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Synthesis of Antenna Array Based on Hybrid Improved Sparrow Search Algorithm and Convex Programming

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ABSTRACT: This paper presents a hybrid method that combines the improved sparrow search algorithm (ISSA) and convex programming (CP) to synthesize sparse arrays under multiple design constraints. The proposed method initiates with introducing ISSA, which establishes a sparse array layout while effectively reducing peak sidelobe levels (PSLLs) through optimizing nonuniform element positions. Subsequently, when the position is fixed, the subproblem of minimizing PSLL is transformed into a convex problem with beamwidth constraint, employing CP to determine optimal excitation amplitudes. The PSLL serves as the fitness function in ISSA to simultaneously optimize both element position and excitation amplitude, to achieve PSLL reduction of sparse array. Afterward, some examples of linear and rectangular planar arrays with low sidelobe are simulated and discussed in detail. Numerical experiments show the effectiveness and reliability of ISSA-CP, which can further reduce PSLL while saving array elements. Ultimately, utilizing the numerical simulation results as a foundation, a full-wave simulation is undertaken to verify the practicality of the novel hybrid method.

1. INTRODUCTION

Nowadays, in many radar and wireless communication systems, the synthesis of nonuniform array antennas is a challenging and significant problem that attracts even more attention [1–4]. The nonuniform array antenna with unequal spacing has more outstanding advantages than the uniform array. For example, nonuniform array can save the quantity of array elements and control system costs. At the same time, it obtains a low sidelobe easily, achieves better directivity, and reduces the influence of mutual coupling. Moreover, there are two categories of nonuniform arrays, including thinned arrays and sparse arrays. Generally, when optimizing the element position, the sparse array offers an enhanced degree of freedom to achieve better radiation characteristics than thinned arrays [5].

According to previous research, by optimizing the excitation phase, excitation amplitude, and position of array elements, an array antenna pattern with low peak sidelobe level (PSLL) can be obtained under multiple constraints [6]. Over the past several decades, many methods have emerged for antenna array synthesis. It can be divided into three main categories: numerical synthesis techniques, intelligent optimization algorithms, and convex programming (CP). Among most general numerical synthesis techniques, representative methods include fast Fourier transform (FFT) [7], matrix pencil method (MPM) [8], etc. Such methods have attracted great attention from antenna designers. However, if a good initial point is not used, it may lead to convergence to the local minimum, and then the optimal solution cannot be guaranteed [9].

Currently, advancements in numerical optimization technology have facilitated the increasing adoption of intelligent optimization algorithms for array pattern synthesis, such as differential evolution (DE) [10–12], genetic algorithm (GA) [13–17], go-caterpillar mutation (GCM) [18], simulated annealing (SA) [19], and sparrow search algorithm (SSA) [20, 21]. These methods are biologically inspired algorithms and use population-based random search to exhibit outstanding performance in synthesizing sparse arrays, effectively obtaining optimal solutions. For example, Chen et al. [22] proposed a modified real GA (MGA) to transition the space from actual distance to Chebyshev distance between sparse array elements. To improve the degree of freedom of element distribution, Dai et al. [23] proposed DE with an asymmetric mapping method (DE-AMM) to transform element position optimization into an optimization problem of variable matrix **X**, **Y**, and state matrix **W**. In addition, Zhao et al. [18] proposed the GCM optimization algorithm (GCM-OA) to update the array by changing the position of random elements during every iteration to reduce the robustness and speediness of PSLL. As a new algorithm proposed in recent years, SSA has the characteristics of simple generality, strong robustness, and good optimization effect. On this basis, Tian et al. [21] applied density-weighted and chaos SSA (CSSA) in sparse array synthesis optimization to decrease the array PSLL to some extent. However, since these methods are only adjusted for the element position, their performance in achieving low PSLL is relatively limited.

The optimization problem whose objective function and constraint are both convex is called a convex optimization problem. For a constrained optimization problem, once it is convex, the global optimal solution can be obtained quickly and accurately by using CP [24]. Lebret and Boyd [25] showed how to transform the array synthesis problem into a convex optimization problem and applied CP to the array synthesis for the first

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time. In addition, they listed the actual optimization results of the problems such as decreasing sidelobe, beamforming, and broadband synthesis, which laid the foundation for further research. In recent years, as researchers have paid more attention to nonuniform arrays, CP is widely used in array antenna fields because of its high efficiency. By constructing a convex problem with position and excitation as variables, CP is used to solve the problem to achieve the goal of array synthesis. For example, Nai et al. [26] used the iterative weighted l_1 norm method to sparsify the quantity of array elements, thereby indirectly achieving the purpose of optimizing the position of the element. Angeletti and Toso [27] proposed a deterministic method that combined both the positions and excitation amplitudes to design linear aperiodic arrays, which used the weighted L2 norm to optimally fit the pattern of a reference linear continuous aperture. Battaglia et al. [28] presented an innovative mask-constrained beam power synthesis approach that overcomes the constraints of conventional techniques, offering enhanced versatility and flexibility.

Besides, in low sidelobe array synthesis, the subproblem of minimizing PSLL becomes a convex problem when the element position is fixed. Therefore, the global optimization algorithm combined with CP becomes an effective method to solve this problem. Isernia et al. [29] used SA combined with CP (SA-CP) method to optimize the position and excitation amplitude jointly for the linear array synthesis. Moreover, Cui et al. [30] used the DE combined with CP (DE-CP) method to synthesize low PSLL of linear arrays with pencil beams. At present, CP is introduced into sparse array synthesis due to its efficient process and better results of optimization. Furthermore, better array radiation characteristics can be achieved through the intelligent optimization algorithm combined with CP of the element excitation and position.

