)$. Although the swarm of UAVs is being subjected to the same HPRF DEW source it will be assumed that the defeat times of each threat differ, so that the jump process has single step transitions only.

Let ν_k be the jump time of process $\phi(t)$ from state k to state $k - 1$ ($1 \leq k \leq N$). Hence $0 < \nu_N < \nu_{N-1} < \dots < \nu_2 < \nu_1$. Figure 1 provides an illustration of this process, showing the function $\phi(t)$ plotted over time t until it reaches state zero. Through considerations of the definition of the jump times and with reference to Figure 1 it can be argued that

$$\phi(t) = \sum_{k=1}^N k \mathbb{I}[\nu_{k+1} \leq t < \nu_k] \quad (2)$$

where $\mathbb{I}(x)$ is the indicator function, taking unity if the condition x holds and zero otherwise, and ν_{N+1} is defined as zero. Observe that by taking the mean of (2)

$$\mathbb{E}[\phi(t)] = \sum_{k=1}^N k \mathbb{P}[\nu_{k+1} \leq t < \nu_k] \quad (3)$$

which suggests it will be easier to work with means, where \mathbb{E} denotes statistical expectation, and \mathbb{P} denotes probability. If an expression for (3) can be produced, for a given HPRF DEW, then it can

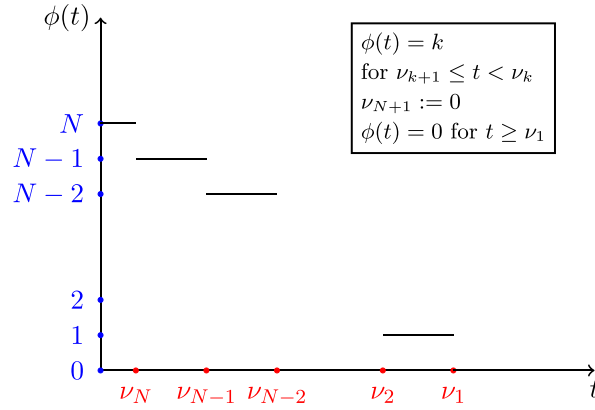


Figure 1. An illustration of the pure death process model for the survivability of the UAVs in the presence of an HPRF DEW. The process starts with N UAVs and gradually reduces to zero, reducing at each jump time ν_j by unity as illustrated.

be used to evaluate the performance of the HPRF DEW system in defeating the swarm of UAVs over time. In particular, this model will permit the determination of minimum power levels at each UAV vulnerability to guarantee a desired level of performance.

2.2. UAV Lifetimes

The relationship between the jump times of $\phi(t)$ and UAV lifetimes is now examined. Suppose that L_1, L_2, \dots, L_N are the lifetimes of each UAV, respectively, where it is assumed that these are independent but not necessarily identically distributed. Then the order statistics $L_{(1)} < L_{(2)} < \dots < L_{(N)}$ enumerate the order in which the swarm members expire. Hence $L_{(1)}$ is the minimum while $L_{(N)}$ is the maximum lifetime. Recall that by definition ν_N is the time the process $\phi(t)$ jumps from state N to $N-1$, through one expired UAV. Hence $\nu_N = \min\{L_1, L_2, \dots, L_N\} = L_{(1)}$. Similarly the time it takes for the process to jump from state N to $N-2$ will be exactly given by the lifetime of the second UAV. Therefore, $\nu_{N-1} = L_{(2)}$. Using similar reasoning one can show that the k th jump time is $\nu_k = L_{(N-k+1)}$, so the time it takes for the process $\phi(t)$ to jump from state k to $k-1$ will be directly determined by the lifetime of the $(N-k+1)$ th individual. Hence it is necessary to determine statistical expressions for the lifetime variables so that the order statistics can be determined, and subsequently the jump times of the process $\phi(t)$.

