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Convergence Determination Method for Uncertainty Analysis Surrogate Models Based on MEAM

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ABSTRACT: In recent years, uncertainty analysis has become a hot topic in the field of Electromagnetic Compatibility (EMC), and nonintrusive uncertainty analysis methods have been widely applied due to their advantage of obtaining results without modifying the original solver. Among them, the Surrogate Model Method has attracted widespread attention from researchers in the field of EMC due to its high computational efficiency and resistance to the curse of dimensionality. However, the issue of determining the convergence of the surrogate models seriously affects the computational efficiency and convenience of this method in practical applications. To address this issue, a convergence determination method for uncertainty analysis surrogate models based on Mean Equivalent Area Method (MEAM) is proposed in this paper. The complete convergence time of the Surrogate Model Method can be accurately determined through iterative calculation by this method, and the effectiveness of the proposed method is verified by calculating parallel cable crosstalk prediction examples from published literature. Finally, based on the proposed convergence determination method, the real-time convergence determination problem of the Surrogate Model Method is also preliminarily discussed in this paper, and by establishing a polynomial relationship, the real-time convergence of the Surrogate Model Method can be roughly determined.

1. INTRODUCTION

Uncertainty analysis is a hot research topic in the field of EMC in recent years. It considers the input parameters of simulation models as uncertainty variables to improve the effectiveness of EMC simulation models or methods. Usually, such uncertainties are caused by various factors in the actual environment, such as motion or vibration, tolerances in the manufacturing process, and cognitive deficiencies of researchers.

The uncertainty analysis methods can be divided into two categories based on whether the original solver needs to be modified: intrusive and non-intrusive. Typical intrusive uncertainty analysis methods include Perturbation Method [1], Stochastic Galerkin Method [2, 3], and Stochastic Testing Method [4]. For solving complex EMC problems, commercial electromagnetic simulation software is often needed to simulate the actual electromagnetic environment [5]. The intrusive uncertainty analysis methods require modifications to the solver's program during the solution process. However, the vast majority of commercial electromagnetic simulation software companies have not fully opened up the core program of the solver, which leads to intrusive uncertainty analysis methods being unable to be used in many cases. In contrast, the non-intrusive uncertainty analysis methods do not require modifying the simulation software solver program, and only a stable deterministic solver is required to perform normal solution calculations, making them more practical than the intrusive uncertainty analysis methods.

Typical non-intrusive uncertainty analysis methods include Monte Carlo Method (MCM) [6], Stochastic Collocation Method (SCM) [7], Stochastic Reduced-Order Models (SROM) [8], Surrogate Model Method [9, 10], etc. MCM is based on the law of weak large numbers and has extremely high computational accuracy, but it has the disadvantage of low computational efficiency, so the results of the MCM are usually used as a standard to validate the accuracy of other efficient uncertainty analysis methods [3, 6, 7].

With the gradual improvement of generalized polynomial chaos theory in the field of computational fluid dynamics, SCM emerged, which has the dual advantages of high computational accuracy and efficiency. However, as the number of random variables increases, the computation time of the SCM will exponentially increase, leading to the curse of dimensionality [7, 11]. To effectively address the curse of dimensionality, SROM is widely applied in EMC uncertainty analysis. SROM has the best applicability to random input mathematical models, but it cannot directly provide accurate worst-case estimation results in uncertainty analysis, which greatly affects its applicability in practical applications [8].

Since 2020, uncertainty analysis methods based on surrogate models have been gradually proposed. The principle is to treat the surrogate model as a black box, and the deterministic simulation results of quantitative times are used as the input and output of the black box to train the surrogate model. Finally, a large number of samples are taken from the randomness of the model input to obtain the final result. This method has high computational efficiency without the curse of dimensionality, and its accuracy is controllable, making it a hot research di-

