(Received 10 August 2024, Accepted 9 October 2024, Scheduled 13 October 2024)

Adaptive Block-Based Krylov Subspace Basis Functions for Solving Bistatic Scattering Problems

Haoran Yuan[1], Zhonggen Wang[1], Yufa Sun[2], and Wenyan Nie[3,*]

[1]*School of Electrical and Information Engineering, Anhui University of Science and Technology, Huainan 232001, China*
[2]*School of Electronics and Information Engineering, Anhui University, Hefei 230039, China*
[3]*School of Mechanical and Electrical Engineering, Huainan Normal University, Huainan 232001, China*

ABSTRACT: This study aims to improve the efficiency of constructing basis functions for solving the electromagnetic scattering problem of objects using the method of moments combined with compressive sensing and Krylov subspace. To this end, a region decomposition method based on a clustering algorithm is proposed to accelerate the construction process of Krylov subspace basis functions. First, the midpoints of the common edges of triangular pairs are used to form a clustered dataset. Then, the initial clustering center is set, and the processes of clustering center updating and regional decomposition of the constructed dataset are completed iteratively. Finally, each subdomain is expanded according to the average distance from data points to the clustering center to ensure the continuity of currents. The numerical computation results show that the proposed method can achieve significant time efficiency.

1. INTRODUCTION

The method of moments (MoM) represents a powerful technique for solving scattering and radiation problems in the electromagnetic (EM) field. In recent years, the combination of the compressed sensing (CS) [1, 2] technique and MoM method, denoted as CS-MoM method [3, 4], has reduced the computational complexity of solving the electromagnetic scattering problem significantly [5–7]. The main principle of the CS-MoM method is to use a part of the impedance matrix equation from the MoM method to construct an underdetermined system that satisfies the structure of the CS technique [8–10]. For the sparse basis construction in CS-MoM, mainly the characteristic basis functions (CBFs) [11–13], characteristic modes (CMs) [14, 15], and Krylov subspace basis functions (KSBFs) [16] are used. Compared to CBFs and CMs, KSBFs have the unique advantage of being used for solving the EM scattering problems in arbitrary dimensions. However, because of the high computational complexity of the sparse basis construction, it is difficult to use KSBFs to solve the scattering problem for objects with comparatively large electrical sizes. In [17], a block-based KSBFs (CS-Krylov-block) method for constructing a sparse basis was proposed to accelerate the sparse basis generation. In the traditional region decomposition method, multiple cubes are represented in the shape of a grid in the resolution space; the object is divided; and subdomains are generated. However, this can cause different block sizes [18] for irregular computational objects, and different numbers of unknowns in too-large and too-small regions can reduce the accuracy and efficiency of the calculation. In addition, this method is a non-adaptive region decomposition method that requires human participation. Therefore, how to perform region decomposition on the computational object adaptively to speed up the calculation process is one of the important issues to address in solving the EM problems.

In recent years, the rapid development of machine learning (ML) [19] has brought a new perspective to computational electromagnetics, introducing a new research direction in the field of intelligent electromagnetics. Because of its powerful big data-driven and computational capabilities, ML has emerged in the EM field [20, 21], and excellent results have been achieved in electromagnetic modeling, simulations, and new materials analysis. This has brought numerous innovative perspectives to the EM field, which has been developing for more than 100 years. Second, the ML-based clustering algorithms were one of the first methods used in pattern recognition and data mining tasks. These methods were used to study large databases in various applications, and clustering algorithms for big data have received increasing attention.

In this paper, a region decomposition method based on a combination of the k-means [22] clustering method and the CS (CS-Krylov-block-k-means) is proposed to accelerate the KSBF generation process. The k-means clustering method is a data grouping method that groups all data with the maximum similarity into clusters, ensuring the minimum similarity of data between the clusters. This method uses the midpoints of common edges of triangular pairs in a dissected grid as clustered data points to form a clustered dataset. A number of points are randomly selected as the initial iteration points of the k-means algorithm; the number of initial iteration points is the same as the number of regions after region decomposition. Then, the clustering center is continuously updated through iterations until the end of region decomposition. This avoids the interference caused by different sizes of regions and improves the computational efficiency of the KSBF generation process. The

* Corresponding author: Wenyan Nie (wynie5240@163.com).

validity and effectiveness of the proposed method are verified through theoretical analyses and numerical simulations.

