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Advanced Numerical Approaches for Magnetic Force Calculations: A Comprehensive Review

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ABSTRACT: Magnetic forces play a significant role in modern engineering applications, from medical imaging, data storage to transportation and industrial machinery. Accurate and efficient computational methods for magnetic force are necessary for engineering design and optimisation. However, different methods are typically based on distinct assumptions and are suited to different application scenarios. To assist researchers and engineers in selecting the most appropriate method for their specific needs, this review provides a comprehensive overview of various numerical approaches for calculating magnetic forces across different magnetic systems. Several key methods such as Dipole Method, Filament Method, Finite Element Method (FEM), Energy Method, Maxwell Tensor Method, Integral Method and Boundary Element Method (BEM) are discussed in detail, demonstrating their fundamental theories, applicable scenarios, advantages, and limitations. Recent advancements and improved versions of these methods are also covered, demonstrating their enhanced accuracy and efficiency. In addition, the potential solutions of these methods and future directions of developing advanced magnetic force computation techniques are also discussed in this paper.

1. INTRODUCTION

Magnetic forces arise from the interaction between magnetic fields and magnetic materials, playing a crucial role in various natural phenomena and technological applications. These forces are fundamental to the behavior of magnets and are described by the laws of electromagnetism [1]. Magnetic forces are ubiquitous in human society, playing a significant role in various applications, such as aligning compass needles with the Earth's magnetic field, powering electric motors and generators, and enabling the operation of various electronic devices [2]. The study of magnetic forces involves understanding the principles of magnetism, the behavior of magnetic materials, and the mathematical models used to calculate these forces. In the realm of technology, magnetic forces are involved in the design of many engineering applications [3]. For example, magnetic resonance imaging (MRI) relies on magnetic forces to produce detailed images of the human body, revolutionizing medical diagnostics [4]. Additionally, magnetic forces are integral to data storage technologies, such as hard drives, enabling the vast amounts of digital information we rely on daily [5]. Beyond technology, magnetic forces influence navigation, renewable energy solutions, and measurement techniques, highlighting their importance in advancing human knowledge and improving quality of life [6–8].

Building on recent advancements, magnetic levitation (maglev) technology has emerged as a widely adopted technique in industrial applications, which allows objects to be suspended without physical contact by harnessing magnetic forces [9]. Since maglev technology was successfully developed in the last century, it has been widely applied across various fields, which is regarded as a mature and reliable technique [10, 11]. Magnetic forces are fundamental to the operation of maglev systems, such as operations of high-speed maglev trains. In maglev train systems, magnetic forces are generated to lift and propel the train, eliminating friction between the train and tracks and allowing for smoother and faster travel. The significance of magnetic forces in maglev systems lies in their ability to provide stable levitation and propulsion, which are essential for the efficient and safe operation of these trains [12–14]. The magnetic forces in maglev systems are influenced by several factors, including the strength of the magnetic field, the self-weight of the train, and the distance between the magnets [15].

Magnetic forces are integral to various magnetic systems, each with unique characteristics and applications. Permanent magnets, such as neodymium magnets and ceramic magnets, are generally popular types of magnets. They can generate a consistent magnetic field without the need for an external power source. These magnets are widely used in applications ranging from electric motors to MRI machines due to their strong and stable magnetic fields [16, 17]. Superconducting magnets, on the other hand, operate at extremely low temperatures to achieve zero electrical resistance, allowing them to generate exceptionally high magnetic fields. These magnets are crucial in applications like particle accelerators and simple maglev systems, where high magnetic field strength and stability are essential [18, 19]. Another significant magnet type, electromagnetic magnets, which rely on electric current to produce a magnetic field. By adjusting the current in coils, the strength of the magnetic field can be varied, making these magnets suitable for applications such as electric cranes and maglev trains [20]. Hy-

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brid magnets combine the properties of permanent magnets and electromagnets to achieve even higher magnetic fields, and this type of magnets is becoming more popular for industrial applications. These systems are used in advanced scientific research and industrial applications where both high field strength and stability are required [21, 22]. To provide a clearer perspective on the relationship between magnet types, their magnetic force characteristics, and typical use cases, a summary of various magnetic systems and their common applications is illustrated in Figure 1.

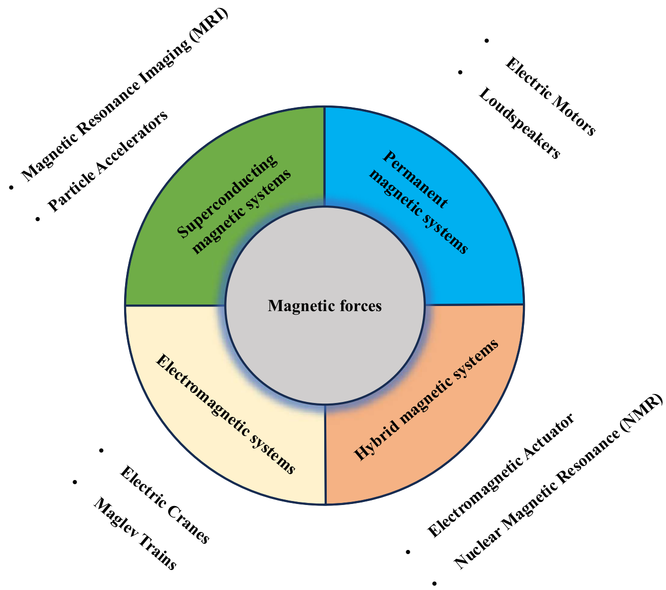

FIGURE 1. Magnetic force within different magnetic systems and their popular applications.

