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Dynamic Resources Management for Integrated Optimized Entanglement in Quantum Repeater Networks

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ABSTRACT: Quantum repeaters are essential for long-distance quantum communication and surmounting challenges like signal attenuation and decoherence. Current quantum repeater networks are constrained by static cutoff times for low-fidelity, suboptimal resource allocation, and the absence of quantum-classical integration. This paper introduces a hybrid quantum-classical method to address these issues by employing dynamic cutoff times entrusted upon real-time fidelity decay and decoherence rates. Markov Decision Process (MDP) is utilized to characterize the system with the aim to optimize the entanglement generation, waiting and swapping processes. In this study, the objective is to reduce the time which is needed to realize end-to-end entanglement while meeting the requirements of classical channel capacity. To manage constraints such as classical user demands, quantum memory limits, and network congestion, Lagrangian optimization has been applied. The combined approach improves the use of both classical resources and quantum, providing a simplified solution that is adaptable to different users' needs and different network conditions. The effectiveness of the model is tested via simulations processes along with the mathematical process. It demonstrated important gains in fidelity preservation resource efficiency and latency minimization compared to the state-of-the-art methods. This study makes a valuable contribution in the development of quantum networks, providing a robust establishment to build a quantum network to support the security in distributed quantum computing and global communications.

1. INTRODUCTION

Quantum repeaters are crucial for extending quantum communication, surpassing no-cloning theorem's constraints [1]. It mitigates issues like signal loss and decoherence by segmenting the channel and regenerating entanglement dynamically [2]. This approach utilizes quantum key distribution (QKD) and enables quantum internet, integrating with a classical system for scalability and efficiency [3, 4]. Its importance is asserted for secure communications intercontinental quantum computing and precise distributed sensing/timing [5]. However, existing quantum repeaters face three main limitations, foremost in connection with entanglement generation and conservation across nodes [6]. Firstly, it is predominated by static cutoff times, ignoring real time fidelity decay due to decoherence, leading to suboptimal resource employ and weak entanglement [7]. Secondly, classical users and quantum repeaters are typically modeled independently regarding resources management, network congestion, and bit rate requirements [8]. Lastly, current architectures struggle to scale for quantum hardware and large complex topologies such as networks and error bearing. An integrated framing considering dynamic cutoff and Bellman equation are founded via cross layer optimization and a classical quantum co-design. This is fundamental for feasible, scalable quantum networks to address varying requirements and constraints [9]. The integration of classical and quantum components is decisive for a functionality of feasible quantum communication networks [10]. Therefore, a hybrid system architecture is encouraged via entanglement management to adapt the quantum layer and information transmission to the classical layer. Subsequently, the practical application should tackle challenges like network topology optimization, scaling, mitigating quantum memory decay, and decoherence. The system utilizes real time dynamic cutoff to obtained high entanglement quality. It must achieve enhanced latency and throughput whilst ensuring durable security which is bolstered by quantum cryptography [11]. This approach supplies scalable and efficient basics for future quantum network progress. This study strived to introduce a hybrid system that improves resources, dynamic cutoff times and classical user co-design in a multi-layer optimized method. It comprises link thresholds, fidelity decay, and decoherence rates employing the fidelity aware formula for cutoff times. In addition, we improved a quantum resource allocation and classical demand realization within an MDP framework by modified Bellman equation considering delivery time, unused entanglement, and quantum resources prorated to classical bit rate demands. The proposed models used Lagrangian multipliers to solve the nonlinear optimization method regarding the quantum memory and demand constraints. Further, we introduced real time adaptability to coincide user requirements and network congestion, thereby diminishing the delivery time. It harmonizes diverse demands by adjusting bit rates according to fidelity thresholds. This feature enhances quantum resource effectiveness in a hybrid network. Furthermore, integrating dynamic cutoff with

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TABLE 1. Thematic related works.

Thematic Group	Limitations
Quantum Repeater Chain Enhancements	Static cutoff times, neglecting the dynamic nature of fidelity decay. No adaptive cutoff policies that respond to real time network conditions [12–19].
Resource Management in Quantum Repeater	Lack integration between quantum repeaters and classical systems, disregarding network congestion and bit rate demands [12, 16, 17, 19, 22, 23].
Optimization Models in Quantum Networks	Not fully integration with real-time adaptation to quantum and classical resource demands. Further, the optimization is not fully explored in existing works [19–21, 24, 25].
Entanglement Distribution and Fidelity Management	Ignore dynamic management of fidelity and rely on static swap [12, 19, 23, 25].
Hybrid Quantum-Classical Network Integration Research Gaps	Fail to address complex network topologies. An integrated approach combining both layers with adaptive policies is essential [8, 20, 24, 25].

classical demand models utilizing Lagrangian optimization is proposed, incorporating fidelity decay physics and balancing a quantum allocation with classical demand amongst congestion.

