(Received 29 August 2023, Accepted 13 December 2023, Scheduled 21 December 2023)

Evaluation of the Reliability of a Magnetic Levitation System by the Intrusive Stochastic Finite Element Method

Zehor Oudni[1,2,*] and Thinhinane Mahmoudi[1]

[1] Electrical Engineering Department, University Mouloud Mammeri of Tizi-Ouzou, Algeria
[2] Renewable Energy Management Laboratory (LMER), Université Abdrahmane Mira, Bejaia, Algeria

ABSTRACT: This work concerns the study of the reliability of a magnetic levitation system. A numerical calculation method based on the introduction of a random variable on the physical property of the materials constituting the levitation system is proposed. The latter is called intrusive stochastic finite element method (ISFEM), and the randomness of physical properties is taken care of, thus modeling in uncertain medium is feasible. The electromagnetic problem is treated with 2D hypotheses for modeling in an uncertain environment. This method was developed in 1991 and used for sensitivity and reliability analysis in the mechanical field; it is extended to the study of applications in linear elasticity and in electromagnetism. The random variable is of Gaussian type. The assessment of the reliability of the levitation system is discussed. The results obtained are compared with those found by the Latin hyper cube method. The intrusive stochastic finite element model provides very conclusive results in a very short time compared to those obtained by Latin hyper cube modeling.

1. INTRODUCTION

The study of the reliability of the electromagnetic systems related to the degradation of the physical properties of the materials constituting them leads to considering the study in uncertain environment. The electromagnetic modelling of these systems in order to carry out a sensitivity analysis and to evaluate the reliability encourages us to consider the physical parameters and random variables [1–4]. Taking into account the randomness of physical properties in the modeling of uncertain environments is essential to improving the accuracy of predictions, reducing the risks associated with these magnetic levitation systems, and better understanding the uncertainties and risks associated with this type of system. Physical properties, such as density and magnetic permeability, can fluctuate significantly from material to material and vary depending on temperature [5] or other factors. These variations must be considered in order to obtain models better representing real conditions and precise predictions under uncertain conditions of the behaviour of the magnetic levitation system.

Knowing that the magnetic forces which keep the object levitating are very sensitive to variations in the physical properties of the materials involved, these uncertainties must be considered in modelling in order to avoid errors in predicting the behaviour of these types of systems since they are used in critical applications, such as magnetic levitation trains, where modelling errors can induce errors and serious consequences.

Experimental methods such as Weibull's law provide answers that tell us about reliability. But the time necessary to obtain the results and the cost of the installations for the experiment are thus exaggerated compared to the methods of simulation in general [6, 7]. ISFEM is a method based on the stochastic finite element method used for sensitivity and reliability analysis in the mechanical and electromagnetic field. It is a powerful method that allows uncertainties in material properties and boundary conditions to be taken into account in numerical simulations. The main steps of ISFEM are the definition of random variables, the propagation of uncertainties, the sensitivity analysis, and the reliability analysis.

ISFEM is used to evaluate the reliability of mechanical systems subjected to uncertain conditions, and the probability of failure is thus determined. ISFEM makes it possible to optimize the design of mechanical systems by integrating uncertainties in material properties and boundary conditions. ISFEM is also used in the characterization and evaluation of defects in conductive and composite materials using the non-destructive eddy current testing (NDT-EC) technique.

The main advantages of ISFEM, compared to other stochastic modelling methods, lie in its precision, efficiency, and flexibility. It allows processing and post-processing in a single step without resorting to the inverse problem.

In short, ISFEM is a powerful stochastic modelling method which allows uncertainties to be taken into account in numerical simulations in the mechanical and electromagnetic domains with very reduced calculation times [8–11].

This method achieves a reduced computation time compared to other simulation methods such as Monte Carlo (MC) and Latin Hypercubic (LHC), as well as approaches based on experimental analysis [12–15].

The present study consists in representing the response of a magnetic levitation system in the form of probability [16–18]. Magnetic levitation is a technology that relies on the use of magnetic fields to suspend an object in the air without any physical support. Practical applications are present in trans-

* Corresponding author: Zehor Oudni (z_mohellebi@yahoo.fr).

port, storage, and suspension for the stabilization of satellites in space, in medicine [19], magnetic bearings to prevent wear, and in the field of scientific research [20].

The context of the research carried out in this article is magnetic levitation applied to Maglev, intended for transport, also called magnetic levitation train. These trains are characterized by traveling at high speeds. This performance is due to the elimination of friction between the wheels and the rails. These trains (Maglev) are efficient and fast, hence their interest in public transport, whose physical property of the constituent material is a random variable. This random variable is expanded in series by Hermite polynomials of a reduced centred Gaussian variable. The answer thus obtained is developed on series of polynomial chaos.

The computer code is produced independently in a Matlab environment, and it is associated with the intrusive stochastic finite element model. The application is a magnetic levitation system whose random variable is the magnetic permeability with the Gaussian type distribution [21, 22].

The choice of the Gaussian distribution also called normal law to model the random variable in magnetic levitation is motivated by its common use in the modelling of continuous random variables. It has symmetry and a bell shape. Magnetic permeability being a physical quantity which varies from one material to another can be considered as a continuous random variable. The magnetic forces that keep the object levitating can be affected by the hazard of this physical property. The Gaussian distribution is characterized by a mean value, and a standard deviation is a good approximation of these effects.