The accurate calculation of magnetic forces is critical for the optimal design, operation, and reliability of a wide range of electromagnetic systems, including electric motors, magnetic bearings, maglev transportation, and other high-precision magnetic devices [23]. These forces directly influence performance characteristics such as torque generation, load capacity, energy efficiency, and dynamic stability. Inaccuracies in force estimation can lead to suboptimal component design, excessive energy consumption, thermal losses, and, in severe cases, mechanical failure due to unintended vibrations or structural fatigue [24]. Beyond conventional electromechanical systems, magnetic force accuracy is paramount in medical technologies, such as MRI machines, where precise control of magnetic fields is essential for achieving high-resolution imaging, patient safety, and diagnostic reliability [25]. Additionally, in industrial applications, accurate magnetic force computations facilitate the development of advanced manufacturing systems and novel magnetic materials [26].

Given its wide-ranging importance, magnetic force modelling has been an active area of research for several decades. This sustained interest has led to the development of several theoretical and numerical techniques, each based on different physical assumptions and offering distinct trade-offs in computational complexity, accuracy, and applicability. For instance, methods such as Filament Method and Integral Method have demonstrated strong performance in both academic studies and engineering practice [27–30]. As modern applications continue to demand greater precision, compactness, and reliability, the need for accurate and efficient magnetic force computation has become increasingly significant. This demonstrates the need for a comprehensive evaluation of available methods, considering not only their mathematical formulations but also their practical implications in diverse application domains.

While there are several comprehensive reviews regarding popular magnetic devices [31–37] and magnetic materials [38–42], as well as application-focused surveys on systems such as maglev trains and magnetic bearings [13, 14, 43–47], there remains a notable gap in the literature, which is a systematic review of the fundamental numerical methods for magnetic force calculation. Each approach, based on distinct theoretical principles, presents different strengths and limitations depending on system geometry, material properties, and boundary conditions. Consequently, no single method consistently outperforms the others across all scenarios. Addressing this gap requires a consolidated assessment that compares numerical approaches on a common basis, linking their theoretical foundations to their performance in specific magnetic configurations. Such a review would help researchers and engineers with clearer guidance for selecting the most suitable method for their needs, ultimately supporting more effective design and optimisation of magnetic technologies.

This paper presents a comprehensive review of widely used numerical approaches for magnetic force calculation across various magnetic systems. The review is organized into seven core sections, each dedicated to a specific method: Dipole Method, Filament Method, FEM, Energy Method, Maxwell Tensor Method, Integral Method, and BEM. For each approach, fundamental principles, typical application scenarios, and inherent limitations are systematically analysed. Additionally, a detailed comparative analysis is provided at the end of each section to highlight the advantages of the selected methods. The final section summarizes the review and discusses future potential directions for advancing magnetic force computation techniques.

2. DIPOLE METHOD

Dipole Method is a widely used approach for calculating magnetic forces, particularly in systems involving small magnetic particles or dipoles. This method models each magnetic entity as a magnetic dipole, simplifying the complex interactions into more manageable calculations. The fundamental theory behind the Dipole Method involves magnetic dipole moment **m** and magnetic field **B**. The force **F** on a magnetic dipole for the separated magnetic charge model is given by:

$$\mathbf{F} = (m \cdot \nabla)\,\mathbf{B} \tag{1}$$

It describes how the magnetic dipole interacts with magnetic field, leading to forces that depend on the gradient of the magnetic field and the orientation of the dipole [48, 49]. In 1998, an analytical solution was developed that explicitly defines the relationship between the magnetic force and distance term, expressed as:

$$\mathbf{F}(\mathbf{d}, \mathbf{m}_1, \mathbf{m}_2) = \frac{3\mu_0}{4\pi d^5}\left[(\mathbf{m}_1\cdot\mathbf{d})\,\mathbf{m}_2 + (\mathbf{m}_2\cdot\mathbf{d})\,\mathbf{m}_1 + (\mathbf{m}_1\cdot\mathbf{m}_2)\,\mathbf{d} - \frac{5(\mathbf{m}_1\cdot\mathbf{d})(\mathbf{m}_2\cdot\mathbf{d})}{d^2}\mathbf{d}\right] \quad (2)$$

where d is the distance-vector from dipole moment $\mathbf{m}_1$ to dipole moment $\mathbf{m}_2$ [50]. Several improved versions of this method based on the same concept have also been developed by the following researchers [51, 52].

