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Time Domain Hybrid Method for the Coupling Analysis of Parallel Traces on PCB Excited by Ambient Wave

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Abstract—Currently, numerical methods used for the coupling analysis of printed circuit board (PCB) traces excited by ambient wave are still rare. In this work, a time domain hybrid method is presented for the coupling simulation of parallel traces of PCB efficiently, which is consisted of the finite-difference time-domain (FDTD) method, transmission line (TL) equations, and subgridding technique. Within this method, the coupling model of parallel traces on PCB is constructed by using TL equations firstly. Then, the per-unit-length (p.u.l) inductance and capacitance parameters of the traces are calculated by the empirical formulas obtained by the fitting of measurement data in the literature. And the FDTD method combined with the subgridding technique is applied to model the structures of PCB substrate and ground plate to obtain the excitation fields of the traces, which are introduced into TL equations as equivalent source terms. Finally, the central difference scheme of FDTD is utilized to discretize the TL equations to obtain the transient responses on the terminal loads of the traces. The significant features of this presented method are that it can realize the synchronous calculations of electromagnetic field radiation and transient responses on the traces and avoid modeling the fine structures of the traces directly. The accuracy and efficiency of this presented method have been verified via the numerical simulations of multiple parallel traces on PCB in free space and inside a shielded cavity by comparing with the Baum-Liu-Tesche (BLT) equation and electromagnetic software CST.

1. INTRODUCTION

With the rapid development of wireless communications, the integration degrees of electronic devices are becoming higher and higher. The semiconductor elements, microwave and millimeter wave chips, and other electronic elements are integrated together on one printed circuit board (PCB), and the data transmission between these elements and chips is carried out through these traces. When the PCB is exposed to the complex electromagnetic environment, strong interference signals should be generated and propagated through the traces, which may cause damage to the terminal circuits. Therefore, studying the coupling problem of space electromagnetic fields to the PCB traces is necessary for the electromagnetic safety assessment of the PCB.

In the current stage, some efficient numerical methods have been proposed for the coupling analysis of PCB traces, such as full wave algorithms [1–3], BLT equation [4–6], and FDTD-SPICE [7–9]. Full wave algorithms, such as FDTD method [1], method of moments (MOM) [2], and finite element method (FEM) [3], need amount of computer memory and computation time for the coupling simulation of PCB traces, because the fine structures of PCB traces must be meshed in these methods. BLT equation uses nodes and pipes to construct the relationship between the space electromagnetic fields and the

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PCB traces with their terminal loads, and obtains the voltage and current responses on the terminal loads of the traces. Although BLT equation can avoid meshing the structures of the traces directly, it is a frequency domain algorithm and has low efficiency in dealing with the coupling problems caused by broadband signals. FDTD-SPICE method is a time domain hybrid algorithm. Within this method, the FDTD is used to calculate the excitation fields around the traces, and the SPICE software is applied to build the SPICE equivalent circuit representing the coupling of the traces, so as to obtain the transient responses on the terminal loads of the traces. However, the calculation processes of the space electromagnetic fields and the simulation of the SPICE equivalent circuit are separated. When the calculation time is increased, the efficiency of this method will be decreased sharply.

Therefore, an efficient time domain hybrid method, consisting of FDTD method with subgridding technique and TL equations, is proposed for the coupling analysis of PCB traces, which can realize the synchronous computation of space electromagnetic fields and transient responses on the traces and their terminal loads. The remaining of this work is organized as follows. In Section 2, the theory of the proposed method is discussed in detail. The correctness and efficiency of the proposed method are verified by some numerical examples in Section 3. Te conclusion of this work is given in Section 4.

2. THEORY OF THE HYBRID METHOD

The typical coupling model of PCB traces excited by ambient wave is shown in Fig. 1. In this work, the PCB traces with the configuration of parallel and equal length are considered. These traces with the same width and height are located on the substrate of PCB, and the terminal loads of the traces to the PEC ground plate are connected with resistance.