2. LITERATURE SURVEY

In [12], a static cutoff time-based bipartite entanglement distribution method via quantum repeater chains is presented, eschewing dynamic time adjustments. In [13], quantum repeater chain management strategies are introduced for entanglement distribution, neglecting explicit resource optimization using Bellman's equation. The study of [14] optimizes repeater parameters for benchmark-matching long-distance quantum communication, but omits temporal fidelity considerations and dynamic cutoff adjustments. The authors in [15] considered entanglement distribution methods based on network topology and coherence times, yet their approach lacks dynamic cutoff mechanisms and static swap frequency analysis. Conversely, [16] focused on reducing entanglement pair generation time with optimized resource utilization, without integrating fidelity loss prevention in deteriorating links through adaptive cutoff times. In [17], the focus was on steady-state entanglement distribution and continuous operation, neglecting dynamic management and classical integration. In contrast, the authors of [18] refined quantum repeater analysis using Markov chains to compute transmission rates and waiting times with fixed cutoffs. This differs from the proposed adaptive approach requiring dynamic cutoff adjustments according to fidelity deterioration. Meanwhile, [19] analyzed fidelity degradation and optimization in linear fundamental repeater chains with static policies. After that, [20] presented a method for predicting user requests to boost entanglement generation, focusing on immediate latency enhancements. Ref. [21] utilized queuing theory to compare sequential and parallel quantum state distribution in one-way networks but did not address resource constraints and fidelity degradation. Subsequently, [22] examined a star-topology quantum entanglement switch, analyzing decoherence effects and cutoff times with a basic model, overlooking dynamic adjustments based on fidelity decay. In [23], the study analyzed noise and imperfections' effects on quantum repeater networks' entanglement distribution rates and fidelity, highlighting limitations but not suggesting dynamic adaptability or hybrid operation solutions. Ref. [24] introduced an MDP-based theoretical framework for assessing near-term quantum networks, concentrating on fundamental communication and steady-state dynamics while neglecting multi-layered hybrid architecture complexity testing. Another study [25] addressed long-distance entanglement distribution challenges via probabilistic entanglement swaps, assuming static or non-existent cutoffs. Finally, in [26], the study concentrated on enhancing multiplexing strategies and entanglement distillation via quasi-local policies to elevate fidelity and minimize waiting times. However, this work missed using dynamic cutoff times and testing the hybrid integration. The term "state-of-the-art" (SOTA) refers to the current leading edge of quantum communication techniques [27], which serve as benchmarks for comparison with the proposed framework, such as enhancing entanglement distribution and swapping while countering decoherence over large distances, as discussed in [12, 13, 18]. Current research often relies on static cutoff times without considering real-time fidelity decay or quantum-classical bit rate interdependence [12, 16, 19]. While some works propose handling classical and quantum resource demands concurrently [15, 22], they lack adaptive allocation strategies. Our novelty lies in the dynamic cutoff mechanism for entangled links, which adjusts to real-time conditions [7, 12]. This contrasts with the static cutoffs prevalent in earlier studies [13, 19]. Subsequently, the paper introduces a dynamic framework to enhance resource allocation, addressing limitations in current studies such as static cutoff policies, quantum-classical communication disintegration, scalability inefficiencies, and fidelity degradation. It features adaptive cutoff times, integrated quantum-classical resource management, and efficient scalability with fault tolerance, aiming to fill research gaps for practical, scalable, and reliable quantum communication network implementation. Table 1 provides a thematic discussion about the provided related works.

3. METHODOLOGY

The methodology proposed a dynamic quantum repeater network model integrating quantum and classical resources. It em-

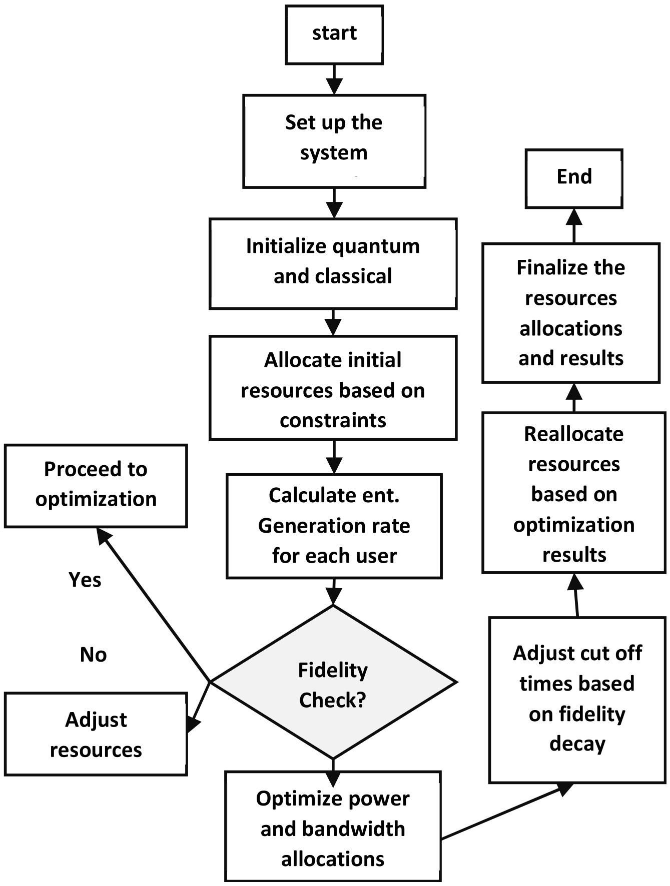

FIGURE 1. The optimization process and system model.

ploys an MDP with Lagrangian optimization, managing entangled links with dynamic cutoff times. This hybrid system enhances performance by optimizing decisions in fluctuating conditions and enabling real-time adjustments to user demands and network variations, while preserving a usable fidelity threshold. Figure 1 provides a flowchart of the proposed methodology for the system model and optimization process.

3.1. Dynamic Cutoff Times in Quantum Repeater

This proposed framework targets improved durability of entangled links via adaptive cutoff times. It involves a temporal state vector $s(t) = (g_{ij}(t))$, with $g_{ij}(t)$ denoting link age in discrete intervals. A dynamic cutting rule is introduced, discarding links where $g_{ij}(t) \geq t_{cut}(g_{ij}(t))$ to maintain robustness. Fidelity decay is modeled by equation:

$$F_{ij}(t) = F_{new} \exp\left(-\frac{t}{\tau}\right) \tag{1}$$

where F_{new} represents the new link fidelity, while τ is the decoherence constant. Resource optimization and fidelity preservation are integrated through a formulated dynamic cutoff time, which is a critical aspect as follows [12]:

$$t_{cut}\left(g_{ij}\right) = \tau \ln\left(\frac{3}{4F_{\min}} - \frac{1}{F_{new}}\right)^{\frac{1}{n-1}} \tag{2}$$

where $F_{\min}$ is the defined threshold for usable entanglement, and exponent $n-1$ scales the cutoff time linearly with network size n. We have developed a methodology addressing entangled link aging, with link age $g_{ij}(t)$ incrementally evolving as:

$$\begin{aligned} &g_{ij}(t+1) = g_{ij}(t) + 1, \\ &g_{ij}(t) \leq t_{cut}\left(g_{ij}\right) \ \text{(condition)} \end{aligned} \tag{3}$$

This is cast into an MDP framework for optimal decision-making under uncertainty in quantum repeater chains. The optimized policy $\pi(s)$ aims to minimize $T_\pi(s)$, with considering all possible $g_{ij}(t)$ configurations within state space S, and function J encapsulates the essence of the proposed approach by integrating three vital components, as follows

$$\begin{aligned} J \ &= \ T_\pi(s) = \min\left(\sum_{ij} g_{ij}(t) - t_{cut}\left(g_{ij}(t)\right)\right. \\ &\left. +\alpha \sum_i q_i(t) + \beta T_{delivery}\right) \end{aligned} \tag{4}$$

where $\sum_{ij} g_{ij}(t) - t_{cut}(g_{ij}(t))$ prioritizes entanglement links approaching dynamic cutoff, and $q_i(t)$ represents node i's quantum resources weighted by parameter α. $T_{delivery}$ is also significant, adjusted by factor β. The method adheres to constraints: $q_i(t) \leq Q_{\max} \forall I$, ensuring sufficient resources, $g_{ij}(t) \geq t_{cut}(g_{ij}(t))$ for efficient entanglement utilization, and respects classical user bit rate demands $R_u(t)$ via high-fidelity links as the use of an indicator function $\Pi(\cdot)$, i.e., which equals 1 if the fidelity of the entangled link surpasses the minimum threshold $F_{\min}$, and if the condition is not met, the function is 0. Lagrangian multiplier method efficiently encapsulates constraints, leading to the construction of the composite function L, which integrates the original objective J with constraint terms.