Dipole Method is a widely adopted analytical and numerical approach for calculating magnetic forces, particularly in systems comprising multiple small magnetic entities. This method simplifies complex magnetic interactions by approximating each particle as a magnetic dipole, effectively reducing the problem to a series of dipole-dipole interactions. The primary advantage of this approach lies in its computational efficiency, as it allows for the rapid estimation of magnetic forces in large-scale particle systems without solving Maxwell's equations in their full form. Dipole approximation assumes that each particle is sufficiently small and well separated from its neighbors, so that higher-order multipole contributions can be neglected. Under these conditions, the magnetic field produced by a particle is treated as equivalent to that of an ideal dipole, and the resultant forces can be derived from the interaction energy between such dipoles. This assumption is valid in dilute systems or when particles are much smaller than the characteristic length scale of the system. However, in densely packed configurations or for particles of larger dimensions, dipole approximation may introduce significant errors, as it does not account for near-field effects, magnetic shape anisotropy, or collective field interactions.

Dipole Method is especially applicable in scenarios involving paramagnetic colloids, ferrofluids, and magnetic nanoparticles, where particle sizes are typically in the nanometer to micrometer range, and interactions are weak enough to justify the dipole model. Recent advancements in this field include the development of micro-mutual-dipolar (MMD) model, which extends the traditional dipole framework by incorporating mutual dipole-dipole interactions within particle aggregates [53]. This enhancement enables a more accurate representation of interparticle forces, particularly in cases where magnetic coupling effects cannot be neglected. MMD model improves predictive accuracy for force and torque calculations in complex, clustered systems of paramagnetic particles, offering greater fidelity in simulating the behavior of magnetic fluids. Therefore, while the classical Dipole Method provides a computationally tractable solution for modeling magnetic forces in dispersed systems, its limitations in accuracy under specific conditions have prompted the development of more refined models such as MMD. These ongoing improvements continue to expand the applicability and precision of dipole-based force calculation techniques in nanoscale and mesoscale magnetic system.

3. FILAMENT METHOD

Filament Method is a widely used approach for calculating magnetic forces between current-carrying conductors. This method simplifies the complex problem of magnetic force calculation by representing conductors as a series of filaments, each carrying a portion of the total current. By breaking down conductors into smaller elements, the method allows for a more manageable and precise calculation of magnetic interactions [54]. The fundamental theory behind Filament Method involves calculating the mutual inductance between these filaments and using this information to determine the resulting magnetic forces. The governing equations for Filament Method are based on the Biot-Savart law and Ampère's circuital law. Magnetic field **B** at a point due to a current element $I \cdot dI$ is given by the Biot-Savart law, as shown in Equation (3):

$$\mathbf{B} = \frac{\mu_0}{4\pi}\int \frac{I \cdot dI \times \mathbf{r}}{|\mathbf{r}|^3} \quad (3)$$

where μ_0 is the permeability if free space, I the current, and $\mathbf{r}$ the position vector from the current element to the point of interest [55–57]. In Equation (4), the force **F** between two current-carrying filaments can then be calculated using the Lorentz force law [58, 59].

$$\mathbf{F} = I \cdot dI \times \mathbf{B} \quad (4)$$

By applying these laws, Filament Method calculates the magnetic field generated by each filament and then determines the mutual inductance between pairs of filaments. Mutual inductance is a measurement of how the magnetic field produced by one filament induces a voltage in another filament. This mutual inductance is then used to calculate the magnetic forces between filaments, which can be summed together to find the total magnetic force between these conductors.

3.1. Coaxial Magnetic System

The magnetic force between two current-carrying circular coils with current values I_1 and I_2 can be derived from the general expression for their mutual inductance M with respect to distance Z [60].

$$F = I_1 I_2 \frac{\partial M}{\partial Z} \quad (5)$$

The Filament Method for calculating magnetic forces in coaxial systems was initially proposed to determine the magnetic force generated by a filament coil and a circular coil under coaxial conditions. This method leverages the fundamental theory of mutual induction, simplifying the problem by representing the conductors as a series of filaments. This approach was first applied to calculate the magnetic forces between a filament coil and a circular coil, providing a straightforward solution for systems with simple geometries [61]. Then, the method was adapted to handle more complex systems, such as calculating magnetic forces between coils with axial length rather than just single circular coils. For coaxial thin coil pairs, where the coils are assumed to have no radial thickness, the method simplifies the problem by representing each coil as a series of filaments for analysis. This simplification has been used in some literature, which allows for accurate force calculations in systems with simple geometries, making it a useful tool for analyzing magnetic interactions in such configurations [62].

When dealing with thick coil pairs, where the coils possess significant radial thickness, the traditional Filament Method for calculating magnetic forces requires modifications to account for the added complexity. The presence of radial thickness introduces additional factors that must be considered to ensure accurate force calculations. Specifically, radial thickness affects the distribution of the magnetic field and resulting forces, which requires a more sophisticated approach. To address these challenges, the Filament Method was enhanced by incorporating the effects of the coil's cross-sectional area. This enhancement involved the use of semi-analytical expressions that include complete elliptic integrals and Heuman's Lambda function. These mathematical tools provide a more precise representation of the magnetic interactions within thick coil systems, capturing the nuances introduced by the coil's geometry. The improved method allows for the accurate calculation of magnetic forces in a broader range of practical applications, including those involving thick coils with substantial radial dimensions. By incorporating these advanced mathematical techniques, the method can effectively handle the additional complexity introduced by the coil's geometry, ensuring reliable and precise results. The coil system is illustrated in Figure 2, which provides a visual representation of the coil geometries. The magnetic force on the thick coil can be calculated using the improved method, as described by Equation (6):