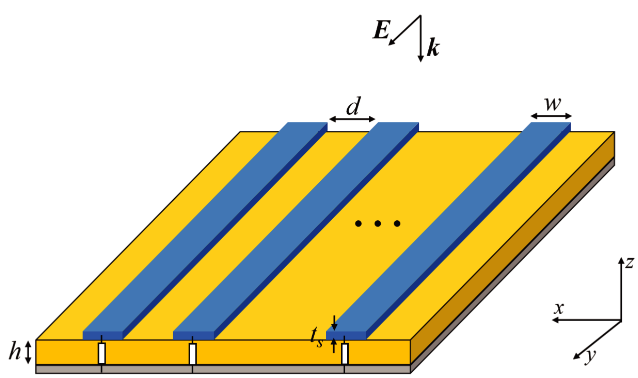


Figure 1. Typical coupling model of PCB traces excited by ambient wave.

Generally, the distances between the traces and the ground plane are less than the minimum wavelength of incident wave. Due to the image theory, the values of the currents on the traces and their image currents are the same, while the directions of these currents are different. Hence, the radiation of the traces can be ignored. Besides, the wave propagation mode between the traces and the ground plane is assumed as TEM mode, and the loss of the traces is neglected. Under these circumstances, the coupling of parallel traces excited by ambient wave can be represented by the time domain transmission line equations [10] as

$$\frac{\partial}{\partial y}\mathbf{V}(y,t) + \mathbf{L}\frac{\partial}{\partial t}\mathbf{I}(y,t) = \mathbf{V}_F(y,t) \tag{1}$$

$$\frac{\partial}{\partial y}\mathbf{I}(y,t) + \mathbf{C}\frac{\partial}{\partial t}\mathbf{V}(y,t) = \mathbf{I}_F(y,t) \tag{2}$$

where $\mathbf{C}$ and $\mathbf{L}$ denote the p.u.l capacitance and inductance matrices of the traces, respectively. $\mathbf{V}(y,t)$ and $\mathbf{I}(y,t)$ represent the voltage and current vectors of the traces, respectively. $\mathbf{V}_F(y,t)$ and $\mathbf{I}_F(y,t)$

are the distributed voltage and current sources of the traces, respectively, which can be expressed as

$$\mathbf{V}_F(y,t) = -\frac{\partial}{\partial y}\mathbf{E}_T(y,t) + \mathbf{E}_L(y,t) \tag{3}$$

$$\mathbf{I}_F(y,t) = -\mathbf{C}\frac{\partial}{\partial t}\mathbf{E}_T(y,t) \tag{4}$$

where $\mathbf{E}_T(y,t)$ and $\mathbf{E}_L(y,t)$ are obtained from the space electromagnetic fields, which are written as

$$[\mathbf{E}_T(y,t)]_i = \int_0^{h_s} \mathbf{e}_z^{ex}(x_i,y,z,t)\,dz \tag{5}$$

$$[\mathbf{E}_L(y,t)]_i = \mathbf{e}_y^{ex}(x_i,y,h_s,t) - \mathbf{e}_y^{ex}(x_i,y,0,t) \tag{6}$$

where i stands for the i-th trace, and h_s is the thickness of the PCB substrate. e_y^{ex} and e_z^{ex} represent the incident electric fields along and perpendicular to the traces, respectively. Thus, the traces can be removed when these incident electromagnetic fields surrounding the traces are calculated, because the equivalent sources of TL equations are not relevant to the scattering electromagnetic fields of the traces.

Obviously, the precision of TL equations is determined by the accurate calculations of the p.u.l distribution parameters and equivalent sources of the traces, which will be introduced in detail as follows.

2.1. Per Unit Length Distribution Parameter Calculation of Parallel Traces

According to the empirical formulas given in [11], as long as the highest frequency of the interference signal is less than 3 GHz, and the size of parallel traces meets the proportion requirements, i.e., $0.1\,\text{mm} < h_s < 1.5\,\text{mm}$, $0.017\,\text{mm} < t_s < 0.0685\,\text{mm}$, $0.25 < d/h_s < 3.75$, $0.1 < w/h_s < 5$ and $0.14 < d/w < 4$, the elements of the p.u.l inductance and capacitance matrices of parallel traces can be calculated as