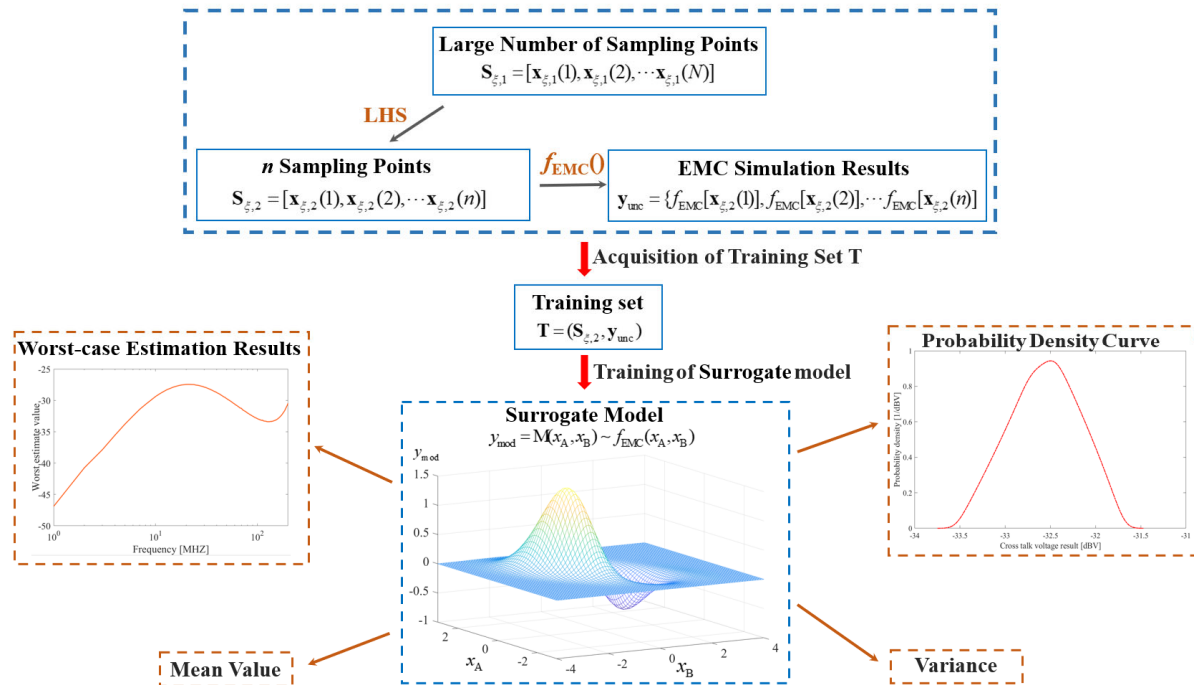* Corresponding author: Yujia Song (songyujia@dlut.edu.cn).

FIGURE 1. Schematic diagram of the Surrogate Model Method.

rection in recent times [9]. However, because users often cannot predict the number of deterministic simulations required to train the surrogate model during computation, when the algorithm stops computing before it is fully converged, there will be errors in the results. If too many computations are used to ensure algorithm convergence, it will lead to a significant waste of computing resources. So how to accurately determine the convergence of the surrogate model is a key issue that affects the computational efficiency of this method, and a convergence determination method for uncertainty analysis surrogate models is proposed in this paper which can accurately determine the convergence of the surrogate models, thereby improving the computational efficiency of Surrogate Model Method and reducing the waste of computational resources. Based on this method, a universal mathematical polynomial relationship for surrogate models is also constructed in this paper to study and discuss the real-time convergence determination problem of surrogate models.

The structure of this paper is as follows. Section 2 introduces the process of the Surrogate Model Method. Section 3 introduces the convergence determination method for uncertainty analysis surrogate models based on MEAM. Section 4 validates the proposed method using the parallel cable crosstalk example. Section 5 provides a preliminary discussion on the real-time convergence determination problem of surrogate models. Section 6 summarizes the entire text.

2. INTRODUCTION TO THE SURROGATE MODEL METHOD

When uncertainty analysis problems are dealt with, random variables can be used to model random events, assuming that

the uncertainty analysis problem can be represented by the following formula:

$$y(\xi) = f_{\text{EMC}}[x_A(\xi_1), x_B(\xi_2)] \qquad (1)$$

Among them, $f_{\text{EMC}}()$ is a stable EMC simulation solver; $x_A(\xi_1)$ and $x_B(\xi_2)$ are random variables with $\xi_1 \sim \text{pdf}(\xi_1)$, $\xi_2 \sim \text{pdf}(\xi_2)$; and $y(\xi)$ is the uncertainty analysis result, which is also a random variable.