In addition, the Gaussian distribution is easy to use and understand, and it facilitates data analysis and modelling complex phenomena.

The development coefficients are determined by the projection method where the random variable follows the Gaussian type constitutive law. The developed computer code allows both sensitivity and reliability to be analyzed in a single step. The application-related system response is magnetic levitation force [22–26].

The reliability index β is deduced from the random solution obtained with the random magnetic permeability input data considered. The reliability status of the maglev system is achieved, and the results are given. A confrontation between the Latin hypercube method [13] and ISFEM is given for a single air gap or heightening of the load.

The probability of failure as a function of the reliability index is also determined by a correlation using a numerical method, the Lagrange polynomial.

A comparison between a reference curve obtained by the FORM simulation method [27] and that obtained by the ISFEM method is presented in Figure 9.

2. STOCHASTIC FINITE ELEMENTS FORMULATION

2.1. Deterministic Electromagnetic Equation

Modeling electromagnetic problems in an uncertain environment can be carried out by considering the following specific 2D assumptions: constant cross section, short wavelength, and slow variation. The first hypothesis assumes that the geometry of the environment is constant in the direction of propagation of the electromagnetic wave. The second assumes that the wavelength of the electromagnetic wave is much smaller than the size of the environment[28].

The third assumes that the electromagnetic properties of the environment vary slowly with respect to the wavelength of the electromagnetic wave.

There are limitations to using 2D assumptions. Indeed, they may not be accurate for electromagnetic problems which present significant 3D effects such as the effects of the propagation of electromagnetic waves in the direction perpendicular to the 2D plane and electromagnetic problems which deal with complex structures or heterogeneous materials. In these cases, 3D models are necessary to obtain accurate results [29].

Our study system verifies at least one of the 2D hypotheses, hence the consideration of a 2D electromagnetic problem whose unknown is represented by the magnetic vector potential A. To illustrate the application of the finite element method intrusive stochastics (ISFEM) in electromagnetic calculation, we will consider a magnetostatic problem represented by a partial derivative equation having the form below [2, 13]:

$$\frac{\partial}{\partial x}\left(\frac{1}{\mu}\frac{\partial A}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{1}{\mu}\frac{\partial A}{\partial y}\right)=-J_{sz} \tag{1}$$

μ: Magnetic permeability [H/m]

J_{sz}: Source current density along z direction [A/m^2]

The finite elements formulation of (1) using Galerkin method [30–32], by considering homogeneous Dirichlet boundary conditions, leads to the following matrix system:

$$[M]\,[A]=[F] \tag{2}$$

Elements of matrix and vector are defined by general terms below:

$$M_{ij}=\iint_{\Omega}\frac{1}{\mu}\left(\frac{\partial\alpha_i}{\partial x}\cdot\frac{\partial\alpha_j}{\partial x}+\frac{\partial\alpha_i}{\partial y}\cdot\frac{\partial\alpha_j}{\partial y}\right)dxdy \tag{3}$$

$$F_i=\iint_{\Omega}J_{sz}\alpha_i dxdy \tag{4}$$

$$[A]=[A_1,A_2,...,A_n]^T$$

$[A]$: Unknowns vector

α_i: Shape function at node i

α_j: Projection function which is chosen identical to shape function α_i.

2.2. Hermite Polynomials

We consider X a real random variable of finite variance. We can develop variable X on the basis of the Gaussian Hermite polynomials [33, 34, 24].

$$X=\sum_{i=0}^{\infty}a_iH_i\left(\xi\right) \tag{5}$$

$\{H_i(x),\, i = 0, ..., \infty\}$ are Hermite polynomials, and $\{a_i,\, i = 0, ..., \infty\}$ are coefficients of Hermite polynomials, where ξ is a random variable Gaussian centred reduced (r.v.g.c.r) of density of probability [35–38]:

$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \tag{6}$$

The Hermite polynomials are expressed by:

$$H_i(x) = (-1)^i e^{\frac{x^2}{2}} \frac{d}{dx}\left(e^{\frac{-x^2}{2}}\right) \tag{7}$$

We notice that all Hermite polynomials are orthogonal between them with regard to the Gaussian measure [15, 33]:

$$E\left[H_n(\xi(\theta))\, H_m(\xi(\theta))\right] = 0, \text{ if } n \neq m \tag{8}$$

where $E[\cdot]$ denotes the mathematical expectation which in the case of Gaussian random variable can be expressed by:

$$E\left[g(\xi)\right] = \int_{-\infty}^{+\infty} g(x)\phi(x)\, dx \tag{9}$$

$g(\xi)$ is a function of real variable. We consider X as input data (X is then a vector of dimension M). It is necessary to consider the Hermite polynomials of multivariables $\Psi_n(\xi_1(\theta), ..., \xi_n(\theta))$ defined by:

$$\Psi_n(\xi_1(\theta), ..., \xi_M(\theta)) = \prod_{i=1}^{M} H_{\alpha j}(\xi_i(\theta)) \tag{10}$$

For the computation of the coefficients of Hermite polynomials two methods could be used [9, 11]:

- Projection method
- Collocation method

The work develops the method of projection. The method of projection uses the orthogonality of the Hermite polynomials with regard to the Gaussian measure. By using (8), the calculation of the mathematical expectation allows to obtain:

$$E[XH_i(\xi)] = a_i E[H_i(\xi)^2] \tag{11}$$

Knowing that:

$$E[H_i(\xi)^2] = i! \tag{12}$$

then, the Hermite polynomials coefficients are given by:

$$a_i = \frac{E[XH_i(\xi)]}{i!} \tag{13}$$

In (13), a_i are the unknowns' coefficients to be determined. When we use the Iso-probabilistic transformation $X \to \xi$, with the equality verification $F_X(X) = \Phi(\xi)$, we can write:

$$X = F_X^{-1}(\Phi(\xi)) \tag{14}$$

And then,

$$E[XH_i(\xi)] = \int_R F_X^{-1}(\Phi(t))\, H_i(t)\, \phi(t)\, dt \tag{15}$$

From (13) and (15), we deduce the final form of Hermite polynomials coefficients:

$$a_i = \frac{1}{i!}\int_R F_X^{-1}(\Phi(t))\, H_i(t)\, \phi(t)\, dt \tag{16}$$

When X follows a Gaussian law of average μ_t and of standard deviation σ_t, the calculation of the coefficients of the development (8) is as follows:

$$a_0 = \mu_t \tag{17}$$

$$a_1 = \sigma_t \tag{18}$$

$$a_i = 0\ si\ i > 0 \tag{19}$$

The projection of the unknown A in the base of Hermite polynomials allows writing:

$$A^i = \sum_{j=0}^{n_A} A_j^i \Psi j(\xi_1, \cdots, \xi_M) \tag{20}$$

$$A = \sum_{j=0}^{n_A} A_j \Psi j(\xi_1, \cdots, \xi_M) \tag{21}$$

$\{\xi_1, ..., \xi_2\}$ are the Gaussian centred reduced variables.

n_A: Polynomial chaos order

The modelling of Gaussian random variables uses polynomial chaos of order 2 for the decomposition of them into orthogonal Hermite polynomials [39–41].

This process is exploited to model the uncertainties of magnetic permeability as a Gaussian random variable in the context of our study. Taking into account the hazard on the magnetic permeability of the environment makes it possible to assess the impact on the results of the simulation.

The definition of magnetic permeability and source current density as random variables in the base of Hermite polynomials permits to write:

$$\mu = \sum_{i=0}^{n_\mu} \mu_i \Psi_i(\xi_1) \tag{22}$$

$$J = \sum_{i=0}^{n_J} J_i \Psi_i(\xi_2) \tag{23}$$

Then, the mass matrix of random problem can be written.

$$M = \sum_{e=1}^{N_e} \iint_{\Omega_e} M^e\left(\frac{1}{\mu}(\xi_1)\right) d\Omega_e \tag{24}$$

M^e is the random elementary matrix related to element e of solving domain. N_e is the total number of elements of meshed domain.

2.3. Polynomial Chaos

By introducing the development (16), (18) into the system (3), we shall have:

$$M = \sum_{j=0}^{p-1} M_j \Psi_j (\xi_1, \xi_2) \quad (25)$$

$$F = \sum_{j=0}^{p-1} F_j \Psi_j (\xi_1, \xi_2) \quad (26)$$

2.4. Stochastic Problem Construction

We consider $n_A = p - 1$ and minimize the residue:

$$\varepsilon_p = \left(\sum_{i=0}^{p-1} M_i \Psi_i (\xi_1, \xi_2)\right) \left(\sum_{j=0}^{p-1} A_j \Psi_j (\xi_1, \xi_2)\right) - \sum_{j=0}^{p-1} F_j \Psi_j (\xi_1, \xi_2) \quad (27)$$

This minimization in the sense of Galerkin means requiring the orthogonality of the residue with the base of projection $\{\Psi_j, j = 0, ..., p\}$ [31, 35]:

$$E[\varepsilon_p \Psi_k] = 0, \quad k = 0, ..., p-1$$

Then we deduce:

$$\sum_{i=0}^{p-1} \sum_{j=0}^{p-1} M_i A_j E[\psi_i \psi_j \psi_k] = F_k \quad (28)$$

$k = 0, ..., p-1$.

This system represented by (28) whose size is the product of the number of degrees of freedom Nddl of the model in the determinist finite elements (i.e., the size of every vector A_j) by the order of the development P gives vectors $[A_0, A_1, ..., A_{p-1}]$ [12, 26].

$$\sum_{j=0}^{p-1} M_{jk} A_j = F_k \quad (29)$$

The system (29) is developed then in the following way:

$$\begin{bmatrix} \mathbf{M}_{00} & \mathbf{M}_{01} & \mathbf{M}_{02} & \dots & \mathbf{M}_0 & p-1 \\ \mathbf{M}_{10} & \mathbf{M}_{11} & \mathbf{M}_{12} & \dots & \mathbf{M}_1 & p-1 \\ \mathbf{M}_{20} & \mathbf{M}_{21} & \mathbf{M}_{22} & \dots & & \\ \vdots & & & \ddots & & \\ \mathbf{M}_{p-1} & \mathbf{0} & & \dots & \mathbf{M}_{P-1} & p-1 \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 \\ \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \vdots \\ \mathbf{A}_{p-1} \end{bmatrix} = 0 \begin{bmatrix} \mathbf{F}_0 \\ \mathbf{F}_1 \\ \mathbf{F}_2 \\ \vdots \\ \vdots \\ \mathbf{F}_{p-1} \end{bmatrix} \quad (30)$$

2.5. Magnetic Force Calculation

The magnetic force F_m acting on load piece is obtained using magnetic energy W_m variation formulate [15, 18, 23, 31]:

$$F_m = \frac{\partial W_m}{\partial y} \quad (31)$$

$$W_m = \int_\Omega \left(\int_0^B H \cdot dB\right) d\Omega \quad (32)$$

Mechanical equation governing the displacement of the mobile part of the system is given by the related classical equation:

$$m \frac{dv(t)}{dt} = F_m \pm F_g \quad (33)$$

m: Load mass [kg],
$v(t)$: Speed [m/s],
F_g: force due to load weight [N].