$$[L]_{ij} = \begin{cases} \mu_0 \left\{ \begin{aligned} & 3.71\left(\frac{h_s}{w}\right)^{0.041} + 0.018\left(\frac{h_s}{w}\right)^{-0.73} - 3.39\left(\frac{h_s}{t_s}\right)^{-0.0006} \\ & + \exp\left(-1.89\frac{d}{h_s}\right)\cdot\left[0.75\left(\frac{h_s}{w}\right)^{-0.0052} - 0.84\left(\frac{h_s}{t_s}\right)^{-0.026}\right] \end{aligned} \right\} & i = j \\ \mu_0 \left\{ \begin{aligned} & \left[-0.415\left(\frac{h_s}{w}\right)^{-0.16} - 2.38\left(\frac{t_s}{w}\right)^{1.18}\right]\cdot\left(\frac{|i-j|\cdot(d+w)-w}{h_s} + 1.07\right)^{-2.6} \\ & + \left(\frac{|i-j|\cdot(d+w)-w}{h_s} + 0.89\right)^{-2.03}\cdot\left[0.418\left(\frac{h_s}{w}\right)^{0.13} + 1.37\left(\tfrac{t_s}{w}\right)^{1.09}\right] \end{aligned} \right\} & i \neq j \end{cases} \tag{7}$$

$$[C]_{ij} = \begin{cases} \varepsilon_0\varepsilon_r \left\{ \begin{aligned} & \left[1.15\left(\frac{w}{h_s}\right)^{0.963} + 1.07\left(\frac{t_s}{h_s}\right)^{0.049}\right] \\ & + \exp\left(-3.52\frac{d}{h_s}\right)\cdot\left[0.75\left(\frac{w}{h_s}\right)^{0.25} + 2.7\left(\frac{t_s}{h_s}\right)^{1.36}\right] \end{aligned} \right\} & i = j \\ -\varepsilon_0\varepsilon_r \left\{ \begin{aligned} & 1.17\left(\frac{w}{h_s}\right)^{0.083}\cdot\left(\frac{|i-j|\cdot(d+w)-w}{h_s} + 0.402\right)^{-0.78} \\ & + \left(\frac{|i-j|\cdot(d+w)-w}{h_s} + 1.32\right)^{-0.8}\cdot\left[-1.36\left(\frac{w}{h_s}\right)^{-0.037} + 0.227\left(\frac{t_s}{h_s}\right)^{0.98}\right] \end{aligned} \right\} & i \neq j \end{cases} \tag{8}$$

where i and j stand for the i-th and j-th traces, respectively. μ_0, ε_0, and ε_r are the permeability and permittivity of free space and relative permittivity of the material of PCB substrate, respectively. h_s, w, t_s, and d are the height of the dielectric substrate, the width, thickness and distance of these traces, respectively.

2.2. Equivalent Distribution Source Calculation of the Traces Based on the FDTD with Subgridding Technique

FDTD is a mature time domain simulation tool, which can be well applied to model the PCB structure and extract the electric fields along and perpendicular to the traces as the equivalent distribution sources of TL equations. Even though the fine structures of the traces do not need to be modeled, the grid size used by the FDTD to mesh the structures of PCB substrate should be still fine, because the wavelength shortening effect happens in the substrate. Under this circumstance, subgridding technique is one appropriate way to improve the computation efficiency of FDTD method. It means that the PCB substrate is meshed by fine grid, and rest region is meshed by coarse grid. Meanwhile, the thickness of the ground plate is neglected in this work, and it can be modeled by setting the electric fields on the ground plate as zero.

Maintaining the continuity of electric and magnetic field components on the interfaces between coarse and fine grids is the premise to guarantee the late time stability of subgridding technique. In this work, the subgridding technique using separated spatial and temporal subgridding interfaces [12, 13] is employed, which has excellent late time stability. Here, arbitrary cross-section of the computation region is taken as an example to clearly describe the coupling process of the electric and magnetic fields on the temporal and spatial subgridding interfaces, as shown in Fig. 2, where the mesh ratio of the fine grid and coarse grid is 1 : 3.

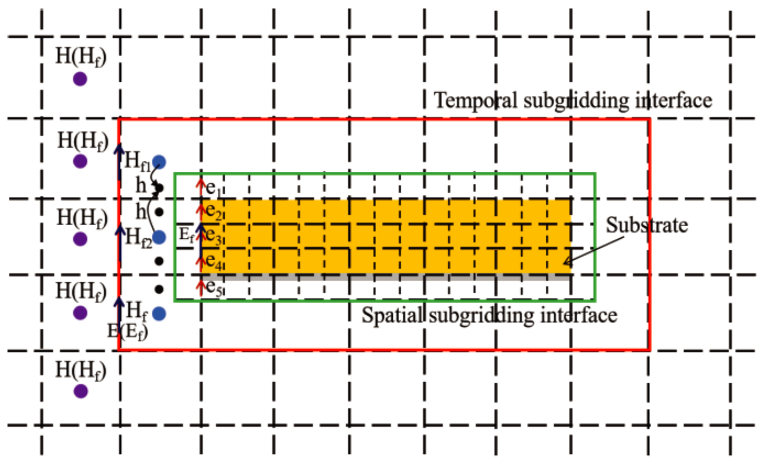


Figure 2. The scheme of the subgridding technique with separated temporal and spatial subgridding interfaces.