Through analysis of the above sparse array synthesis methods, it can be concluded that achieving a better synthesis effect of the array pattern is a complex problem that needs to optimize the array element quantity, position, and excitation. In the realm of array synthesis with low sidelobe levels, the effect is limited by the optimization of only element positions. Hence, a joint approach to optimizing both position and excitation is employed to address the issue of inadequate degrees of freedom in the design of nonuniform arrays. This strategy not only enhances the design flexibility but also reduces the PSLL of the array pattern. In contrast to alternative intelligent optimization algorithms, SSA is highly competitive regarding convergence rate, search accuracy, and stability. However, SSA also has insufficient population diversity, and it is easy to fall into local optimal problems when solving nonlinear problems. In view of these problems, it is necessary to improve SSA. By improving SSA with multiple strategies, the efficiency and search accuracy of SSA in element position optimization in sparse arrays are improved. After that, using the improved SSA to search the element position can improve the efficiency of sparse array optimization and obtain better antenna radiation characteristics. When the position of the array element is determined by SSA optimization, the array factor is a convex function about the excitation amplitude, and the optimal excitation amplitude can be effectively solved by CP, to achieve the reduction of PSLL. At present, there are many mature solvers for solving convex optimization problems, and the MATLAB toolbox CVX [31] is chosen in this paper. Furthermore, by designing the fitness function reasonably, the position and excitation of the array can be obtained quickly, and the PSLL can be reduced effectively.

In this paper, a novel hybrid method combining improved sparrow search algorithm (ISSA) with CP is proposed for the synthesis of linear and rectangular planar arrays under multiple constraints. This novel hybrid method jointly optimizes both the element position and excitation amplitude, effectively minimizing the PSLL of the sparse array. To begin with, the hybrid method is divided into two parts. In the first part, the SSA is improved.

1) The inherent chaotic disorder and ergodic properties in the initialization process are used to enhance the degree of freedom of array position.

2) The adaptive T-distribution variation method is used to perturb individual positions to improve the ability of the algorithm to jump out of the local optimal.

3) When the algorithm is trapped in convergence stagnation, the opposition learning strategy is introduced to help the algorithm jump out of the current solution and find other optimal solutions.

By improving the SSA, the efficiency and search accuracy of SSA in element position optimization in sparse array are improved. In the second part, CP is used to solve the convex problem of excitation amplitude under the minimum PSLL target to search for the global optimal solution. Furthermore, the adoption of a symmetric array layout manages the number of optimization variables efficiently while enhancing the algorithm's convergence rate. Therefore, the hybrid method enables element-count reduction in antenna arrays while maintaining radiation characteristics comparable to the full array, thereby reducing system costs.

The remaining sections of this paper are structured as follows. Section 2 provides the mathematical models for synthesis problems. Section 3 presents the improvement of sparrow search algorithm. Section 4 presents the CP used in the model of antenna array sparse synthesis. Section 5 presents numerical simulation outcomes and comparative analyses to validate the proposed method. Lastly, Section 6 provides conclusions drawn from this paper.

2. MATHEMATIC MODELS FOR SYNTHESIS PROBLEMS

Considering a rectangular planar array antenna of $2L \times 2H$ aperture which is centrosymmetric and placed in the *xoy* plane, the array element quantity is $4N$, and the distribution of elements in a quarter symmetrical array is illustrated in Figure 1.

In a rectangular planar array, the position of the first quadrant element can be represented by the coordinate (x_i, y_i). In order to satisfy the requirements of array aperture, the N-th element coordinate is set to (L, H). The positions of the other

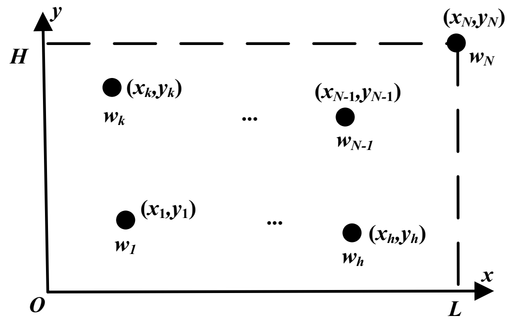

FIGURE 1. Geometry of a quarter sparse planar array.

three quadrants can be obtained symmetrically from the position coordinates of the first quadrant. Suppose that in an array with element quantity $4N$, the i-th element coordinate is $d_i = (x_i, y_i)$, and the excitation of the element is w_i. Therefore, a sparse rectangular planar array factor can be represented as follows:

$$F(u,v) = \sum_{i=1}^{4N} w_i \mathrm{e}^{\mathrm{j}k(x_i u + y_i v)} \tag{1}$$

where $u = \sin\theta\cos\varphi$, $v = \sin\theta\sin\varphi$, θ indicates the pitch angle; φ represents the azimuth angle; k denotes the wave number, $k = 2\pi/\lambda$; λ represents the wavelength. Moreover, when $\varphi = 0$, a linear array factor can be obtained from the rectangular planar array factor, and the linear array is located on the x-axis. Then, the corresponding analysis can be carried out on this basis.