The expression of the speed is then as follows:

$$v(t + \Delta t) = (F_m(t) \pm F_g) \frac{\Delta t}{m} + v(t) \quad (34)$$

Δt: Time step [s].

3. APPLICATION TO MAGNETIC LEVITATION SYSTEM

The device considered in this study to validate the stochastic electromagnetic model is a magnetic levitation system, represented in Figure 1. This same device allowed the determination of the reliability index by the Hypercube Latin method [12, 27].

3.1. Geometrical Characteristics

The geometrical characteristics of the system of levitation are shown in Figure 1.

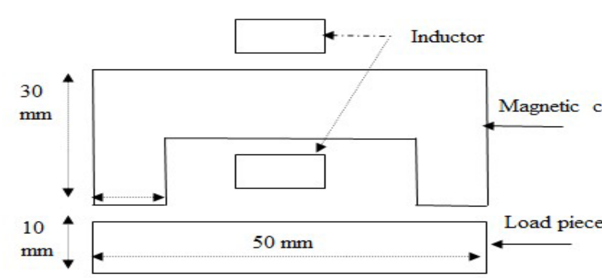

FIGURE 1. Levitation studied system.

3.2. Physical Properties

The physical properties considered are:

1. For the inductor:

Electric Conductivity: $\sigma = 5.9 * 1e + 7 [\Omega \cdot \text{m}]^{-1}$
Magnetic reluctivity: $\mu = \mu_0$ [H/m]

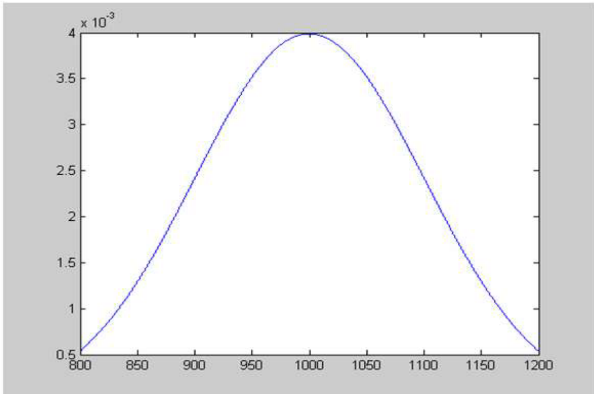

FIGURE 2. Normal distribution of permeability μ_r.

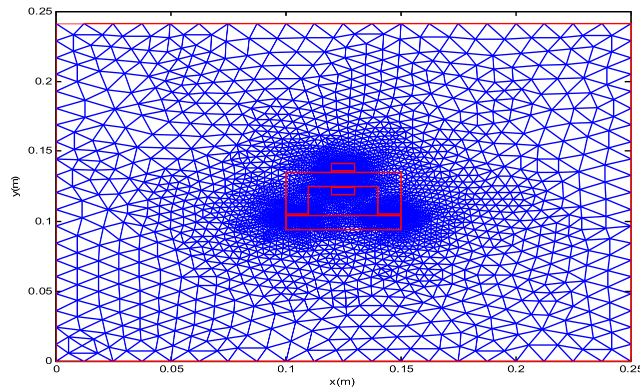

FIGURE 3. Solving domain mesh.

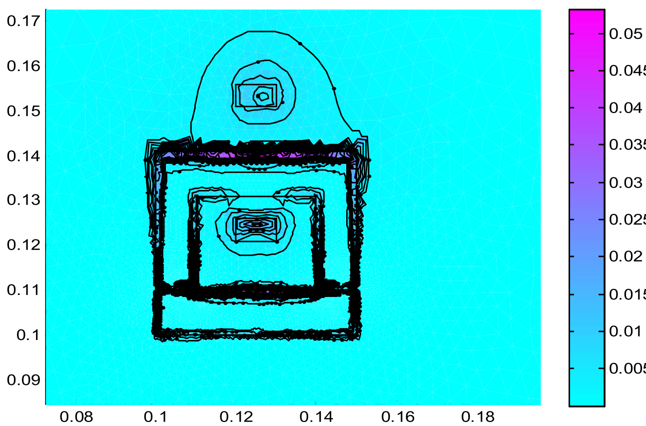

FIGURE 4. Magnetic flux density vectors [T] (solution for $p = 1$).

