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Computation of Spectral-Domain Green's Functions of the Infinitesimal Current Source in a Planar Multilayer Medium

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Abstract—This paper presents a novel theoretical and numerical approach for an infinitesimal current source (ICS) located in a planar isotropic multilayer medium. Using the mixed-potential integral equation (MPIE) formulation for depicting the electromagnetic disturbance created by the ICS, a detailed definition of Green's functions of Lorenz potentials and fields is provided in this paper. The proposed Green's functions are valid for the considered multilayer isotropic medium, which can have arbitrary layer parameters. This paper also analyzes two commonly observed special cases of the multilayer medium — the multilayer soil including air and the multilayer lossless dielectric — and the proposed equations are modified to meet the requirements of the medium. Green's functions can be obtained from the systems of linear equations proposed in this study. In comparison to other approaches, the advantage of the proposed procedure is that the solutions of the equations are immediately obtained in any field layer of the multilayer medium. In addition, the proposed system of linear equations can be solved easily using well-known numerical computation methods. Furthermore, this paper offers an alternative way of obtaining Green's functions, which are closed-form expressions for the kernels of spectral-domain Green's functions.

1. INTRODUCTION

An infinitesimal current source (ICS) creates an electromagnetic disturbance in its surroundings, i.e., creates an electric and magnetic field. The accurate distribution of these fields can be determined by solving the full set of Maxwell equations directly with the electric field intensity and magnetic field intensity. However, instead of solving the full set of Maxwell equations for the electric field intensity and magnetic field intensity, it is possible to reduce the number of unknown values using auxiliary functions such as electromagnetic potentials. Therefore, by introducing these auxiliary functions, the order of the differential equations to be solved is increased, and the number of unknown quantities — now the potentials — is decreased. There are several different ways to express these potential functions using various gauge conditions. However, regardless of the gauge condition or the combination of potential functions, the electric field intensity and magnetic field intensity produced by the ICS must remain the same in all cases. This paper considers scalar electric potential (SEP) and vector magnetic potential (VMP), connected with the Lorenz gauge condition, hence the potentials used in this paper can be called Lorenz potentials. The obtained formulation of equations is called mixed-potentials integral equation (MPIE) formulation [1–4]. The MPIE formulation is important in this theoretical approach since VMP is used to define all other essential quantities: SEP, electric field intensity, and magnetic field intensity. SEP is also important in some cases, such as grounding grid analysis based on the finite element technique [5]. In that analysis, once the SEP of the grounding system is determined, the current

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distribution through the conductors can be computed. Consequently, the potentials and fields can be determined at any field point in the surrounding medium.

The authors use the term infinitesimal current source instead of the often-used term electric dipole because the observed multilayer medium can be conductive. In the case of a conducting medium, an infinitesimal current source leaks the current into the surrounding medium. However, in an ideal dielectric medium, the infinitesimal current source leaks the displacement current into the surrounding medium and can be represented as an electric dipole.

The ICS can be located in the various geometries of the surrounding medium. This paper considers a planar isotropic multilayer medium in which the effect of layer boundaries must be taken into account. Two special cases are the multilayer soil with air and multilayer lossless dielectric. The equations can be modified to meet the requirements, depending on the medium. Since the observed current source is infinitesimal, the differential equations are described by the spatial-domain Green's functions. Satisfying the boundary conditions on the layer boundaries leads to a system of linear equations in which kernel functions of spectral-domain Green's functions are the unknowns. A system of linear equations can be solved in two ways. First, it can be easily solved using well-known numerical methods for solving the system of linear equations, e.g., Gaussian elimination, LU decomposition, and singular value decomposition. This paper also provides an alternative method since it offers closed-form expressions for the kernel functions [6, 7], which can be modified to meet the requirements of any isotropic multilayer medium.

The proposed approach is different from the other approaches dealing with this topic [3, 4, 8–10], in which the closed-form expressions for spectral-domain Green's functions of the Lorenz potentials in the source layer are obtained using double Fourier transformation and generalized reflection coefficients [9, 10] or using transmission line analogy [3, 4, 8]. Subsequently, the spectral-domain Green's functions of the Lorenz potentials in other layers can be computed iteratively starting from the source layer [8–10].

A system of linear equations can be obtained from the boundary conditions for the spectral-domain Green's functions of the Lorenz potentials. This system has more equations than unknowns. However, by introducing the proposed kernels of spectral-domain Green's functions, a new system of linear equations containing the same number of equations and unknowns is obtained, and that system has a unique numerical and analytical solution. The numerical solution for the given parameters can be easily computed, while the proposed analytical solution requires considerable effort to obtain.

With respect to [6] and [7], a multilayer medium is generalized in this paper. This required a generalization of the previously obtained closed-form expression for kernels of the spectral-domain Green's functions of the Lorenz potentials. The system of linear equations for the numerical computation of the SEP of a vertical ICS is also novel. In addition, matrix expressions for computation of the spectral-domain Green's functions of the electric field intensity and magnetic field intensity are also given, which are obtained from the spectral-domain Green's functions of the Lorenz potentials.

2. THE INFINITESIMAL CURRENT SOURCE IN A PLANAR MULTILAYER MEDIUM

The analysis of the infinitesimal current source (ICS) considered in this paper will be constrained to a planar multilayer medium. This multilayer medium can consist of n layers, each linear, isotropic, and homogeneous, from an electromagnetic standpoint. Therefore, the i-th layer is characterized by its electric conductivity σ_i, electric permittivity ε_i, and magnetic permeability μ_i. The formulas developed in this paper are written in such a way that allows the origin of the coordinate system to migrate along the z-axis. Each i-th layer can also be characterized by its value of the wave propagation constant $\bar{\gamma}_i$ and the complex-valued electrical conductivity $\bar{\kappa}_i$, described by the following equations:

$$\bar{\gamma}_i^2 = j \cdot \omega \cdot \mu_i \cdot \bar{\kappa}_i \tag{1}$$

$$\bar{\gamma}_i = \sqrt{\frac{\omega \cdot \mu_i}{2}} \cdot \left(\frac{\sigma_i}{N} + j \cdot N\right); \quad N = \sqrt{\omega \cdot \varepsilon_i + \sqrt{(\omega \cdot \varepsilon_i)^2 + \sigma_i^2}} \tag{2}$$

$$\bar{\kappa}_i = \sigma_i + j \cdot \omega \cdot \varepsilon_i \tag{3}$$

where N is an auxiliary function, ω the circular frequency of the time-harmonic current, and j the imaginary unit.

In the case of a homogeneous and unbounded medium, the orientation of the ICS is irrelevant. However, in the case where layer boundaries exist, the orientation of the ICS becomes important. To satisfy the boundary conditions, an arbitrarily oriented ICS must be divided into two components: a horizontal infinitesimal current source (HICS) and a vertical infinitesimal current source (VICS). Both of these sources can be located in any layer of the multilayered medium at a point $T' = (x', y', z' \equiv d)$. The key number of the layer in which the current source is located is denoted by index s (Figure 1).

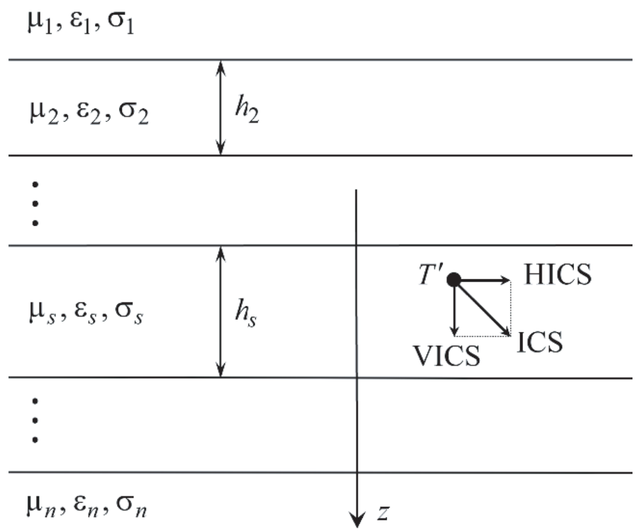

Figure 1. Planar multilayer medium in which the ICS is located.