The MCM needs to sample a total of N points $\mathbf{S}_{\xi,1} = [\mathbf{x}_{\xi,1}(1), \mathbf{x}_{\xi,1}(2), \cdots, \mathbf{x}_{\xi,1}(N)]$ during large-scale sampling, where each point is in the form of $\mathbf{x}_{\xi,1}(i) = [x_{A,1}(i), x_{B,1}(i)]$ and $x_{A,1} \sim \text{pdf}(x_A)$, $x_{B,1} \sim \text{pdf}(x_B)$. After sampling, deterministic EMC simulation $f_{\text{EMC}}[\mathbf{x}_{\xi,1}(i)]$ is performed on each sampling point to describe the uncertainty of model parameters. A total of N simulations are required to ensure convergence, and N is a large value. When the single EMC simulation time is long, the efficiency of the MCM is very low. Compared with MCM, Surrogate Model Method can obtain uncertainty analysis results with less computational resources. Its schematic diagram is shown in Figure 1.

The Latin hypercube sampling can be utilized by Surrogate Model Method to obtain n sampling points $\mathbf{S}_{\xi,2} = [\mathbf{x}_{\xi,2}(1), \mathbf{x}_{\xi,2}(2), \cdots, \mathbf{x}_{\xi,2}(n)]$ from a large number of sampling points $\mathbf{S}_{\xi,1} = [\mathbf{x}_{\xi,1}(1), \mathbf{x}_{\xi,1}(2), \cdots, \mathbf{x}_{\xi,1}(N)]$ mentioned above, where $\mathbf{x}_{\xi,2}(i) = [x_{A,2}(i), x_{B,2}(i)]$, and n is much smaller than N. These n sampling points are input into the same EMC solver $f_{\text{EMC}}()$, and the simulation result is $\mathbf{y}_{\text{unc}} = \{f_{\text{EMC}}[\mathbf{x}_{\xi,2}(1)], f_{\text{EMC}}[\mathbf{x}_{\xi,2}(2)], \cdots, f_{\text{EMC}}[\mathbf{x}_{\xi,2}(n)]\}$. The simulation result is combined with $\mathbf{S}_{\xi,2}$ to form a training set $\mathbf{T} = (\mathbf{S}_{\xi,2}, \mathbf{y}_{\text{unc}})$, and the form of a single point in training set $\mathbf{T} = (\mathbf{S}_{\xi,2}, \mathbf{y}_{\text{unc}})$ is $\mathbf{T}(i) = \{[x_{A,2}(i), x_{B,2}(i)], \mathbf{y}_{\text{unc}}(i)\}$.

n training set pairs $\mathbf{T}(i) = \{[x_{A,2}(i), x_{B,2}(i)], \mathbf{y}_{unc}(i)\}$ are used as inputs and outputs to train a surrogate model, and the resulting surrogate model is:

$$y_{mod} = M(x_A, x_B) \sim f_{EMC}(x_A, x_B) \qquad (2)$$

The sampling points of the MCM are input to estimate the result $y_{mod} (\mathbf{S}_{\xi,1})$ of all points, with the aim of approximating $f_{EMC}(\mathbf{S}_{\xi,1})$. Finally, the mean, variance, and probability density curves are obtained. Only n deterministic EMC simulations are conducted using the Surrogate Model Method, which is much smaller than the N simulations in MCM. When the cost of a single EMC simulation is high, the efficiency of the Surrogate Model Method is much higher than that of MCM. However, since the value of n cannot be obtained in advance in actual calculations, relying solely on the user's experience to determine the approximate value of n will have a significant impact on the computational efficiency of this method.

3. CONVERGENCE DETERMINATION METHOD FOR UNCERTAINTY ANALYSIS SURROGATE MODELS BASED ON MEAM

In mathematics, there are many statistical measures used to characterize the results of uncertainty analysis, such as mean value and variance. However, among all the characterization quantities of uncertainty analysis results, probability density curve is the most representative. The probability density curve can display all the details of the uncertainty analysis results, and all other important statistical measures can also be obtained based on the probability density curve.