The implementation procedure of this subgridding technique contains four steps:

Firstly, sub-grid magnetic fields H_f on the temporal subgridding interface are obtained by the coarse grid magnetic fields H expanded in Taylor series, which is expressed as

$$H_f^{n+v} = H_f^n + Av + Bv^2/2 \tag{9}$$

where $A = (H^{n+1} - H^{n-1})/2$, $B = H^{n+1} - 2H^n + H^{n-1}$. v is selected as 1/3, 2/3, and 1, respectively.

Secondly, fine grid magnetic fields h on the spatial subgridding interface are obtained by the sub-grid magnetic fields H_f via linear interpolation schemes, which is expressed as

$$h = \alpha H_{f1}/3 + (1-\alpha) H_{f2}/3 \tag{10}$$

where α can be 1 or 2. H_{f1} and H_{f2} are the adjacent coarse magnetic fields on the sub grid region, as shown in Fig. 2.

Thirdly, sub-grid electric field E_f near the spatial subgridding interface is modified by the linear interpolation schemes of fine grid electric fields $e_1 \sim e_5$ after each fine grid time step, which is written as

$$E_f = e_1/9 + 2e_2/9 + e_3/3 + 2e_4/9 + e_5/9 \tag{11}$$

where e_1, e_2, e_3, e_4, and e_5 are the fine grid electric fields, as shown in Fig. 2.

Finally, the electric fields E_f at sub-grid region are assigned to update the coarse grid electric fields E at the same position after each coarse grid time step.

The detailed introduction of the FDTD subgridding method can be found from [12] and [13].

2.3. The Solutions of TL Equations

After the transmission line equations are established, the difference scheme of FDTD is applied to discretize the TL equations as

$$\frac{\mathbf{V}^n(k+1)-\mathbf{V}^n(k)}{\Delta y}+\mathbf{L}\frac{\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)-\mathbf{I}^{n-\frac{1}{2}}\left(k+\frac{1}{2}\right)}{\Delta t}=-\frac{\mathbf{E}_{\mathbf{T}}^n(k+1)-\mathbf{E}_{\mathbf{T}}^n(k)}{\Delta y}+\mathbf{E}_{\mathbf{L}}^n\left(k+\frac{1}{2}\right) \tag{12}$$

$$\frac{\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)-\mathbf{I}^{n+\frac{1}{2}}\left(k-\frac{1}{2}\right)}{\Delta y}+\mathbf{C}\frac{\mathbf{V}^{n+1}(k)-\mathbf{V}^n(k)}{\Delta t}=-\mathbf{C}\frac{\mathbf{E}_T^{n+1}(k)-\mathbf{E}_T^n(k)}{\Delta t} \tag{13}$$

Equations (12) and (13) are arranged to obtain the iteration formulas of the currents and voltages on the traces, which can be expressed as

$$\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)=\mathbf{I}^{n-\frac{1}{2}}\left(k+\frac{1}{2}\right)-\left(\frac{\mathbf{L}}{\Delta t}\right)^{-1}\left[\begin{array}{l}\dfrac{\mathbf{V}^n(k+1)-\mathbf{V}^n(k)}{\Delta y}\\ +\dfrac{\mathbf{E}_T^n(k+1)-\mathbf{E}_T^n(k)}{\Delta y}-\mathbf{E}_L^n\left(k+\dfrac{1}{2}\right)\end{array}\right] \tag{14}$$

$$\mathbf{V}^{n+1}(k)=\mathbf{V}^n(k)-\left(\frac{\mathbf{C}}{\Delta t}\right)^{-1}\left[\frac{\mathbf{I}^{n+\frac{1}{2}}\left(k+\frac{1}{2}\right)-\mathbf{I}^{n+\frac{1}{2}}\left(k-\frac{1}{2}\right)}{\Delta y}\right]-\left[\mathbf{E}_T^{n+1}(k)-\mathbf{E}_T^n(k)\right] \tag{15}$$

3. NUMERICAL SIMULATION

To verify the accuracy and efficiency of the proposed method, two cases about PCB traces excited by ambient wave in the environments of free space and shielded enclosure are simulated by this method, BLT equation, and electromagnetic software CST Microwave Studio (MWS) based on finite integration technique (FIT), which are compared in the calculation results and consumption of computation time.