$$\begin{aligned} L \ &= \ J + \sum\nolimits_i \lambda_i \left(q_i(t) - Q_{\max}\right) \\ &+ \sum\nolimits_u \mu_u \left(R_u(t) - \sum\nolimits_{ij} \Pi\left(F_{ij}(t) \geq F_{\min}\right)\right. \\ &\left. \times \mathit{Bit\,Rate}(g_{ij}(t))\right) + v\left(C(t) - C_{\max}\right) \end{aligned} \tag{5}$$

In this formulation, λ_i, μ_u, and v are coefficients that quantify the importance of adhering to each constraint. Then, the proposed approach involves taking partial derivatives of the Lagrangian in relation to the variables of interest $(q_i(t), R_u(t))$, where with respect to $q_i(t)$, the partial derivative of the Lagrangian is given by:

$$\frac{\partial L}{\partial q_i(t)} = -\alpha \cdot \frac{\partial T_{delivery}\left(s, a\right)}{\partial q_i(t)} + \lambda_i \tag{6}$$

where α is a Lagrange multiplier at each node i. Setting this derivative equal to zero, we obtain:

$$\lambda_i = \alpha \cdot \frac{\partial T_{delivery}\left(s, a\right)}{\partial q_i(t)} \tag{7}$$

where λ_i represents the optimal quantum resource allocation for node i. Regarding bit rate demand $R_u(t)$, the corresponding partial derivative is expressed as:

$$\frac{\partial L}{\partial R_u(t)} = -\eta + \mu_u \frac{\partial}{\partial R_u(t)} \sum\nolimits_{ij} \Pi\left(F_{ij}(t) \geq F_{\min}\right) \times \mathit{Bit\,Rate}(g_{ij}(t)) \tag{8}$$

Here, η is a Lagrange multiplier that accounts for the cost of not meeting classical user demands, and μ_u is the congestion price, where equating this derivative to zero yields the condition:

$$\mu_u = \eta \cdot \frac{1}{\frac{\partial \sum_{ij} \Pi(F_{ij}(t) \geq F_{\min}) \times \mathit{Bit\ Rate}(g_{ij}(t))}{\partial R_u(t)}} \tag{9}$$

This equation represents the ideal bit-rate distribution for network resource efficiency. The dynamic cutting method was applied to assess optimization's temporal effect on cross-link aging, comparing pre- and post-optimization scenarios.

3.2. Resources-Demand via Bellman Equation

To enhance the Bellman equation for dynamic quantum resource allocation in a quantum network, we introduce a modified MDP incorporating resource availability and demand optimization. The state $s(t)$ encapsulates quantum resources and demand, defined by entangled link age $g_{ij}(t)$, fidelity $F_{ij}(t)$, and quantum resources $q_i(t)$ at node i, along with end-to-end entanglement progress $E(t)$, denoted as: $S(t) = (\{g_{ij}(t), F_{ij}(t), q_i(t)\}_{ij}, E(t))$. Actions $a(t)$ include entanglement generation, swapping, or waiting, with state transitions $P(s'/s(t), a)$ reflecting resource dynamics and demand satisfaction. The reward function $R(s, a)$ aims to minimize $T_{\mathit{delivery}}(s, a)$ for end-to-end entanglement and maximize fidelity, optimizing overall resource utilization, so the reward is expressed as:

$$R(s,a) = -\alpha T_{\mathit{delivery}}(s,a) - \beta\Big(\sum\nolimits_{ij} g_{ij}(t) - t_{\mathit{cut}}\left(g_{ij}(t)\right) - y\sum\nolimits_i q_i(t)\Big) \tag{10}$$

where we utilize positive constants α, β, y to adjust the significance of individual components. As a result, the original Bellman equation is adapted for the proposed model as follows:

$$T_\pi(s(t)) = \min_{a \in A}\left[R\left(s(t), a\right) + \sum_{s'} P\left(s'/s(t), a\right) \cdot T_\pi\left(s'(t)\right)\right] \tag{11}$$

Here, $T_\pi(s(t))$ denotes the expected entanglement delivery time under policy π for state $s(t)$. $R(s(t), a)$ signifies the reward from action a in state $s(t)$, with $P(s'/s(t), a)$ representing the transition probability to next state $s'(t)$, considering all possible next states s'. To handle resource constraints, we impose a non-negativity condition for qubit availability $q_i(t) \geq 0$, for all i and a demand satisfaction constraint $E(t) = \sum_{i,j}\left(1 - \frac{F_{ij}(t)}{F_{\min}}\right)$, and $E(t)$ quantifies the system's proximity to meeting end-to-end entanglement demand. This framework seeks an optimized quantum network for efficient entanglement management, balancing resource availability and demand.

3.3. Classical Oriented Quantum Repeaters

To extend the system's practicality, it was aimed to integrate classical users with specific bit rate requirements and existing classical infrastructure. This involved creating a model that combines quantum and classical elements. At time t, the system state encompasses both quantum and classical aspects: quantum resources are denoted by $g_{ij}(t)$, $F_{ij}(t)$, and $q_i(t)$, while classical user i's bit rate demand is represented by $U_i(t)$ and the total system demand by $R(t)$. The overall system state is thus defined as $s(t)$.