The paper is organised as follows. First, the theoretical principles of the proposed method and the comparative literature are described in Section 1. The computational complexity of the proposed method is analysed in Section 2, and numerical validation of the proposed method is carried out in Section 3, while conclusions are given in Section 4.

2. THEORY

2.1. Traditional CS-Krylov-Block Method

According to the MoM principle, the integral equation can be converted to an impedance matrix equation as follows:

$$\mathbf{Z}_{N\times N}\mathbf{I}_{N\times 1}=\mathbf{E}_{N\times 1}, \tag{1}$$

where $\mathbf{Z}$ represents the impedance matrix; N is the number of unknowns; $\mathbf{I}$ is the current coefficient to be solved; $\mathbf{E}$ is the excitation vector.

In the CS-Krylov-block theory, the original object is first divided into a number of smaller blocks, whose number is calculated by $p=0.9N^{1/3}$ [23], where p represents the number of blocks; therefore, (1) can be converted to:

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \cdots & \mathbf{Z}_{1p} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \cdots & \mathbf{Z}_{2p} \\ \vdots & \vdots & \cdots & \vdots \\ \mathbf{Z}_{p1} & \mathbf{Z}_{p2} & \cdots & \mathbf{Z}_{pp} \end{bmatrix} \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \\ \vdots \\ \mathbf{J}_p \end{bmatrix} = \begin{bmatrix} \mathbf{E}_1 \\ \mathbf{E}_2 \\ \vdots \\ \mathbf{E}_p \end{bmatrix}, \tag{2}$$

where $\mathbf{Z}_{ii}$ has a size of $N_i\times N_i$, and $\mathbf{Z}_{ij}$ $(i,j=1,2,\ldots,p; i\neq j)$ is the self-impedance matrix and mutual impedance matrix, respectively; $\mathbf{J}_i$ and $\mathbf{E}_i$ denote the current coefficient vector and excitation vector on a block i, respectively; $\mathbf{J}_i$ and $\mathbf{E}_i$ have the same size of $N_i\times 1$, where N_i is the number of unknowns on block i.

Since blocking leads to discontinuities in the edge currents, it is necessary to extend a certain distance on each block as a buffer and then construct the KSBFs on each block respectively, which can be mathematically expressed by:

$$\mathbf{K}_{k_b}^{i,e}(\mathbf{Z}_{ii}^{e},\mathbf{E}_{ii}^{e})=\text{span}\{\mathbf{V}_{\text{i}}^{\text{e}},\mathbf{Z}_{\text{ii}}^{\text{e}}\mathbf{E}_{\text{i}}^{\text{e}},(\mathbf{Z}_{\text{ii}}^{\text{e}})^2\mathbf{E}_{\text{i}}^{\text{e}},\ldots,(\mathbf{Z}_{\text{ii}}^{\text{e}})^{\text{k}_\text{b}-1}\mathbf{E}_{\text{i}}^{\text{e}}\}. \tag{3}$$

where e is the extension symbol, and then a set of orthogonal bases is generated using the partial orthogonal method to obtain $\mathbf{Q}_i$ as follows:

$$\mathbf{Q}_i=\begin{bmatrix}\mathbf{J}_i^1 & \mathbf{J}_i^2 & \cdots & \mathbf{J}_i^{k_b}\end{bmatrix}. \tag{4}$$

A KSBF $\mathbf{J}_i^{\text{KSP}}$ is constructed on each block i respectively, and the sparse basis matrix formed after its construction is defined as follows:

$$\mathbf{J}^{\text{KSP}}=\begin{bmatrix} \mathbf{J}_1^{\text{KSP}} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_2^{\text{KSP}} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{J}_p^{\text{KSP}} \end{bmatrix}, \tag{5}$$

Therefore, the current can be expressed as $\mathrm{I}=\mathbf{J}^{\text{KSP}}\alpha$, where the dimension of α is $k_b\times 1$, and k_b is the number of basis functions after blocking; α can be obtained by the one-time least-square.