The first case is the coupling analysis of PCB with multiple traces excited by ambient wave, as shown in Fig. 3. The dimension of PCB substrate is $L_c \times W_c \times H_c = 4\,\text{cm} \times 4\,\text{cm} \times 0.2\,\text{cm}$, and the dielectric constant of substrate material is 4.4. The size of the ground plate is $L_c \times W_c = 4\,\text{cm} \times 4\,\text{cm}$. Three parallel traces with length 4 cm, width 0.6 mm, thickness 0.1 mm, distance 2 mm, are located on the substrate. The terminal loads of the traces are $Z_1 = Z_2 = Z_3 = 50\,\Omega$ and $Z_4 = Z_5 = Z_6 = 100\,\Omega$, respectively. A Gaussian pulse, expressed as $E_0(t) = E_0 \exp[-4\pi(t-t_0)^2/\tau^2]$, where $E_0 = 1000\,\text{V/m}$, $\tau = 2\,\text{ns}$, and $t_0 = 1.6\,\text{ns}$, is perpendicular to the PCB.

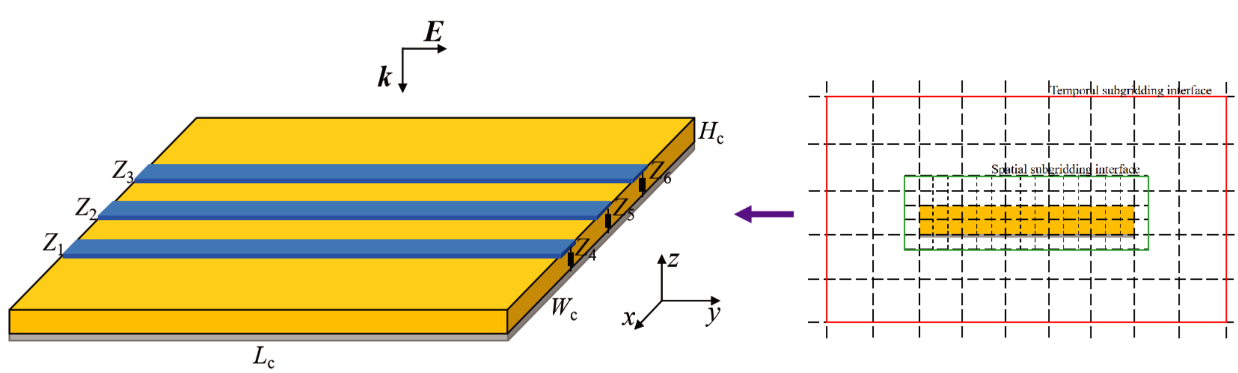


Figure 3. Coupling model of the parallel traces in free space excited by ambient wave.

The coarse grid and fine grid selected by the hybrid method are 3 mm and 1 mm respectively in absence of modeling the fine structures of the traces directly. And the cross-section of the sub-grid region can be found from Fig. 3. Because the whole structure of the PCB should be meshed in the CST MWS, the minimum grid selected by the CST must be less than 0.1 mm, which ensures that the fine structures of the traces can be meshed. Obviously, our proposed method should save more mesh number than that of CST; in other words, our method needs much less computation time. By the way, the electric fields needed to construct the source terms of the BLT equation are obtained via the Inverse Fast Fourier Transform (IFFT) operation of the electric fields calculated by the hybrid method. Here, the voltage responses on the loads Z_2 and Z_6 obtained from the proposed method, BLT equation, and CST are given in Fig. 4. It can be seen clearly that the results of the three methods are almost in perfect agreements. Moreover, the results of this method and BLT equation are almost completely consistent.