Because the area enclosed by the probability density curve and the horizontal axis of the coordinate system is always 1, this characteristic makes it more suitable to use the "common area" of the area enclosed by the two probability density curves and the horizontal axis as the evaluation criterion for the validity of the simulation uncertainty analysis results. The closer the "common area" is to 1, the closer the simulation data results are to the standard results. MEAM is a method for evaluating the effectiveness of uncertain simulation results. Its idea is to apply an equivalent uniform distribution probability density to replace the probability density function of standard data and simulation data. The area of the "common area" can be converted into a rectangular area, and the effectiveness evaluation result can be converted into the area of a rectangle [12, 13]. MEAM can quantify the difference between standard data and simulation results, and is particularly suitable for evaluating the effectiveness of uncertainty analysis results in computational electromagnetics. Based on MEAM, a convergence determination method for uncertainty analysis of the Surrogate Model Method is proposed in this paper. The specific process of this method is as follows:

Step 1: Latin hypercube sampling is utilized to obtain Num_1 sampling points $\mathbf{W}_1 = [\mathbf{x}_1(1), \mathbf{x}_1(2), \cdots, \mathbf{x}_1(Num_1)]$ from a large number of sampling points $\mathbf{S}_{\xi,1}$ described in Section 2. These Num_1 sampling points are also input into the EMC solver $f_{EMC}(\ )$, and the simulation results are combined with $\mathbf{W}_1$ to form a training set $\mathbf{T}_1$, which is used to construct the initial surrogate model $Y_{mod,1}$. $\mathbf{S}_{\xi,1}$ is input into $Y_{mod,1}$, and the

probability density curve result L_1 is obtained through calculation. In this step, the value of Num_1 increases with the complexity of the uncertainty analysis problem, and users need to judge the value of Num_1 based on experience, usually $6 \leq Num_1 \leq 24$.

Step 2: On the basis of Step 1, Latin hypercube sampling is used to sample $\frac{Num_1}{3}$ more points from the sampling space, and these $\frac{Num_1}{3}$ sampling points are also input into EMC solver $f_{EMC}(\ )$. Combined with $\mathbf{T}_1$, a training set $\mathbf{T}_2$ with $Num_2 = Num_1 + \frac{Num_1}{3} = \frac{4Num_1}{3}$ sets of data is obtained, and a surrogate model $Y_{mod,2}$ is constructed using $\mathbf{T}_2$ to obtain the probability density curve result L_2. Among the Num_2 sets of data in $\mathbf{T}_2$, $\frac{Num_2}{2}$ sets of data are randomly selected and used as the new training set $\mathbf{t}_2$. $\mathbf{t}_2$ is used to construct the surrogate model $y_{mod,2}$ and obtain the probability density curve result l_2. The MEAM result M_1 between L_1 and L_2 and the MEAM result m_1 between l_2 and L_2 are calculated respectively.

Step 3: Calculate $F = M_1 + m_1$. If F is greater than Q_1 and m_1 greater than Q_2, the algorithm is considered to have fully converged. Otherwise, repeat step 2 and use Latin hypercube sampling to sample $\frac{Num_1}{3}$ points again, obtaining results such as $\mathbf{T}_3$ and L_3. The steps are iteratively repeated until the algorithm meets the judgment requirements for complete convergence. At this point, the probability density curve result L_i is the final uncertainty analysis result. Since MEAM uses the "common area" between two probability density curves to represent similarity, literature suggests that algorithm convergence can be determined when this "common area" value is greater than 0.95. Therefore, in the algorithm proposed in this paper, $Q_1 = 1.90$ and $Q_2 = 0.98$ can be taken to ensure convergence.

Through the above steps, the method proposed in this paper can iteratively judge the convergence of the surrogate models when the user cannot predict the required number of deterministic simulations for training the surrogate model. This method can minimize the waste of computing resources and provide great convenience for users, especially beginners, to avoid situations where the accuracy of uncertainty analysis results is insufficient due to too few deterministic simulations.

