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Application of Non-Embedded Uncertainty Analysis Methods in Worst Case Estimation of the EMC

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Abstract—In recent years, the non-embedded uncertainty analysis method has been widely used in the field of Electromagnetic Compatibility due to its wide application range. In this paper, from the perspective of the practical application of uncertainty analysis methods, four non-embedded uncertainty analysis methods are applied to the worst-case estimation of Electromagnetic Compatibility, which are the Monte Carlo Method, Stochastic Collocation Method, Stochastic Reduced-Order Models, and Kriging surrogate model method. The performances of four uncertainty analysis methods in terms of computational accuracy, computational efficiency, and ability to deal with complex problems are compared in detail by using the parallel cable crosstalk prediction example in the existing literature and the uncertainty analysis example of self-constructed optimization test function, which provides a theoretical basis for uncertainty analysis method to guide the actual Electromagnetic Compatibility design.

1. INTRODUCTION

Uncertainty analysis method has been attracting much attention in the field of Electromagnetic Compatibility (EMC) in recent years. In order to improve the credibility of simulation results, stochastic mathematical model is constructed to describe the uncertainty factors in the actual electromagnetic environment [1, 2].

The non-embedded uncertainty analysis method means that only a black box solver with good stability is needed in the process of realizing uncertainty analysis, without any rewriting of the algorithm inside the solver. For both EMC researchers and EMC engineering designers, it is an effective EMC prediction method to construct complex geometric models and perform finite element analysis with commercial electromagnetic simulation software [3, 4]. The underlying code of the electromagnetic analysis algorithm in commercial electromagnetic simulation software is usually not open source, so researchers in the field of EMC will pay more attention to the performance improvement research and development application of non-embedded uncertainty analysis methods. It is worth noting that the accuracy of the black box solver will directly determine the reliability of the deterministic EMC simulation results and then affect the credibility of the uncertainty analysis results, so it is very important to choose it reasonably.

The Monte Carlo Method (MCM) is the most widely used non-embedded uncertainty analysis method. It has almost the highest computational accuracy, but also the lowest computational efficiency. In the process of practical engineering application, the MCM is often unable to be used because of the long simulation time. However, in theoretical research, the results provided by the MCM are standard data to judge the accuracy of other uncertainty methods [5, 6]. The Stochastic Collocation Method (SCM) is an uncertainty analysis method based on generalized polynomial chaos theory. The convergence of the SCM is excellent, so it has the dual advantages of high computational efficiency and

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high computational accuracy. However, as the number of random variables describing the uncertainty input of EMC simulation increases, the computational efficiency of the SCM will decrease exponentially, which is the dimensional disaster problem [7–9]. In order to effectively solve the dimensional disaster problem, the Stochastic Reduced-Order Model (SROM) [10] and Kriging surrogate model method [11] have been applied to the uncertainty analysis of EMC simulation in recent years. The SROM has the best application range for the random input mathematical model, but it can only provide the mean prediction value and variance prediction value in the uncertainty analysis results. The Kriging surrogate model method is proposed based on the continuity assumption. The disadvantage is that its accuracy is poor when the EMC simulation nonlinearity is large.

In the application process of these non-embedded uncertainty analysis methods in the field of EMC, more attention is paid to the calculation of the quantitative transfer process in the simulation model. In other words, the goal is to obtain the uncertainty analysis results in the form of probability density curve, which is closer to the research in the mathematical sense. The field of EMC is a discipline that focuses more on practical applications, so the worst-case estimation form uncertainty analysis results are more meaningful, which is the focus of the "gene" level of EMC. For example, the maximum crosstalk results of cable bundles and the worst-case prediction of shielding effectiveness are typical worst-case estimation applications in the field of EMC. In this case, from the perspective of EMC prediction practicability, this paper compares the performance of different non-embedded uncertainty analysis methods in the worst-case estimation application and provides a theoretical basis for the practical application of uncertainty analysis methods in the field of EMC.