$$S(t) = \Big(\{g_{ij}(t), F_{ij}(t), q_i(t)\}_{ij}, \{U_i(t)\}, R(t)\Big).$$

Moreover, in the extended MDP framework, at any given time t, we have a set of actions $a(t)$ to choose from, including entanglement generation, swapping, or waiting when there are inadequate resources or low classical user demand. Then, the decision-making process is governed by transition probabilities $P(s'/s(t), a)$, where these probabilities are broken down into two components, which are $P(s'_{\text{quantum}}/s_{\text{quantum}}, a)$ and $P(s'_{\text{classical}}/s_{\text{classical}}, a)$. As a result, the mathematical representation of the proposed reward function is:

$$R(s,a) = -\alpha T_{\mathit{delivery}}(s,a) - \beta\left(\sum_{ij} g_{ij}(t) - t_{\mathit{cut}}\left(g_{ij}(t)\right) - y\sum_i q_i(t) - \delta R(t)\right) \tag{12}$$

where $T_{\mathit{delivery}}(s, a)$ is the time to achieve end-to-end entanglement from state (s) with action a; $(\sum_{ij} g_{ij}(t) - t_{\mathit{cut}}(g_{ij}(t)))$ measures the unused potential of an entangled link before reaching the cutoff time; $R(t)$ represents the current bit rate demand; α, β, y, δ are positive weights adjusting the importance of each objective. Thus, the modified Bellman is expressed as:

$$T_\pi\left(s(t)\right) = \min_{a \in A}\left[R\left(s(t), a\right) + \sum_{s'} P\left(s'/s(t), a\right) \cdot T_\pi\left(s'(t)\right)\right] \tag{13}$$

Also, the model suggests constraints on classical communications, with the total demand $R(t)$ bounded by the sum of high-fidelity entangled links $R(t) \leq \sum_{i,j} \Pi(F_{ij}(t) \geq F_{\min})$ for all pairs (i, j). Given a hybrid system, resource optimization seeks to minimize $T_\pi(s(t))$ required for entanglement distribution, by optimizing the rate function $R(s(t), a)$, as follows:

$$R(s,a) = -\alpha T_{\mathit{delivery}}(s,a) - \beta\left(\sum_{ij} g_{ij}(t) - t_{\mathit{cut}}\left(g_{ij}(t)\right) - y\sum_i q_i(t) - \delta R(t)\right) \tag{14}$$

Then, we introduce Lagrange multipliers to handle constraints effectively, for the quantum resources, so the Lagrangian for these quantum resource constraints is $L_q = \sum_i \lambda_i q_i(t)$, and moving on to the classical communication demand, the Lagrangian for the classical communication demand constraint is:

$$L_{classical} = \mu \left(R(t) - \sum_{ij} \Pi \left(F_{ij}(t) \geq F_{\text{min}} \right) \right) \tag{15}$$

As a result, we combine the original reward function with these Lagrangian terms to form the full Lagrangian L, as follows:

$$\begin{aligned} L \;=\; & -\alpha T_{delivery}(s,a) - \beta \Big(\sum_{ij} g_{ij}(t) - t_{cut}\left(g_{ij}(t)\right) \\ & - y \sum_i q_i(t) - \delta R(t) \Big) + \sum_i \lambda_i q_i(t) \\ & + \mu \left(R(t) - \sum_{ij} \Pi \left(F_{ij}(t) \geq F_{\text{min}} \right) \right) \end{aligned} \tag{16}$$

In addition in the quest for an optimal policy, we must differentiate the Lagrangian concerning the decision variables ($q_i(t)$ and $R(t)$), and differentiating L with respect to $q_i(t)$ and equating to zero, we obtain:

$$\frac{\partial L}{\partial q_i(t)} = -y + \lambda_i = 0 \tag{17}$$

At the optimal policy, this multiplier is equivalent to the cost penalty, $\lambda_i = y$. Similarly, differentiating L with respect to $R(t)$ and setting to zero gives us:

$$\frac{\partial L}{\partial R(t)} = -\delta + \mu = 0 \tag{18}$$

where $\mu = \delta$, and the demand for classical communication is harmonized with the cost penalty δ at the optimal state. Note that hybrid systems rely on elective criteria for selecting hyperparameters to ensure high-quality entanglements, expedite delivery, and optimize resource allocation. This adaptability is vital in the face of fluctuating network environments. Network design can utilize hyperparameters in various ways, tailored to the researcher's specific performance metric. While some may adopt random parameter selection for a general model, others may prefer increasing β to enhance delivery efficiency. It is noteworthy that certain parameters may hold greater significance in achieving particular objectives.

3.4. Multi-Layered Hybrid Quantum-Classical System

The proposed approach involves designing multi-layer hybrid quantum-classical systems addressing individual user bit rate demands $R_u(t)$ within total load $R(t)$, expressed as:

$$C(t) = \sum_u \frac{R_u(t)}{R(t)} \tag{19}$$

This framework aims to handle network congestion $C(t)$ efficiently. Additionally, the work introduces a dynamic multilayered network with a composite state vector $s(t)$ representing the comprehensive network state at any given time t, as follows: $s(t) = (\{g_{ij}(t), F_{ij}(t), q_i(t)\})\{R_u(t)\}_u C(t)$. As a consequence, the overall transition from $s(t)$ to $s'(t)$ due to action $a(t)$ involves a quantum-classical amalgamation, as indicated by equation:

$$\begin{aligned} P_{(S'/S,a)} \;=\; & P\left(s'_{\text{quantum}}/s_{\text{quantum}}, a\right) P\left(s'_{\text{classical}}/s_{\text{classical}}, a\right) \\ & P_{\text{congestion}}\left(C(t)\right) \end{aligned} \tag{20}$$

Here, P denotes transition probabilities. The goals are to fence quantum resource utilization with classical user requirements, improving delivery time and fidelity. Parameters α, β, y, δ, η, and k are employed to balance these factors effectively in the design of R, as follows:

$$\begin{aligned} R(s,a) = & -\alpha T_{delivery}(s,a) - \beta \left(\sum_{ij} g_{ij}(t) - t_{cut}\left(g_{ij}(t)\right) \right) \\ & - y \sum_i q_i(t) - \delta C(t) - \eta R(t) - k P_{error}\left(s(t), a\right) \end{aligned} \tag{21}$$

Here, $P_{error}(s(t), a)$ indicates a probability of error during entanglement generation or swapping, potentially influenced by deep-rooted quantum hardware imperfections or injurious network conditions. This is pivotal for assessing quantum protocol robustness and reliability amongst such challenges. So, the modified Bellman equation:

$$T_\pi(s(t)) = \min_{a \in A} \left[R(s(t), a) + \sum\nolimits_{s'} P\left(s'/s(t), a\right) \cdot T_\pi\left(s'(t)\right) \right] \tag{22}$$