Sampling M points uniformly for the space angle u and v, then the array factor $F(u,v)$ can be defined as the following matrix form:

$$F(u,v) = \mathbf{A}(u,v)\mathbf{w} \tag{2}$$

where the matrix $\mathbf{A}$ is defined as:

$$\begin{aligned}\mathbf{A}(u,v) = &\Big[\mathrm{e}^{\mathrm{j}k(x_1 u + y_1 v)}, \mathrm{e}^{\mathrm{j}k(x_2 u + y_2 v)}, \\ &\ldots, \mathrm{e}^{\mathrm{j}k(x_{N-1} u + y_{N-1} v)}, \mathrm{e}^{\mathrm{j}k(x_N u + y_N v)}\Big]\end{aligned} \tag{3}$$

The excitation amplitude values of N elements form vector $\mathbf{w}$:

$$\mathbf{w} = [w_1, w_2, \ldots, w_{N-1}, w_N]^T \tag{4}$$

For the convenience of calculation, the matrix coordinate vectors $[x_1, x_2, \ldots, x_N]$ and $[y_1, y_2, \ldots, y_N]$ are converted into two matrix position matrices $\mathbf{X} \in \mathbf{R}^{P\times Q}$ and $\mathbf{Y} \in \mathbf{R}^{P\times Q}$:

$$\mathbf{X} = \begin{bmatrix} x_{1,1} & \ldots & x_{1,Q} \\ \vdots & & \vdots \\ x_{P,1} & \ldots & x_{P,Q} \end{bmatrix}, \quad \mathbf{Y} = \begin{bmatrix} y_{1,1} & \ldots & y_{1,Q} \\ \vdots & & \vdots \\ y_{P,1} & \ldots & y_{P,Q} \end{bmatrix} \tag{5}$$

The formula for calculating P and Q is as follows:

$$\begin{cases} \max\left\{\frac{P\times Q - N}{P_{\max}\times Q_{\max}} + \frac{L-(P-0.5)d_{\min}}{L} + \frac{H-(Q-0.5)d_{\min}}{H}\right\} \\ s.t. \quad P\times Q \geq N, \quad P,Q \in Z^+ \\ \qquad P \leq P_{\max}, \quad Q \leq Q_{\max} \\ \qquad P_{\max} = \lfloor L/d_{\min}\rfloor, \quad Q_{\max} = \lfloor H/d_{\min}\rfloor \end{cases} \tag{6}$$

where $d_{\min}$ means the minimum distance between two adjacent elements, and $\lfloor * \rfloor$ means round down. When $P \times Q = N$, both $\mathbf{X}$ and $\mathbf{Y}$ are full arrays. When $P\times Q > N$, $P\times Q - N$ elements will be sparse to ensure that element quantity is N.

Under multiple constraints, the array element position and excitation amplitude are optimized to achieve the optimization goal of minimizing PSLL.

In a rectangular planar array, minimizing PSLL is used as the optimization objective. The fitness function is defined as the sum of two PSLLs about $\varphi = 0$ and $\varphi = \pi/2$ planes:

$$\mathrm{PSLL}(\mathbf{X},\mathbf{Y},\mathbf{w}) = \max\left\{\left|\frac{F\,|_{(\theta,0)}}{F_{\max}}\right| + \left|\frac{F\,|_{(\theta,\pi/2)}}{F_{\max}}\right|\right\} \tag{7}$$

The fitness function can also be defined as the PSLL of the full plane:

$$\mathrm{PSLL}(\mathbf{X},\mathbf{Y},\mathbf{w}) = \max\left\{\left|\frac{F\,|_{(\theta,\varphi)}}{F_{\max}}\right|\right\} \tag{8}$$

where $F_{\max}$ represents the mainlobe maximum value. Moreover, the values of pitch angle θ and azimuth angle φ are the sidelobe regions of the array radiation pattern.

For the synthesis problem of low sidelobe unequal spacing array with a symmetrical pattern under multiple constraints, the optimization model is as follows:

$$\begin{cases} \min\limits_{\mathbf{X},\mathbf{Y},\mathbf{w}} \ \mathrm{PSLL}(\mathbf{X},\mathbf{Y},\mathbf{w}) \\ s.t. \quad 0 \leq w_i \leq 1, \quad 1 \leq i \leq N \\ \qquad \sqrt{(x_{k,g}-x_{j,h})^2 + (y_{k,g}-y_{j,h})^2} \geq d_{\min} \\ \qquad 1 \leq k,\ j \leq P;\ 1 \leq g,\ h \leq Q;\ (k,g) \neq (j,h) \\ \qquad 0 \leq x_{k,g},\ x_{j,h} \leq L;\ 0 \leq y_{k,g},\ y_{j,h} \leq H; \\ \qquad x_{P,Q} = L;\ y_{P,Q} = H \end{cases} \tag{9}$$

The element position matrix $\mathbf{X}$, $\mathbf{Y}$, and excitation amplitude $\mathbf{w}$ are set as the optimization variables of this problem. Moreover, the minimization of PSLL in the array radiation pattern is achieved through solving optimal $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{w}$.