2.3. Distribution of Lifetimes

Next the relationship between UAV lifetime and power density functions is explored, in order to produce a statistical expression for it. Suppose that the i th UAV has a series of n_i vulnerabilities at a wavelength of $\lambda_{(i;j)}$ and with corresponding disruption threshold $\tau_{(i;j)}$, for $j \in \{1, 2, \dots, n_i\}$. If we let $P_{(i;j)}(t)$ be the power density the HPRF DEW is applied to UAV i at vulnerability j , then the UAV will be disrupted if this exceeds the threshold $\tau_{(i;j)}$. Hence considering all vulnerabilities of UAV i , it will follow that it will be disrupted if at least one threshold is exceeded by the relevant power density function. Equivalently, the UAV's lifetime will exceed time t provided the power density level incident on the target at time t is smaller than the corresponding vulnerability threshold for each of the n_i UAV vulnerabilities. Mathematically it follows that $L_i > t$ is equivalent to $P_{(i;j)}(t) < \tau_{(i;j)}$ for each $j = 1, 2, \dots, n_i$.

If it is assumed that these vulnerabilities are independent, then one can produce an expression for the complementary distribution function of the lifetime, given by

$$\mathbb{P}(L_i > t) = \prod_{j=1}^{n_i} \mathbb{P}(P_{(i;j)}(t) < \tau_{(i;j)}) = \prod_{j=1}^{n_i} \mathbb{P}\left(\frac{\kappa_{(i;j)}}{R_{(i;j)}^2(t)} < \tau_{(i;j)}\right), \quad (4)$$

where $\kappa_{(i;j)}$ is given by

$$\kappa_{(i;j)} = \frac{P_{T_{(i;j)}} A_T A_{R_j}}{\lambda_{(i;j)}^2} \quad (5)$$

and can be interpreted as the time-independent component of the power density function, incident on UAV i and at vulnerability j , for $j \in \{1, 2, \dots, n_i\}$. In expression (5) the term $P_{T_{(i;j)}}$ is the power the HPRF DEW applied at wavelength $\lambda_{(i;j)}$ through a directive antenna with area A_T and to a target with vulnerability surface area A_{R_j} for vulnerability j .

Under the assumption that the elements of $\kappa_{(i;j)}$ are deterministic, but the threshold $\tau_{(i;j)}$ is stochastic, the final expression in (4) may be expressed in terms of the complementary distribution function of $\tau_{(i;j)}$. One potential choice for this is to assume that it has a distribution that is exponential. The rationale for this choice has been based upon the fact that this distribution can be used to model electronic components lifetimes.

Suppose that for UAV i the j th threshold has an exponential distribution with parameter $\mu_{(i;j)}$. Then (4) is reduced to

$$\mathbb{P}(L_i > t) = e^{-\sum_{j=1}^{n_i} \frac{\kappa_{(i;j)} \mu_{(i;j)}}{R_{(i;j)}^2(t)}}. \quad (6)$$

If it is supposed that the threats are a swarm of small-sized UAVs, one may assume that the distance between the HPRF DEW and each vulnerability is roughly the same, especially in view of the comparison between engagement distances (> 300 m) and localised vulnerability distances on the threat (< 5 cm on a commercial-off-the-shelf (COTS) UAV). Thus each $R_{(i;j)}(t) = R_i(t)$ and (6) is reduced to

$$\mathbb{P}(L_i > t) = e^{-\sum_{j=1}^{n_i} \frac{\kappa_{(i;j)} \mu_{(i;j)}}{R_i^2(t)}}. \quad (7)$$

Note that since the time-varying component $R_i(t)$ in the right hand side of (7) is independent of j , it may be removed from the summation. This is useful from the point of view of developing a simpler statistical expression for the complementary distribution function of the lifetime of the i th UAV, as defined in the left hand side of (7). The inverse cumulative distribution function method can be used to simulate L_i from (7) as follows. Suppose that U_i is uniformly distributed on the unit interval. Then in order to simulate L_i one must solve

$$\mathbb{P}(L_i > t) = U_i, \quad (8)$$

and the solution to (8) in terms of t will be a realisation of L_i . By applying (7) to (8) one can show that the solution is given by

$$R_i^2(t) = \frac{\sum_{j=1}^{n_i} \kappa_{(i;j)} \mu_{(i;j)}}{\log(U_i^{-1})}. \quad (9)$$