$$F = \frac{\mu_0 N_1 N_2 I_1 I_2}{12\left(R_4-R_3\right)\left(R_2-R_1\right)\left(Z_4-Z_3\right)\left(Z_2-Z_1\right)} \cdot \sum_{n=1}^{n=16} (-1)^{n-1} \varphi_n \quad (6)$$

where N_1 and N_2 are the number of turns of both coils; R_1–R_4 are the inner and outer radii of both coils; Z_1–Z_4 are the cylindrical coordinates; and φ_n represents a complex magnetic term that involves both the first and second kind elliptic integrals in the evaluation of multiple integrals. Detailed derivation and calculation procedures are available in the original literature [63]. In addition, Integral Method for coaxial systems has evolved significantly, providing accurate and efficient solutions for calculating magnetic forces in both simple and complex geometries. Several advanced methods based on the same concept have been proposed for solving magnetic force in coaxial systems, further demonstrating its reliability and accuracy [64, 65]. By leveraging these advanced techniques, the Integral Method can effectively handle a wide range of practical applications, from simple coaxial coil systems to more complex configurations involving multiple coils and varying geometries.

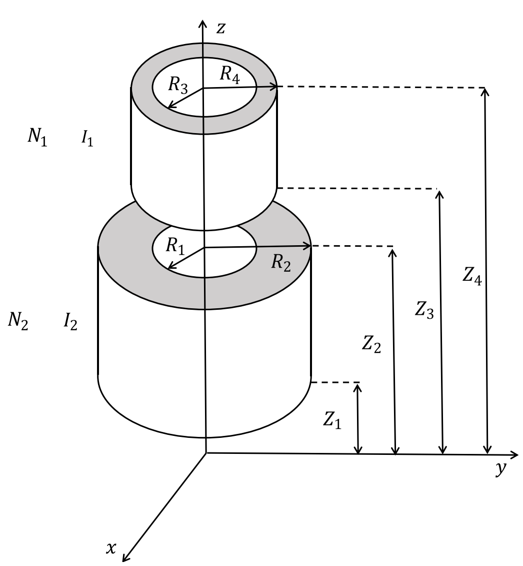

FIGURE 2. A schematic of two coaxial thick coils.

3.2. Misaligned Magnetic System

Even though magnets or coils are often designed to be coaxial to achieve maximum force along the coils' centerlines, achieving an ideal coaxial condition is extremely challenging. This difficulty arises due to offsets incurred during installation, vibrational influences, or other extraneous perturbations, which result in the occurrence of radial force components within magnetic systems [66]. These misalignments can significantly affect the performance and efficiency of electromagnetic devices, making it crucial to accurately calculate the resulting forces. Filament Method has also been extensively used to calculate magnetic forces in misaligned coils, addressing the challenges posed by lateral and angular displacements.

The investigation of restoring forces between non-coaxial circular coils was first conducted by Kim et al., who proposed a calculation method for these forces and analyzed their characteristics. This pioneering work provided valuable insights into the behavior of radial magnetic forces between two current-carrying coils, highlighting the complexities introduced by misalignment. Their method involved calculating the mutual inductance between misaligned coils and using this information to determine the resulting forces. This approach laid the groundwork for further research into the effects of misalignment on magnetic systems [67]. The effects of coil misalignments on the magnetic field and magnetic force components between circular filaments were further investigated. Detailed results were obtained using advanced models and SCILAB application software, which allowed for a comprehensive analysis of the impact of misalignments on magnetic interactions. These studies demonstrated that even small misalignments could lead to significant changes in the magnetic forces, underscoring the importance of accurate force calculations in misaligned systems [68].

Additionally, advanced methods using Filament Method with Grover's formulas have been proposed to calculate the mutual inductance and magnetic forces between misaligned coils with parallel axes. These methods have demonstrated high accuracy and applicability, particularly in the context of superconducting magnets. By incorporating Grover's formulas, the Filament Method can account for the mutual inductance between misaligned coils, providing a more precise calculation of the resulting magnetic forces. In this method, Coil 1 is discretized into $(2m+1)\times(2n+1)$ cells, while Coil 2

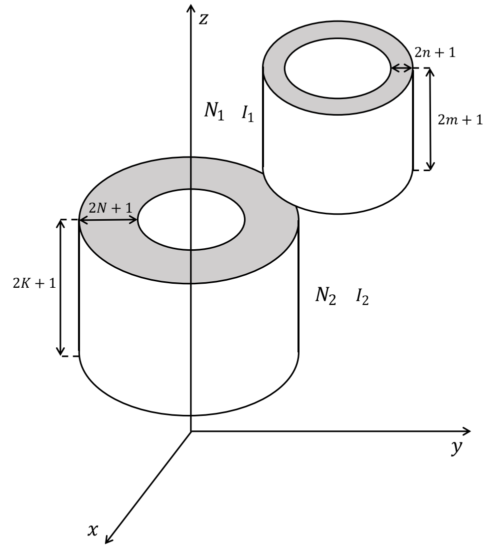

FIGURE 3. A schematic of two misaligned thick coils.