The cube-based block generation process in the KSBFs involves placing multiple grid-shaped cubes in a resolution space. An object is divided according to each cube to generate subdomains. For irregular objects, the divided subdomains are not uniform, which can prolong the calculation time of matrix operations required to form the KSBFs. To address the mentioned problems, the k-means algorithm is used in this paper.

2.2. Proposed Method

The k-means is a clustering algorithm for large-scale datasets which divides a dataset into p clusters such that each data point belongs to the nearest cluster, and the cluster's center denotes the average of all data points in the cluster.

Dividing the given N data $(X_1,X_2,\ldots,X_N)$ into p groups $(S_1,S_2,\ldots,S_p)$ and satisfying $p\leq N$, it is essentially grouping the data to minimize equation (6):

$$\arg\min_{S}\sum_{i=1}^{p}\sum_{X\in S_i}\parallel X-\mu_i\parallel^2, \tag{6}$$

The main principle of the proposed method is to use the midpoints of the common edges of triangle pairs as clustering data points and constitute a clustering dataset; the number of blocks is calculated using the method introduced in [23]. The proposed method randomly selects points with the same number of blocks as the initial iteration points of the k-means clustering method and updates the clustering center through iterations to complete the region decomposition.

As shown in Figs. 1(a) and 1(b), the region decomposition of a cone sphere with a gap is performed using the two methods. The histogram of the number of unknowns on each block after region decomposition of the cube-based method and the k-means clustering-based method is plotted in Fig. 2. As shown in Fig. 2, the number of unknowns in the decomposed region of the proposed method is more uniform than the traditional region decomposition method. The calculation times of the two methods for the same number of blocks are presented in Table 1. Note that the calculation time here refers to the time needed to complete the region decomposition, where it can be seen that for the same irregular object, the proposed method is more advantageous than the traditional one, regardless of the number of blocks.

Further, the subdomain needs to be extended to ensure the continuity of the edge currents after the region decomposition. This extension is based on the average distance from the clustered data points to the cluster center in the subdomain, as shown in Fig. 3.

3. COMPLEXITY ANALYSIS

The main innovation of this study refers to the application of the clustering algorithm to the region decomposition process to obtain a more uniform sub-region. The computational com-

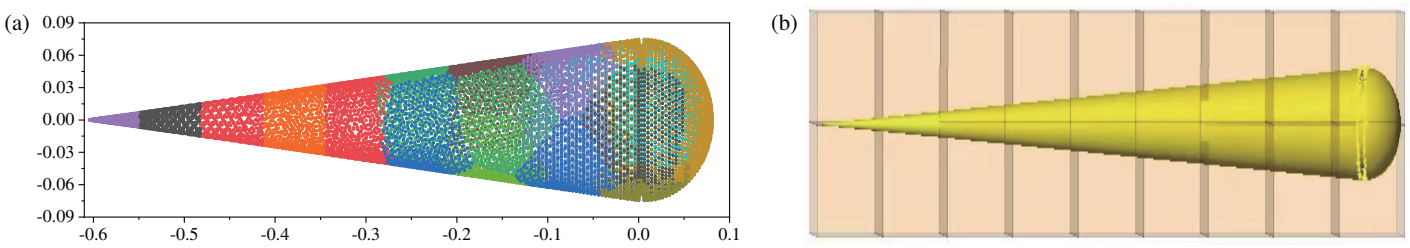

FIGURE 1. Configuration of the block generation for the cone sphere with a gap. (a) k-means clustering-based method. (b) Cube-based method.