As remarked earlier it will be assumed that the UAVs travel at the same constant speed, denoted as ν metres per second, and it will also be supposed that the i th UAV will take T_i seconds to reach the HPRF DEW if not disrupted. Hence $R_i(t) = \nu(T_i - t)$ for $0 \leq t \leq T_i$. Applying this to (9) and simplifying one can solve for t and then deduce that L_i may be generated through

$$L_i \stackrel{d}{=} T_i - \frac{1}{\nu} \sqrt{\frac{\sum_{j=1}^{n_i} \kappa_{(i;j)} \mu_{(i;j)}}{\log(U_i^{-1})}}, \quad (10)$$

where $X \stackrel{d}{=} Y$ implies that the two random variables are equivalent from a statistical perspective. For a particular swarm and HPRF DEW system one can then apply (10) to generate lifetime realisations, which can then be used to generate lifetime order statistics and subsequently used to determine realisations of the jump times ν_k . Finally, these may be applied to evaluate (3) for expected performance prediction in terms of number of threats still active by time t . This process is illustrated for the reader in Figure 2.

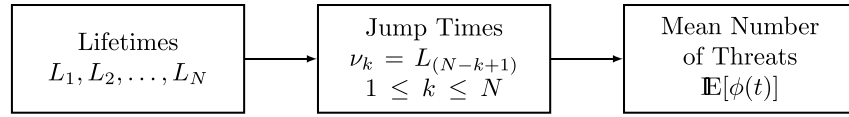


Figure 2. Block diagram illustration of the modelling process, beginning with threat lifetimes and producing the expected number of threats to survive over time.

In the next section the application of these results will be illustrated with a specific example, and it will be shown how insights into HPRF DEW system requirements can be acquired.

3. APPLICATION

Consider the following scenario where Red Force deploys a swarm of UAVs directed towards a ground-based Blue Force base protected by a fixed HPRF DEW designed for regional defence. Suppose that there are $N = 10$ UAVs in total, from two different categories of COTS drones, consisting of 4 and 6 members respectively. Hence in the notation of the previous section i ranges from 1 to 10. Figure 3 provides an illustration of the combat scene. These two sets of UAVs will be referred to as Type 1 and 2 throughout, and their vulnerability characteristics will be discussed subsequently. Both types of UAVs are assumed to be deployed 500 m from the Blue HPRF DEW and travel at a constant speed of 10 m/s directly towards it. Hence, the engagement will conclude at time $t = 50$ s. It is assumed that the swarm appears entirely at time $t = 0$ and is within the HPRF DEW's operational range at this time.

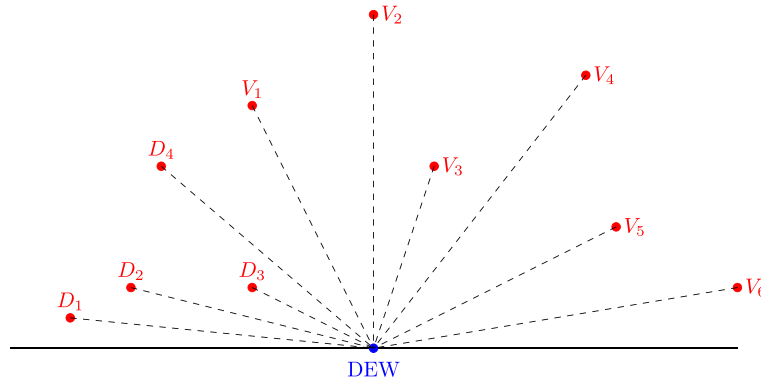


Figure 3. An illustration of the combat scene used in the example. The single HPRF DEW is at the centre of the combat space, viewed as a 2-dimensional surface. The two classes of threats are labelled D_1, D_2, D_3, D_4 and V_1, V_2, \dots, V_6 respectively.

Assume that three distinct vulnerabilities exist for both UAV types: the remote communication device, internal global positioning system (GPS), and microcontroller. Consequently, utilising the notation introduced in the previous section, it follows for each i that j will range from 1 to 3. However, since there are two sets of homogeneous UAVs, each with three similar vulnerabilities, one may relax the notation in the following to facilitate the discussion.