Here, $T_\pi(s(t))$ betokens the expected time to meet classical user needs given state $s(t)$ and policy π. This study extends prior formulations of the objective function J, which quantifies quantum resources and classical requirements under dynamic network conditions through system state $s(t)$. The objective function J is defined as:

$$\begin{aligned} J \;=\; \min \Big[& -\alpha T_{delivery}(s,a) - \beta \Big(\sum_{ij} g_{ij}(t) - t_{cut}\left(g_{ij}(t)\right) \\ & - y \sum_i q_i(t) - \delta C(t) \Big) - \eta R(t) \Big] \end{aligned} \tag{23}$$

Then, we formulated the constraints which are expressed mathematically as follows:

$q_i(t) \leq Q_{\text{max}}$ for all nodes i (Quantum Resources).

$$R_u(t) \leq \sum_{ij} \Pi\left(F_{ij}(t) \geq F_{\text{min}}\right) \times \mathit{Bit\,Rate}\left(g_{ij}(t)\right).$$

$$C(t) = \sum\nolimits_u \frac{R_u(t)}{R(t)} \leq C_{\text{max}} \text{ (Network Congestion)}.$$

$g_{ij}(t) \leq t_{cut}\left(g_{ij}(t)\right)$ (Quantum Fidelity Decay and

TABLE 2. Model symbols.

Symbol	Description	Units/Remarks
n	Number of nodes in the network	-
T	Decoherence time constant	sec
$F_{\min}$	Minimum fidelity threshold	Dimensionless (0–1)
F_{new}	Fidelity of newly entanglement	Dimensionless (0–1)
$s(t)$	State vector at time t	-
$a(t)$	Action vector at time t	-
$\pi(s)$	Policy function in MDP	-
J	Reward function/Objective function	-
$\alpha, \beta, \gamma, \delta, \eta$	coefficients in the reward	-
g_{ij}	Weight at node i	-
$R_i(t)$	Bit rate demand of user i at time t	Bits/sec
$R(t)$	Aggregated classical bit rate	Unit: bits/unit time
$F(t)$	Fidelity of an entangled link at time t	-
$age(t)$	Age of an entangled link	Time unit
λ_i, μ, ν	Lagrange multipliers for constraints	-
$s'(t)$	Next state after action $a(t)$	Composite state vector
$(P(s's, a))$	Transition probability	-
Δt	Time step in simulations	sec

Cutoff Time).

Then, we introduce Lagrange multipliers λ_i, μ_u and v, as follows:

$$L = J + \sum_i \lambda_i (q_i(t) - Q_{\max}) + \sum_u \mu_u \Bigg(R(t) - \sum_{ij} \Pi$$

$$(F_{ij}(t) \geq F_{\min}) \times \mathit{Bit\,Rate}\,(g_{ij}(t)) \Bigg) + v\,(C(t) - C_{\max}) \quad (24)$$

We derive the Lagrangian for the system, considering $q_i(t)$, $R_u(t)$ and $C(t)$ as variables, as follows:

$$\frac{\partial L}{\partial q_i(t)} = -y \cdot \frac{\partial T_d}{\partial q_i(t)} + \lambda_i = 0 \quad (25)$$

This yields:

$$\lambda_i = y \frac{\partial T_{delivery}(s, a)}{\partial q_i(t)} \quad (26)$$

$$\frac{\partial L}{\partial R_u(t)} = -\eta + \mu_u$$

$$\frac{\partial \left(\sum_{ij} \Pi \left(F_{ij}(t) \geq F_{\min} \right) \times \mathit{Bit\,Rate}\,(g_{ij}(t)) \right)}{\partial R_u(t)} = 0 \quad (27)$$

This gives us:

$$\mu_u = \eta \cdot \frac{1}{\frac{\partial \left(R(t) - \sum_{ij} \Pi(F_{ij}(t) \geq F_{\min}) \times \mathit{Bit\,Rate}(g_{ij}(t)) \right)}{\partial R_u(t)}} \quad (28)$$

$$\frac{\partial L}{\partial C(t)} = -v \cdot \frac{\partial \left(\sum_u \frac{R_u(t)}{R(t)} \right)}{\partial C(t)} = 0 \quad (29)$$

This results in:

$$v = \frac{\partial \left(\sum_u \frac{R_u(t)}{R(t)} \right)}{\partial C(t)} \quad (30)$$

Hence, by adhering to Lagrangian-derived conditions, we achieve a balance in quantum resource allocation, meet classical user demands, and improve efficient network flux, ensuring that all constraints are satisfied for fulfillment maximization. Table 2 provides description of the symbols used in this work.

3.4.1. Lagrangian Formulation, Solvers, and Convergence Strategy

The objective of the optimization problem is to enhance a composite performance metric encompassing Quality of Service (QoS), Energy Efficiency (EE), cost efficiency, and fairness within a multi-user wireless communication system. This endeavor is subject to resource constraints, namely bandwidth, power, and minimum Signal-to-Interference-plus-Noise Ratio (SINR) thresholds for each user, which are essential for adhering to the system's physical and operational limitations.

A Lagrangian approach is employed to address this multifaceted challenge, introducing Lagrange multipliers that facilitate the transformation of the problem into a more manageable form suitable for numerical optimization. This technique effectively combines the objective function, representing the performance metrics, with the constraints that guarantee the solution's feasibility within the specified system constraints. MATLAB's fmincon solver is utilized to navigate the optimization

TABLE 3. Simulation parameters.

Parameter	Value/Description
Network Topology	Hybrid quantum-classical system with multiple nodes and interconnected links.
Number of Nodes (n)	5 nodes ($n = 5$).
Number of Classical Users	10 users, each with specific bit rate demands.
Decoherence Time Constant (τ)	1 time unit ($\tau = 1$).
Minimum Fidelity Threshold ($F_{\min}$)	0.2, ensuring operational fidelity for entangled links.
Initial Fidelity (F_{new})	Randomly initialized values between 0.2 and 1.0.
Quantum Resource Weights (g_{ij})	Defined as weights for resource usage (dimensionless), typically set to 1.
Bit Rate Demand ($R(t)$)	Each user has a random bit rate demand between 0 and 10.
Action Set ($a(t)$)	Actions include entanglement generation, swapping, or waiting based on available resources.
State Transitions ($s(t)$)	States are defined by age of entangled links, fidelity, and available quantum resources.
Reward Function (J)	Balances delivery time, fidelity, and quantum resource utilization based on user demands.