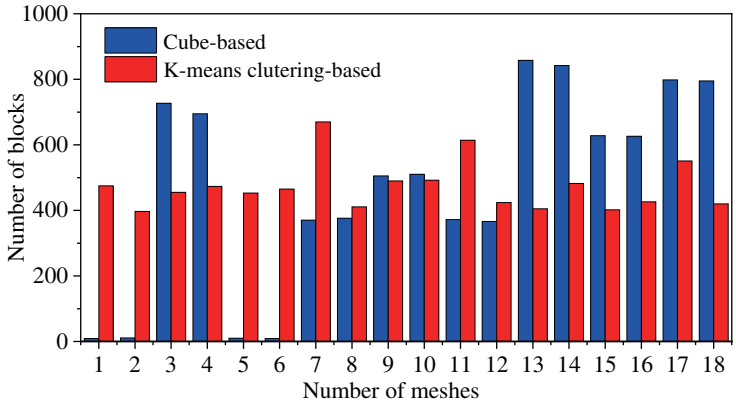

FIGURE 2. Mesh distribution histogram of the cut cone sphere with a gap.

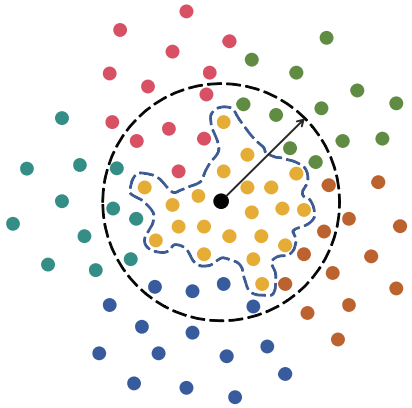

FIGURE 3. Illustration of the k-means-based clustering block expansion.

TABLE 1. The simulating time and error.

	Blocks	10	14	18	22	26	30
Calculation time (s)	k-means clustering-based	6.6	8.1	9.8	11.1	12.4	14.4
	Cube-based	10.8	11.7	12.7	14.1	15.7	18.1
Errors (%)	k-means clustering-based	1.5	1.5	1.6	1.7	1.6	1.8
	Cube-based	1.6	1.6	1.7	1.8	1.9	2.2

plexity of the traditional region decomposition method and the proposed method in filling the impedance matrix and reconstructing the current coefficient is the same. Therefore, there is only a necessity to analyze the computational complexity of the two methods in constructing the basis functions. In the regional decomposition process of an object, having a total of N unknowns, the object is divided into p blocks. The computational complexity of each block is $O((N/p)^2)$, and the computational complexity of a block i is $O(N_i^2)$; thus, the total computational complexity is $O(\sum(N_1^2 + N_2^2 + \ldots + N_p^2))$. The minimum value of $O(\sum(N_1^2 + N_2^2 + \ldots + N_p^2))$ must meet the condition of $N_1 = N_2 = \ldots = N_p$. Based on the above situation, as shown in Fig. 2, the proposed method is more uniformly blocked than CS-Krylov-block, which satisfies the condition of $N_1 \approx N_2 \approx \ldots \approx N_p$. In summary, the proposed method has lower computational complexity than the traditional region decomposition in terms of constructing the basis functions. Therefore, the proposed method can significantly reduce the calculation time compared to the traditional methods.

4. NUMERICAL RESULTS

Several numerical simulations were performed to compare the proposed method with the CS-Krylov-block and MoM methods. The relative error in the radar cross section (RCS) was used to compare the accuracy of the methods, and it was calculated by:

$$\text{Err} = (\|\sigma_{\text{CS}} - \sigma_{\text{MoM}}\|_2 / \|\sigma_{\text{MoM}}\|_2) \times 100\%, \quad (7)$$

where σ_{CS} and σ_{MoM} denote the results calculated by the proposed method and traditional MoM, respectively.