3. IMPROVED SPARROW SEARCH ALGORITHM

3.1. Sparrow Search Algorithm

SSA is an intelligent optimization algorithm that searches for optimal solutions based on the sparrows behavior of foraging and anti-predation. Assume that N_p sparrows exist in the dim-dimensional search space given in SSA, and the i-th sparrow position is represented by $d_{i,g} = x_{i,g} + j \cdot y_{i,g}$, where i is an integer from 1 to N_p, g is an integer from 1 to dim, and the sparrow position distribution can be defined by a matrix as follows:

$$\mathbf{d} = \begin{bmatrix} x_{1,1}, y_{1,1} & x_{1,2}, y_{1,2} & \ldots & x_{1,dim}, y_{1,dim} \\ x_{2,1}, y_{2,1} & x_{2,2}, y_{2,2} & \ldots & x_{2,dim}, y_{2,dim} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N_P,1}, y_{N_P,1} & x_{N_P,2}, y_{N_P,2} & \ldots & x_{N_P,dim}, y_{N_P,dim} \end{bmatrix} \tag{10}$$

The SSA process is as follows.

Step 1: Initialize the population and related parameters, and calculate the fitness value of the initial population.

Step 2: Update the position of explorers:

$$d_{i,g}^{it} = \begin{cases} d_{i,g}^{it} \cdot \exp(\frac{-i}{\eta \cdot I_{ter}}), & R_1 < ST \\ d_{i,g}^{it} + Q_r \cdot \mathbf{L}_1, & R_1 \geq ST \end{cases} \quad (11)$$

where it represents the number of current iterations; I_{ter} denotes the maximum iterations; $d_{i,g}^{it}$ represents the i-th sparrow position in the g dimension of the it generation; η denotes the uniform random number between $(0, 1]$; Q_r represents a random number that conforms to a standard normal distribution; $\mathbf{L}_1$ denotes a matrix with all 1 units; $R_1 \in (0, 1)$ indicates the warning value; and ST denotes the safety value. When $R_1 < ST$ indicates that the current population is safe in this search environment, explorers can expand the space on this basis, and then lead the population to achieve higher fitness. On the other hand, when $R_1 < ST$ indicates that the current search environment is dangerous, the population immediately moves towards a safe area.

Step 3: Update the position of followers:

$$d_{i,g}^{it} = \begin{cases} Q_r \cdot \exp(\frac{d_{worst}^{it} - d_{i,j}^{it}}{i^2}), & i > n/2 \\ d_p^{it+1} + \left| d_{i,g}^{it} - d_p^{it+1} \right| \cdot A^+ \cdot \mathbf{L}_1, & i \leq n/2 \end{cases} \quad (12)$$

where d_p denotes the explorer's current optimal position, and d_{worst} denotes the global worst individual position at present. When $i > n/2$, it means that the i-th follower is in a situation of low fitness currently. So the follower will go to other areas for foraging behavior to obtain higher fitness. On the contrary, when $i \leq n/2$, it means that the i-th follower will find a position randomly for foraging which is near d_p.

Step 4: Update aware of dangerous sparrow positions:

$$d_{i,g}^{it+1} = \begin{cases} d_{best}^{it} + \beta \cdot \left| d_{i,g}^{it} - d_{best}^{it} \right|, & f_i > f_g \\ d_{i,g}^{it} + K_r \cdot \left(\frac{\left| d_{i,g}^{it} - d_{worst}^{it} \right|}{(f_i - f_w) + \varepsilon_0} \right), & f_i = f_g \end{cases} \quad (13)$$

where d_{best} represents the global optimal position at present, and β denotes a random number subject to a normal distribution whose variance is 1, and average is 0. $K_r \in [-1, 1]$ represents a random number. f_i, f_g, and f_w denote the fitness values of each individual, current global optimal individual, and worst individual, respectively. ε_0 is set to avoid having a zero in the denominator. In the initial stage, the position of this part of the sparrow is random. When $f_i > f_g$, it is defined that the periphery sparrow in population should move closer to the safe area. When $f_i = f_g$, it means that the central individual in the population needs to move towards others to avoid being attacked by predators, so as to achieve position adjustment.

Step 5: Update the current best solution.

3.2. Tent Chaotic Model

It is of vital importance that the position of an initial group will affect the overall development direction of the whole group to a certain extent. By introducing tent chaotic disturbance into SSA, the initial solution can have better ergodic uniformity. An improved tent chaotic model is chosen to initialize the population for its relatively fast convergence and good ergodic uniformity. The self-mapping expression for tent chaos is:

$$z_{i+1} = \begin{cases} 2z_i + \text{rand}(0,1)/N_p I_{ter}, & 0 \leq z \leq 1/2 \\ 2(1 - z_i) + \text{rand}(0,1)/N_p I_{ter}, & 1/2 < z \leq 1 \end{cases} \quad (14)$$

After the Bernoulli transformation, the expression is:

$$z_{i+1} = (2z_i) \bmod 1 + \text{rand}(0,1)/N_p I_{ter} \quad (15)$$

where $\text{rand}(0, 1)$ denotes the random number between $[0, 1]$.

3.3. T Distribution

The adaptive t distribution variation adjusted by dynamic selection probability is introduced in the iteration process, and the iteration number it is used as the degree of freedom parameter of t distribution, which can improve the probability of the adaptive t distribution mutation operator to disturb the sparrow position in the early iteration stage, so as to enhance the ability of the algorithm to jump out of the local optimal solution in the early iteration stage. At the same time, in the late iteration, the excellent local development ability of the original algorithm is brought into play, and the small probability of t distribution variation is used as a supplement to improve the convergence speed of the algorithm. The specific location update method is as follows:

$$\begin{cases} d_{i,g}^{it+1} = d_{i,g}^{it} + d_{i,g}^{it} \cdot t(it) \\ p = w_1 - w_2 \times (I_{ter} - it)/I_{ter} \end{cases} \quad (16)$$

where w_1 determines the upper limit of the dynamic selection probability, and w_2 determines the amplitude of change of the dynamic selection probability. Based on $d_{i,g}^{it}$, the proposed updated formula adds a random interference term $d_{i,g}^{it} \cdot t(iter)$, which can not only make full use of the current position information, but also add random interference information, which can avoid the algorithm falling into local optimal.