4. PARALLEL CABLE CROSSTALK PREDICTION EXAMPLE CONSIDERING GEOMETRIC RANDOMNESS

This section uses the parallel cable crosstalk prediction example considering geometric randomness shown in Figure 2 to verify the effectiveness of the method proposed in this paper. The deterministic simulation solver for modulus conversion method is implemented in the MATLAB environment in this example, and the parallel cable crosstalk calculation in mature literature is replicated to simulate the capacitance coupling effect and inductance coupling effect between cables, in order to accurately calculate the remote crosstalk voltage. After comparison, the deterministic simulation results are consistent with the results in [13, 14], ensuring the reliability and representativeness of the results. Assuming that the height of parallel cables is an uncertain input parameter, the example is described by the following
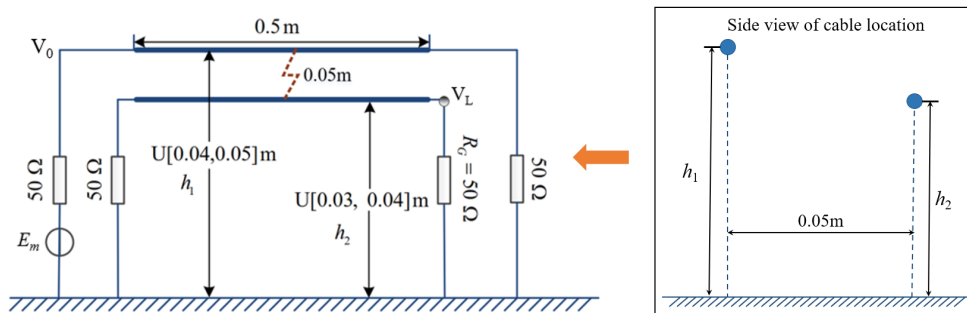
FIGURE 2. Parallel cable crosstalk prediction example considering geometric randomness.
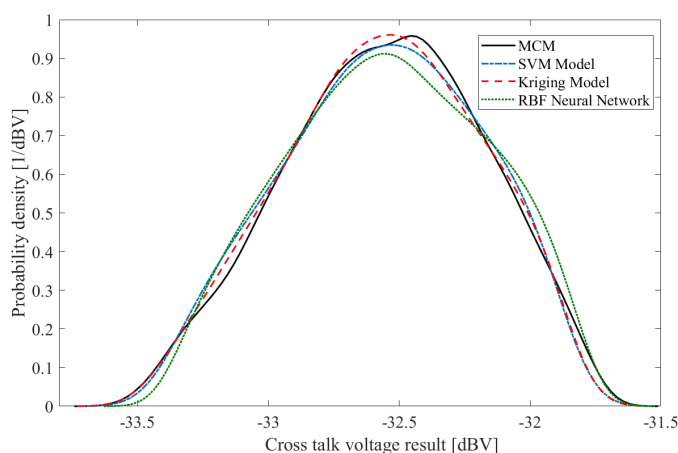

FIGURE 3. Uncertainty analysis results when 3 methods are fully converged.

FIGURE 4. Uncertainty analysis results when 3 methods are almost fully converged.

random variable model:

$$
\begin{cases}
h_1(\xi_1) = 0.045 + 0.005 \times \xi_1 \ [\mathrm{m}] \\
h_2(\xi_2) = 0.035 + 0.005 \times \xi_2 \ [\mathrm{m}]
\end{cases}
\tag{3}
$$

Among them, ξ_1 and ξ_2 are uniformly distributed random variables within $[-1, 1]$. The horizontal distance between two cables is 0.05 m. The output result calculated in this example is the far end crosstalk voltage V_{dB} at 75 MHz, and the calculation formula for V_{dB} is as follows:

$$
V_{\mathrm{dB}} = 20 \log_{10} \frac{|V_L|}{|V_0|}
\tag{4}
$$

In order to verify the effectiveness of the proposed method in determining the convergence of the surrogate models, 3 representative models (SVM model [15], Kriging model [9], and RBF neural network model [16, 17]) and MCM are used in this paper to calculate parallel cable crosstalk prediction examples. MCM is used as the standard result and is calculated 5000 times in total. Figure 3 shows the probability density curves of the 3 models when they converge completely. SVM model is calculated 36 times, Kriging model calculated 46 times, and RBF neural network model is calculated 64 times. Table 1 shows the MEAM results between the 3 methods and MCM when they fully converge. It can be seen from the table that the MEAM values of all 3 methods are greater than 0.95, indicating that they are fully converged.
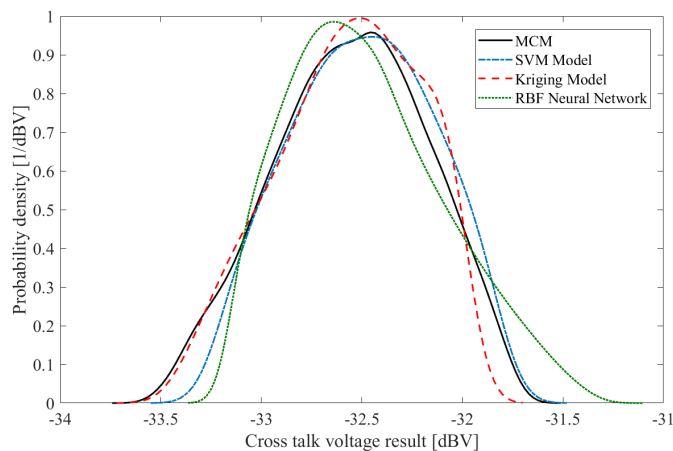
TABLE 1. The required number of deterministic simulations and the MEAM values for complete convergence of the 3 methods.