problem, employing a trio of algorithms: Sequential Quadratic Programming (SQP), Active-Set Algorithm, and Interior-Point Algorithm. These methods are selected for their proficiency in exploring the solution space and pinpointing the optimal allocation of resources. Initiating the optimization process, feasible initial guesses are provided for the decision variables, such as bandwidth and power assignments for each user. These initial conditions are decisive in determining the convergence speed and the solver's might to recognize the best possible outcome. The convergence of the optimization process was assessed using two major parameters: the stability of the objective function and a pledge to Karush Kuhn-Tucker (KKT) conditions. The convergence of the optimization was initially estimated founded on the stability of the objective function J, which embraces entanglement delivery time, employed entanglement, and quantum resource availability. The optimization process was deemed converged when the variation in the objective function J between consecutive iterations became less than a predefined threshold (ε), signalizing that those further iterations would not harvest substantial improvements. Posteriorly, given the presence of constraints like classical bit rate demands, quantum memory limits, and network congestion, the KKT conditions played a pivotal role in ensuring that these constraints were esteemed throughout the optimization. The system was deemed to have converged once Karush-Kuhn-Tucker (KKT) conditions were satisfied within an acceptable tolerance, confirming that the optimal solution was attained without violating any constraints. In addition, computational efficiency was factored into the convergence account. The convergence tolerance was set to strike an equilibrium between fidelity and computational practicality. For large scope systems with expanded quantum repeaters and classical users, heuristics or approximations as value or policy iteration were utilized to ensure near optimal solutions within a feasible time frame.

4. RESULTS AND ANALYSIS

The system fills by dynamically allocating quantum resources with classical communication bit rate requirements. It combines a classical and quantum system for efficacious entanglement distribution which harmonize varying user demands. The underlying traffic models incorporate demand-based adjustments, considering fidelity thresholds and network congestion. These parameters are pivotal in simulating real-time adaptability, optimizing resource utilization, and maintaining system performance amidst fluctuating conditions. They are consistently applied across the optimization framework and reflected in the simulation outcomes. Table 3 shows the simulation parameters and critical details used in the model.

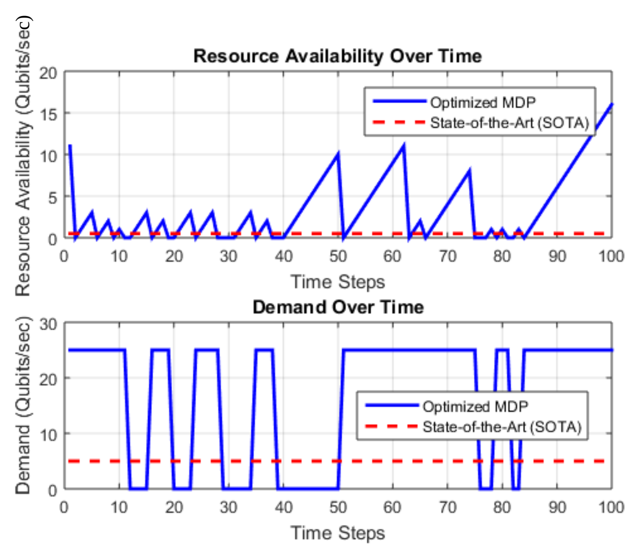

FIGURE 2. Resource management and demand satisfaction in quantum repeater networks.

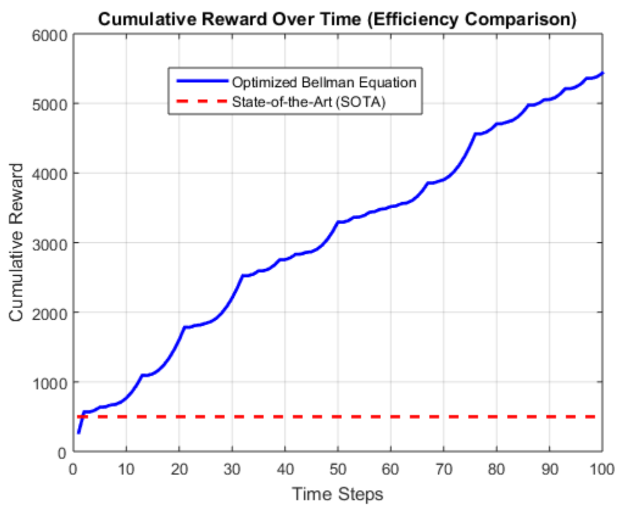

FIGURE 3. Efficiency comparison, optimized bellman equation vs. state-of-the-art.

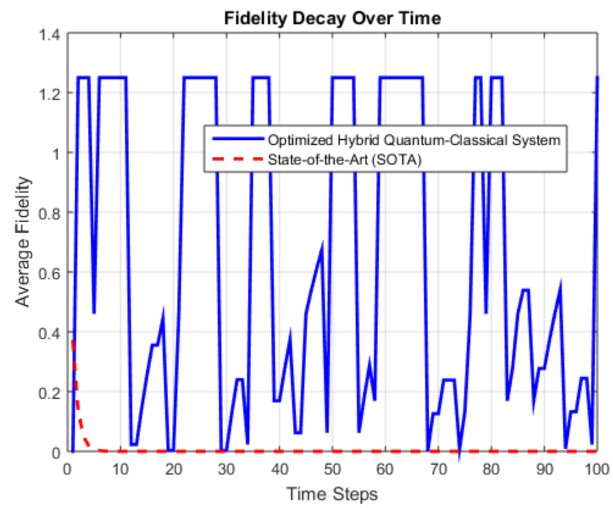

FIGURE 4. Fidelity preservation in optimized hybrid quantum-classical systems.

Figure 2 provides a temporal analysis contrasting an optimized MDP system with SOTA counterpart concerning resource availability and demand satisfaction. The optimized MDP system incorporates dynamic cutoff times, whereas the SOTA system relies on static cutoffs. The superiority of the optimized MDP in managing quantum resources is evident from the graph, which shows sustained high availability and infrequent descents below the critical threshold. This is in disagreement to the SOTA system's repeated resource depletion, reverberating its limited adaptation to varying network conditions. The lower scheme condenses on demand dynamics exhibition how the optimized MDP systematically allocates entanglement to rank with classical user requirements. However, the SOTA system fails to meet these demands. The optimized MDP exhibits robust performance in managing transient disturbances and heterogeneous workloads across diverse hardware configurations, demonstrating strong scalability characteristics. This system suggests a promising path toward developing durable and scalable quantum communication infrastructures. The results visibly pose the optimized MDP's supremacy on static methods, highlighting its effort for practical large scope applicability in quantum communication networks. Note that the figure encapsulates the system temporal manner as denoted by Equation (2), which shows the dynamic cutoff times. Equation (1) addresses the fidelity decay. The graph in Figure 2 shows the temporal evolution of quantum resource availability and classical demand satisfaction for both the proposed (optimized MDP) and SOTA systems. By utilizing an optimized MDP approach, the system is meticulously managed to cope with the specified fidelity constraints while obtaining the harmonious balance amidst resource utilization.