In the general case, the equation for a longitudinal current $\bar{I}^\ell$ through the ICS can be described by:

$$\bar{I}^\ell \cdot d\vec{\ell} = \bar{I}^\ell_x \cdot d\ell \cdot \vec{i} + \bar{I}^\ell_y \cdot d\ell \cdot \vec{j} + \bar{I}^\ell_z \cdot d\ell \cdot \vec{k} \tag{4}$$

where $d\ell$ is the infinitesimal length of the ICS, whereas the longitudinal current components $\bar{I}^\ell_x$, $\bar{I}^\ell_y$, $\bar{I}^\ell_z$, and the unit-vector $\vec{\ell}_0$ are, respectively:

$$\bar{I}^\ell_x = \bar{I}^\ell \cdot \ell_{0x}; \quad \bar{I}^\ell_y = \bar{I}^\ell \cdot \ell_{0y}; \quad \bar{I}^\ell_z = \bar{I}^\ell \cdot \ell_{0z} \tag{5}$$

$$\vec{\ell}_0 = \ell_{0x} \cdot \vec{i} + \ell_{0y} \cdot \vec{j} + \ell_{0z} \cdot \vec{k} \tag{6}$$

where ℓ_{0x}, ℓ_{0y}, and ℓ_{0z} are x-, y-, and z-components of the ICS unit-vector, respectively. In the case of the HICS, the longitudinal current $\bar{I}^\ell_h$ can be defined as:

$$\bar{I}^\ell_h = \bar{I}^\ell \cdot \ell_{0h}; \quad \ell_{0h} = \sqrt{\ell^2_{0x} + \ell^2_{0y}} \tag{7}$$

where ℓ_{0h} is the length of the unit-vector of the HICS.

Except for longitudinal current $\bar{I}^\ell$, which flows along the current source, there is also a leakage current that flows from the source into the surrounding medium. This can be defined by a leakage current density $\bar{\tau}$:

$$\bar{\tau} = -\frac{\partial \bar{I}^\ell}{\partial \ell} = \bar{\tau}_x + \bar{\tau}_y + \bar{\tau}_z = \bar{\tau}_h + \bar{\tau}_z \tag{8}$$

and its components are defined by:

$$\bar{\tau}_x = \bar{\tau} \cdot \ell^2_{0x}; \quad \bar{\tau}_y = \bar{\tau} \cdot \ell^2_{0y}; \quad \bar{\tau}_z = \bar{\tau} \cdot \ell^2_{0z} \tag{9}$$

$$\bar{\tau}_h = \bar{\tau}_x + \bar{\tau}_y = \bar{\tau} \cdot \ell^2_{0h} \tag{10}$$

where $\bar{\tau}_x$, $\bar{\tau}_y$, and $\bar{\tau}_z$ are x-, y-, and z-components of the leakage current density, respectively, and $\bar{\tau}_h$ is the leakage current density of the HICS.

In the case of a homogeneous and unbounded medium, the longitudinal current through the ICS creates a VMP with the same orientation as the current source. However, in the case of a planar multilayer medium, the longitudinal current can also create an additional VMP that can have a different orientation from the ICS. The VMP can be obtained by satisfying the boundary conditions between the layers, and these conditions are the continuity of tangential components of the electric field intensity and magnetic field intensity. Once the VMP is obtained, other essential quantities can be obtained as well. Derivation of Lorenz potential expressions, as well as the fields of the ICS, will be explained in the following sections.

3. VERTICAL INFINITESIMAL CURRENT SOURCE (VICS)

3.1. VMP Green's Functions for VICS

As previously mentioned, the ICS can be divided into VICS and HICS. Since the current source is infinitesimal, the Lorenz potentials, as well as the electric field intensity and magnetic field intensity, can be described using spatial-domain Green's functions for each layer of the medium. The infinitesimal VMP of the VICS $\underline{\vec{A}}_i^v$ in the i-th layer has only one component with which the boundary conditions can be satisfied, and it can be written in the matrix form for the i-th layer:

$$\left\{ \begin{array}{c} d\bar{A}_i^{vx} \\ d\bar{A}_i^{vy} \\ d\bar{A}_i^{vz} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \\ \bar{G}_i^{Azz} \end{array} \right\} \cdot \bar{I}^{\ell} \cdot \ell_{0z} \cdot d\ell \tag{11}$$

where $\bar{G}_i^{Azz}$ is the z-component of the VMP spatial-domain Green's function in the i-th layer; $d\bar{A}_i^{vx}$, $d\bar{A}_i^{vy}$, and $d\bar{A}_i^{vz}$ are x-, y-, and z-components of the infinitesimal VMP of the VICS, respectively. The spatial-domain Green's function of the VMP for VICS can be described by the following Helmholtz differential equation [1, 6]:

$$\Delta\, \bar{G}_i^{Azz} - \bar{\gamma}_i^2 \cdot \bar{G}_i^{Azz} = -\,\mu_i \cdot \delta\left(\vec{R} - \vec{R}'\right) \tag{12}$$

where $\delta\,(\vec{R} - \vec{R}')$ is the Dirac delta function, $\vec{R}$ the radius-vector of the field point with its coordinates:

$$\vec{R} = \{x,\ y,\ z\} \tag{13}$$

and $\vec{R}'$ the radius-vector of the source point with its coordinates:

$$\vec{R}' = \left\{x',\ y',\ z' = d\right\} \tag{14}$$

The solution of (12), obtained by the separation of variables [11], can be written as follows [6]:

$$\bar{G}_i^{Azz} = \frac{\mu_i}{4 \cdot \pi} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Avv}\right) \tag{15}$$

where $\bar{S}_0(\tilde{\bar{G}}_i^{Avv})$ represents the Sommerfeld integral (SI) of VMP spectral-domain Green's function for the i-th layer. The definition of SIs is given in Appendix A. Hence, the VMP defined in (11) can be written as:

$$\left\{ \begin{array}{c} d\bar{A}_i^{vx} \\ d\bar{A}_i^{vy} \\ d\bar{A}_i^{vz} \end{array} \right\} = \left\{ \begin{array}{c} 0 \\ 0 \\ \ell_{0z} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Avv}\right) \end{array} \right\} \cdot \frac{\mu_i \cdot \bar{I}^{\ell} \cdot d\ell}{4 \cdot \pi} \tag{16}$$

where the VMP spectral-domain Green's function $\tilde{\bar{G}}_i^{Avv}$ can be written as [6, 10, 11]:

$$\tilde{\bar{G}}_i^{Avv} = \tilde{\bar{G}}_i^{Avv}\left(\bar{\lambda}, z,\ d\right) = \delta_{i,s} \cdot e^{-\bar{\alpha}_s \cdot |z-d|} + \bar{\Theta}_i^{Avv} \cdot e^{-\bar{\alpha}_i \cdot (z - z_{i-1})} + \bar{\mathrm{X}}_i^{Avv} \cdot e^{-\bar{\alpha}_i \cdot (z_i - z)} \tag{17}$$

whereas $\delta_{i,s}$ is the Kronecker delta symbol; z_{i-1} and z_i are the z-coordinates of the upper and lower boundaries of the i-th layer, respectively; $\bar{\Theta}_i^{Avv}$ and $\bar{\mathrm{X}}_i^{Avv}$ are kernel functions [6, 11], also known as coefficients of up-going and down-going waves [1]. The complex-valued parameter:

$$\bar{\alpha}_i = \sqrt{\bar{\lambda}^2 + \bar{\gamma}_i^2} \tag{18}$$

is a function of the SI integration variable $\bar{\lambda}$ and the wave propagation constant $\bar{\gamma}_i$ of the i-th layer.

The boundary conditions of the VMP spatial-domain Green's functions obtained from the continuity of the tangential components of the electric field intensity and magnetic field intensity can be written as [1]:

$$\bar{G}_i^{Azz}\Big|_{z=z_i} = m_i \cdot \bar{G}_{i+1}^{Azz}\Big|_{z=z_i}; \quad i = 1,\ 2,\ \ldots,\ n-1 \tag{19}$$

$$\left.\frac{\partial \bar{G}_i^{Azz}}{\partial z}\right|_{z=z_i} = m_i \cdot \bar{q}_i \cdot \left.\frac{\partial \bar{G}_{i+1}^{Azz}}{\partial z}\right|_{z=z_i}; \quad i = 1,\ 2,\ \ldots,\ n-1 \tag{20}$$

where:

$$m_i = \mu_i/\mu_{i+1}; \quad \bar{q}_i = \bar{\kappa}_i/\bar{\kappa}_{i+1} \tag{21}$$

From (19) and (20), the expressions for the boundary conditions of the VMP spectral-domain Green's functions can be derived as follows:

$$\tilde{\bar{G}}_i^{Avv}\Big|_{z=z_i} = \bar{p}_i \cdot \tilde{\bar{G}}_{i+1}^{Avv}\Big|_{z=z_i}; \quad i = 1,\ 2,\ \ldots,\ n-1 \tag{22}$$

$$\left.\frac{\partial \tilde{\bar{G}}_i^{Avv}}{\partial z}\right|_{z=z_i} = \bar{p}_i \cdot \bar{q}_i \cdot \left.\frac{\partial \tilde{\bar{G}}_{i+1}^{Avv}}{\partial z}\right|_{z=z_i}; \quad i = 1,\ 2,\ \ldots,\ n-1 \tag{23}$$

where

$$\bar{p}_i = \bar{\alpha}_i/\bar{\alpha}_{i+1} \tag{24}$$

The unknowns in this approach are the VMP kernel functions $\bar{\Theta}_i^{Avv}$ and $\bar{\mathrm{X}}_i^{Avv}$ from (17), which can be computed from the system of linear equations derived from (22) and (23) [6]:

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avv} + \bar{\mathrm{X}}_i^{Avv} - \bar{p}_i \cdot \bar{\Theta}_{i+1}^{Avv} - \bar{p}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avv} = \bar{f}_i^v \tag{25}$$

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avv} - \bar{\mathrm{X}}_i^{Avv} - \bar{q}_i \cdot \bar{\Theta}_{i+1}^{Avv} + \bar{q}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avv} = \bar{g}_i^v \tag{26}$$

where $i = 1,\ 2, \ldots,\ n-1$, whereas:

$$\bar{\Theta}_1^{Avv} = \bar{\mathrm{X}}_n^{Avv} = 0 \tag{27}$$

$$\bar{\nu}_i = e^{-\bar{\alpha}_i \cdot h_i} \tag{28}$$

and h_i is the thickness of the i-th layer of the medium. The system of linear equations (25) and (26) is the same as that in the case of planar multilayer soil, where the magnetic permeability of each layer is equal to the permeability of the vacuum μ_0 [6]. This system of equations, containing $2 \cdot n - 2$ number of equations, can be written in the matrix form:

$$\left[\bar{D}^v\right] \cdot \left\{\begin{array}{c} \bar{\mathrm{X}}_1^{Avv} \\ \bar{\Theta}_2^{Avv} \\ \bar{\mathrm{X}}_2^{Avv} \\ \vdots \\ \bar{\Theta}_{n-1}^{Avv} \\ \bar{\mathrm{X}}_{n-1}^{Avv} \\ \bar{\Theta}_n^{Avv} \end{array}\right\} = \left\{\begin{array}{c} \bar{f}_1^v \\ \bar{g}_1^v \\ \bar{f}_2^v \\ \vdots \\ \bar{g}_{n-2}^v \\ \bar{f}_{n-1}^v \\ \bar{g}_{n-1}^v \end{array}\right\} \tag{29}$$

where

$$\bar{f}_i^v = \begin{cases} \bar{p}_{s-1} \cdot e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

$$\bar{g}_i^v = \begin{cases} -\bar{q}_{s-1} \cdot e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{31}$$

The kernel functions $\bar{\Theta}_i^{Avv}$ and $\bar{\mathrm{X}}_i^{Avv}$ can be easily computed using well-known numerical algorithms for the computation of a system of linear equations (29). Another procedure using the closed-form kernel functions is described in detail in [6]. In both cases, the VMP of the VICS for each layer of the medium can be determined.

3.2. SEP Green's Functions for VICS

The scalar electric potential (SEP) is a function of the leakage current flowing into the medium. The infinitesimal SEP of the VICS $d\bar{\varphi}_i^v$ in the i-th layer can be defined as:

$$d\bar{\varphi}_i^v = \bar{G}_i^{\varphi v} \cdot \bar{\tau} \cdot \ell_{0z}^2 \cdot d\ell = \bar{G}_i^{\varphi v} \cdot \bar{\tau}_z \cdot d\ell \tag{32}$$

where $\bar{\tau}_z$ is the leakage current density along the z-axis. The SEP spatial-domain Green's function $\tilde{\bar{G}}_i^{\varphi v}$ can be written as:

$$\bar{G}_i^{\varphi v} = \frac{1}{4 \cdot \pi \cdot \bar{\kappa}_i} \cdot \bar{S}_0 \left(\tilde{\bar{G}}_i^{\varphi v} \right) \tag{33}$$

where $\bar{S}_0(\tilde{\bar{G}}_i^{\varphi v})$ represents the SI of SEP spectral-domain Green's function for the i-th layer given in Appendix A. The SEP spectral-domain Green's function, similar to VMP, can be written in a form using kernel functions [6]:

$$\tilde{\bar{G}}_i^{\varphi v} = \tilde{\bar{G}}_i^{\varphi v} \left(\bar{\lambda},\, z,\, d \right) = \delta_{i,s} \cdot e^{-\bar{\alpha}_s \cdot |z-d|} + \bar{\Theta}_i^{\varphi v} \cdot e^{-\bar{\alpha}_i \cdot (z - z_{i-1})} + \bar{\mathrm{X}}_i^{\varphi v} \cdot e^{-\bar{\alpha}_i \cdot (z_i - z)} \tag{34}$$

The connection between the VMP and SEP spectral-domain Green's functions is given in the following equation [10]:

$$\frac{\partial \tilde{\bar{G}}_i^{\varphi v}}{\partial z'} = \frac{\partial\, \tilde{\bar{G}}_i^{\varphi v}}{\partial d} = -\frac{\partial\, \tilde{\bar{G}}_i^{Avv}}{\partial z} \tag{35}$$

from which the SEP kernel functions can be defined as [7]:

$$\bar{\Theta}_i^{\varphi v} = \frac{\bar{\alpha}_i}{\bar{\alpha}_s} \cdot \bar{\Theta}_i^a; \quad \bar{\Theta}_i^a = \frac{1}{\bar{\alpha}_s} \cdot \frac{\partial \bar{\Theta}_i^{Avv}}{\partial d} \tag{36}$$

$$\bar{\mathrm{X}}_i^{\varphi v} = -\frac{\bar{\alpha}_i}{\bar{\alpha}_s} \cdot \bar{\mathrm{X}}_i^a; \quad \bar{\mathrm{X}}_i^a = \frac{1}{\bar{\alpha}_s} \cdot \frac{\partial \bar{\mathrm{X}}_i^{Avv}}{\partial d} \tag{37}$$

It can be seen from (36) and (37) that SEP Green's functions can be computed from VMP Green's functions. However, the auxiliary functions $\bar{\Theta}_i^a$ and $\bar{\mathrm{X}}_i^a$ are also the solutions of another system of linear equations:

$$\left[\bar{D}^v \right] \cdot \left\{ \begin{array}{c} \bar{\mathrm{X}}_1^a \\ \bar{\Theta}_2^a \\ \bar{\mathrm{X}}_2^a \\ \vdots \\ \bar{\Theta}_{n-1}^a \\ \bar{\mathrm{X}}_{n-1}^a \\ \bar{\Theta}_n^a \end{array} \right\} = \left\{ \begin{array}{c} \bar{f}_1^{\varphi v} \\ \bar{g}_1^{\varphi v} \\ \bar{f}_2^{\varphi v} \\ \vdots \\ \bar{g}_{n-2}^{\varphi v} \\ \bar{f}_{n-1}^{\varphi v} \\ \bar{g}_{n-1}^{\varphi v} \end{array} \right\} \tag{38}$$

where the linear equation system matrix $[\bar{D}^v]$ is the same as in (29), whereas:

$$\bar{f}_i^{\varphi v} = \begin{cases} -\bar{p}_{s-1} \cdot e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{39}$$

$$\bar{g}_i^{\varphi v} = \begin{cases} \bar{q}_{s-1} \cdot e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{40}$$

The right-hand side in (38) can also be computed from (30) and (31) as:

$$\bar{f}_i^{\varphi v} = \frac{1}{\bar{\alpha}_s} \cdot \frac{\partial \bar{f}_i^v}{\partial d}; \quad \bar{g}_i^{\varphi v} = \frac{1}{\bar{\alpha}_s} \cdot \frac{\partial \bar{g}_i^v}{\partial d} \tag{41}$$

Therefore, the kernel functions $\bar{\Theta}_i^{\varphi v}$ and $\bar{\mathrm{X}}_i^{\varphi v}$ can be easily computed using the well-known numerical algorithms for the computation of a system of linear equations (38), and after that, Equations (36) and (37). Another procedure with the SEP closed-form kernel functions is based on Equations (36) and (37), where SEP closed-form kernel functions can be easily obtained from VMP closed-form kernel functions [6, 7]. In both cases, the SEP of the VICS for each layer of the medium can be determined.