Surrogate model	Number of simulations	MEAM values
SVM model	36	0.9700
Kriging model	46	0.9665
RBF neural network model	64	0.9608

In order to demonstrate the effectiveness of the method proposed in this paper more clearly, the uncertainty analysis results of the 3 methods that are about to achieve complete convergence are calculated respectively. At this time, SVM model is calculated 29 times, Kriging model calculated 37 times, and RBF neural network model calculated 52 times, all of which are 80% of the deterministic simulation times required for complete convergence. The probability density curves of the 3 methods are shown in Figure 4, and Table 2 shows the MEAM results between the 3 methods and MCM at this time. It can be seen that at this point, the MEAM results of the 3 methods are all below 0.85, and they have not fully converged. Moreover, there is a significant difference between the results of the above 3 methods and the MCM results. If the results at this point are used as the final uncertainty analysis results, it will result in significant errors. This is an inherent problem in uncertainty analysis using surrogate models. The algorithm proposed in this paper

TABLE 2. The number of deterministic simulations and MEAM values required for the 3 methods to converge almost completely.

Surrogate model	Number of simulations	MEAM values
SVM model	29	0.8474
Kriging model	37	0.8377
RBF neural network model	52	0.8301

can avoid similar errors in Surrogate Model Methods and ensure complete convergence of the algorithm.

The above results can prove that although the convergence speed of the 3 surrogate models varies, the method proposed in this paper can accurately determine the convergence of these 3 different surrogate models, indicating that this method solves the convergence determination problem of Surrogate Model Method.

5. RESEARCH ON REALTIME CONVERGENCE DETERMINATION METHOD OF SURROGATE MODELS

When using Surrogate Model Method for uncertainty analysis, it is also meaningful to provide users with a real-time convergence judgment result, which can help users determine the approximate convergence progress of uncertainty analysis calculations at any time and provide a reference for users. In response to this issue, a preliminary study is conducted on the real-time convergence determination method of the surrogate models in this section.

Although Surrogate Model Method has certain differences in convergence speed due to the use of different models, there are certain rules in the convergence process of different model results when calculating EMC uncertainty analysis problems. Therefore, if a universal real-time convergence determination method for different models can be proposed, it will provide great convenience for users. On the basis of the convergence determination method proposed in Section 3, a real-time convergence determination method for surrogate models is proposed in this section, and a universal convergence determination polynomial relationship for different surrogate models is fitted. This polynomial relationship can be used to approximate the real-time convergence of different Surrogate Model Methods. The fitting process of this polynomial relationship is as follows:

Step 1: Similar to Step 1 in Section 3, Latin hypercube sampling is utilized to obtain Num_1 sampling points from a large number of sampling points $\mathbf{S}_{\xi,1}$. The value of Num_1 also needs to be determined by the user based on experience. For the ease of description, it is assumed that $Num_1 = 6$. After deterministic EMC simulation, the training set $\mathbf{T}_1$ can be obtained, and a surrogate model $Y_{\text{mod},1}$ can be constructed using $\mathbf{T}_1$. Finally, the probability density curve result L_1 is obtained.

Step 2: Among the 6 sets of data in $\mathbf{T}_1$, $6/2 = 3$ sets of data are randomly selected, resulting in $C_6^3 = 20$ different combinations. These combinations are used as new training sets $[\mathbf{t}_1(1), \mathbf{t}_1(2), \cdots, \mathbf{t}_1(20)]$, and 20 surrogate models $[y_{\text{mod},1}(1), y_{\text{mod},1}(2), \cdots, y_{\text{mod},1}(20)]$ are constructed using

these training sets to obtain 20 sets of probability density curve results $[l_1(1), l_1(2), \cdots, l_1(20)]$.