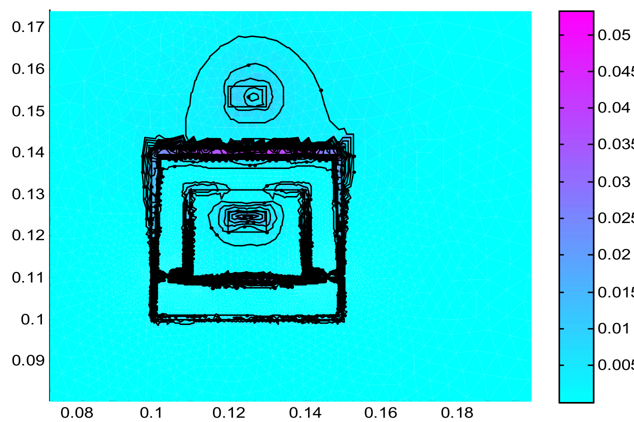

FIGURE 5. Magnetic flux density vectors [T] (solution for $p = 2$).

Power frequency: $f = 50\,\text{Hz}$

2. For the levitated piece:

Electric conductivity: $\sigma = 1e + 6\,[\Omega \cdot \text{m}]^{-1}$
Magnetic permeability (random variable): $\mu = \mu_1$ [H/m]

The distribution of the random variable of the magnetic permeability μ_1 is of Gaussian type, characterized by a mean value and a standard deviation [13, 30]. This is shown in Figure 2.

3.3. Mesh Domain

The mesh of solving domain is as shown in Figure 3.

The mesh of the solving domain is characterized by 4533 nodes and 8984 triangles.

4. APPLICATION AND RESULTS

4.1. Random Solution of Stochastic Problem

The resolution allows obtaining three solutions A_0, A_1, and A_2 of dimension np corresponding to the order of the considered Hermite polynomials. The results obtained for the distribution of the magnetic vector potential A make it possible to deduce the distribution of magnetic flux density vectors which are represented in Figures 4, 5, and 6.

The obtained results show that these last ones depend on the order p of Hermite polynomials used where for every order $p = 1$, $p = 2$, and $p = 3$ the distribution of the magnetic vector potential differs. In order to be able to validate the model of calculation by stochastic finite elements method applied to a system of magnetic levitation, an analysis of reliability was undertaken.

4.2. Reliability Analysis

The study of reliability of the system of magnetic levitation was realized by proceeding to the calculation of the function of limit state given by:

$$g_{\text{ISFEM}} = (F_g) - \sum_{j=0}^{p-1} F_j^{i0} \Psi_j \left(\xi_1, \xi_2\right)_{\max} \tag{35}$$

The obtained solution allows to deduct the indication of reliability from the calculation of g_{ISFEM} which is the function of limit state [9, 21, 36, 37].

The obtained solution makes it possible to deduce the reliability index from the calculation of g_{ISFEM} which is the limit state function [9, 35].

$$\beta = \text{Min}\sqrt{g_{\text{ISFEM}}} \tag{36}$$

air gap (m)	0.001	0.002	0.003
g_{ISFEM}	0.4905	0.4430	0.4346
β_{computed}	0.7004	0.6656	0.6592
β_{HLC}	0.7089	-	-

TABLE 1. Comparative values of the reliability index.

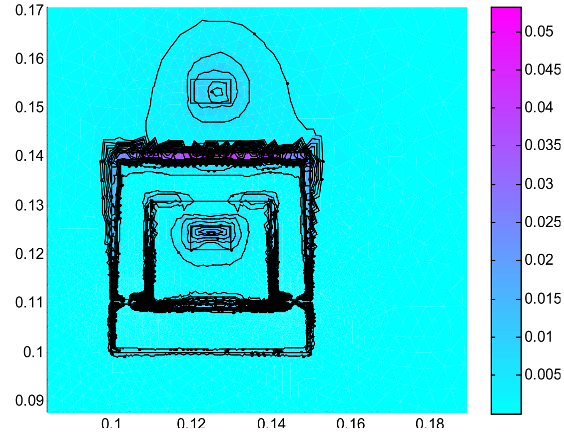

FIGURE 6. Magnetic flux density vectors [T] (solution for $p = 3$).

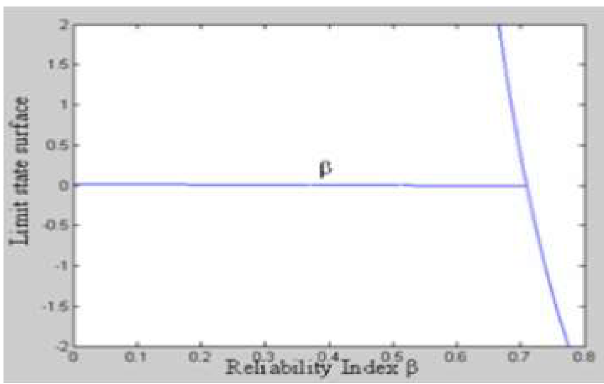

FIGURE 7. The limit state surface in the centered plane reduces with the reliability index.

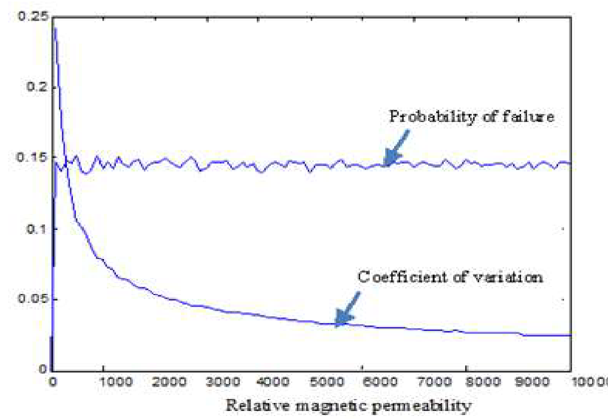

FIGURE 8. Probability of failure as a function of the relative magnetic permeability of the $\mu_{1\text{average}}$.