4.1. Perfect Electrical Conductor Cone Sphere with Gap

First, a cone sphere with a gap consisting of a cone, a cylinder, and a hemisphere was constructed and irradiated by a plane wave with a frequency of 2.7 GHz; 8,505 unknowns were obtained by discretizing the Rao-Wilton-Glisson basis functions. According to $p = 0.9N^{1/3}$, the cone sphere with a gap was divided into 18 blocks, and the number of KSBFs on each block was set to 150; thus, the total number of KSBFs was

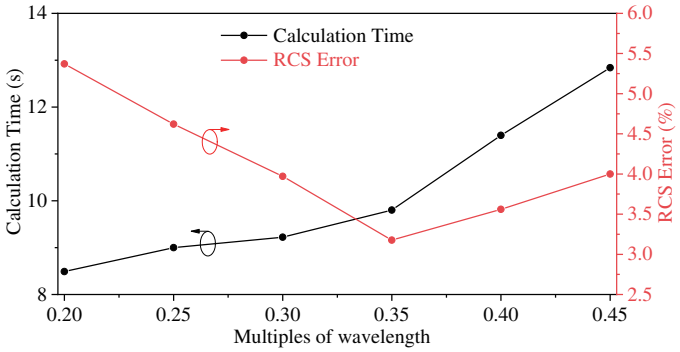

FIGURE 4. Calculation time and RCS error against extended size.

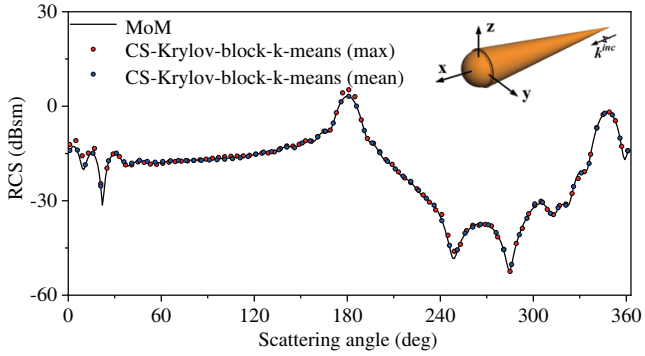

FIGURE 5. Comparison chart of different expansion methods.

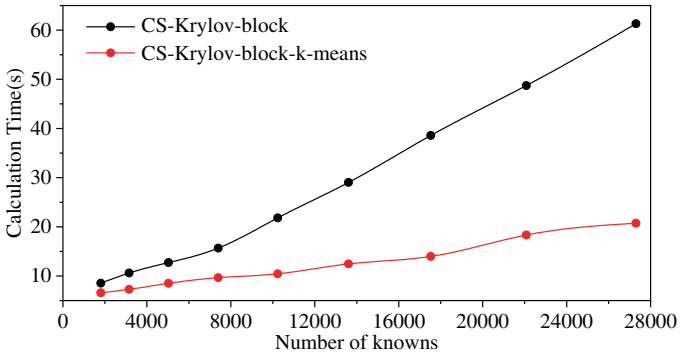

FIGURE 6. Time comparison with different unknowns.

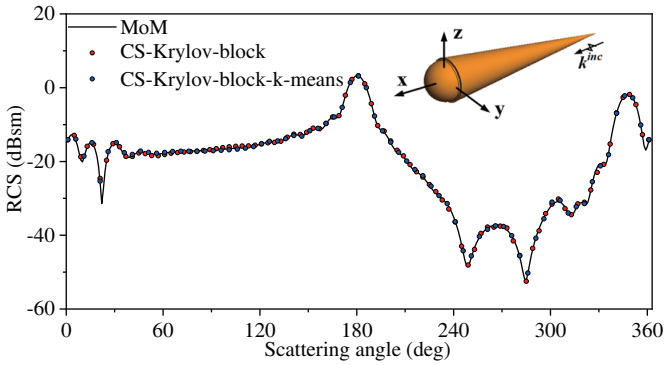

FIGURE 7. Bistatic RCS of cone sphere with gap in horizontal polarization.