Step 3: The MEAM result $\mathbf{U}_1 = [ME_{\text{test},1}(1), ME_{\text{test},1}(2), \cdots, ME_{\text{test},1}(20)]$ between the 20 sets of probability density curve results $[l_1(1), l_1(2), \cdots, l_1(20)]$ and L_1 is calculated respectively. Then, the mean result X_N and variance result Y_N of the 20 numbers in $\mathbf{U}_1$ are calculated. According to Section 3, the required number of deterministic calculations for different surrogate models to fully converge is Num_C, where $Z_N = \frac{Num_1}{Num_C}$ and $\mathbf{T}_F(1) = [X_N, Y_N, Z_N]$.

Step 4: Similar to Section 3, Latin hypercube sampling is used to sample 2 more points from the sampling space, with $Num_2 = Num_1 + 2$. Repeat the above steps until $Z_N = \frac{Num_i}{Num_C} = 1$. Finally, the data in $\mathbf{T}_F$ is used to fit the corresponding polynomial $Z_N = F(X_N, Y_N)$.

The polynomial $Z_N = F(X_N, Y_N)$ reflects the corresponding relationship between the mean result X_N, variance result Y_N, and convergence percentage Z_N. When conducting uncertainty analysis, X_N and Y_N can be calculated. By substituting them into $Z_N = F(X_N, Y_N)$, the convergence percentage Z_N of the surrogate model under the current number of sampling points can be obtained. Z_N is a rough estimate and not entirely accurate, but it can provide some reference for users. The SVM model, Kriging model, and RBF neural network model are used for the above calculations, and 3 corresponding polynomials are obtained. The polynomial fitted by the SVM model is:

$$\begin{aligned} Z_{N,1} = & F_1(X_{N,1}, Y_{N,1}) = -2.169 + 4.27 \times X_{N,1} \\ & + 87.68 \times Y_{N,1} - 0.7008 \times X_{N,1}^2 \\ & - 102.3 \times X_{N,1} \times Y_{N,1} - 532.4 \times Y_{N,1}^2 \end{aligned} \quad (5)$$

The polynomial fitted by the Kriging model is:

$$\begin{aligned} Z_{N,2} = & F_2(X_{N,2}, Y_{N,2}) = -2.023 + 6.183 \times X_{N,2} \\ & + 64.94 \times Y_{N,2} - 2.948 \times X_{N,2}^2 \\ & - 116.4 \times X_{N,2} \times Y_{N,2} \end{aligned} \quad (6)$$

The polynomial fitted by the RBF neural network model is:

$$\begin{aligned} Z_{N,3} = & F_3(X_{N,3}, Y_{N,3}) = 7.606 - 22.86 \times X_{N,3} \\ & - 150.4 \times Y_{N,3} + 18.07 \times X_{N,3}^2 \\ & + 211.2 \times X_{N,3} \times Y_{N,3} \end{aligned} \quad (7)$$