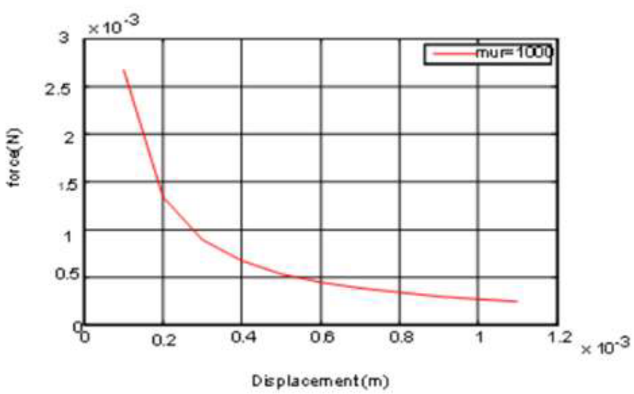

FIGURE 9. Evolution of the magnetic levitation force as a function of displacement.

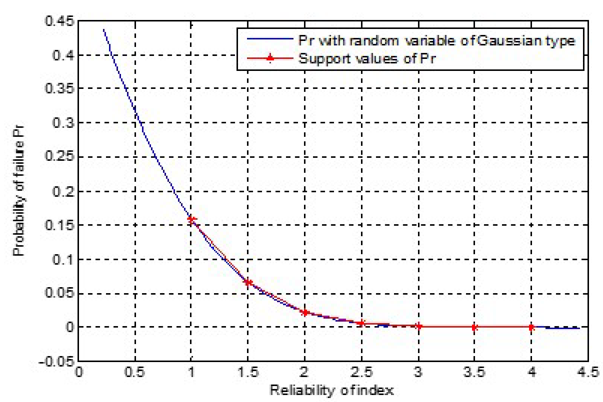

FIGURE 10. Probability of failure as a function of the reliability index.

The Latin hypercube simulation method is proposed as a confrontation method with the ISFEM. The different results obtained are presented below. It should be noted that this method is based on several draws. Considering a single air gap, the calculations take a lot of time, and the operation requires great rigor to achieve conclusive results [12, 13].

The Latin hypercube method is a non-intrusive method of stochastic modelling. It is used for sensitivity and reliability analysis in the mechanical field. This is based on the finite element method (FEM) which takes into account uncertainties in material properties during numerical simulations.

The main steps of the Latin hypercube method are the definition of random variables, generation of a set of random design points (Samplings), numerical simulation for each design point, sensitivity analysis, and analysis of reliability [42].

The criteria for comparing results between the Latin hypercube method and ISFEM depend on the objectives of the analysis. In our case, efficiency, precision, and calculation times are the comparison criteria.

The Latin hypercube method is a slower and heavier method, and presents a lack of precision in terms of probability of failure compared to ISFEM which is faster and more precise. However, it can be an alternative to ISFEM in certain cases.

The results obtained from simulations of the ISFEM method for different air gaps and by Hypercube Latin for a single air gap shown in Figure 7 are summarized in Table 1.

Table 1 of the results obtained by the ISFEM and the Latin hypercube illustrates that the execution time of the calculation code for three air gaps for the ISFEM only allows results to be obtained for a single air gap with the hyper cube Latin. For ISFEM, the simulation time is 28.43 seconds for a number of mesh nodes 4533, and the number of triangles is 8984 with a 3.4 GHz CPU single core and 218 MB of RAM. On the other hand, the simulation time for the Latin hypercube is practically three times more.

The probability of failure and the coefficient of variation [12, 13] are shown in the same Figure 8. We see that the value of the probability of failure is approximately 15%, for a $\mu_r = 1000$. We find the same value of $Pr = 15\%$, with the ISFEM method, for a reliability index equal to 1, this corresponds to the random permeability characterized by an average value $\mu_{1\text{average}} = 1000$ and a standard deviation of 0.85.

Figure 9 shows the evolution of the magnetic levitation force as a function of displacement for a magnetic permeability $\mu_{1\text{average}} = \mu_r = 1000$, standard deviation 1.

Figure 10 illustrates the probability of failure as a function of the reliability index obtained by the FORM simulation method [3, 15] and ISFEM method.

5. CONCLUSION

The intrusive stochastic finite element method (ISFEM) implemented in this work was applied to the study of a magnetic levitation system whose application is magnetic levitation train (Maglev). The used model is based on the consideration of magnetic permeability as a Gaussian random variable characterized by a mean value and a standard deviation; it is developed on the polynomial chaos of order 2.

This process made it possible to model uncertainties of the magnetic permeability as a continuous random variable. The impact on the simulation results is highlighted.

The random response of the stochastic problem obtained in orders 0, 1, and 2 makes it possible to deduce the limit state function and consequently the reliability index corresponding to the magnetic levitation force as well as the probability of failure.

The stochastic finite element model is confronted with the Latin hypercube simulation method, for the reliability index and the evaluation of the probability of failure, and the obtained results agree strongly. The reliability of the levitation system is highlighted by the presented results. It is noted that for the reliability index of 0.7, the probability of failure is significant of the order 26%. This tells us about the state of instability of the levitation system.