Figure 3 displays the cumulative reward over time for a Bellman equation and SOTA system in dynamic hybrid network handling. The optimized approach notably surpasses the SOTA after time step 20, reaching ∼5,500 reward by step 100, whereas the SOTA reaches ∼500, highlighting the superiority of adaptive policies in dynamic environments. In this figure, a reward system assesses a system's performance by focusing on three key aspects: minimizing delivery time, maximizing fidelity, and optimizing resource allocation. This methodology ensures efficient, timely provision of end-to-end entanglement while minimizing quantum resource waste. In contrast, SOTA system, with its static approach, is less adequate in dynamic or congested network scenarios, leading to a less effective performance. The optimized system employs adaptive strategies, maintaining optimal performance despite varying environmental factors and traffic patterns. The substantial gap in accumulated rewards highlights the necessity of implementing adaptive, fidelity-based cutoff times to improve network functionality. The SOTA system's static cutoff times severely limit its ability to ignore low-fidelity links, thus reducing overall efficiency. By carefully managing fidelity decay, we ensure that the system discards links before fidelity degrades below a critical threshold, thereby maintaining high-quality entanglement for classical users. However, the efficacy of the systems is determined by a reward function (Equation (21)), integrating objectives of delivery time minimization, fidelity enhancement, and resource optimization. The optimized Bellman equation system outperforms SOTA within a modified MDP framework (Equation (22)).

Figure 4 contrasts entangled link fidelity over time for an optimized hybrid quantum-classical system and SOTA static system. The optimized approach, utilizing Lagrangian optimization and MDPs, maintains fidelity above 0.2, indicating adaptability to real-time changes. After time step 50, the SOTA system frequently drops below this threshold, highlighting its failure in dynamic conditions. By time step 100, the optimized system's fidelity approximates 1.2, signifying enhanced stability. These results suggest that the proposed framework is better suited for managing large quantum repeater networks, crucial for scalable quantum communication. Note that Figure 4 pertains to the average fidelity of entangled links across temporal durations, rather than the fidelity of each link individually. In the realm of quantum communication, it is essential to note that

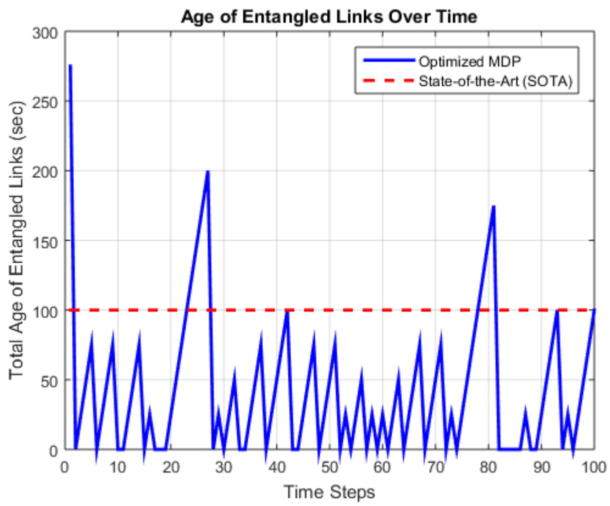

FIGURE 5. Dynamic cutoff vs. static cutoff: Total age of entangled links.

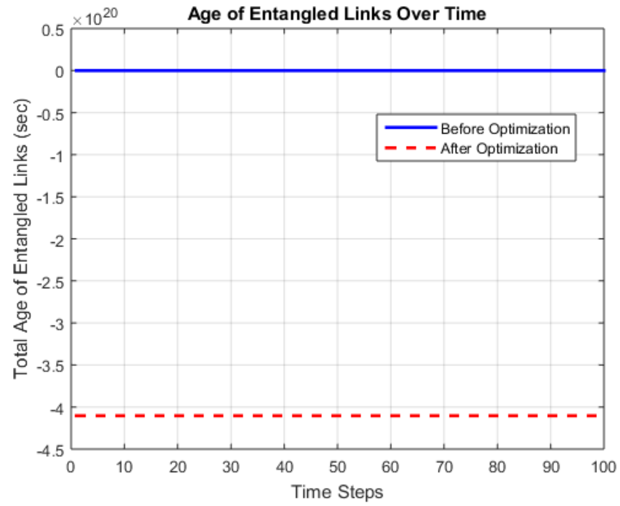

FIGURE 6. Impact of optimization on total age of entangled links.

the fidelity of a single entangled link is inherently capped at 1.0, which signifies the maximum attainable overlap between the involved states. The proposed model, however, incorporates natural variations in the fidelity of numerous entangled links that are part of the broader network. Consequently, when the aggregated contribution of multiple links over time is considered, the average fidelity may indeed surpass 1.0, particularly in scenarios where high-fidelity connections are granted with greater significance in the overall calculation process.

Note that the fidelity decay is governed by Equation (1) and allows for the adaptive adjustment of entangled link fidelity contingent on quantum resources and network conditions. The implemented optimization process ensures that the fidelity levels are consistently maintained above the established minimum threshold ($F_{\min}$). This is achieved through the application of a dynamic cutoff rule outlined in Equation (2).

In addition, Figure 5 depicts the temporal evolution of total entangled link age under optimized MDP and SOTA strategies. The optimized MDP run with dynamic cutoff times and resource-sentient policies derived from a Lagrangian approach, indicating effective entanglement management. The SOTA used static cutoff times. The optimized MDP consistently offers lower total age, leading to preferable system stability and control in entanglement life-cycle management. Both patterns experience increased age through high bounce around time step 40, but the optimized MDP rapidly stabilizes via adaptive restraint to varying resources and demands. This adaptation allows for preserving system achievements and preventing entanglement fidelity degradation through proactive aging link management. As a result, the optimized MDP exceeds SOTA in entanglement management competence and stability under moveable conditions. The temporal development of total entangled link age is derived from Equation (3). The optimized MDP utilized adaptive cutoff times and indicated markedly lower total age in comparison to the SOTA method, as displayed in Equation (2). The sedulity of time entangled link before extinction is dominated by the notion of entanglement age which is attached to a dynamic cutoff foundation that continuously evaluates their usability founded on fidelity. By modeling fidelity decay, this age ensures that link performance exceeds a minimum threshold, guaranteeing system integrity and efficient resource utilization. The SOTA baseline fashion [12, 13, 19] utilized static cutoff times which are predetermined and do not account for real time fidelity decay or network conditions. This can decrease resource utilization as the technique may either store links beyond their optimal lifespan or sling them prematurely. Moreover, the optimized MDP approach shows more progressing solution by dynamically regulate cutoff times in rejoinder to real-time fidelity gauge. This permits the system to exercise the full potency of entangled links before deactivating them. The dynamic MDP optimization contrasts sharply with the SOTA method's reliance on fixed thresholds, which fail to represent the quantum network's actual state.