3.4. Opposition-Based Learning

Opposition learning is a commonly used strategy to jump out of the local optimal solution position. In the process of algorithm iteration, an opposing learning strategy is introduced for convergence stop detection. If the ratio of fitness function value f_i of individual sparrow to optimal fitness function value f_g in population approaches 1 after it_0 iterations (30 times in this paper) and the algorithm does not terminate, then the algorithm is considered to be in convergence stagnation. When the algorithm falls into convergence stagnation, the opposing learning strategy is introduced in the reconnaissance part of the population to increase the diversity of sparrow population and improve iteration efficiency. The mathematical expression is as follows:

$$d_{i,g}^{it+1} = lb + ub - \beta \cdot d_{i,g}^{it} \quad (17)$$

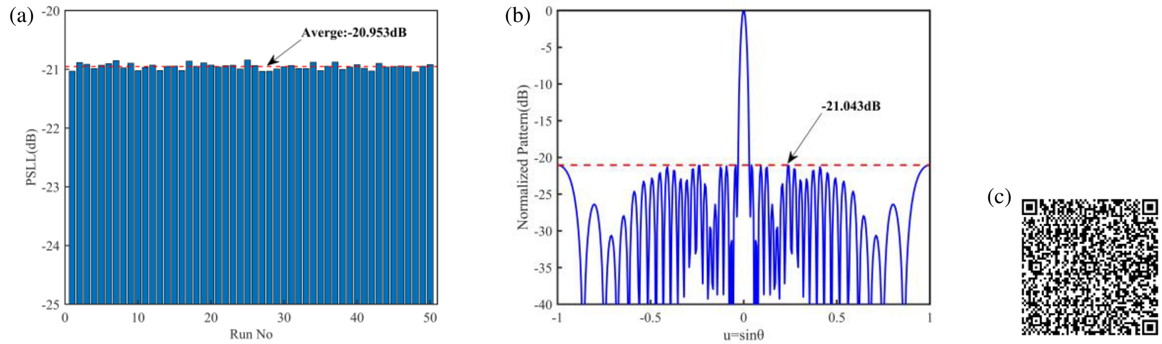

FIGURE 2. Results of 37-elements sparse linear array obtained by the ISSA method: (a) results of 50 independent experiments, (b) radiation pattern, (c) position.

where lb and ub are the lower and upper bounds of the search space, respectively.

4. CONVEX PROGRAMMING

In a convex optimization problem, achieving a local optimal solution coincides with attaining the global optimal solution. By converting the original non-convex formulation into a convex problem, an effective solution path can be provided for the original problem, which can simplify the calculation process. Furthermore, by transforming the problem into a strictly convex form, the uniqueness of the global optimal solution is guaranteed, and the fuzziness of the optimization results is eliminated.

In the synthesis problem of the array antenna pattern, there are objective functions or constraints derived from the array antenna pattern $F(u, v)$. Observe in (1) that if the excitation **w** and position **d** of all elements are taken as variables at the same time, then $F(u, v)$ is highly nonlinear and non-convex with respect to **w** and **d**. However, Equation (1) is partially linear for the excitation amplitude **w**. Specifically, for the determined position **d**, $F(u, v)$ is the linear weight of the element factor $e^{\mathrm{j}k(x_i u+y_i v)}$. The weight of each element is the corresponding excitation w_i, which means that $F(u, v)$ is a linear function concerning **w** (and also a convex function). Therefore, when the position of the array element is obtained by ISSA, the corresponding array synthesis problem about the variable **w** can be converted into a convex problem. Then, the array excitation amplitude of the desired antenna pattern is optimized by convex optimization theory.

In this paper, the position **d** and excitation amplitude **w** are optimized to minimize the PSLL. For any array of determined element position **d**, the sub-problem of (7) and (8) is the convex problem about the excitation amplitude **w**, which is expressed as follows:

$$\begin{cases} \min\limits_{\mathbf{w}} & \mathrm{PSLL}(\mathbf{d}, \mathbf{w}) \\ s.t. & |A(u_k, v_k)\mathbf{w} - F_d(u_k, v_k)| \leq \varepsilon \end{cases} \tag{18}$$

where ε denotes an approximate error, and it is usually set to 0.01; $F_d(u_k, v_k)$ represents the expected main beam shape. The CVX solver can effectively solve the above problems, calculate the excitation amplitude **w**, and determine the corresponding PSLL.

5. ANALYSIS OF SIMULATION RESULTS

To verify the validity of the proposed hybrid synthesis method, various simulations and discussions are carried out in this part. In all calculation examples, the population size is 200; the number of iterations is 300; the discoverer probability is set to $P_N = 0.2$; the alert probability is set to $S_N = 0.2$; and the safe value is set to $ST = 0.8$.

5.1. Symmetric Linear Array Synthesis with ISSA Optimized Array Position

In this experiment, only ISSA is used to optimize the position of the element, without considering the influence of the excitation amplitude of the element, so as to prove the effectiveness of ISSA.