For modeling magnetic levitation systems, the ISFEM method is widely used. We do not have sufficient experimental evidence to say that the results obtained from ISFEM align with real-world observations or experimental data. However, we have presented comparative results which are consistent, emphasizing a very good agreement between the two stochastic methods, one intrusive and the other non-intrusive.

However, the ISFEM method was used for the characterization and evaluation of defects in conductive and composite materials by the technique of non-destructive testing by eddy currents (NDT-EC) [15, 35], and the results were validated by experimental data which align with real-world observations.

When carrying out the work, the ISFEM method devotes a very short time compared to the Latin hypercube simulation method, which amounts to the fact of implementing several codes for sampling, on the other hand, the ISFEM method, the analysis of the system, and the study of the reliability in only one step with a sequence of direct calculation.

The implementation of ISFEM in calculation code under Matlab environment was a challenge, and some source codes had to be modified in order to be able to simulate the stochastic aspect represented by the stochastic matrix. In the context of this study, there is a single random variable, and the size of the system to be simulated is very practical. However, if the random variables were to multiply, a matrix reduction system would become necessary in order to reduce the size of the stochastic matrix system. Structures requiring a 3D study could also use the reduction system.

If we have to address future applications or potential areas of research where the ISFEM approach could be exploited, it would be the mechanics of fluids, structures, solids, materials, acoustics, electromagnetism particularly in NDT-EC, structural dynamics, in short any system requiring the resolution of partial differential equations in uncertain environments.

We can conclude that the ISFEM method offers a great opportunity and tools to evaluate the reliability of a system evolving in an uncertain environment.

REFERENCES

[1] Bancel, F., "Magnetic nodes," *Journal of Physics D: Applied Physics*, Vol. 32, No. 17, 2155–2161, 1999.

[2] Furlani, E. P., *Permanent Magnet and Electromechanical Devices: Materials, Analysis, and Applications*, 1st ed., Academic Press, 2001.

[3] Lemaire, M., "Sensitivity and reliability analysis in an uncertain environment," in *5th European Conference on Numerical Methods in Electromagnetism, NUMELEC'06*, Lille, 2006.

[4] Logesh, K. and S. R. Madane, "Improved CLC routing protocol with node classification algorithm for MANET," *Applied Mathematics & Information Sciences*, Vol. 12, No. 5, 1013–1019, 2018.

[5] Al-Qrinawi, M. S., T. M. El-Agez, M. S. Abdel-Latif, and S. A. Taya, "Capacitance-voltage measurements of hetero-layer OLEDs treated by an electric field and thermal annealing," *International Journal of Thin Film Science and Technology*, Vol. 10, 217–226, 2021.

[6] Yonnet, J.-P. and H. Allag, "Three-dimensional analytical calculation of permanent magnet interactions by magnetic node representation," *IEEE Transactions on Magnetics*, Vol. 47, No. 8, 2050–2055, 2011.

[7] Yonnet, J., "Permanent magnet bearings and couplings," *IEEE Transactions on Magnetics*, Vol. 17, No. 1, 1169–1173, 1981.

[8] Berkache, A., J. Lee, and E. Choe, "Evaluation of cracks on the welding of austenitic stainless steel using experimental and numerical techniques," *Applied Sciences*, Vol. 11, No. 5, Mar. 2021.

[9] Berveiller, M., "Stochastic finite element analysis: Intrusive and non-intrusive approaches for reliability analysis," Ph.D. dissertation, Blaise Pascal-Clermont II University, 2005.

[10] Haldar, A. and S. Mahadevan, *Probability, Reliability, and Statistical Methods in Engineering Design*, 1st ed., Wiley, 1999.

[11] Pranesh, S. and D. Ghosh, "A FETI-DP based parallel hybrid stochastic finite element method for large stochastic systems," *Computers and Structures*, Vol. 195, 64–73, Jan. 2018.

[12] Fang, K. T. and C. X. Ma, "Wrap-around L2-discrepancy of random sampling, Latin hypercube and uniform designs," *Journal of Complexity*, Vol. 17, No. 4, 608–624, Dec. 2001.

[13] Fang, K. T., D. Maringer, Y. Tang, and P. Winker, "Lower bounds and stochastic optimization algorithms for uniform designs with three or four levels," *Mathematics of Computation*, Vol. 75, No. 254, 859–878, 2006.

[14] Lyskawinski, W., C. Jedryczka, D. Stachowiak, P. Lukaszewicz, and M. Czarnecki, "Finite element analysis and experimental verification of high reliability synchronous reluctance machine," *Eksploatacja i Niezawodnosc-Maintenance and Reliability*, Vol. 24, No. 2, 386–393, 2022.

[15] Oudni, Z., M. Feliachi, and H. Mohellebi, "Assessment of the probability of failure for EC nondestructive testing based on intrusive spectral stochastic finite element method," *Eur. Phys. J. Appl. Phys.*, Vol. 66, No. 5, 133–137, 2014.

[16] Furlani, E. P., "A formula for the levitation force between magnetic disks," *IEEE Transactions on Magnetics*, Vol. 29, No. 6, 4165–4169, Nov. 1993.

[17] Furlani, E. P., *Permanent Magnet and Electromechanical Devices: Materials, Analysis, and Applications*, 1st ed., Academic Press, 2001.