4. HORIZONTAL INFINITESIMAL CURRENT SOURCE (HICS)

4.1. VMP Green's Functions for HICS

The HICS lies in the $z = z' = d$ plane of the coordinate system, which means that it can have x- and y-components. Therefore, the longitudinal current of the HICS generates the infinitesimal VMP $\underline{\vec{A}}_i^h$ in the i-th layer which can be written as:

$$\begin{Bmatrix} d\bar{A}_i^{hx} \\ d\bar{A}_i^{hy} \\ d\bar{A}_i^{hz} \end{Bmatrix} = \left[\bar{G}_i^{Ah}\right] \cdot \begin{Bmatrix} \ell_{0x} \\ \ell_{0y} \\ 0 \end{Bmatrix} \cdot \bar{I}^\ell \cdot d\ell \tag{42}$$

where the matrix of the VMP spatial-domain Green's functions is:

$$\left[\bar{G}_i^{Ah}\right] = \begin{bmatrix} \bar{G}_i^{Axx} & 0 & 0 \\ 0 & \bar{G}_i^{Ayy} & 0 \\ \bar{G}_i^{Azx} & \bar{G}_i^{Azy} & 0 \end{bmatrix} \tag{43}$$

From (43) it can be seen that both x- and y-components require an additional component with which the boundary conditions can be satisfied. In the literature, the usual and easiest way is to take z-component as an additional component, although it can be chosen differently [12]. The VMP spatial-domain Green's functions in (43) can be obtained from the Helmholtz differential equations related to the x- and y-components of the longitudinal current:

$$\Delta \bar{G}_i^{Axx} - \bar{\gamma}_i^2 \cdot \bar{G}_i^{Axx} = -\mu_i \cdot \delta\left(\vec{R} - \vec{R}'\right) \tag{44}$$

$$\Delta \bar{G}_i^{Azx} - \bar{\gamma}_i^2 \cdot \bar{G}_i^{Azx} = 0 \tag{45}$$

$$\Delta \bar{G}_i^{Ayy} - \bar{\gamma}_i^2 \cdot \bar{G}_i^{Ayy} = -\mu_i \cdot \delta\left(\vec{R} - \vec{R}'\right) \tag{46}$$

$$\Delta \bar{G}_i^{Azy} - \bar{\gamma}_i^2 \cdot \bar{G}_i^{Azy} = 0 \tag{47}$$

The solutions of the system of equations are the VMP spatial-domain Green's functions written in the following form:

$$\bar{G}_i^{Axx} = \bar{G}_i^{Ayy} = \frac{\mu_i}{4 \cdot \pi} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \tag{48}$$

$$\bar{G}_i^{Azx} = \cos\phi \cdot \bar{G}_i^{Azh}; \quad \bar{G}_i^{Azy} = \sin\phi \cdot \bar{G}_i^{Azh} \tag{49}$$

$$\bar{G}_i^{Azh} = \frac{\mu_i}{4\cdot\pi}\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{Avh}\right) \tag{50}$$

$$\cos\phi = \frac{x-x'}{r};\quad \sin\phi = \frac{y-y'}{r} \tag{51}$$

where r is the distance between the source point and field point, whereas $\bar{S}_0(\tilde{\bar{G}}_i^{Ahh})$ and $\bar{S}_1(\tilde{\bar{G}}_i^{Avh})$ are SIs defined in Appendix A. From (48)–(51) it can be seen that y-component expressions can be easily described by the x-component expressions. Therefore, the final expressions for both components can be obtained based on the satisfaction of the boundary conditions only for the x-component.

The infinitesimal VMP of HICS from (42) can now be written as:

$$\left\{\begin{array}{c} d\bar{A}_i^{hx} \\ d\bar{A}_i^{hy} \\ d\bar{A}_i^{hz} \end{array}\right\} = \left\{\begin{array}{c} \ell_{0x}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ \ell_{0y}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ f_{zh}\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{Avh}\right) \end{array}\right\}\cdot\frac{\mu_i\cdot\bar{I}^\ell\cdot d\ell}{4\cdot\pi} \tag{52}$$

where:

$$f_{hz} = \ell_{0x}\cdot\cos\phi + \ell_{0y}\cdot\sin\phi \tag{53}$$

whereas the VMP spectral-domain Green's functions can be described by the following equations:

$$\tilde{\bar{G}}_i^{Ahh} = \delta_{i,s}\cdot e^{-\bar{\alpha}_s\cdot|z-d|} + \bar{\Theta}_i^{Ahh}\cdot e^{-\bar{\alpha}_i\cdot(z-z_{i-1})} + \bar{\mathrm{X}}_i^{Ahh}\cdot e^{-\bar{\alpha}_i\cdot(z_i-z)} \tag{54}$$

$$\tilde{\bar{G}}_i^{Avh} = \bar{\Theta}_i^{Avh}\cdot e^{-\bar{\alpha}_i\cdot(z-z_{i-1})} + \bar{\mathrm{X}}_i^{Avh}\cdot e^{-\bar{\alpha}_i\cdot(z_i-z)} \tag{55}$$

Obtaining the unknown VMP kernel functions $\bar{\Theta}_i^{Ahh}$, $\bar{\mathrm{X}}_i^{Ahh}$, $\bar{\Theta}_i^{Avh}$, and $\bar{\mathrm{X}}_i^{Avh}$ for HICS can start with the boundary conditions of VMP spatial-domain Green's functions:

$$\left.\bar{G}_i^{Axx}\right|_{z=z_i} = \left.\bar{G}_{i+1}^{Axx}\right|_{z=z_i};\quad i=1,\ 2,\ \ldots,\ n-1 \tag{56}$$

$$\left.\frac{\partial\bar{G}_i^{Axx}}{\partial z}\right|_{z=z_i} = m_i\cdot\left.\frac{\partial\bar{G}_{i+1}^{Axx}}{\partial z}\right|_{z=z_i};\quad i=1,\ 2,\ \ldots,\ n-1 \tag{57}$$

$$\left.\bar{G}_i^{Axx}\right|_{z=z_i} = m_i\cdot\left.\bar{G}_{i+1}^{Axx}\right|_{z=z_i};\quad i=1,\ 2,\ \ldots,\ n-1 \tag{58}$$

$$\left.\left(\frac{\partial\bar{G}_i^{Axx}}{\partial x}+\frac{\partial\bar{G}_i^{Azx}}{\partial z}\right)\right|_{z=z_i} = m_i\cdot\bar{q}_i\cdot\left.\left(\frac{\partial\bar{G}_{i+1}^{Axx}}{\partial x}+\frac{\partial\bar{G}_{i+1}^{Azx}}{\partial z}\right)\right|_{z=z_i} \tag{59}$$

where $i=1,2,\ldots,n-1$. The boundary conditions of VMP spectral-domain Green's functions can be obtained from (56)–(59):

$$\left.\tilde{\bar{G}}_i^{Ahh}\right|_{z=z_i} = \frac{\bar{p}_i}{m_i}\cdot\left.\tilde{\bar{G}}_{i+1}^{Ahh}\right|_{z=z_i} \tag{60}$$

$$\left.\frac{\partial\tilde{\bar{G}}_i^{Ahh}}{\partial z}\right|_{z=z_i} = \bar{p}_i\cdot\left.\frac{\partial\tilde{\bar{G}}_{i+1}^{Ahh}}{\partial z}\right|_{z=z_i} \tag{61}$$

$$\left.\tilde{\bar{G}}_i^{Avh}\right|_{z=z_i} = \bar{p}_i\cdot\left.\tilde{\bar{G}}_{i+1}^{Avh}\right|_{z=z_i} \tag{62}$$

$$\left.\frac{\partial\tilde{\bar{G}}_i^{Avh}}{\partial z}\right|_{z=z_i} - \bar{p}_i\cdot\bar{q}_i\cdot\left.\frac{\partial\tilde{\bar{G}}_{i+1}^{Avh}}{\partial z}\right|_{z=z_i} = (\bar{q}_i\cdot m_i-1)\cdot\left.\tilde{\bar{G}}_i^{Ahh}\right|_{z=z_i} \tag{63}$$

where $i=1,2,\ldots,n-1$.