For this experiment, Equation (8) is chosen as the fitness function for the linear array. The proposed ISSA method is applied to the synthesis of the 37-elements linear array with a maximum aperture of 21.996λ. The minimum distance between two adjacent elements is set to 0.5λ. Figure 2(a) shows the PSLLs obtained after 50 independent experiments. Figure 2(b) shows the radiation pattern synthesized by this work. The PSLL obtained by this work is -21.043 dB, lower than the results of MGA [14] (-20.560 dB) and IGA [17] (-20.850 dB). At the same time, Figure 2(c) shows the QR code which obtains the corresponding element position at the lowest PSLL. In order to better analyze the convergence effect of the algorithm, Table 1 provides a comparison of the results of this work with the other methods mentioned above. It can be seen that compared with MGA and IGA, the optimal value, worst value, and average value of PSLL obtained by ISSA have obvious advantages.

TABLE 1. Comparison of different algorithms to optimize the PSLL of 37-elements sparse linear array.

Method	N	PSLL (dB)		
		Optimal	Average	Worst
MGA [14]	37	-20.560	-20.340	-20.180
IGA [17]	37	-20.850	-20.80	-20.660
This work	37	-21.043	-20.953	-20.842

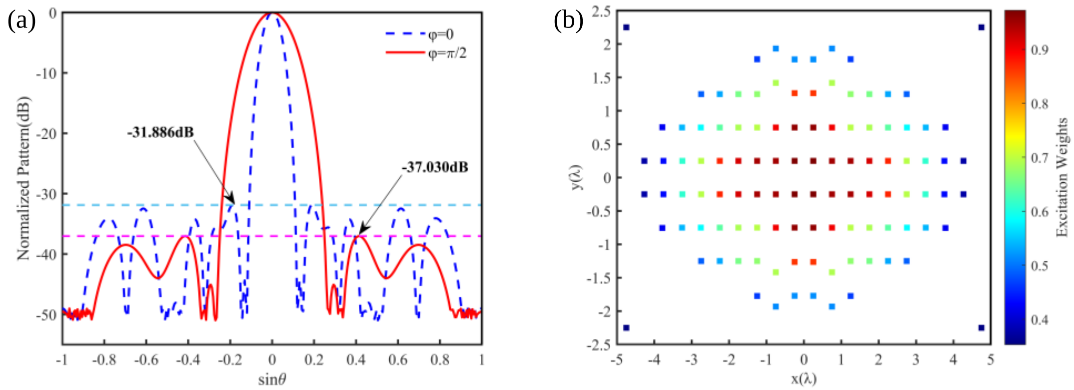

FIGURE 3. Results of example A obtained by the ISSA-CP method: (a) radiation pattern in $\varphi = 0$ and $\varphi = \pi/2$ planes, (b) position and excitation amplitude of the array elements.

5.2. Synthesis Based on ISSA-CP Joint Optimization of Array Position and Excitation Amplitude

Based on the analysis of the advantages of ISSA, a hybrid ISSA-CP method is proposed. The optimal array positions are obtained through the iterative process of the ISSA. Subsequently, based on the obtained position, CP is utilized to optimize the excitation amplitude for the sparse array.

5.2.1. Example A

For this experiment, the total element quantity of arrays is set to $4N = 108$, and the sum of PSLL on two planes $\varphi = 0$ and $\varphi = \pi/2$ is selected as fitness. Since the positions of the other quadrants of the array can be obtained symmetrically from the positions of the first quadrant, we only consider the first quadrant here.

About the optimization of the same model using MGA [22], DE-AMM [23], CSSA [21], and GCM-OA [18], it can be seen that in order to ensure the consistency of array aperture when optimizing a sparse array, it is of vital importance to ensure that the optimized array has elements in the four corners of the whole aperture. Besides, $P = 9$, $Q = 4$ can be calculated from (6). The uniform planar array of 9×4 is used as the original array's first quadrant, and the thinned array of $N = 27$ weighted by the method mentioned in [21] is selected as the initial array. Then, the ISSA-CP method is used to optimize the element position and excitation amplitude of the array and carry out the subsequent simulation design.

Figure 3(a) shows the normalized pattern of the $\varphi = 0$ plane and $\varphi = \pi/2$ plane. The experimental results show that the optimal PSLL obtained by this work is -68.916 dB (-31.886 dB in the $\varphi = 0$ plane and -37.030 dB in the $\varphi = \pi/2$ plane), which is lower than MGA (-45.456 dB) [22], DE-AMM (-61.454 dB) [23], CSSA (-62.429 dB) [21], and GCM-OA (-66.862 dB) [18]. Figure 3(b) shows the position and excitation amplitude of the array elements corresponding to the optimal PSLL obtained by this method. Because the position and excitation amplitude of the array are symmetric with respect to the x-axis and y-axis, the position and excitation amplitude distributions of the other three quadrants can be obtained from the array elements in the first quadrant. The QR code in Figure 3(c) lists the coordinates corresponding and excitation amplitude to the first quadrant of the array, where it can be seen that the spacing between adjacent elements meets the minimum spacing of not less than 0.5λ in the array.

5.2.2. Example B

For this experiment, the total element quantity of the array is set to $4N = 100$. Equation (8) is chosen as the optimization target, and ISSA-CP is used to optimize the full-plane PSLL. Besides, $P = 9$, $Q = 4$ can be calculated from (6). By using the density-weighted method mentioned in [21], the original array's first quadrant is thinned by 11 elements. Then, the ISSA-CP method is used to optimize the element position and excitation amplitude of the array, aiming to meet the simulation design targets.