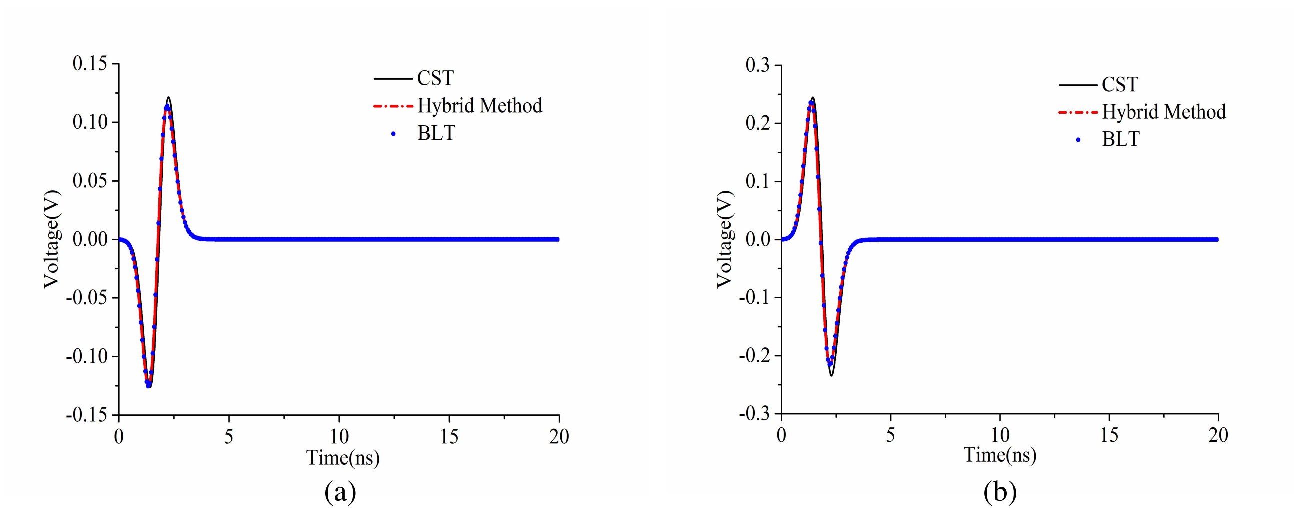


Figure 4. Voltage responses on the terminal loads of the traces for the first case. (a) Voltages on Z_2, (b) voltages on Z_6.