is divided into $(2K+1) \times (2N+1)$ cells in the cross-section, as illustrated in Figure 3. Axial and radial components of the magnetic force are then calculated using the following equations:

$$F_{\text{Axial}} = \frac{N_1 N_2 \sum_{g=-K}^{g=k} \sum_{h=-N}^{h=N} \sum_{p=-m}^{p=m} \sum_{l=-n}^{l=n} F_1}{(2K+1)(2N+1)(2m+1)(2n+1)} \quad (7)$$

$$F_{\text{Radial}} = \frac{N_1 N_2 \sum_{g=-K}^{g=k} \sum_{h=-N}^{h=N} \sum_{p=-m}^{p=m} \sum_{l=-n}^{l=n} F_2}{(2K+1)(2N+1)(2m+1)(2n+1)} \quad (8)$$

where F_1 represents the axial magnetic force between a single pair of circular elements, and F_2 represents the radial magnetic force between a single pair of circular elements. Their detailed calculation methods can be referred to the original literature [69]. In 2010, a similar method was also proposed to analyze the mechanical stability of superconducting magnets with misaligned coils, which proved the method's effectiveness in handling complex geometries and misalignment scenarios [70]. In addition, Filament Method has been extended to coils with arbitrary relative orientations and positions, improving the precision of magnetic force calculations in complex configurations [71]. More recently, a sensitivity analysis has also been conducted to support the design of misaligned coil systems [72]. Beyond force analysis, a numerical approach, has also been developed to calculate both the magnetic force and torque between circular coils with nonparallel axes [73]. However, existing literature on magnetic force or torque analysis in more complex magnetic system configurations remains limited, and the accuracy of applying Filament Method in these configurations requires further validation.

Filament Method is particularly useful in scenarios where conductors have simple geometries, such as coaxial coils, solenoids, and circular loops. The method is less effective for conductors with highly irregular shapes or varying cross-sections. Furthermore, the accuracy of results depends on the number of filaments used, with higher accuracy requiring more filaments and increased computational effort. This method has been commonly applied in the design and analysis of electromagnetic devices like inductors and transformers. Moreover, one significant recent improvement to Filament Method includes segmentation approach, which enhances the accuracy and efficiency of calculations. This approach involves interpolating complex curves with a set of line segments, allowing for more precise force calculations between filaments of arbitrary shapes. However, extending the method to three-dimensional problems can be challenging and may require additional computational resources [74].

Filament Method's ability to handle misaligned coils has made it a significant tool in various applications, including magnetic levitation systems, magnetic bearings, and electromagnetic actuators. In magnetic levitation systems, for example, lateral or angular misalignments can significantly reduce levitation force and compromise stability. Filament Method provides a practical way to quantify these effects and guides control system design. Similarly, in magnetic bearings, small displacements between rotor and stator coils can introduce uneven forces, leading to increased instability, making accurate misalignment analysis essential. This method has also been used in electromagnetic actuators and contactless energy transfer systems, where force sensitivity to coil displacement must be well understood for reliable operation. Extensions of Filament Method, often combined with integral formulations, now allow the accurate and efficient computation of forces in both coaxial and non-coaxial configurations. By incorporating advanced mathematical techniques and refined discretization schemes, Filament Method continues as a valuable tool for engineers and researchers working with complex electromagnetic systems where geometric accuracy and misalignment effects play a decisive role.

4. FINITE ELEMENT METHOD

Unlike Filament Method, which represents conductors as a series of current-carrying filaments and focuses on the mutual inductance and forces between these filaments, FEM simplifies the calculation process by discretizing the entire problem domain into smaller, finite elements. This approach focuses on the mutual inductance and forces between these elements, making calculations more straightforward. The magnetic force between these elements is then calculated using complete elliptic integrals. The governing equations in this method are Gauss' law of magnetism and Ampere's law with Maxwell's correction:

$$\nabla \cdot \mathbf{B} = 0 \quad (9)$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial D}{\partial t}$$

$$(\text{Always simplify as } \nabla \times \mathbf{H} = \mathbf{J}) \quad (10)$$

where **B** is the magnetic flux density, **H** the magnetic field intensity, and **J** the current density. FEM solves these equations by breaking down the domain into elements, applying boundary conditions, and using interpolation functions to approximate the field variables within each element.

One of the primary advantages of this method is its ability to provide faster computation times than traditional methods.

This makes it particularly suitable for real-time applications where quick and efficient calculations are essential. However, the method has its limitations. It is less effective for conductors with highly irregular shapes or varying cross-sections, as the simplification to elements may not accurately represent the actual geometry. Additionally, the accuracy of results depends on the number of elements used, and higher accuracy requires more elements, which in turn increases the computational effort.