The structure of this paper is as follows. In Section 2, the worst-case estimation applications of different non-embedded uncertainty analysis methods are given. In Section 3, the worst-case estimation performance comparison is carried out by using the parallel cable crosstalk example. In Section 4, the uncertainty analysis problem is constructed by the test function, and the performance of each uncertainty analysis method is discussed in depth. In Section 5 gives the conclusion of this paper.

2. APPLICATION OF NON-EMBEDDED UNCERTAINTY ANALYSIS METHOD IN WORST-CASE ESTIMATION

In order to show the worst-case acquisition methods of different non-embedded uncertainty analysis methods, it is assumed that the uncertainty input of EMC simulation is a model of two random variables, namely $\xi = \{\xi_1, \xi_2\}$.

2.1. Worst-Case Estimation Based on the MCM

The MCM is proposed based on the weak law of large numbers. It directly samples the random variable vector $\{\xi_1, \xi_2\}$ and generates a large number of sampling points $\{x_i, y_i\}$. The deterministic EMC simulation is carried out on each sampling point, and the simulation result $U_i = \mathrm{EMC}(x_i, y_i)$ is obtained. Finally, the maximum or minimum value that meets the worst-case condition can be selected by direct statistics.

2.2. Worst-Case Estimation Based on the SCM

According to the generalized polynomial chaos theory, the chaotic polynomial corresponding to the random variable $\xi = \{\xi_1, \xi_2\}$ is derived. Take the zero points $\{a_1, \ldots, a_n\}$ of the chaotic polynomial, and then arrange it in the form of tensor product to obtain the collocation points, as shown below.

$$p(\xi) = \{a_1, \ldots, a_n\} \otimes \{a_1, \ldots, a_n\}\,. \tag{1}$$

Deterministic EMC simulation is performed on the collocation points $p(\xi)$, and multidimensional Lagrange interpolation is performed to obtain the random variable polynomial result $U_{\mathrm{SCM}}(\xi)$, as shown below.

$$U_{\mathrm{SCM}}(\xi) = \sum_{i=0}^{M} \mathrm{EMC}\left[p_i(\xi)\right] L_i\left[p_i(\xi), p(\xi), \xi\right] \tag{2}$$

Among them, EMC $[p_i(\xi)]$ refers to the result of deterministic EMC simulation at each collocation point $p_i(\xi)$. $L_i\,[p_i(\xi), p(\xi), \xi]$ is the result of the multidimensional Lagrange interpolation polynomial at the collocation point $p_i(\xi)$, and the random variable ξ is used as the function.

The polynomial result $U_{\text{SCM}}(\xi)$ is equivalent to a surrogate model, which directly brings in a large number of sampling points $\{x_i, y_i\}$ representing the random variable vector $\{\xi_1, \xi_2\}$, $U_i = U_{\text{SCM}}(x_i, y_i)$. Finally, the statistical U_i is selected to meet the maximum or minimum value of the worst-case condition.

It is worth noting that the calculation amount of the process of sampling points into polynomial $U_{\text{SCM}}(\xi)$ is negligible, so the SCM only needs to perform n^2-order deterministic EMC simulation, and the calculation efficiency is much higher than the MCM.

2.3. Worst-Case Estimation Based on the SROM

The first step of the SROM is the same as the MCM. The random variable vector $\{\xi_1, \xi_2\}$ is statistically sampled as $\{x_i, y_i\}$, and the similarity between sampling points is defined by Euclidean distance, namely $\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$. Applying the intelligent optimization algorithm, the center clustering is realized to select N representative sampling points $q_k = \{x_{q,k}, y_{q,k}\}$, and the weight $w_{q,k}$ of the whole represented by the representative sampling points is calculated. Deterministic EMC simulation is performed on representative sampling points, and the mean result $E(U)$ and variance result $\sigma(U)$ are calculated using statistical principles. The results are as follows.

$$E(U) = \sum_{k=1}^{N} w_{q,k} \times \text{EMC}(q_k) \tag{3}$$

$$\sigma(U) = \sum_{k=1}^{N} w_{q,k} \times [\text{EMC}(q_k) - E(U)]^2 \tag{4}$$

The SROM cannot directly obtain the worst-case estimation results. According to the nature of Gaussian distribution, "mean ±3 times standard deviation" can be used to replace the worst-case estimation, which is 99.73% confidence interval demarcation point.