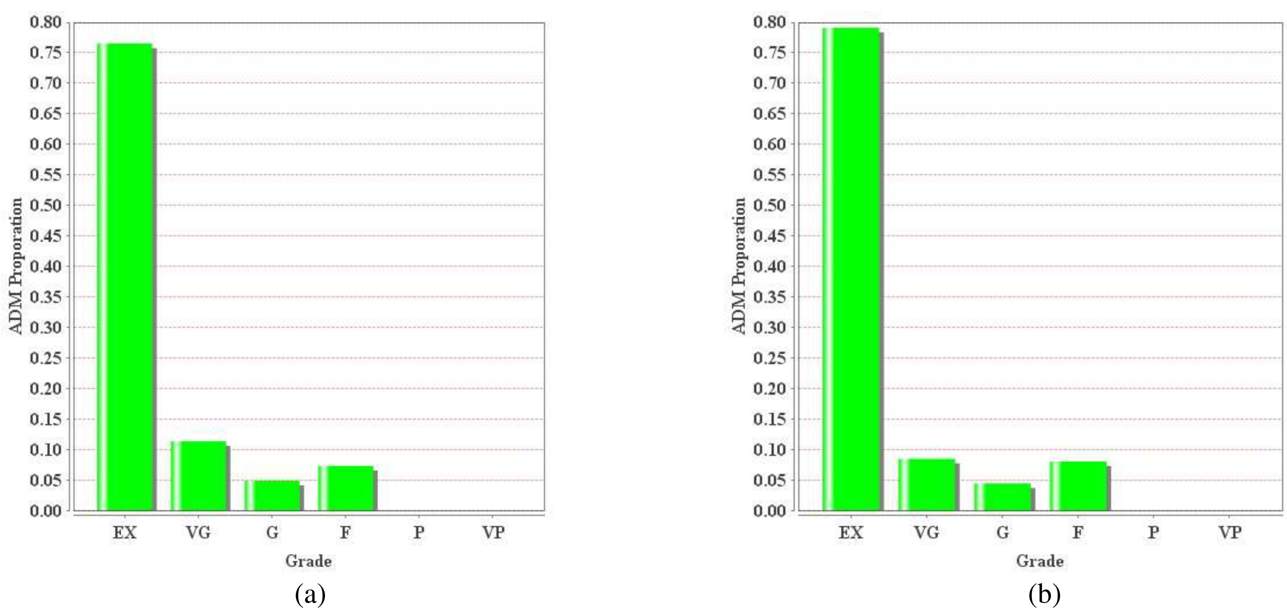


Figure 5. ADM assessment of the voltage responses on the loads for the first case. (a) ADM of the voltages on Z_2, (b) ADM of the voltages on Z_6.

To further evaluate the precision of the proposed method, the feature selective validation (FSV) technique [14, 15] is applied to analyze the errors of the results obtained by the hybrid method and CST. The amplitude difference measure (ADM) of the FSV is utilized to assess the results of the two methods, as shown in Fig. 5, where EX, VG, G, F, P, and VP stand for Excellent, Very Good, Good, Fair, Poor, and Very Poor grades, respectively. It can be seen that our method has the same computational precision as the full-wave simulation of CST, because the proportion of reaching and exceeding the good grade is greater than 92%; meanwhile, the relative difference error of the peak values of the two results is about 3.6%.

To verify the capacity of this method to deal with the coupling analysis of PCB traces in complex environment, the simulation case of PCB traces located in a shielded enclosure and excited by ambient wave is taken into account, where the PCB structure, terminal loads of traces, and incident wave are the same as the first case, as shown in Fig. 6. The dimension of the shielded enclosure is $L_s \times W_s \times H_s = 10\,\text{cm} \times 8\,\text{cm} \times 4\,\text{cm}$, and thickness is 3 mm. There are six slots with size $L_d \times W_d = 4.5\,\text{cm} \times 0.4\,\text{cm}$ on the top of the enclosure.

In the same way, the voltage responses on the loads Z_2 and Z_6 computed by the proposed method, BLT equation, and CST are shown in Fig. 7. The ADM assessment of the results obtained by the

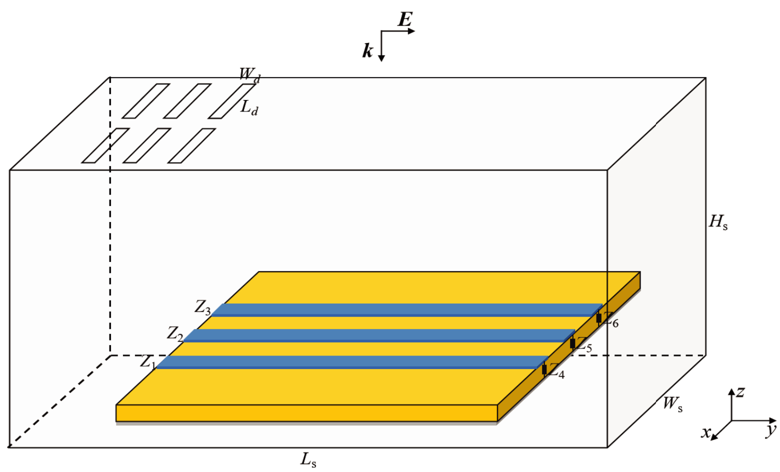


Figure 6. Coupling model of the parallel traces in shielded enclosure excited by ambient wave.