FEM can be effectively applied in scenarios involving cylindrical coils, where the geometry lends itself well to the FEM approximation. It is also useful for calculating forces in systems with complex shapes by segmenting the conductors into elements. This segmentation allows for a more manageable and precise calculation of magnetic interactions. In 2009, a new and fast procedure for calculating magnetic force between cylindrical coils was proposed. As shown in Figure 4, two coils are divided into $n_{r1} \times n_{a1}$ cells and $n_{r2} \times n_{a2}$ cells, respectively, and the force between them can be calculated with the following equation:

$$\mathbf{F} = \sum_{k=0}^{n_{r2}-1} \sum_{j=0}^{n_{r1}-1} \sum_{l=0}^{n_{a2}-1} \sum_{i=0}^{n_{a1}-1} \mathbf{a}_z \cdot \left(\frac{\mu_0 N_1 I_1 N_2 I_2 Z_{il} k'}{2\sqrt{r_k r_j}\,(1-k'^2)\, n_{r1} n_{r2} n_{a1} n_{a2}} \right) \cdot \left[\begin{array}{c} \left(1-k'^2\right) K\left(k'\right) \\ -\left(1-\frac{1}{2}k'^2\right) E\left(k'\right) \end{array} \right] \tag{11}$$

where $\mathbf{a}_z$ is a unit vector, and other geometric parameters can be referred to the original literature. The new methodology for calculating magnetic forces using concentric rings method has been validated through experimental tests and simulations, demonstrating its effectiveness in various practical applications. By leveraging finite element analysis, this method avoids the need to analyze complicated geometric details, significantly enhancing calculation efficiency. This improvement increases the speed of the calculation process without compromising the level of accuracy achieved, making it a highly efficient approach for real-time applications [75].

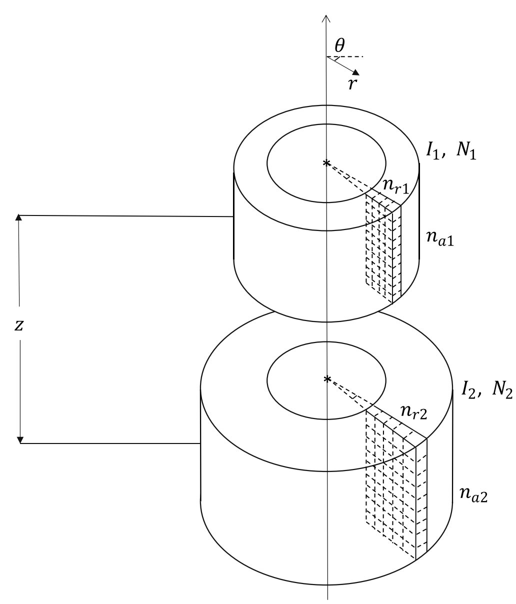

FIGURE 4. Division of the coils into different meshes to calculate the force between them.

In addition to its application in cylindrical coils, this method has also been successfully applied to magnetic force calculations between planar spiral coils, which are another significant type of coil used in engineering designs. Planar spiral coils are commonly used in applications such as inductive charging systems, wireless power transfer, and various types of sensors. The ability to accurately calculate magnetic forces in these coils is crucial for optimizing their performance and ensuring reliable operation [76, 77]. FEM, with its enhanced efficiency and accuracy, provides a practical solution for engineers and researchers working with both cylindrical and planar spiral coils. Recent studies have demonstrated its capability across a variety of electromagnetic systems, such as modelling running magnetic resistance in superconducting maglev systems using COMSOL Multiphysics, evaluating vector and scalar potential formulations for dipole magnet modelling, and designing permanent magnetic holding devices as well as active magnetic bearings in ANSYS Maxwell [78–80]. These examples highlight FEM's ability to handle nonlinear material properties, three-dimensional geometries, and coupled multi-physics problems with high fidelity. By simplifying the problem of mutual inductance and forces between elements, FEM-based approaches can still be computationally efficient while retaining flexibility to accommodate complex geometries. Therefore, Filament Method is a simpler and more efficient approach for systems with straightforward geometries and thin coils, while FEM remains a more versatile and accurate method suitable for complex and detailed analyses.

5. ENERGY METHOD

Energy Method is another powerful approach for calculating magnetic forces, and it involves determining the force by analyzing the change in energy of the system as a function of position. This method is based on the principle of virtual work, which states that the work done by the forces during a virtual displacement is equal to the change in energy. For magnetic systems, this can be expressed as:

$$F = -\frac{\partial W}{\partial x} \tag{12}$$

where W is the magnetic energy, and x is the position. In addition, the magnetic energy in a system can be calculated using magnetic field $\mathbf{B}$ and magnetic permeability μ:

$$W = \frac{1}{2\mu} \int \mathbf{B}^2 dV \tag{13}$$

where the integral is taken over the volume V of the magnetic field [81, 82]. Once the magnetic energy is known as a function of position, the force can be found by taking the derivative of the energy with respect to the position. Energy Method is widely used in the design and analysis of electromechanical devices such as motors, generators, and actuators. Energy Method provides a straightforward way to calculate forces without the need to directly solve magnetic field equations, and it can be applied to a wide range of magnetic systems, including those with complex geometries and material properties. However, the accuracy of the force calculation highly depends on the accuracy of the magnetic field distribution [83, 84].

Several studies have highlighted the effectiveness and versatility of the Energy Method for magnetic force calculations. For instance, a recent study discussed the advantages of using energy approach to determine magnetic force in complex geometries, demonstrating its applicability in various scenarios [85]. Another work focused on the accuracy and applicability of the Energy Method (also called virtual work principle) for noise and vibration assessment, providing an analytic comparison on magnetic force computation between the Energy Method and other methods, and the relative error of magnetic force calculation between different methods is as shown in Figure 5 [86].