Equations (60)–(63) yield a system of $4 \cdot n - 4$ linear equations, where the unknowns are VMP kernel functions:

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Ahh} + \bar{\mathrm{X}}_i^{Ahh} - \frac{\bar{p}_i}{m_i} \cdot \bar{\Theta}_{i+1}^{Ahh} - \frac{\bar{p}_i}{m_i} \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Ahh} = \bar{f}_i^h \tag{64}$$

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Ahh} - \bar{\mathrm{X}}_i^{Ahh} - \bar{\Theta}_{i+1}^{Ahh} + \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Ahh} = \bar{g}_i^h \tag{65}$$

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avh} + \bar{\mathrm{X}}_i^{Avh} - \bar{p}_i \cdot \bar{\Theta}_{i+1}^{Avh} - \bar{p}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avh} = 0 \tag{66}$$

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avh} - \bar{\mathrm{X}}_i^{Avh} - \bar{q}_i \cdot \bar{\Theta}_{i+1}^{Avh} + \bar{q}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avh} + \frac{\bar{q}_i \cdot m_i - 1}{\bar{\alpha}_i} \cdot \left(\bar{\nu}_i \cdot \bar{\Theta}_i^{Ahh} + \bar{\mathrm{X}}_i^{Ahh}\right) = -\delta_{i,s} \cdot \frac{\bar{q}_s \cdot m_s - 1}{\bar{\alpha}_s} \cdot e^{-\bar{\alpha}_s \cdot (z_s - d)} \tag{67}$$

whereas:

$$\bar{\Theta}_1^{Ahh} = \bar{\mathrm{X}}_n^{Ahh} = \bar{\Theta}_1^{Avh} = \bar{\mathrm{X}}_n^{Avh} = 0 \tag{68}$$

$$\bar{f}_i^h = \begin{cases} (\bar{p}_{s-1}/m_{s-1}) \cdot e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{69}$$

$$\bar{g}_i^h = \begin{cases} -e^{-\bar{\alpha}_s \cdot (d - z_{s-1})} & \text{for} \quad i = s-1 \\ -e^{-\bar{\alpha}_s \cdot (z_s - d)} & \text{for} \quad i = s \\ 0 & \text{otherwise} \end{cases} \tag{70}$$

The system of linear equations (64)–(67) can be computed using well-known numerical computation methods, and as a result, the unknown kernel functions are obtained in all layers of the medium. This system of equations can also be divided into two subsystems of equations where some similarities to the VICS equations occur. Equations (64) and (65) are related only to the x-component and can be solved independently from (66) and (67). This subsystem of equations can be written in the matrix form:

$$\left[\bar{D}^h\right] \cdot \left\{ \begin{matrix} \bar{\mathrm{X}}_1^{Ahh} \\ \bar{\Theta}_2^{Ahh} \\ \bar{\mathrm{X}}_2^{Ahh} \\ \vdots \\ \bar{\Theta}_{n-1}^{Ahh} \\ \bar{\mathrm{X}}_{n-1}^{Ahh} \\ \bar{\Theta}_n^{Ahh} \end{matrix} \right\} = \left\{ \begin{matrix} \bar{f}_1^h \\ \bar{g}_1^h \\ \bar{f}_2^h \\ \vdots \\ \bar{g}_{n-2}^h \\ \bar{f}_{n-1}^h \\ \bar{g}_{n-1}^h \end{matrix} \right\} \tag{71}$$

and it has a similar form as (29) for VICS. The solution of this subsystem of equations can also be obtained in two ways: using numerical computation methods or following the procedure with the closed-form expressions of kernel functions $\bar{\Theta}_i^{Ahh}$ and $\bar{\mathrm{X}}_i^{Ahh}$ given in detail in [6], with certain generalizations given in Appendix B. When the kernel functions $\bar{\Theta}_i^{Ahh}$ and $\bar{\mathrm{X}}_i^{Ahh}$ are known, they can be included in the second subsystem of linear equations on the right-hand side of the subsystem:

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avh} + \bar{\mathrm{X}}_i^{Avh} - \bar{p}_i \cdot \bar{\Theta}_{i+1}^{Avh} - \bar{p}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avh} = 0 \tag{72}$$

$$\bar{\nu}_i \cdot \bar{\Theta}_i^{Avh} - \bar{\mathrm{X}}_i^{Avh} - \bar{q}_i \cdot \bar{\Theta}_{i+1}^{Avh} + \bar{q}_i \cdot \bar{\nu}_{i+1} \cdot \bar{\mathrm{X}}_{i+1}^{Avh} = -\bar{\Psi}_i \tag{73}$$

where $i = 1, 2, \ldots, n-1$ and:

$$\bar{\Psi}_i = \frac{\bar{q}_i \cdot m_i - 1}{\bar{\alpha}_i} \cdot \left(\delta_{i,s} \cdot e^{-\bar{\alpha}_s \cdot (z_s - d)} + \bar{\nu}_i \cdot \bar{\Theta}_i^{Ahh} + \bar{\mathrm{X}}_i^{Ahh}\right) \tag{74}$$

This subsystem of equations, containing $2 \cdot n - 2$ number of equations, can also be written in matrix form:

$$\left[\bar{D}^v\right]\cdot\left\{\begin{array}{c}\bar{\mathrm{X}}_1^{Avh}\\ \bar{\Theta}_2^{Avh}\\ \bar{\mathrm{X}}_2^{Avh}\\ \vdots\\ \bar{\Theta}_{n-1}^{Avh}\\ \bar{\mathrm{X}}_{n-1}^{Avh}\\ \bar{\Theta}_n^{Avh}\end{array}\right\}=\left\{\begin{array}{c}0\\ -\bar{\Psi}_1\\ 0\\ \vdots\\ -\bar{\Psi}_{n-2}\\ 0\\ -\bar{\Psi}_{n-1}\end{array}\right\}\tag{75}$$

where the linear equation system matrix $[\bar{D}^v]$ is the same as that for VICS in (29) and (38). When the remaining kernel functions $\bar{\Theta}_i^{Avh}$ and $\bar{\mathrm{X}}_i^{Avh}$ are obtained, the VMP of the HICS can be obtained.

Therefore, the kernel functions $\bar{\Theta}_i^{Ahh}$, $\bar{\mathrm{X}}_i^{Ahh}$, $\bar{\Theta}_i^{Avh}$, and $\bar{\mathrm{X}}_i^{Avh}$ can be easily computed using well-known numerical algorithms for the computation of systems of linear equations (71) and (75). Another procedure with the closed-form kernel functions, which is given in detail in [6], was developed for multilayer soil (special case), and the generalization of these closed-form kernel functions is given in Appendix B. In both cases, the VMP of the HICS for each layer of a general planar multilayer medium can be determined.

4.2. SEP Green's Functions for HICS

The infinitesimal SEP is a function of the leakage current, and in the i-th layer, it can be defined as:

$$d\bar{\varphi}_i^h = d\bar{\varphi}_i^x + d\bar{\varphi}_i^y = \bar{G}_i^{\varphi h}\cdot\bar{\tau}\cdot\ell_{0h}^2\cdot d\ell = \bar{G}_i^{\varphi h}\cdot\bar{\tau}_h\cdot d\ell \tag{76}$$

where $d\bar{\varphi}_i^h$ is the infinitesimal SEP of the HICS, whereas $d\bar{\varphi}_i^x$ and $d\bar{\varphi}_i^y$ are the x- and y-components, respectively.

Using the Lorenz gauge condition [11], the SEP spatial-domain Green's function for HICS can be written as:

$$\bar{G}_i^{\varphi h} = \bar{G}_i^{\varphi x} = \bar{G}_i^{\varphi y} = \frac{1}{4\cdot\pi\cdot\bar{\kappa}_i}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{\varphi h}\right) \tag{77}$$

where $\bar{S}_0(\tilde{\bar{G}}_i^{\varphi h})$ represents the SI given in Appendix A, whereas the SEP spectral-domain Green's function can be described by the following equations [6]:

$$\tilde{\bar{G}}_i^{\varphi h} = \tilde{\bar{G}}_i^{\varphi h}\left(\bar{\lambda}, z, d\right) = \delta_{i,s}\cdot e^{-\bar{\alpha}_s\cdot|z-d|} + \bar{\Theta}_i^{\varphi h}\cdot e^{-\bar{\alpha}_i\cdot(z-z_{i-1})} + \bar{\mathrm{X}}_i^{\varphi h}\cdot e^{-\bar{\alpha}_i\cdot(z_i-z)} \tag{78}$$

The SEP spectral-domain Green's function can be obtained using the VMP spectral-domain Green's functions [6, 10]:

$$\tilde{\bar{G}}_i^{\varphi h} = \tilde{\bar{G}}_i^{Ahh} + \frac{\partial\tilde{\bar{G}}_i^{Avh}}{\partial z} \tag{79}$$

From (54), (55), (78), and (79), the SEP kernel functions for HICS can be obtained using the following equations [6]:

$$\bar{\Theta}_i^{\varphi h} = \bar{\Theta}_i^{Ahh} - \bar{\alpha}_i\cdot\bar{\Theta}_i^{Avh};\quad i = 2,\ 3,\ldots,\ n \tag{80}$$

$$\bar{\mathrm{X}}_i^{\varphi h} = \bar{\mathrm{X}}_i^{Ahh} + \bar{\alpha}_i\cdot\bar{\mathrm{X}}_i^{Avh};\quad i = 1,\ 2,\ldots,\ n-1 \tag{81}$$

$$\bar{\Theta}_1^{\varphi h} = \bar{X}_n^{\varphi h} = 0 \tag{82}$$

The SEP of the HICS can now be determined using the VMP of the HICS.