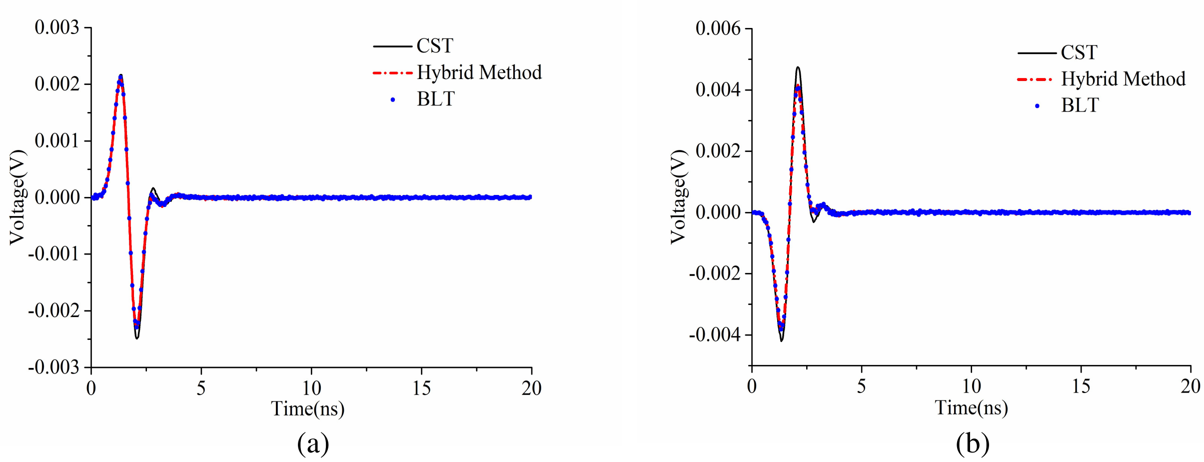
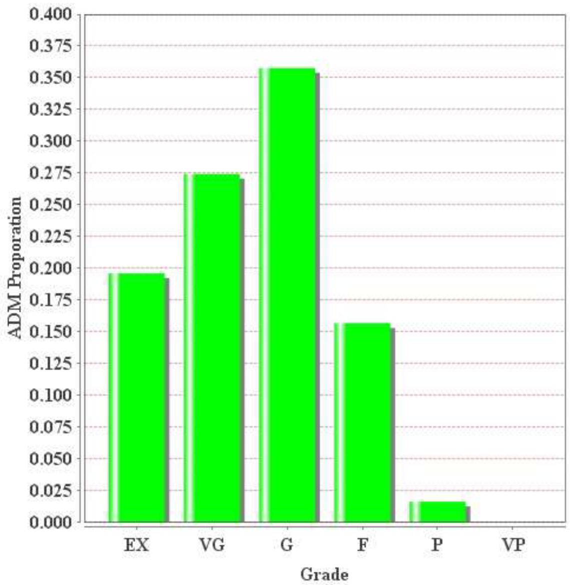
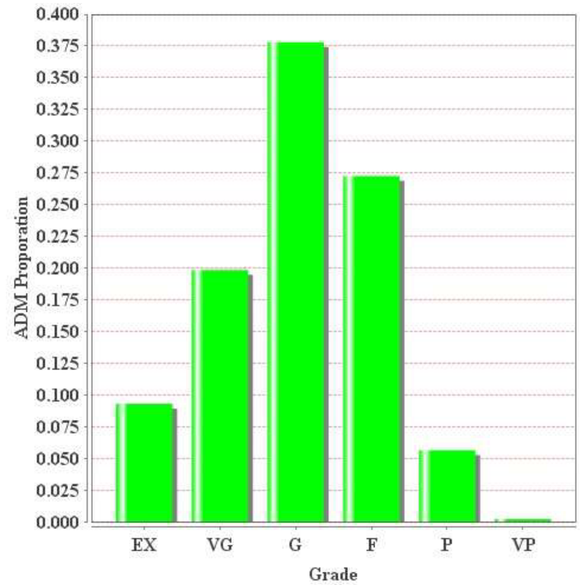


Figure 7. Voltage responses on the terminal loads of the traces for the second case. (a) Voltages on Z_2, (b) voltages on Z_6.



(a)



(b)

Figure 8. ADM assessment of the voltage responses on the loads for the second case. (a) ADM of the voltages on Z_2, (b) ADM of the voltages on Z_6.

proposed method and CST is shown in Fig. 8. We can also see that the results of the two methods still have good coincidence, because the proportion of the poor grade is less than 6%, and the relative difference error of the peak values of the two results is about 10.4%.

4. CONCLUSION

In this work, an efficient time domain hybrid method consisting of FDTD with subgridding technique and transmission line equations is presented to solve the coupling problem of parallel traces on the PCB excited by ambient wave. Within this method, FDTD combined with the subgridding technique separating the temporal and spatial subgridding interfaces is employed to improve the calculation precision of the excitation fields of the parallel traces. Meanwhile, the coupling process of the parallel traces is represented by the transmission line equations and solved by the FDTD to obtain the transient responses on the traces, which can avoid direct meshing the fine structures of the traces and realize the synchronous calculations of electromagnetic field radiation and transient responses on the traces.

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