2.4. Worst-Case Estimation Based on the Kriging Surrogate Model Method

Latin hypercube sampling is performed within the value range of the random variable vector $\{\xi_1, \xi_2\}$, and deterministic EMC simulation is performed at each sampling point $\{m_{x,i}, m_{y,i}\}$, which is recorded as $\text{EMC}(m_{x,i}, m_{y,i})$. Similar to the idea of the SCM, these deterministic simulation results are applied to construct a surrogate model, as shown below.

$$U(\xi) = \text{Kriging}\,[\text{EMC}(m_{x,i}, m_{y,i}), \xi] \tag{5}$$

Formula (5) is only a simplified version of the principal formula, and the detailed construction process of the Kriging surrogate model method can be referred to [11].

Similarly, the random variable vector $\{\xi_1, \xi_2\}$ is sampled in large quantities, and the calculation result U_i is obtained by taking the sampling point $\{x_i, y_i\}$ into the surrogate model of formula (5). Finally, the maximum or minimum value that meets the worst-case condition is selected.

In summary, the MCM needs to perform deterministic EMC simulation point by point on a large number of sampling points $\{x_i, y_i\}$, so the computational efficiency is extremely low, while the other three uncertainty analysis methods only need to simulate at a specific selected point. In addition, the SROM cannot directly obtain the worst-case estimation results, while the other three methods do not have this problem.

3. PARALLEL CABLE CROSSTALK PREDICTION EXAMPLE CONSIDERING GEOMETRIC RANDOMNESS

The parallel cable crosstalk prediction considering geometric randomness is the benchmark example in references [12] and [13], and its schematic diagram is shown in Figure 1.

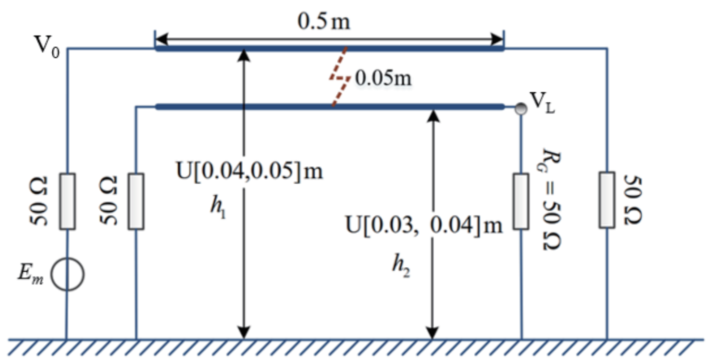

Figure 1. Example of parallel cable crosstalk prediction in references [12] and [13].

The height of two parallel cables has geometric randomness, which can be modeled by the uniform distribution random variables ξ_1 and ξ_2 in the range of $[-1, 1]$, as shown below.

$$h_1(\xi_1) = 0.045 + 0.005 \times \xi_1 \, [\text{m}] \tag{6}$$

$$h_2(\xi_2) = 0.035 + 0.005 \times \xi_2 \, [\text{m}] \tag{7}$$

The frequency range of the example is from 1 MHz to 200 MHz, and the far-end crosstalk voltage V_{dB} to be simulated is presented in the form of decibels.

$$V_{\text{dB}} = 20 \log_{10} \frac{|V_L|}{|V_0|} \tag{8}$$

Additional information about the example can be found in detail in [13].

When the cable is in a moving object such as a car or an aircraft, the geometric position of the cable will show randomness as in Formulas (6) and (7), and the geometric position changes in real time. At this time, it is clear that the maximum crosstalk value has the greatest impact on the disturbed cable, that is, the maximum crosstalk value meets the electromagnetic protection requirements, and the entire system will be in a compatible and stable state. Therefore, for this uncertainty analysis example, the worst-case estimation results are more important than other results such as mean, standard deviation, and probability density curve.