[18] Meeker, D. C., "Finite element method magnetics, version 4.0.1 (03déc2006 build)," http://femm.foster-miller.net, 2003.

[19] Elhadary, A. A., A. El-Zein, M. Talaat, G. El-Aragi, and A. El-Amawy, "Studying the effect of the dielectric barrier discharge non-thermal plasma on colon cancer cell line," *International Journal of Thin Film Science and Technology*, Vol. 10, 161–168, 2021.

[20] Thota, S. and S. D. Kumar, "A new reduction algorithm for differential-algebraic systems with power series coefficients," *Information Sciences Letters*, Vol. 10, No. 1, 59–66, 2021.

[21] Marsden, G., "Magnetic levitation kit," http://www.arttec.net/Levitation/Gallery/Levitation_Applications.htm.

[22] Kosmas, K. and E. Hristoforou, "The effect of magnetic anomaly detection technique in eddy current non-destructive testing," *International Journal of Applied Electromagnetics and Mechanics*, Vol. 25, No. 1-4, 319–324, 2007.

[23] Baghli, L., "Magnetic levitation," http://www.baghli.com/dspic_levitation.

[24] Baghli, L. and A. Rezzougm, "Magnetic levitation, an object-project approach," https://doi.org/10.1051/j3ea/2010002, 2010.

[25] Baghli, L. and A. Rezzoug, "Actionneurs linéaires, MRVlin et MSlin, un projet pédagogique," in *CETSIS 2007*, Vol. 7, 1638–1963, 2007.

[26] Marsden, G., "Levitation flotter objets in servocontrolled magnetic field," *Nuts & Volt Magazine*, Sep. 2003.

[27] Allag, H., J.-P. Yonnet, and M. E. H. Latreche, "Analytical calculation of the torque exerted between two perpendicularly magnetized magnets," *Journal of Applied Physics*, Vol. 109, No. 7, 07E701–07E701–3, Apr. 2011.

[28] Ali, A. R., N. T. M. Eldabe, A. E. H. A. E. Naby, M. Ibrahim, and O. M. Abo-Seida, "EM wave propagation within plasma-filled rectangular waveguide using fractional space and LFD," *The European Physical Journal Special Topics*, 2023.

[29] Abo-Seida, O. M., N. T. M. El-Dabe, A. R. Ali, and G. A. Shalaby, "Cherenkov FEL reaction with plasma-filled cylindrical waveguide in fractional D-dimensional space," *IEEE Transactions on Plasma Science*, Vol. 49, No. 7, 2070–2079, Jul. 2021.

[30] Da Silva Junior, C. R. A., A. T. Beck, and E. d. Rosa, "Solution of the stochastic beam bending problem by Galerkin method and the Askey-Wiener scheme," *Latin American Journal of Solids and Structures*, Vol. 6, No. 1, 51–72, Mar. 2009.

[31] Berveiller, M., B. Sudret, and M. Lemaire, "Comparison of methods for computing the response coefficients in stochastic finite element analysis," *Proc. Asranet*, Vol. 2, 51–72, 2004.

[32] Mahmuda Maya, M. U., M. N. Alam, and A. R. Ali, "Influence of magnetic field on MHD mixed convection in lid-driven cavity with heated wavy bottom surface," *Scientific Reports*, Vol. 13, No. 1, 18959, 2023.

[33] Carpenter, C. J., "Surface-integral methods of calculating forces on magnetized iron parts," *Proceedings of the IEE-Part C: Monographs*, Vol. 107, No. 11, 19–28, 1960.

[34] Jacques, R., *Precis of Probabilities and Statistics for the Use of Reliability*, Octares Edition, 1996.

[35] Oudni, Z., A. Berkache, H. Mehaddene, H. Mohellebi, and J. Lee, "Comparative study to assess reliability in the presence of two geometric defect shapes for non-destructive testing," *Przeglad Elektrotechniczny*, Vol. 95, No. 12, 48–52, 2019.

[36] Sudret, B., M. Berveiller, and M. Lemaire, *Stochastic Finite Elements in Linear Elasticity*, Elsevier, 2004.

[37] Sudret, B., M. Berveiller, and M. Lemaire, "Stochastic finite elements: New perspectives," in *16th French Mechanical Congress*, Nice, Sep. 2003.

[38] Harisson, A. J., "An optimised 50 Nms momentum wheel utilising magnetic repulsion bearings," in *Proceedings of ADCS Conference*, 389–393, Noordwijk, 1977.

[39] Yang, X.-J., A. A. Alsolami, and A. R. Ali, "An even entire function of order one is a special solution for a classical wave equation in one-dimensional space," *Thermal Science*, Vol. 27, No. 1, 491–495, 2023.

[40] Islam, S., B. Halder, and A. R. Ali, "Optical and rogue type soliton solutions of the (2+1) dimensional nonlinear Heisenberg ferromagnetic spin chains equation," *Scientific Reports*, Vol. 13, No. 1, Jun. 2023.

[41] Kumar, S., J. Cao, and M. Abdel-Aty, "A novel mathematical approach of COVID-19 with non-singular fractional derivative," *Chaos Solitons & Fractals*, Vol. 139, Elsevier, 2020.

[42] Dutta, S. and A. H. Gandomi, "Design of experiments for uncertainty quantification based on polynomial chaos expansion metamodels," *Handbook of Probabilistic Models*, 2020.