2,700. Moreover, the number of newly generated vectors and the number of partially generated vectors from earlier periods were set to 100 and 30, respectively; this was similar to the quasi-generalized minimum residual (QGMRES)] [24] for the two methods.

To analyze the impact of buffers on calculation accuracy and efficiency, this study uses different sizes of extension steps in the proposed method, and the calculation time and accuracy at the corresponding expansion step size are recorded. The expansion in the proposed method was based on the average distance of the clustered data points in each class to the cluster's center, as shown in Fig. 3. The comparison results of the three methods regarding the RCS error and calculation time for different expansion steps are shown in Fig. 4. The calculation time denoted the sum of the blocking time and the time required to generate KSBFs. As presented in Fig. 4, the calculation accuracy and time increased with the buffer size. To balance the calculation efficiency and time, the expansion step size was set to 0.35λ, where λ denotes the incident wave's wavelength.

After determining the expansion step, in order to illustrate the choice of the proposed method in the expansion method, two different expansion methods are numerically verified in this paper. One is to expand according to the maximum distance from the cluster data points to the cluster centre, and the other is to expand according to the average distance from the cluster data points to the cluster centre. After comparison, the difference between the two methods in the total computation time is only 0.3 s, and the latter's computational accuracy is higher, so the latter is chosen as the expansion method, as shown in Fig. 5.

To further investigate the effectiveness of the proposed method, the calculation times of CS-Krylov-block and CS-Krylov-block-k-means methods were counted respectively for different incident wave frequencies, and comparison results are shown in Fig. 6. The results indicated that the frequency increased, which meant that the cone sphere with the gap had larger electrical dimensions and more unknowns. As shown in Fig. 6, with the increase in the number of unknowns, the calculation time of the CS-Krylov-block-k-means method increased slower, and the calculation efficiency was much better than that of the CS-Krylov-block method. Finally, the proposed method could achieve high calculation accuracy, as shown in Fig. 7.

4.2. Perfect Electrical Conductor Missile

To verify the effectiveness of the proposed method for complex geometries, the bistatic RCS of a perfect electrical conductor missile was calculated using the three methods. The incident wave frequency was set to 3.8 GHz, and the surface of the missile was dissected into 66,998 triangular surface elements, resulting in 100,497 unknowns. The missile was divided into 42 blocks, each of which was extended by 0.35λ. The computational order of the Krylov subspace on each block was set to 150; the number of computational vectors generated most recently and the number of partial basis vectors gener-

TABLE 2. The simulating time.

Model	Method	Filling time (s)	Generating basis functions time (s)	Solving time (s)	Blocking time (s)	Total time (s)
Cone sphere with gap	CS-Krylov-block-k-means	35.2	9.3	2.7	9.8	57
	CS-Krylov-block	35.2	17.2	3.5	12.7	68.6
Missile	CS-Krylov-block-k-means	611.6	112.4	623.6	49.6	1397.2
	CS-Krylov-block	611.6	378.8	649.2	66.3	1705.9

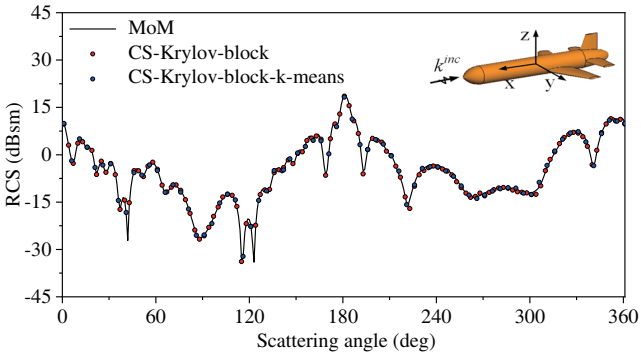

FIGURE 8. Bistatic RCS of missile in horizontal polarization.

ated earlier by the two methods were set to 100 and 80, respectively. As shown in Fig. 8, the calculation accuracy of the CS-Krylov-block-k-means method was within the usable range and matched well with those of the MoM and CS-Krylov-block methods.