5. THE ELECTRIC FIELD INTENSITY OF THE INFINITESIMAL CURRENT SOURCE (ICS)

Once the VMP and SEP are computed, the electric field intensity can be determined using the following equation:

$$\underline{\vec{E}}_i = -\nabla\bar{\varphi}_i - j\cdot\omega\cdot\underline{\vec{A}}_i \tag{83}$$

where $\underline{\vec{E}}_i$ is the phasor of the electric field intensity vector in the i-th layer, and ∇ is the del operator, whereas $\bar{\varphi}_i$ and $\underline{\vec{A}}_i$ are the SEP and VMP in the i-th layer, respectively. The electric field intensity of the ICS can be obtained as the sum of the contribution of VICS and HICS to the electric field intensity:

$$\begin{Bmatrix} d\bar{E}_i^x \\ d\bar{E}_i^y \\ d\bar{E}_i^z \end{Bmatrix} = \begin{Bmatrix} d\bar{E}_i^{vx} \\ d\bar{E}_i^{vy} \\ d\bar{E}_i^{vz} \end{Bmatrix} + \begin{Bmatrix} d\bar{E}_i^{hx} \\ d\bar{E}_i^{hy} \\ d\bar{E}_i^{hz} \end{Bmatrix} \tag{84}$$

The vertical component of the electric field intensity, according to (16), (32), (33), and (83), can be described by the following equation:

$$\begin{Bmatrix} d\bar{E}_i^{vx} \\ d\bar{E}_i^{vy} \\ d\bar{E}_i^{vz} \end{Bmatrix} = -\begin{Bmatrix} \ell_{0z}^2 \cdot \cos\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi v}\right) \\ \ell_{0z}^2 \cdot \sin\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi v}\right) \\ \ell_{0z}^2 \cdot \bar{S}_0\left(\dfrac{\partial \tilde{\bar{G}}_i^{\varphi v}}{\partial z}\right) \end{Bmatrix} \cdot \frac{\bar{\tau} \cdot d\ell}{4 \cdot \pi \cdot \bar{\kappa}_i} - \begin{Bmatrix} 0 \\ 0 \\ \ell_{0z} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Avv}\right) \end{Bmatrix} \cdot \frac{j \cdot \omega \cdot \mu_i \cdot \bar{I}^\ell \cdot d\ell}{4 \cdot \pi} \tag{85}$$

The horizontal component of the electric field intensity, according to (52), (53), (76), (77), and (83), can be described by the following equation:

$$\begin{Bmatrix} d\bar{E}_i^{hx} \\ d\bar{E}_i^{hy} \\ d\bar{E}_i^{hz} \end{Bmatrix} = -\begin{Bmatrix} \ell_{0h}^2 \cdot \cos\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi h}\right) \\ \ell_{0h}^2 \cdot \sin\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi h}\right) \\ \ell_{0h}^2 \cdot \bar{S}_0\left(\dfrac{\partial \tilde{\bar{G}}_i^{\varphi h}}{\partial z}\right) \end{Bmatrix} \cdot \frac{\bar{\tau} \cdot d\ell}{4 \cdot \pi \cdot \bar{\kappa}_i} - \begin{Bmatrix} \ell_{0x} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ \ell_{0y} \cdot \bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ f_{hz} \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{Avh}\right) \end{Bmatrix} \cdot \frac{j \cdot \omega \cdot \mu_i \cdot \bar{I}^\ell \cdot d\ell}{4 \cdot \pi} \tag{86}$$

where the definition of SIs can be found in Appendix A.

6. THE MAGNETIC FIELD INTENSITY OF THE INFINITESIMAL CURRENT SOURCE (ICS)

The magnetic field intensity of the ICS can be obtained from the VMP using the following equation:

$$\begin{Bmatrix} \bar{H}_i^x \\ \bar{H}_i^y \\ \bar{H}_i^z \end{Bmatrix} = \frac{1}{\mu_i} \cdot \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \bar{A}_i^x & \bar{A}_i^y & \bar{A}_i^z \end{vmatrix} \tag{87}$$

Similar to the electric field intensity, the magnetic field intensity can be obtained as the sum of the contribution of VICS and HICS to the magnetic field intensity:

$$\begin{Bmatrix} d\bar{H}_i^x \\ d\bar{H}_i^y \\ d\bar{H}_i^z \end{Bmatrix} = \begin{Bmatrix} d\bar{H}_i^{vx} \\ d\bar{H}_i^{vy} \\ d\bar{H}_i^{vz} \end{Bmatrix} + \begin{Bmatrix} d\bar{H}_i^{hx} \\ d\bar{H}_i^{hy} \\ d\bar{H}_i^{hz} \end{Bmatrix} \tag{88}$$

According to (16) and (87), the vertical component of the magnetic field intensity can be described by the following equation:

$$\begin{Bmatrix} d\bar{H}_i^{vx} \\ d\bar{H}_i^{vy} \\ d\bar{H}_i^{vz} \end{Bmatrix} = \begin{Bmatrix} \ell_{0z} \cdot \sin\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{Avv}\right) \\ -\ell_{0z} \cdot \cos\phi \cdot \bar{S}_1\left(\tilde{\bar{G}}_i^{Avv}\right) \\ 0 \end{Bmatrix} \cdot \frac{\bar{I}^\ell \cdot d\ell}{4 \cdot \pi} \tag{89}$$

The horizontal component of the magnetic field intensity, according to (52), (53), and (87), can be described by the following equation:

$$\left\{\begin{array}{c} d\bar{H}_i^{hx} \\ d\bar{H}_i^{hy} \\ d\bar{H}_i^{hz} \end{array}\right\} = \left\{\begin{array}{c} f_{hz}\cdot\sin\phi\cdot\bar{S}_2\left(\tilde{\bar{G}}_i^{Avh}\right) - \ell_{0y}\cdot\bar{S}_0\left(\dfrac{\partial\tilde{\bar{G}}_i^{Ahh}}{\partial z}\right) \\ \ell_{0x}\cdot\bar{S}_0\left(\dfrac{\partial\tilde{\bar{G}}_i^{Ahh}}{\partial z}\right) - f_{hz}\cdot\cos\phi\cdot\bar{S}_2\left(\tilde{\bar{G}}_i^{Avh}\right) \\ \left(\ell_{0y}\cdot\cos\phi - \ell_{0x}\cdot\sin\phi\right)\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{Ahh}\right) \end{array}\right\}\cdot\frac{\bar{I}^{\ell}\cdot d\ell}{4\cdot\pi} \tag{90}$$

where the definition of SIs can be found in Appendix A.

7. THE ADJUSTMENT OF LORENZ POTENTIALS

The Lorenz potentials are fundamental parts of the MPIE formulation and can be modified according to the requirements because they are auxiliary functions. An important aspect must be respected: the fields must remain unchanged during the modification of the Lorenz potentials. The SEP is essential in grounding grid analysis; therefore, it has to be computed [5]. The SEP of ICS can be obtained as the sum of the SEP of VICS and HICS. The problem is that the two mentioned SEPs cannot be summed up in their current forms due to their incompatibility [13, 14], and thus they have to be adjusted. The SEP of HICS is continuous between the layers of the medium, while the SEP of the VICS is not [2]. Therefore, it is reasonable to choose that the SEP of the HICS is the SEP of the ICS:

$$d\bar{\varphi}_i^h = \bar{G}_i^{\varphi h}\cdot\bar{\tau}\cdot\ell_{0h}^2\cdot d\ell = \bar{G}_i^{\varphi h}\cdot\bar{\tau}_h\cdot d\ell \tag{91}$$

$$d\bar{\varphi}_i^{v\to h} = \bar{G}_i^{\varphi h}\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell = \bar{G}_i^{\varphi h}\cdot\bar{\tau}_z\cdot d\ell \tag{92}$$

$$d\bar{\varphi}_i = d\bar{\varphi}_i^{v\to h} + d\bar{\varphi}_i^h = \bar{G}_i^{\varphi h}\cdot\bar{\tau}\cdot d\ell \tag{93}$$

where $d\bar{\varphi}_i^{v\to h}$ is the adjusted infinitesimal SEP of the VICS, $d\bar{\varphi}_i^h$ the infinitesimal SEP of the HICS, $\bar{G}_i^{\varphi h}$ the SEP spatial-domain Green's function for HICS described by (77), and $\bar{\tau}$ the leakage current density described by (8). The SEP correction for VICS can be written as:

$$d\bar{\varphi}_i^{vc} = d\bar{\varphi}_i^{v\to h} - d\bar{\varphi}_i^v = \bar{G}_i^{\varphi vc}\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell = \left(\bar{G}_i^{\varphi h} - \bar{G}_i^{\varphi v}\right)\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell \tag{94}$$

where SEP spatial-domain Green's function for VICS correction is:

$$\bar{G}_i^{\varphi vc} = \frac{1}{4\cdot\pi\cdot\bar{\kappa}_i}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{\varphi vc}\right) \tag{95}$$