Parameters concerning the vulnerabilities of both UAVs types are detailed in Table 1. It illustrates the frequency at which each vulnerability occurs, namely 2.4, 2.05, and 1.2 GHz, respectively. Corresponding disruption thresholds are modelled by exponential random variables with means of 1000, 175, and 25 W/m² for the first class of UAVs and 1000, 150, and 100 W/m² for the second. These model parameterisations have been motivated by discussions in [7, 13–15]. The inherent target surface area vulnerability (A_R in (1)) is selected arbitrarily ranging from 0.05 m² to 0.1 m². These choices are designed to be consistent with small target coupling cross sections.

The power output at each UAV's vulnerable frequency is dependent on the HPRF DEW waveform and power source. For an ultra-wideband system, only a proportion of the total power is distributed

Table 1. Summary of the vulnerable frequency (Hz), corresponding power level (W), disruption threshold (W/m^2) and surface area vulnerability (m^2) for each vulnerability and UAV given a HPRF DEW operating at 100 kW with the power distribution specified in the preceding discussion.

UAV	Parameter	Remote Comms	GPS	Microcontroller
Type 1	frequency	2.4 GHz	2.05 GHz	1.2 GHz
	power distribution	10 kW	20 kW	35 kW
	disruption threshold	$1 \text{ kW}/\text{m}^2$	$175 \text{ W}/\text{m}^2$	$25 \text{ W}/\text{m}^2$
	surface area vulnerability	0.05 m^2	0.05 m^2	0.1 m^2
Type 2	frequency	2.4 GHz	2.05 GHz	1.2 GHz
	power distribution	10 kW	20 kW	35 kW
	disruption threshold	$1 \text{ kW}/\text{m}^2$	$150 \text{ W}/\text{m}^2$	$100 \text{ W}/\text{m}^2$
	surface area vulnerability	0.1 m^2	0.1 m^2	0.05 m^2

at the vulnerabilities' frequencies, while the remainder will be distributed across non-vulnerable target frequencies. For the purposes of the example considered in this section, arbitrary distributions of power will be utilised. For a real HPRF DEW, if one has knowledge of the underlying waveform, then its power spectral density may be examined to determine the actual power distribution over frequencies of interest.

If the HPRF DEW is powered by an x kW source, suppose that $0.1x$ kW is outputted at 2.4 GHz, $0.2x$ kW at 2.05 GHz and $0.35x$ kW at 1.0 GHz. Throughout the HPRF DEW antenna effective area is assumed to be 1 m^2 . Under the assumption that the DEW has a power level of 100 kW the power distribution above is given, together with other parameters, in Table 1.

Under the described engagement, the effectiveness of the HPRF DEW at any given range is dictated by the power attributed to the vulnerable frequencies. Hence, to understand the minimum power requirements in order to achieve a disruptive effect at a given range, the formulae introduced in the previous section can be utilised. Consider a typical member of both classes of UAVs individually, reducing their lifetimes given by (7), to each of the vulnerabilities. If a disruptive effect is desired at a range of 250 m, then the susceptibility of each vulnerability and the UAV as a whole can be determined. This is illustrated in Figure 4 for both Type 1 (subplot a) and Type 2 (subplot b) UAVs. The power plot along the x -axis is that of the HPRF DEW source. This is then distributed over each of the