Figure 2 shows the worst-case estimation results of the far-end crosstalk voltage of four non-embedded uncertainty analysis methods, namely MCM, SCM, SROM, and Kriging. The simulation results of MCM are regarded as standard data, and the accuracy of the simulation results of other uncertainty analysis methods is evaluated by the Feature Selection Verification (FSV) method. The results are shown in Table 1. The relevant content of the FSV method is detailed in [14, 15].

Table 1. The effectiveness evaluation results provided by the FSV method.

Non-embedded uncertainty analysis method	FSV values
SROM	0.0447
SCM	8.6811×10^{-4}
Kriging	0.0097

According to the results in Table 1, the SCM, SROM, and Kriging all belong to the "Excellent" level, but the SROM is slightly worse than the other two uncertainty analysis methods. The SROM is a highly accurate uncertainty analysis method, but it cannot directly obtain the worst-case estimate, so the error is introduced by the "mean ± 3 times standard deviation" representation.

The MCM uses 8000 deterministic EMC simulations to ensure convergence. In order to compare the accuracy, the other three uncertainty analysis methods use 16 EMC simulations, which show that

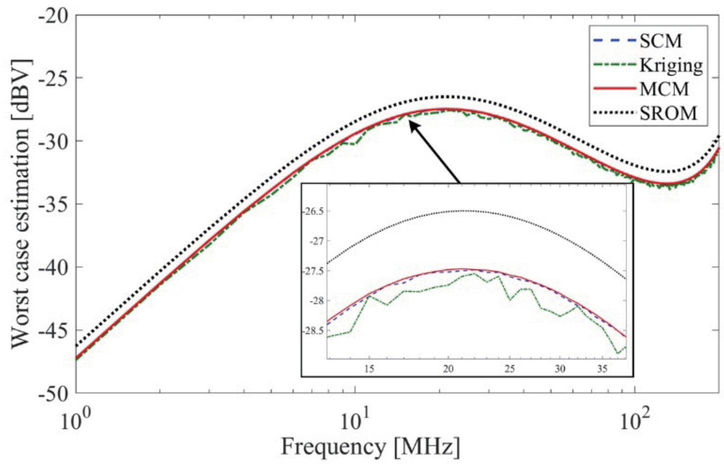

Figure 2. The worst-case prediction result of the far-end crosstalk voltage is estimated.

the calculation efficiency of the MCM is far worse than that of other methods. In summary, in the case of parallel cable crosstalk prediction considering geometric randomness, the SCM and Kriging perform well in accuracy and computational efficiency. The accuracy of the SROM is slightly worse, and the computational efficiency of the MCM is poor.

Figure 3 shows the far-end crosstalk voltage results of all MCM sampling points at 75 MHz. The overall result value presents a plane, which shows that the relationship between the far-end crosstalk voltage V_{dB} and the ground distance of the two cables is an approximate linear function. Obviously, the relationship between the input and output of the EMC simulation is relatively easy, that is, the parallel cable crosstalk prediction example considering geometric randomness is a simple uncertainty analysis problem. Therefore, in order to better show the performance of non-embedded uncertainty analysis methods, it is necessary to construct more complex EMC simulation uncertainty analysis problems.

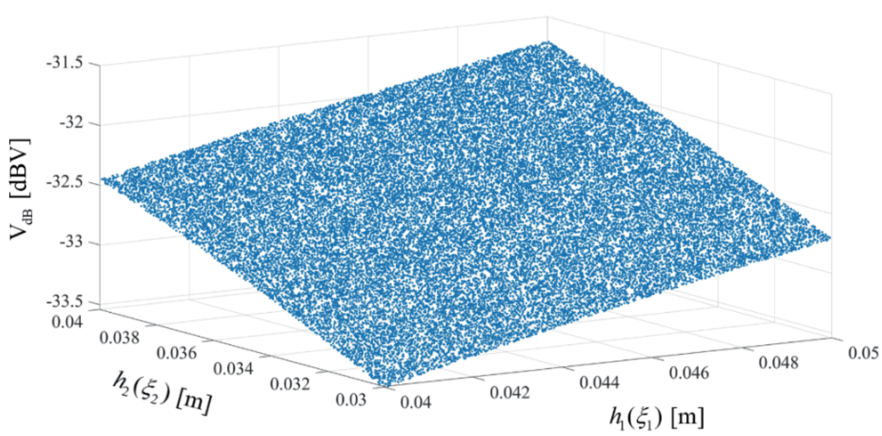

Figure 3. Uncertainty analysis results of the MCM at 75 MHz.