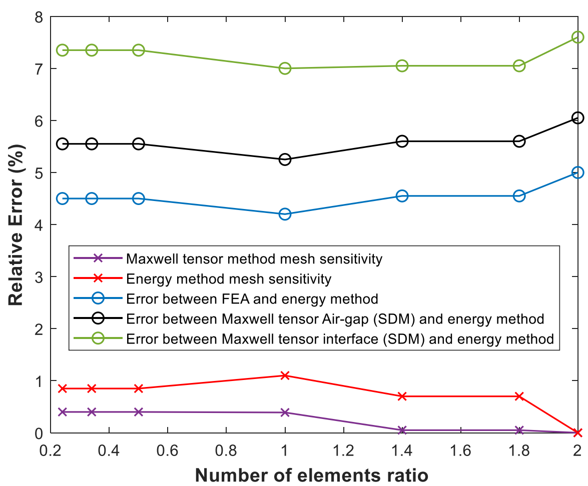

FIGURE 5. The relative error between VWP and Maxwell tensor and the discretization error. Reproduced from [86].

Furthermore, a detailed analysis of energy and co-energy methods was provided in 2002, which focuses on the evaluation of forces in magnetic materials. The expressions of magnetic energy in magnetic system driven by current sources and magnetic system driven by a permanent magnet are obtained respectively:

$$W_{\text{Current}} = \iiint_0^{\Omega_\infty} \left(\int_0^H B \cdot dH \right) d\Omega \tag{14}$$

$$W_{Pm} = \iiint_0^{\Omega_\infty - \Omega_{pm}} \left(\int_0^B H(B,T) \cdot dB \right) d\Omega + \iiint_0^{\Omega_{pm}} \left(\int_{B_0}^B H(B,T) \cdot dB \right) d\Omega \tag{15}$$

where Ω is the volume of the magnetic field, W_{Current} the magnetic energy in the current source magnetic system, and W_{Pm} the magnetic energy in the permanent magnet magnetic system [87]. These applications of Energy Method demonstrate its accuracy and efficiency in different scenarios and prove its broad applicability in various magnetic systems. By analyzing the change in energy of the system, this method provides a clear and precise way to determine the resulting magnetic forces, making it become an essential tool in the field of electromagnetics.

Energy Method is based on the principle that the magnetic force can be derived from the rate of change of stored magnetic energy or co-energy with respect to displacement. This makes it particularly effective in systems where the magnetic field distribution is well characterized, allowing force evaluation without the need to directly compute local field stresses. Traditional formulations are often limited by the difficulty of obtaining accurate energy distributions in systems with complex geometries or nonlinear material properties. To address this, improved versions of Energy Method frequently incorporate advanced numerical techniques such as finite element analysis (FEA), which enables precise modelling of the magnetic field and energy density. By integrating energy over the computational domain, the method provides reliable force predictions even in complicated configurations [86]. This combined approach has been successfully applied in the design of electromagnetic actuators, solenoids, and magnetic bearings, where force-displacement relationships are critical for performance optimization. In magnetic levitation systems and wireless power transfer coils, Energy Method is particularly valuable for analysing stability and efficiency, as it allows designers to evaluate how small positional changes influence the stored magnetic energy and, consequently, the restoring or coupling forces. Through these applications, Energy Method — especially when being enhanced with numerical simulation — demonstrates both versatility and robustness as a tool for modern electromagnetic system design.

6. MAXWELL TENSOR METHOD

Maxwell Stress Tensor Method is a powerful tool in electromagnetism for calculating the forces and stresses in a magnetic field. Maxwell stress tensor T is a second-order tensor that represents the interaction between electromagnetic forces and mechanical momentum. The governing equation for the Maxwell stress tensor in a magnetic field is:

$$T_{ij} = \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right) \tag{16}$$

where B_i and B_j are the components of the magnetic flux density $\mathbf{B}$; μ_0 is the permeability of free space; and δ_{ij} is the Kronecker delta. The force $\mathbf{F}$ on a body can then be calculated by integrating the Maxwell stress tensor over a closed surface S surrounding the body:

$$\mathbf{F} = \oint \mathbf{T} \cdot \mathbf{n} dA \tag{17}$$

where $\mathbf{n}$ is the unit normal vector to the surface, and A is the surface area [88–90]. Recently, a modified Maxwell stress tensor for more accurate local force calculations in magnetic bodies has been proposed, which give a detailed guide on applying

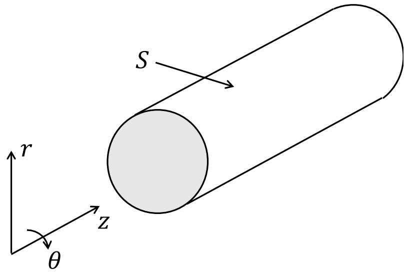

FIGURE 6. Surface of Maxwell stress tensor integration with a demonstration of magnetic force in different directions. Reproduced from [91].

this method for force calculation in different directions [91]. For magnetic forces in different directions (Figure 6), they can be expressed as:

$$F_z = \frac{1}{\mu_0} \oint B_z B_r dA \tag{18}$$

$$F_\theta = \frac{1}{\mu_0} \oint B_\theta B_r dA \tag{19}$$

$$F_r = \frac{1}{\mu_0} \oint \frac{B_r^2 - B_\theta^2 - B_z^2}{2} dA \tag{20}$$

Similar to Energy Method, the final form of magnetic force equations derived from Maxwell Stress Tensor Method can vary significantly depending on the specific application scenario. This method provides precise force calculations by considering the entire magnetic field distribution, making it highly accurate for a wide range of applications. However, computation can be expensive for systems with highly complex geometries, and the accuracy of the results depends on the precise calculation of the magnetic field distribution. This reliance on detailed field calculations can be a limitation in scenarios where the magnetic field is difficult to model and calculate accurately.