The SI in (95) is defined in Appendix A, and it contains the SEP spectral-domain Green's function:

$$\tilde{\bar{G}}_i^{\varphi vc} = \tilde{\bar{G}}_i^{\varphi h} - \tilde{\bar{G}}_i^{\varphi v} = \tilde{\bar{G}}_i^{Ahh} + \frac{\partial\tilde{\bar{G}}_i^{Avh}}{\partial z} - \tilde{\bar{G}}_i^{\varphi v} \tag{96}$$

which can be defined by kernel functions:

$$\tilde{\bar{G}}_i^{\varphi vc} = \bar{\Theta}_i^{\varphi vc}\cdot e^{-\bar{\alpha}_i\cdot(z-z_{i-1})} + \bar{\mathrm{X}}_i^{\varphi vc}\cdot e^{-\bar{\alpha}_i\cdot(z_i-z)} \tag{97}$$

$$\bar{\Theta}_i^{\varphi vc} = \bar{\Theta}_i^{\varphi h} - \bar{\Theta}_i^{\varphi v} = \bar{\Theta}_i^{Ahh} - \bar{\alpha}_i\cdot\bar{\Theta}_i^{Avh} - \bar{\Theta}_i^{\varphi v} \tag{98}$$

$$\bar{\mathrm{X}}_i^{\varphi vc} = \bar{\mathrm{X}}_i^{\varphi h} - \bar{\mathrm{X}}_i^{\varphi v} = \bar{\mathrm{X}}_i^{Ahh} + \bar{\alpha}_i\cdot\bar{\mathrm{X}}_i^{Avh} - \bar{\mathrm{X}}_i^{\varphi v} \tag{99}$$

The correction of the SEP demands the correction of the VMP for VICS since they are coupled by the Lorenz gauge condition. Therefore, the infinitesimal VMP of the ICS can be obtained as the sum of the horizontal contribution, vertical contribution, and correction:

$$\left\{\begin{array}{c} d\bar{A}_i^x \\ d\bar{A}_i^y \\ d\bar{A}_i^z \end{array}\right\} = \left\{\begin{array}{c} d\bar{A}_i^{hx} \\ d\bar{A}_i^{hy} \\ d\bar{A}_i^{hz} \end{array}\right\} + \left\{\begin{array}{c} 0 \\ 0 \\ d\bar{A}_i^{vz} \end{array}\right\} + \left\{\begin{array}{c} d\bar{A}_i^{vcx} \\ d\bar{A}_i^{vcy} \\ d\bar{A}_i^{vcz} \end{array}\right\} \tag{100}$$

where the corrected infinitesimal VMP of the VICS is:

$$\left\{\begin{array}{c} d\bar{A}_i^{vcx} \\ d\bar{A}_i^{vcy} \\ d\bar{A}_i^{vcz} \end{array}\right\} = -\frac{1}{j\cdot\omega}\cdot\left\{\begin{array}{c} \dfrac{\partial\bar{G}_i^{\varphi vc}}{\partial x} \\ \dfrac{\partial\bar{G}_i^{\varphi vc}}{\partial y} \\ \dfrac{\partial\bar{G}_i^{\varphi vc}}{\partial z} \end{array}\right\}\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell \tag{101}$$

The correction can also be described using SIs:

$$\left\{\begin{array}{c} d\bar{A}_i^{vcx} \\ d\bar{A}_i^{vcy} \\ d\bar{A}_i^{vcz} \end{array}\right\} = -\left\{\begin{array}{c} \cos\phi\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi vc}\right) \\ \sin\phi\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi vc}\right) \\ \bar{S}_0\left(\dfrac{\partial\tilde{\bar{G}}_i^{\varphi vc}}{\partial z}\right) \end{array}\right\}\cdot\frac{\mu_i\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell}{4\cdot\pi\cdot\bar{\gamma}_i^2} \tag{102}$$

hence the infinitesimal VMP of the ICS can be described by:

$$\left\{\begin{array}{c} d\bar{A}_i^{x} \\ d\bar{A}_i^{y} \\ d\bar{A}_i^{z} \end{array}\right\} = \left\{\begin{array}{c} \ell_{0x}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ \ell_{0y}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{Ahh}\right) \\ f_{zh}\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{Avh}\right)+\ell_{0z}\cdot\bar{S}_0\left(\tilde{\bar{G}}_i^{Avv}\right) \end{array}\right\}\cdot\frac{\mu_i\cdot\bar{I}^\ell\cdot d\ell}{4\cdot\pi} - \left\{\begin{array}{c} \cos\phi\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi vc}\right) \\ \sin\phi\cdot\bar{S}_1\left(\tilde{\bar{G}}_i^{\varphi vc}\right) \\ \bar{S}_0\left(\dfrac{\partial\tilde{\bar{G}}_i^{\varphi vc}}{\partial z}\right) \end{array}\right\}\cdot\frac{\mu_i\cdot\bar{\tau}\cdot\ell_{0z}^2\cdot d\ell}{4\cdot\pi\cdot\bar{\gamma}_i^2} \tag{103}$$

where all SIs are defined in Appendix A.

8. THE SPECIAL CASES: THE MULTILAYER SOIL AND THE MULTILAYER LOSSLESS DIELECTRIC

This paper presents a theory for the computation of electromagnetic phenomena in the case of a general planar multilayer medium. Usually, the literature deals with specific multilayer media, such as the multilayer soil with air in the case of grounding grid analysis, or the multilayer lossless dielectric that can be considered in the printed circuit technology. Both cases have their characteristics, and the equations proposed in the previous sections can be modified for both special cases.

8.1. The Multilayer Soil and the Air

In the case of planar multilayer soil and the air, the magnetic permeability of each layer is equal to the permeability of the vacuum:

$$\mu_i = \mu_0; \quad m_i = 1 \tag{104}$$

This modification simplified the equations in the previous sections, although it did not affect the kernel functions of the spectral-domain Green's functions. This case is described in detail in [6, 7].

8.2. The Multilayer Lossless Dielectric

The stratified multilayer lossless dielectric is a medium in which the electric conductivity of each layer is equal to zero:

$$\sigma_i = 0 \tag{105}$$

In this case, ICS becomes a Hertzian electric dipole; VICS becomes a vertical electric dipole (VED); and HICS becomes a horizontal electric dipole (HED). The kernel functions of the spectral-domain Green's functions for VED and HED can be computed from the kernels for VICS and HICS using the following substitutions:

$$\bar{\kappa}_i \rightarrow j \cdot \omega \cdot \varepsilon_i \tag{106}$$

$$\bar{\gamma}_i \rightarrow j \cdot k_i; \quad k_i = \omega \cdot \sqrt{\mu_i \cdot \varepsilon_i} \tag{107}$$

$$\bar{\tau} \rightarrow j \cdot \omega \cdot \bar{\tau}^q \tag{108}$$

where k_i is the wave number of the i-th layer, and $\bar{\tau}^q$ is the linear charge density on the axis of the Hertzian electric dipole.

9. CONCLUSION

This paper depicts the electromagnetic disturbances created by the ICS in its surroundings. The ICS is located in a planar multilayer medium, and it generates the electric field intensity and magnetic field intensity in all layers of the medium. The Lorenz potentials were introduced using the MPIE formulation. The paper describes in detail the definition and computation of Green's functions for Lorenz potentials and fields, as well as the procedure for the adjustment of Lorenz potentials for VICS and HICS. In other words, a detailed procedure for obtaining the Lorenz potentials and fields generated by the ICS in a planar isotropic multilayer medium is given. The paper also considers two special cases of the general multilayer medium usually considered in the literature: the multilayer soil with air and multilayer lossless dielectric. The proposed expressions for Green's functions were modified to meet the requirements and characteristics of each medium.

A novel approach proposed in this paper is different from the other approaches dealing with this topic. The other methods deliver solutions only in the s-th layer where the current source is located, and the solutions in other layers need to be computed iteratively using recursive expressions, going between layers until the field-point layer is reached. Green's functions are obtained by the double Fourier transformation and generalized reflection coefficients, or by a transmission line analogy. Green's functions obtained by the above-mentioned algorithms in the source layer, for chosen parameters, yield the same numerical results as Green's functions obtained by the algorithm proposed in this paper. Therefore, the numerical validation of the proposed algorithm is verified. On the other hand, it is very difficult to reduce one form of Green's functions to another, and therefore analytically validate the proposed approach, except in the case of a two-layer medium. This validation, in the case of planar multilayer soil, is given in detail in [6].

The proposed approach introduces kernels of the spectral-domain Green's function, used for the forming of systems of linear equations obtained from the boundary conditions. This novelty approach, where the source and field layers are arbitrarily positioned, contains systems of equations that can be easily solved using well-known numerical computation algorithms. This is not a numerically demanding task since the number of equations is relatively small because it depends only on the number of medium layers. This paper also offers an analytical solution to the system of linear equations containing recursive functions that required considerable effort to obtain. The difference between them is that numerical computation gives values of the kernels of the spectral domain Green's functions of the Lorenz potentials in all layers, while closed-form expressions describe kernels for the observed field layer.