The calculation times in Figs. 7 and 8 are counted in Table 2, which indicates that the generating basis function time of the CS-Krylov-block-k-means was significantly reduced compared to the CS-Krylov-block by 45.9% and 70.3%, respectively, and the total time efficiency improved by 16.91% and 18.1%, respectively.

5. CONCLUSION

In this paper, a clustering algorithm for optimizing the traditional region decomposition is proposed. It is shown that the combination of the clustering algorithm and Krylov subspace can improve the calculation efficiency of the CS-Krylov-block method. The proposed method uses the ability of the k-means clustering algorithm to generate highly uniform regions adaptively while effectively avoiding common drawbacks, such as the high consumption of memory resources and reduction in the calculation efficiency caused by the non-uniformity of conventional region decomposition. In addition, to attenuate the current discontinuity caused by blocking, the proposed method extends each region as a buffer according to the average distance from the clustered data points to the cluster's center, which improves the calculation efficiency. The results of theoretical analyses and numerical simulations show that the proposed method can effectively reduce the time needed to solve the bistatic scattering problem compared to the conventional CS-Krylov-block method.

ACKNOWLEDGEMENT

This work was supported in part by the Natural Science Foundation of Anhui Provincial Education Department under Grant No. 2022AH051583 and No. 2022AH052138, in part by the National Natural Science Foundation of China under Grant No. 62071004 and No. 61401003, and in part by the Anhui Province Graduate Academic Innovation Project under grant No. 2023xscx074.

REFERENCES

[1] Donoho, D. L., "Compressed sensing," *IEEE Transactions on Information Theory*, Vol. 52, No. 4, 1289–1306, Apr. 2006.

[2] Candes, E. J. and J. R. T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Transactions on Information Theory*, Vol. 52, No. 2, 489–509, Feb. 2006.

[3] Harrington, R. F., "Field computation by moment methods," *The Macmillan Comp.*, Vol. 130, No. 6, 276–280, 1968.

[4] Wang, Z., B.-Z. Wang, and W. Shao, "Efficient construction and solution of MoM matrix equation with compressed sensing technique," *Journal of Electromagnetic Waves and Applications*, Vol. 29, No. 5, 683–692, Mar. 2015.

[5] Chai, S.-R. and L.-X. Guo, "A new method based on compressive sensing for monostatic scattering analysis," *Microwave and Optical Technology Letters*, Vol. 57, No. 10, 2457–2461, Oct. 2015.

[6] Kong, M., M. Chen, X. Cao, L. Zhang, Q. Qi, and X. Wu, "Fast analysis of local current distribution for electromagnetic scattering problems of electrically large objects," *IEEE Access*, Vol. 8, 127 640–127 647, Jul. 2020.

[7] Kong, M., M. S. Chen, X. Y. Cao, J. B. Zhu, X. J. Kuang, Q. Qi, and X. L. Wu, "Fast electromagnetic scattering analysis of inhomogeneous dielectric objects over a wide incident angle," *IEEE Antennas and Wireless Propagation Letters*, Vol. 20, No. 8, 1527–1531, Aug. 2021.

[8] Chai, S.-R. and L.-X. Guo, "Integration of CS into MoM for efficiently solving of bistatic scattering problems," *IEEE Antennas and Wireless Propagation Letters*, Vol. 15, 1771–1774, Feb. 2016.

[9] Chai, S.-R. and L.-X. Guo, "Fast analysis of bistatic scattering problems with compressive sensing technique," *Journal of Electromagnetic Waves and Applications*, Vol. 30, No. 13, 1755–1762, Sep. 2016.

[10] Kong, M., M. S. Chen, B. Wu, and X. L. Wu, "Fast and stabilized algorithm for analyzing electromagnetic scattering problems of bodies of revolution by compressive sensing," *IEEE Antennas and Wireless Propagation Letters*, Vol. 16, 198–201, 2016.