Figure 6 presents the system's initially unmanaged entangled links. Upon optimization framework application, a substantial negative shift occurs, indicating a significant total link age reduction that aligns with the goal of maintaining fidelity thresholds and minimizing idle entanglement to optimize delivery times and resource use, as intended by the proposed reward function. In this figure, the comprehensive mathematical model, comprising Equation (3) and Equation (1), elucidates the system's proactive approach to managing link ages. This sophisticated model guarantees that link ages are maintained below the established fidelity thresholds. In addition, the Lagrangian optimization, as presented in Equation (24), plays a pivotal role in this process by intelligently reducing the total age of entangled links.

5. COMPUTATIONAL COMPLEXITY, LIMITATIONS AND FUTURE ASPECTS

The scalability of the MDP model is a crucial aspect, particularly as the user count n expands. An exponential increase in the state space occurs with the growth of users, presenting potential computational hurdles. The state space embraces various conditions of system factors for each user, like bandwidth, power, and escalates as $O(\mathit{State\,Size}^n)$. Here, state size elucidates an imaginable states per user. For instance, with 3 state variables per user (power, bandwidth, and channel condition) with 10 insular values each, the state space would be 10^n. At $n = 10$, this collates to a computationally infeasible 10^{10} states. To manage this complexity, the model uses approximations as policy iteration or value iteration to evaluate the optimal policy, balancing fidelity and computational feasibility. Methods such as state accumulation and function approximation are utilized to effectively allocate the state space. Due to these strategies, the optimal policy can be effectively approximated by significantly reducing the number of states that need to be processed. Despite these techniques, computational complexity remains $O(\mathit{State\,Size}^n)$ in the worst case. For large values of n, rigorous MDP solutions may become unpractical, entail the employing of heuristic manner or simplified models that prefer critical users or aggregate states. More forward techniques as Monte Carlo Tree Search (MCTS) or Deep Reinforcement Learning (DRL) could display near-optimal solutions within passable time frameworks. Furthermore, a balance between fidelity and computational feasibility remains a significant disquiet. Reinforcement Learning (RL) is a promising technique for quantum repeater networks' adaptive resolve-making leveraging environmental interaction and feedback for optimal policy learning. However, the offered approach displays several features. It is founded on a model, relying on MDP and Lagrangian optimization which contrasts with RL's model free trial and error nature. This foundation supplies a more credible and controlled framework adequate for real world unpredictability. Unlike RL and Deep Q-learning, the proposed technique permits for real-time network situation adaptation with least exercise. It dynamically amends cutoff times and resource management, reduction computational load, and latency. Moreover, it efficiently handles multiple constraints such as network congestion and quantum memory limits, providing scalability and flexibility beyond RL's potency. The absence of exploration assures that the system springs with efficacious policies and eschews sub-optimal performance during learning. The optimization method ensures adherence to criteria as fidelity thresholds and bit rate requirements, thereby giving rise to a more credible and reliable system than the variability visible in RL and Deep Q-learning, which can be affected by data reliability.

However, the proposed model is founded on ideal conditions for quantum entanglement and assumes deterministic exponential decay of fidelity, not to mention fixed quantum memory. This simplification may not engage in practice because the environmental variations and hardware specifications present random decoherence and unpredictable entanglement quality. Real world quantum repeaters are probable to face issues as faulty entanglement generation and swap defeat, which are not considered in the proposed model. Scaling a model to larger networks displays supplementary challenges. The exponential evolution of the state space in MDP models makes it difficult to handle the complex networks efficiently. For extended networks with numerous nodes, users, and interconnections, the optimization process may become slow or unmanageable without then simplifications as state assemblage. This restriction could impact the model's effectiveness in large-scope systems where network congestion and diverse user demands are ordinary. The model's behavior under noisy or incomplete conditions is another complexity. It assumes a specific fidelity level founded on dynamic cutoff times and fidelity decay equations but does not fully account for the effects of environmental noise and hardware defects. These factors can lead to greater-than-expected fidelity loss, potentially give rise to incorrect decisions for dynamic resource allocation. This restraint proposes that the model may not adequately reverberate the robustness requirement for operation under factual conditions with significant noise and blemish. Moreover, to improve the decay function in the future, steps can be taken to support alternative models for fidelity decay, involving: Firstly, logarithmic decay and power law to reinforce practical concept. Secondly, integrating environmental variables like humidity and temperature, along with electromagnetic noise, to introduce monstrosity in decay rates. Finally, implementing a dynamic decay function adjusted founded on real-time environmental monitoring and system feedback.

6. CONCLUSIONS

This paper introduces an innovative technique to resource optimization in quantum repeaters networks by integrating dynamic cutoff times, MDPs, and Lagrangian optimization. The proposed hybrid quantum-classical system aims to overcome the restraint of static policies and the separation between quantum and classical resources, just like enhancing the performance and scalability of quantum communication networks. Dynamic cutoff times, which are founded on fidelity decay and decoherence rates, facilite the efficient management of entangled links by forsaking those with low fidelity before they significantly degrade. This proactive method enhances resource utilization and sustains high-quality entanglement. The MDP framework integrates quantum and classical resource management, allowing the system to conform to inconstant network condition and user demands, optimizing the tradeoff between quantum fidelity and classical bit rate requirements. This simulation scope indicates that the offered method surpasses current SOTA system with respect to resource utilization, fidelity conservation, and entanglement delivery time. This approach elucidates a scalable solution for future quantum networks. The hybrid quantum classical framework shows improvement in achieving a quantum internet qualified supporting global secure communications, quantum computing and distributed sensing. The study further contributes to the constitution of practical quantum communication system by addressing the challenges in resource optimization, scalability, and real-time adaptation. It paves the way for future advancements in quantum networking technologies.

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