Figure 4(a) shows the normalized pattern of quarter, from which the optimal PSLL of -23.171 dB can be obtained in full-plane optimization. Besides, Figure 4(b) shows the normalized pattern in the uv plane, and Figure 4(c) shows three sections of the array pattern. Figure 4(d) illustrates both the positional layout and excitation amplitude of the array elements corresponding to the optimal PSLL. Because the position and excitation amplitude of the array are symmetric with respect to the x-axis and y-axis, the position and excitation amplitude distributions of the other three quadrants can be obtained from the array elements in the first quadrant. The corresponding position coordinates and excitation amplitude are shown in the QR code in Figure 4(d). In the full-plane PSLL optimization, the optimal result by ISSA-CP is -23.171 dB, which is lower than MGA (-18.840 dB) [22], DE-AMM (-21.886 dB) [23], CSSA (-22.195 dB) [21], and GCM-OA (-22.078 dB) [18]. Table 2 provides comparisons of the results of example A and example B with the other methods mentioned above.

It can be seen from the above two rectangular plane examples that the ISSA-CP method shows good effectiveness in finding the optimal PSLL. At the same time, compared with other advanced methods, the ISSA-CP method combined with the optimization of array position and excitation amplitude can better suppress PSLL and has certain advantages.

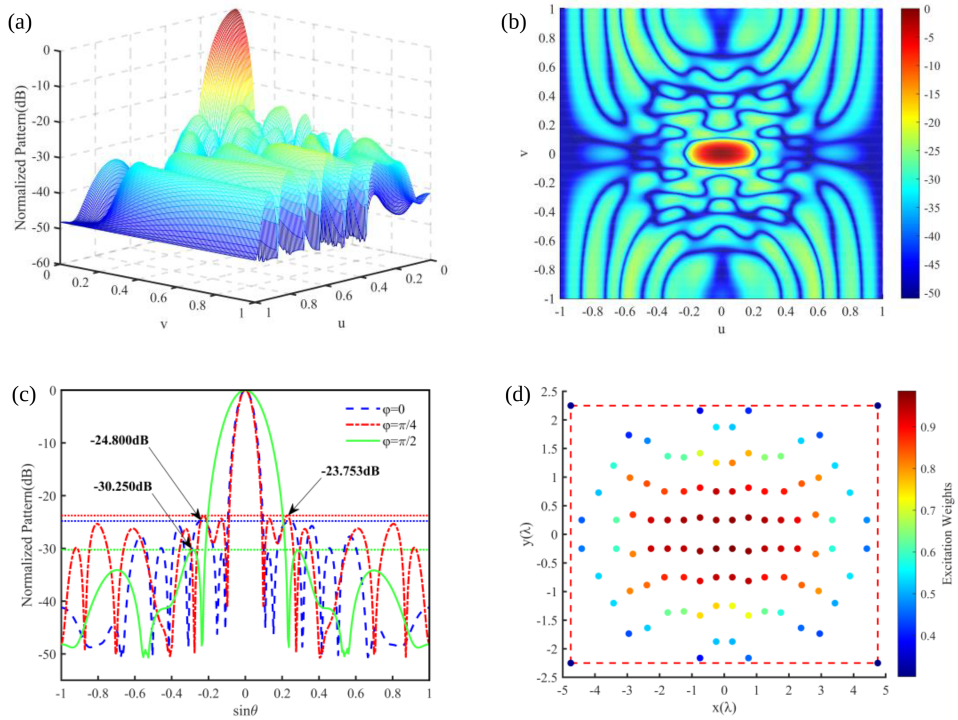

FIGURE 4. Results of example B obtained by ISSA-CP method: (a) quarter of 3D pattern, (b) normalized pattern in the uv plane, (c) radiation pattern in $\varphi = 0$, $\varphi = \pi/4$ and $\varphi = \pi/2$ planes, (d) position and excitation amplitude of the array elements.

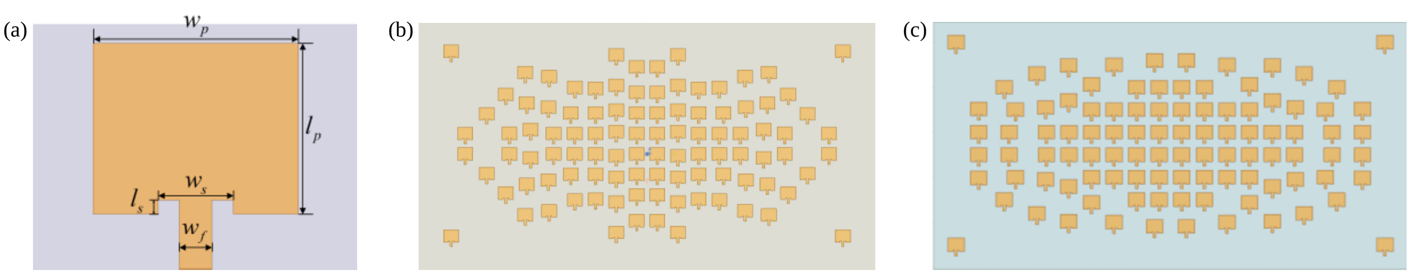

FIGURE 5. The simulation model: (a) antenna element, (b) sparse array analyzed by ISSA-CP, (c) sparse array analyzed by CSSA [21].