A detailed validation of results for the planar multilayer soil is described in [6, 7]. The proposed algorithms for a generalized multilayer medium proposed in this paper were validated by a comparison of the numerical and analytical solutions, which yielded identical results.

In addition to [6], this paper also gives matrix expressions for infinitesimal electric and magnetic field intensity, obtained from Lorenz potentials. The novelty of this paper is forming of the system of linear equations for the numerical computation of the SEP of VICS. Therefore, the SEP of VICS can be easily obtained using well-known numerical matrix-solving methods. The paper also provides a procedure for the adjustment of Lorenz potentials since the obtained SEP of VICS and HICS are incompatible. The adjustment is important in the case of the arbitrary-oriented ICS in which the SEPs of VICS and HICS should be compatible since their contributions have to be summed up.

APPENDIX A. DEFINITION OF THE SOMMERFELD INTEGRALS

In the case of ICS in a planar multilayered medium, scientists have to deal with the so-called Sommerfeld integrals (SIs). The SIs are notorious and well known in the electromagnetic community due to their complicated and time-consuming computation process. They appear in the equations that describe the spatial-domain Green's functions for VMP, SEP, electric field intensity, and magnetic field intensity. In this paper, they appear in three different types due to the order of derivation inside the integral. The zero-order SI, which appears in (15), (16), (33), (48), (52), (77), (85), (86), and (90), can be described by:

$$\bar{S}_0\left(\tilde{\bar{G}}_i\right) = \int_0^\infty \tilde{\bar{G}}_i \cdot \frac{\bar{\lambda}}{\bar{\alpha}_i} \cdot \bar{J}_0\left(\bar{\lambda} \cdot r\right) \cdot d\bar{\lambda} \tag{A1}$$

where $\tilde{\bar{G}}_i$ is the spectral-domain Green's function, α_i a parameter defined by (18), $\bar{\lambda}$ an integration variable, $\bar{J}_0(\bar{\lambda} \cdot r)$ the Bessel function of the first kind of order zero, and r the distance between the source point and the field point defined by:

$$r = \sqrt{\left(x - x'\right)^2 + \left(y - y'\right)^2} \tag{A2}$$

In (A1), the order of derivation of the Bessel function is zero. The first-order SI, which appears in (50), (52), (85), (86), (89), and (90), can be described by:

$$\bar{S}_1\left(\tilde{\bar{G}}_i\right) = \frac{\partial \bar{S}_0\left(\tilde{\bar{G}}_i\right)}{\partial r} = \int_0^\infty \tilde{\bar{G}}_i \cdot \frac{\bar{\lambda}}{\bar{\alpha}_i} \cdot \frac{\partial \bar{J}_0\left(\bar{\lambda} \cdot r\right)}{\partial r} \cdot d\bar{\lambda} \tag{A3}$$

where the first-order derivation of the Bessel function can also be expressed as:

$$\frac{\partial \bar{J}_0\left(\bar{\lambda} \cdot r\right)}{\partial r} = \bar{\lambda} \cdot \bar{J}_1\left(\bar{\lambda} \cdot r\right) \tag{A4}$$

where $\bar{J}_1(\bar{\lambda} \cdot r)$ is the Bessel function of the first kind of the first order. The second-order SI, which appears in (90), can be described by:

$$\bar{S}_2\left(\tilde{\bar{G}}_i\right) = \frac{\partial \bar{S}_1\left(\tilde{\bar{G}}_i\right)}{\partial r} = \int_0^\infty \tilde{\bar{G}}_i \cdot \frac{\bar{\lambda}}{\bar{\alpha}_i} \cdot \frac{\partial^2 \bar{J}_0\left(\bar{\lambda} \cdot r\right)}{\partial r^2} \cdot d\bar{\lambda} \tag{A5}$$

where:

$$\frac{\partial^2 \bar{J}_0\left(\bar{\lambda} \cdot r\right)}{\partial r^2} = \frac{\bar{\lambda} \cdot \bar{J}_1\left(\bar{\lambda} \cdot r\right)}{r} - \bar{\lambda}^2 \cdot \bar{J}_0\left(\bar{\lambda} \cdot r\right) \tag{A6}$$

The second-order SI can be defined alternatively:

$$\bar{S}_2\left(\tilde{\bar{G}}_i\right) = -\bar{S}_0\left(\bar{\lambda}^2 \cdot \tilde{\bar{G}}_i\right) - \frac{1}{r} \cdot \bar{S}_1\left(\tilde{\bar{G}}_i\right) \tag{A7}$$

SIs do not have an analytical solution, so the solution has to be provided using numerical methods. As previously mentioned, the computation of the SIs is demanding because they are highly oscillatory due to the Bessel function in the integrand, slowly convergent with the semi-infinite boundaries of the integral, and in some cases even divergent. The computation of SIs is usually divided into two parts: the first part of SIs and the so-called "tails" of SIs. Each part possesses its specifics; therefore, computational methods and algorithms were developed separately for each part. Numerous studies have dealt with this topic, in which efficient algorithms for the computation of SIs can be found [15–19].

APPENDIX B. GENERALIZATION OF THE VMP CLOSED-FORM SPECTRALDOMAIN GREEN'S FUNCTIONS

This paper offers systems of linear equations, obtained from the boundary conditions, from which the VMP kernel functions of the spectral-domain Green's functions for VICS and HICS can be obtained. One can choose numerical computation or closed-form expressions that avoid forming and solving the system of linear equations. In [6, 7], closed-form spectral-domain Green's functions were introduced for the multilayered soil and air, where the magnetic permeabilities of all layers are $\mu_i = \mu_0$. This paper considers the arbitrary permeability of the layers, so the closed-form expressions need to be generalized.

The VICS expressions do not need to be generalized because the permeability of the layers disappears from the kernel functions equations. Therefore, the VMP closed-form spectral-domain Green's functions for VICS in [6] are valid for any planar multilayer medium.

The generalization of HICS expressions involves several HICS equations in [6], which are listed sequentially. To distinguish the equations between the current paper and [6], the authors used curly brackets {} for the equations from [6].

- Instead of the equations {1} and {3} in [6], expressions (1) and (2) from Section 2 are valid, in which the permeability becomes $\mu_0 = \mu_i$.
- A new parameter m_i needs to be introduced (Equation (21)).
- Instead of {44} in [6], expression (74) in Section 4 is valid.

The additional modifications of the equations in [6] should be done as follows:

- In {49}:

$$\bar{\alpha}_{s-1}/\bar{\alpha}_s \to (\mu_s/\mu_{s-1}) \cdot (\bar{\alpha}_{s-1}/\bar{\alpha}_s) \tag{B1}$$

- In {50}:

$$\bar{\alpha}_i/\bar{\alpha}_s \to (\mu_s/\mu_i) \cdot (\bar{\alpha}_i/\bar{\alpha}_s) \tag{B2}$$

- Equation {52} becomes:

$$\bar{T}_i^h = \frac{2 \cdot \bar{\alpha}_{i+1} \cdot \mu_i}{\bar{\alpha}_{i+1} \cdot \mu_i + \bar{\alpha}_i \cdot \mu_{i+1}} = \bar{T}_{i+1,\,i}^{TE} \tag{B3}$$

- Equation {$B4$} becomes:

$$\bar{R}_i^h = \frac{\bar{\alpha}_{i+1} \cdot \mu_i - \bar{\alpha}_i \cdot \mu_{i+1}}{\bar{\alpha}_{i+1} \cdot \mu_i + \bar{\alpha}_i \cdot \mu_{i+1}} = \bar{R}_{i+1,i}^{TE} = -\bar{R}_{i,i+1}^{TE} \tag{B4}$$

- In {C5}:

$$(\bar{q}_s - 1) \quad \to \quad (\bar{q}_s \cdot m_s - 1) \tag{B5}$$

- In {C6}:

$$(\bar{q}_i - 1) \to (\bar{q}_i \cdot m_i - 1) \tag{B6}$$

- In {C7}:

$$\frac{\bar{q}_{s-1} - 1}{\bar{\alpha}_s} \quad \to \quad \frac{\mu_s}{\mu_{s-1}} \cdot \frac{\bar{q}_{s-1} \cdot m_{s-1} - 1}{\bar{\alpha}_s} \tag{B7}$$

- In {C8}:

$$\frac{\bar{q}_i - 1}{\bar{\alpha}_s} \quad \to \quad \frac{\mu_s}{\mu_i} \cdot \frac{\bar{q}_i \cdot m_i - 1}{\bar{\alpha}_s} \tag{B8}$$

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