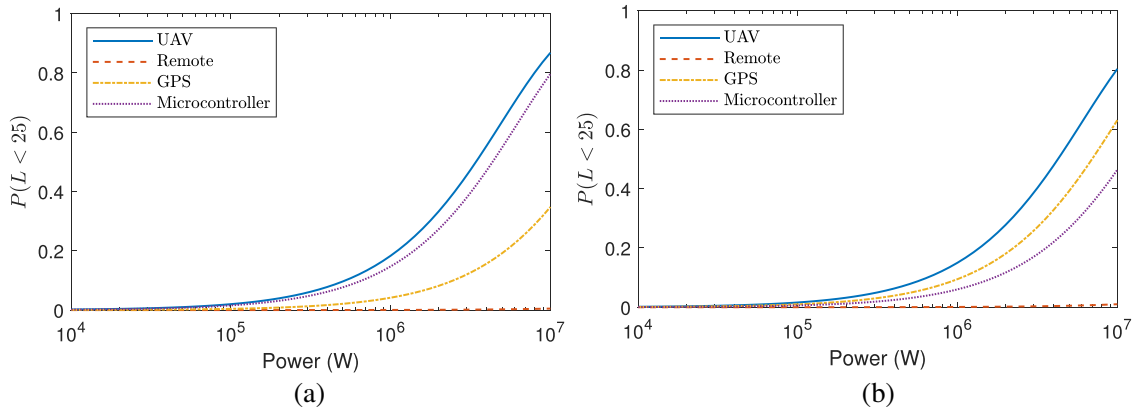


Figure 4. Illustration of the power required to disrupt individual vulnerabilities at their vulnerability wavelength at a distance of 250 m. This is compared to the probability at least one of the vulnerabilities is affected. The power value along the x -axis is that of the source such that a proportion is emitted at each relevant frequency. On the y -axis L refers to lifetime under consideration of all vulnerabilities (denoted UAV) or lifetime when only a single vulnerability is considered.

vulnerability frequencies with the proportions outlined earlier. Evidently, to achieve a 0.8 probability of disruption at a distance of 250 m, a 10 MW power source is required.

To understand the influence of different power distributions at the engagement level, three different distributions of power at vulnerability frequencies are considered. Assuming that the HPRF DEW is powered by an x kW source, distribution 1 assumes that $0.1x$ kW is outputted at 2.4 GHz, $0.2x$ kW at 2.05 GHz, and $0.35x$ kW at 1.0 GHz, consistent with the previously outlined power distribution. Distribution 2 is more heavily concentrated towards smaller frequencies such that $0.05x$ kW are delivered at 2.4 GHz, $0.1x$ kW at 2.05 GHz, and $0.5x$ kW at 1.0 GHz. Alternatively, configuration 3 is biased towards larger frequencies with $0.25x$, $0.25x$, and $0.2x$ kW outputted at 2.4, 2.05, and 1.0 GHz, respectively. This is applied in Figure 5 illustrating the expected number of active UAVs with respect to HPRF DEW's power source. Monte Carlo simulation with 10^5 runs has been used to evaluate (3) at $t = 25$. Evidently, the configuration which concentrates power to smaller frequencies results in the smallest expected number of UAVs at a distance of 250 m. This observation is consistent with the lifetimes pictured in Figure 4 where the microcontroller is highly susceptible in both systems, but overly dominant in Type 1 UAVs.

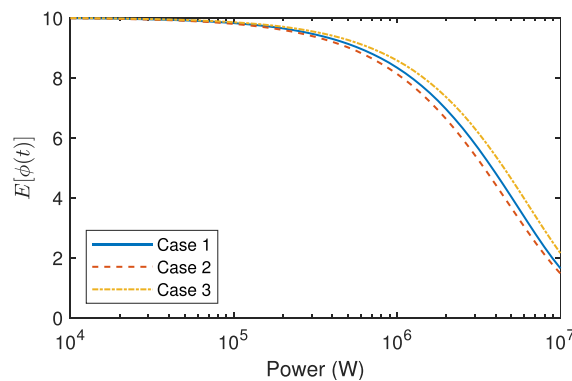


Figure 5. Plot of the expected number of active UAVs under 3 different HPRF DEW power distributions, at $t = 25$.

4. CONCLUSIONS

The purpose of this paper was to introduce a mathematical modelling framework where the performance of an HPRF DEW could be assessed when being applied to counter UAV applications. In particular, it was shown how UAV lifetimes could be expressed in terms of their vulnerability to HPRF DEW at various frequencies, and a stochastic process counting the number of surviving UAVs in a swarm was produced. An example illustrated how these results could be used to determine the minimum HPRF DEW power distribution over various sensitive frequencies in UAVs. These results can be used by the HPRF DEW system designer to determine power levels necessary to ensure threat defeat in practice.

There are a number of avenues for further work and extension of the results introduced in this paper. In the first instance, the exponential model adopted for thresholds needs to be validated with real data. Secondly, there may be more appropriate models for thresholds, which should arise from experimental analysis. Thirdly, it would be of considerable interest to undertake field trials to compare predicted results with real applications of HPRF DEWs applied to various UAV swarms.

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