4. UNCERTAINTY ANALYSIS EXAMPLES CONSTRUCTED BY THE OPTIMIZED TEST FUNCTIONS

The optimization test function is usually used to test the performance of intelligent optimization algorithms. The uncertainty analysis problem constructed in reverse based on optimization test function is complex, so it can be more effectively used to test the performance of uncertainty analysis methods. By using the boundary [A, B] of the optimization test function, the model of uniformly distributed random variables is established, as follows.

$$D(\xi_3) = \frac{A+B}{2} + \frac{A-B}{2}\xi_3 \tag{9}$$

where ξ_3 is a uniformly distributed random variable in the interval $[-1, 1]$. All intervals are modeled by uniformly distributed random variables to form a random variable vector $\xi = \{\xi_3, \xi_4\}$, which constructs an uncertainty analysis problem. The two optimization test functions selected in this section are as follows.

$$z_1(x, y) = -|x| - |y| - |xy|, \qquad -5 \le x \le 5, \ -5 \le y \le 5 \tag{10}$$

$$z_2(x, y) = -4x^2 + 2.1x^4 - \frac{1}{3}x^6 - xy + 4y^2 - 4y^4, \qquad -5 \le x \le 5, \ -5 \le y \le 5 \tag{11}$$

The maximum values are known, namely $z_{1,\max} = 0$ and $z_{2,\max} = 1.032$, which are the answers to the worst-case estimates in uncertainty analysis. The uncertainty analysis based on the MCM is realized. The results are shown in Figure 4 and Figure 5, respectively. It can be seen that the uncertainty analysis problem is more complicated than the parallel cable crosstalk prediction example.

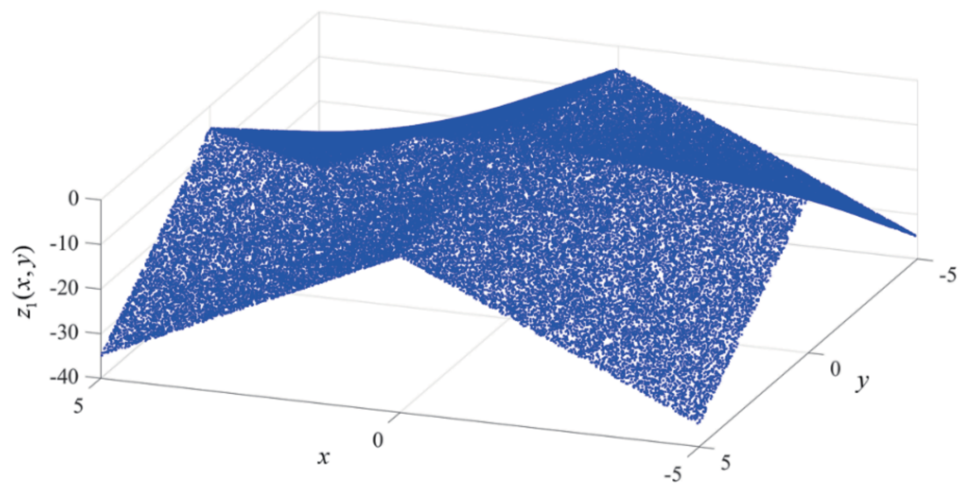

Figure 4. The MCM results of the optimized test function in Formula (10).