[11] Prakash, V. V. S. and R. Mittra, "Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations," *Microwave and Optical Technology Letters*, Vol. 36, No. 2, 95–100, Jan. 2003.

[12] Wang, Z.-G., W.-Y. Nie, and H. Lin, "Characteristic basis functions enhanced compressive sensing for solving the bistatic scattering problems of three-dimensional targets," *Microwave and Optical Technology Letters*, Vol. 62, No. 10, 3132–3138, Oct. 2020.

[13] Wang, P., Z.-G. Wang, Y.-F. Sun, and W.-Y. Nie, "Novel compressive sensing computing model used for analyzing electromagnetic scattering characteristics of three-dimensional electrically large objects," *Acta Physica Sinica*, Vol. 72, No. 3, 54–61, Feb. 2023.

[14] Gao, Y., M. F. Akbar, and G. N. Jawad, "Stabilized and fast method for compressive sensing-based method of moments," *IEEE Antennas and Wireless Propagation Letters*, Vol. 22, No. 12, 2915–2919, Dec. 2023.

[15] Wang, Z., P. Wang, Y. Sun, and W. Nie, "Fast analysis of bistatic scattering problems for three-dimensional objects using compressive sensing and characteristic modes," *IEEE Antennas and Wireless Propagation Letters*, Vol. 21, No. 9, 1817–1821, Sep. 2022.

[16] Cao, X., M. Chen, Q. Qi, M. Kong, J. Hu, L. Zhang, and X. Wu, "Solving electromagnetic scattering problems by underdetermined equations and krylov subspace," *IEEE Microwave and Wireless Components Letters*, Vol. 30, No. 6, 541–544, Jun. 2020.

[17] Wang, Z., H. Yuan, Y. Sun, W. Nie, and P. Wang, "Block-based krylov subspace basis functions for solving bistatic scattering problems," *IEEE Antennas and Wireless Propagation Letters*, Vol. 22, No. 10, 2561–2565, Oct. 2023.

[18] Park, C.-S., Y.-R. Jeong, I.-P. Hong, and J.-G. Yook, "Block size optimization of CBFM for scattering problems," *IEEE Transactions on Antennas and Propagation*, Vol. 66, No. 10, 5370–5377, Oct. 2018.

[19] Zhang, W.-W., D.-H. Kong, X.-Y. He, and M.-Y. Xia, "A machine learning method for 2-D scattered far-field prediction based on wave coefficients," *IEEE Antennas and Wireless Propagation Letters*, Vol. 22, No. 5, 1174–1178, May 2023.

[20] Yao, H. M., L. Jiang, H. H. Zhang, and W. E. I. Sha, "Machine learning methodology review for computational electromagnetics," in *2019 International Applied Computational Electromagnetics Society Symposium — China (ACES)*, Vol. 1, 1–4, Nanjing, China, 2019.

[21] Yao, H. M., L. J. Jiang, and Y. W. Qin, "Machine learning based method of moments (ML-MoM)," in *2017 IEEE International Symposium on Antennas and Propagation & USNC/URSI National Radio Science Meeting*, 973–974, San Diego, CA, USA, 2017.

[22] Kumar, N. and R. Sinha, "Improved structured dictionary learning with adaptive block-size using k-means clustering," in *2018 15th IEEE India Council International Conference (INDICON)*, 1–5, Coimbatore, India, 2018.

[23] Konno, K., Q. Chen, K. Sawaya, and T. Sezai, "Optimization of block size for CBFM in MoM," *IEEE Transactions on Antennas and Propagation*, Vol. 60, No. 10, 4719–4724, Oct. 2012.

[24] Brown, P. N. and Y. Saad, "Hybrid Krylov methods for nonlinear systems of equations," *SIAM Journal on Scientific and Statistical Computing*, Vol. 11, No. 3, 450–481, May 1990.