TABLE 2. Comparison of array PSLL for different algorithms to optimize the PSLL of example A and example B.

Method	MGA [22]	DEAMM [23]	CSSA [21]	GCM-OA [18]	This work
108-elements	−45.456	−61.454	−62.429	−66.862	−68.916
100-elements	18.840	−21.886	−22.195	−22.078	−23.171

5.3. Full-Wave Simulation Verification

To verify the influence of electromagnetic coupling on the actual array, the full-wave electromagnetic field solver HFSS is used to perform a strict full-wave simulation verification process on the optimization results of the rectangular planar array example B. In this experiment, the center frequency of the antenna is set to 30 GHz. The substrate in this experiment is Rogers 5880, featuring a loss tangent of 0.0009, a relative dielectric constant of 2.2, and a thickness measuring 0.635 mm. Figure 5(a) presents the rectangular microstrip patch antenna which is used as the antenna element. The patch has a width w_p of 3.65 mm and a length l_p of 3.04 mm. Moreover, the patch is cut through a slit with a width of 1.34 mm (w_s) and a depth of 0.25 mm (l_s) to achieve better impedance matching.

Through example B, the position coordinate and excitation amplitude of sparse array elements corresponding to the optimal PSLL can be obtained when the element quantity is $4N = 100$. Then, the sparse array obtained by this work is modeled using the antenna elements mentioned above, as shown in Figure 5(b). At the same time, the sparse array obtained in [21] with the same aperture size and element quantity is modeled using the element mentioned above, as shown in Figure 5(c). The radiation patterns obtained by full-wave simulation of the above two sparse arrays are shown in Figure 6(a) and Figure 6(b),

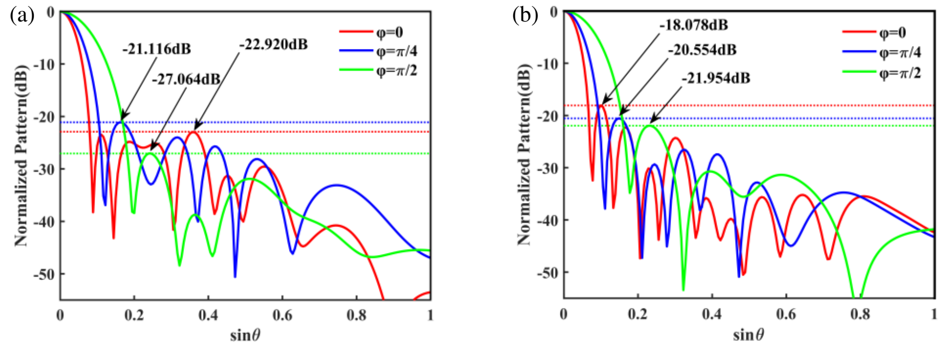

FIGURE 6. The radiation patterns obtained by full-swave simulation: (a) pattern in $\varphi = 0$, $\varphi = \pi/4$ and $\varphi = \pi/2$ planes by ISSA-CP, (b) pattern in $\varphi = 0$, $\varphi = \pi/4$ and $\varphi = \pi/2$ planes by CSSA [21].

TABLE 3. Comparison of full-wave simulation results of CSSA and ISSA-CP.

Method	N	Gain (dBi)	PSLL (dB)			3-dB Beamwidth (deg)		
			$\varphi = 0$	$\varphi = \pi/4$	$\varphi = \pi/2$	$\varphi = 0$	$\varphi = \pi/4$	$\varphi = \pi/2$
CSSA [21]	100	26.763	−18.078	−20.554	−21.954	6	8	13
This work	100	26.218	−22.920	−21.116	−27.064	7	8.5	13.5

respectively. Meanwhile, the radiation characteristics of the above two sparse arrays obtained by full-wave simulation are shown in Table 3. The simulation results show that, when a sparse array is synthesized under the same constraint conditions, such as element quantity and array aperture, the ISSA-CP method has a certain loss in gain compared with CSSA, and the half-power bandwidth (HPBW) is larger, but the obtained PSLL has obvious advantages. The results of the simulation illustrate that this novel method has the advantage of suppressing PSLL by optimizing the array elements position and excitation amplitude, compared with CSSA which only optimizes the position.

6. CONCLUSION

A hybrid ISSA-CP method is proposed to synthesize linear and rectangular planar arrays under multiple constraints. In the hybrid method, the synthesis problem is divided into two parts. One part of the method involves a non-convex problem of optimizing the position of sparse array elements. This problem can be solved effectively by improving the search speed and precision of SSA. The other part involves the convex problem of the array excitation amplitude solved via CP. The proposed method optimizes the position and excitation amplitude of array elements jointly to minimize PSLL. At the same time, the degrees of freedom in optimization are increased by a reasonable selection of optimization models and variables. To prove the feasibility and effectiveness of the proposed method, numerical experiments of linear and rectangular planar arrays are carried out, respectively. Experimental results demonstrate that the proposed method can achieve PSLL reduction of sparse array under different array models and multiple constraints. Beyond the validated linear and rectangular planar arrays, the proposed method demonstrates significant potential for extension to concentric ring arrays and conformal arrays.

Furthermore, with the improvement of computing power and the further development of algorithms, low sidelobe array synthesis technology based on intelligent optimization algorithm and convex optimization will be more mature and popular. The application of these technologies will greatly promote the performance of radar and communication systems. Especially in the complex electromagnetic environment, the array antenna optimized by these technologies will show greater potential.

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