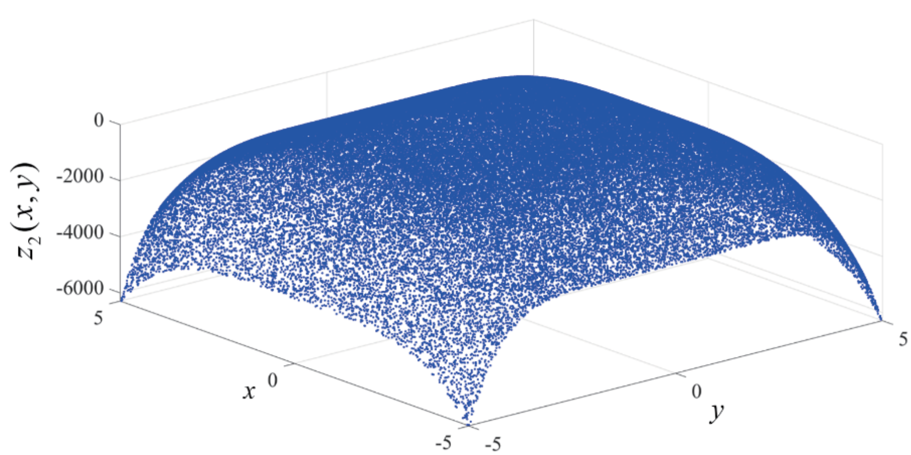

Figure 5. The MCM results of optimizing test functions in formula (11).

Table 2. The worst-case estimation results of two kinds of optimization test function uncertainty analysis examples.

	Simulation times	Problem 1	Problem 2
Answer	/	0	1.032
MCM	80000	-0.0252	1.0299
SCM	49	-0.0052	1.0257
	81	-5.8218×10^{-5}	1.0235
SROM	49	-3.0330	-756.6769
	81	-3.0378	-927.3897
Kriging	49	-0.5625	43.3622
	81	-0.3943	69.2149

Table 2 shows the worst-case estimation results of the MCM, SCM, SROM, and Kriging in the uncertainty analysis problems constructed in Figure 4 and Figure 5. The MCM still uses 80,000 deterministic simulations, and the remaining three non-embedded uncertainty analysis methods give the estimation results under 49 simulations and 81 simulations, respectively.

From the results of Table 2, the difference between the SROM and correct answer is the largest, which is a completely wrong result. The main reason is that the SROM cannot directly give the worst-case estimation results, and the accuracy and stability of the results in the form of “mean ±3 times standard deviation” are poor. Therefore, this representation is not suitable for popularization and application. The accuracy of the Kriging is not as good as that of solving the parallel cable crosstalk prediction example. The reason is that the complexity of the uncertainty analysis problem in Figure 4 and Figure 5 becomes larger, and the disadvantage of poor convergence of the Kriging surrogate model method appears, which affects the accuracy. For the MCM and SCM, their worst-case estimation results are within the acceptable range. The results of the MCM are better in Problem 2, while the results of the SCM are better in Problem 1. The MCM requires much deterministic simulation time. Thus, from the perspective of computational efficiency and accuracy, the SCM is the most suitable uncertainty analysis method for this example.

5. CONCLUSION

For the EMC simulation, this paper realizes the application of the existing non-embedded uncertainty analysis method in the worst-case estimation. The performances of the MCM, SCM, SROM, and Kriging surrogate model method are compared in detail in the parallel cable crosstalk prediction example considering geometric randomness and the uncertainty analysis example of self-built optimization test function, and the following conclusions are drawn. First, the MCM achieves high accuracy in worst-case estimation, but the computational efficiency is extremely low. Second, the SROM cannot directly give the worst-case estimation results, and the accuracy of the results in the form of “mean ±3 times standard deviation” is poor, which is not suitable for this application. Third, the Kriging surrogate model method is not suitable for this application when the EMC simulation problem becomes complicated, and the accuracy is poor. Finally, the SCM has both computational efficiency and computational accuracy, which is most suitable for the worst-case estimation of the EMC simulation.

In order to show the complexity of the EMC simulation uncertainty analysis, this paper adopts two-dimensional random variables model. At this time, the inherent dimension disaster problem of the SCM is not exposed. Therefore, in practical application, we should avoid using the SCM to deal with the worst-case estimation problem in the case of multi-dimensional random variables and strive to alleviate the dimension disaster problem in theoretical research. For the other three non-embedded uncertainty analysis methods, it is necessary to improve the computational efficiency of the MCM as much as possible, to propose the worst-case estimation representation method of the SROM as much as possible, and to improve the ability of the Kriging surrogate model method to deal with nonlinear simulation as much as possible.