The images of the 3 polynomials are shown in Figure 5, Figure 6, and Figure 7, respectively. From the figures, it can be seen that they have the same trend. The Z_N values of the 3 polynomials tend to approach 1 when X_N is at its maximum and Y_N at its minimum. At this point, the surrogate models reach complete convergence. In order to make the polynomial universal, the final polynomial $Z_{N,f} = \frac{F_1(X_N,Y_N)+F_2(X_N,Y_N)+F_3(X_N,Y_N)}{3}$ can be obtained by synthesizing 3 polynomials. This polynomial can provide users with a corresponding convergence percentage Z_N in real time during the calculation of the Surrogate Model Method. Although currently only a rough reference value for the real-time convergence of a surrogate model can be provided by this method, further research on this part in the future is expected to provide assistance for the development and improvement of the functionality of commercial electromagnetic simulation software.
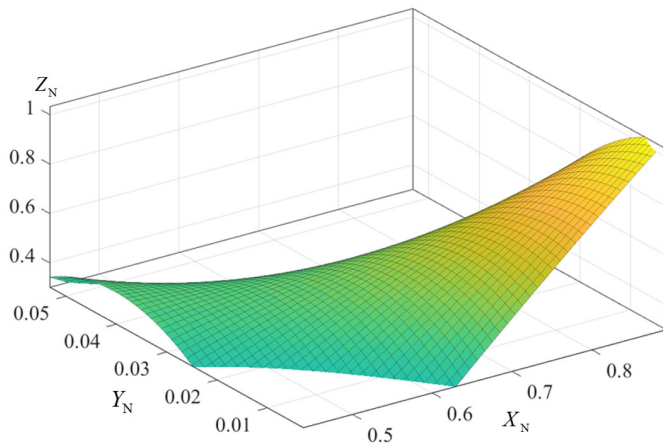
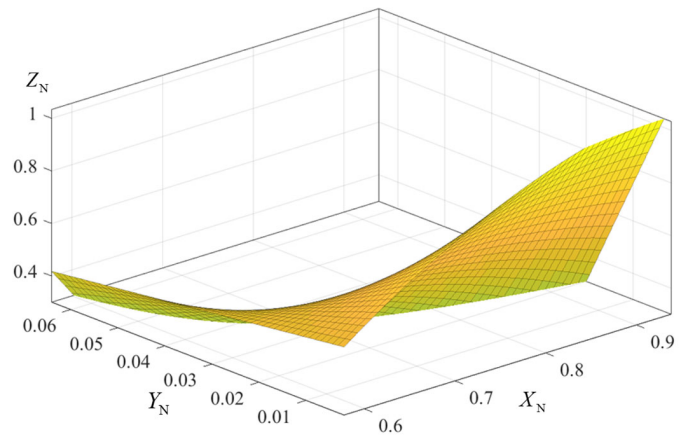**FIGURE 5**. The polynomial figure obtained using SVM model.

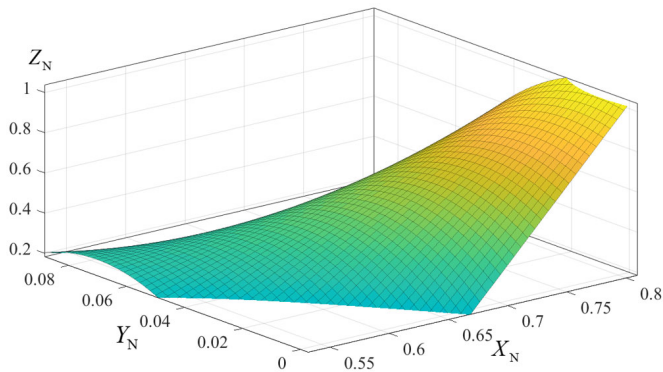**FIGURE 6**. The polynomial figure obtained using Kriging model.

FIGURE 7. The polynomial figure obtained using RBF neural network model.

6. CONCLUSION

Surrogate Model Method is widely used in uncertainty analysis due to its advantages of high computational efficiency and being unaffected by the curse of dimensionality. This method treats the surrogate model as a black box, and the deterministic simulation results of quantitative times are used as the input and output of the black box to train the surrogate model. Finally, a large number of samples are taken from the randomness of the model input to obtain the final result. However, due to the inability of users to predict the number of deterministic simulations required for the convergence of the surrogate models during computation, they can only judge the number of deterministic simulations based on experience in practical applications, resulting in a significant waste of computing resources and seriously affecting the computational efficiency of the Surrogate Model Method. To solve this problem, a convergence determination method for uncertainty analysis surrogate models based on MEAM is proposed in this paper. This method can iteratively calculate the convergence of the surrogate model as the number of training set points increases and repeat the process until the Surrogate Model Method fully converges. The parallel cable crosstalk prediction example is used in this paper to verify the effectiveness of the proposed method, and accurate determination of the convergence of the SVM model, Kriging

model, and RBF neural network model is achieved in this example. Finally, based on the proposed convergence determination method, preliminary research is conducted on the real-time convergence determination method of the Surrogate Model Method in this paper, and a universal polynomial relationship for Surrogate Model Method is fitted. This polynomial relationship can be used to roughly determine the real-time convergence of different Surrogate Model Methods. It is worth noting that the algorithm proposed in this paper is currently based on user's experience to determine the required number of sampling points. In the future, more detailed discussions will be conducted on the precise selection of sampling points to further improve the applicability of this method.

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