REFERENCES

1. Manfredi, P., D. V. Ginste, I. S. Stievano, D. D. Zutter, and F. Canavero, "Stochastic transmission line analysis via polynomial chaos methods: An overview," *IEEE Electromagnetic Compatibility Magazine*, Vol. 6, No. 3, 77–84, 2017.
2. Manfredi, P., D. V. Ginste, D. D. Zutter, and F. Canavero, "Generalized decoupled polynomial chaos for nonlinear circuits with many random parameters," *IEEE Microwave & Wireless Components Letters*, Vol. 25, No. 8, 505–507, Aug. 2015.
3. Spindelberger, C., H. Arthaber, "Improving the performance of direct-conversion SDRs for radiated precompliance measurements," *IEEE Letters on Electromagnetic Compatibility Practice and Applications*, Vol. 5, No. 1, 22–26, 2023.
4. Chen, J., S. Portillo, G. Heileman, G. Hadi, R. Bilalic, M. Martínez-Ramón, S. Hemmady, and E. Schamiloglu, "Time-varying radiation impedance of microcontroller GPIO ports and their dependence on software instructions," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 64, No. 4, 1147–1159, 2022.
5. Pignari, S. A., G. Spadacini, and F. Grassi, "Modeling field-to-wire coupling in random bundles of wires," *IEEE Electromagnetic Compatibility Magazine*, Vol. 6, No. 3, 85–90, 2017.
6. Xie, H., J. F. Dawson, J. Yan, A. Marvin, and M. Robinson, "Numerical and analytical analysis of stochastic electromagnetic fields coupling to a printed circuit board trace," I*EEE Transactions on Electromagnetic Compatibility*, Vol. 62, No. 4, 1128–1135, 2020.
7. Manfredi, P., "A hierarchical approach to dimensionality reduction and nonparametric problems in the polynomial chaos simulation of transmission lines," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 62, No. 3, 736–745, 2019
8. Cui, C. F. and Z. Zhang, "High-dimensional uncertainty quantification of electronic and photonic IC with non-Gaussian correlated process variations," *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, Vol. 39, No. 8, 1649–1661, 2019.
9. Bai, J., G. Zhang, A. Duffy, and L. Wang, "Dimension-reduced sparse grid strategy for a stochastic collocation method in EMC software," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 60, No. 1, 218–224, 2018.
10. Fei, Z., H. Yi, J. Zhou, and X. Qian, "Uncertainty quantification of crosstalk using stochastic reduced order models," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 59, No. 1, 228–239, 2016.
11. Ren, Z., J. Ma, Y. Qi, D. Zhang, and C.-S. Koh, "Managing uncertainties of permanent magnet synchronous machine by adaptive kriging assisted weight index monte carlo simulation method," *IEEE Transactions on Energy Conversion*, Vol. 35, No. 4, 2162–2169, 2020.
12. Bai, J., J. C. Sun, and N. Wang, "Convergence determination of EMC uncertainty simulation based on the improved mean equivalent area method," *The Applied Computational Electromagnetics Society Journal*, Vol. 36, No. 11, 1446–1452, 2021.
13. Bai, J., G. Zhang, D. Wang, A. P. Duffy, and L. Wang, "Performance comparison of the SGM and the SCM in EMC simulation," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 58, No. 6, 1739–1746, 2016.
14. Duffy, A. P., A. Orlandi, and G. Zhang, "Review of the feature selective validation method (FSV). Part I — Theory," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 60, No. 4, 814–821, 2018.
15. Orlandi, A., A. P. Duffy, and G. Zhang, "Review of the feature selective validation method (FSV). Part II — Performance analysis and research fronts," *IEEE Transactions on Electromagnetic Compatibility*, Vol. 60, No. 4, 1029–1035, 2018.