Maxwell Tensor Method evaluates magnetic force and torque by integrating the electromagnetic stress over a closed surface enclosing the region of interest, thereby avoiding the need to directly calculate current-field interactions inside the body. This makes it particularly powerful for systems with complex geometries or distributed fields, as the force can be determined solely from boundary field values. Recent reviews have highlighted the versatility of the Maxwell Tensor Method approach and its widespread use in numerical magnetic force calculations, particularly when being combined with finite element analysis (FEA) for accurate field evaluation [92, 93]. The method has been extensively applied in rotating electrical machines, where electromagnetic torque is obtained by integrating the tensor over circular paths in the air gap to improve numerical stability and accuracy. It is also widely used in the design of magnetic bearings, actuators, transformers, and magnetic levitation systems, where precise knowledge of localised forces is essential for performance and stability. In applications such as superconducting magnets and MRI systems, Maxwell Tensor Method provides a reliable framework for analysing large magnetic forces acting on structural components. Although the method can be computationally intensive and sensitive to the choice of integration surface, coupling it with advanced numerical solvers mitigates these challenges by enabling detailed modelling of field distributions. As a result, Maxwell stress tensor remains one of the most accurate and versatile methods for magnetic force and torque computation across a broad range of electromagnetic systems.

7. INTEGRAL METHOD

Integral Method for magnetic force calculations involves using integral equations to determine the magnetic forces in a system. This method is based on the principle that the magnetic force can be derived from the magnetic field distribution and the properties of the materials involved. The governing equation for the Integral Method is typically expressed as:

$$\mathbf{F} = \int \mathbf{J} \times \mathbf{B} dV \tag{21}$$

where **F** is the magnetic force, **J** the current density, **B** is the magnetic flux density, and V the volume over which the integration is performed [29]. Using Integral Method, Ravaud et al. proposed an analytical solution for expressing the magnetic field generated by a thick coil [94]. This solution provides a detailed and accurate representation of the magnetic field, which is essential for calculating the resulting forces. Subsequently, by assuming an idealized magnet model, an advanced equation was developed to describe the magnetic force between a coil and a permanent magnet as a function of axial displacement [95]. This development allows for precise force calculations in systems where the relative position of the coil and magnet changes, such as in actuators and sensors.

Furthermore, Babic et al. adopted a similar integral approach to present analytic formulas for calculating the mutual inductance and axial magnetic force between a thin wall solenoid and a thick circular coil. These formulas, expressed in terms of complete elliptic integrals and Heuman's Lambda function, provide a simpler and more accurate alternative to previously published methods. The results of their work have been validated through comparisons with numerical simulations and experimental data, demonstrating the effectiveness and accuracy of the Integral Method in various practical applications, as shown in Equation (22) [96]:

$$F = \frac{\mu_0 N_1 N_2 I_1 I_2 R}{(R_2 - R_1)(Z_2 - Z_1)(Z_4 - Z_3)} \times \int_0^\pi \int_{R_1}^{R_2} \int_{Z_1}^{Z_2} \int_{Z_3}^{Z_4} \frac{\left(Z_2' - Z_1'\right) \cos\theta r dr dZ_1' dZ_2' d\theta}{r_0^3} \tag{22}$$

where r, θ, Z are the cylindrical coordinates. Due to the theory of equivalent magnetization, the permanent magnet can be always treated as a coil with equivalent cylindrical surface current density [97–99]. Therefore, to help understand the equation, a 3D representation of the adopted model used in these proposed methods is created as shown in Figure 7.

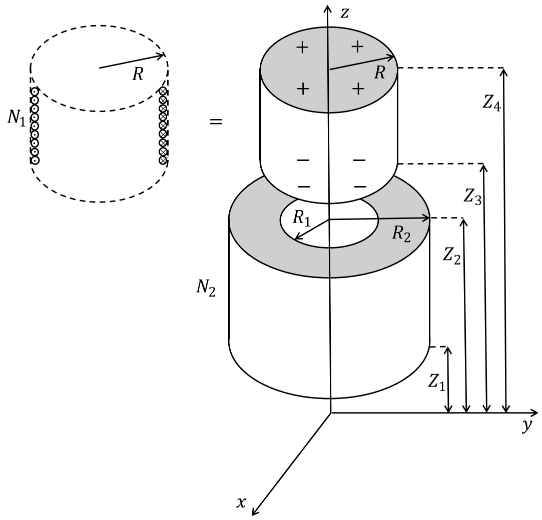

FIGURE 7. A thin wall solenoid and thick coil, with a demonstration of equivalent magnetization of a thin wall solenoid and cylindrical permanent magnet.

In addition, Integral Method has been applied to solve many problems of magnetostatic interactions among magnetic elements with regular shapes [100, 101]. Based on this, another significant work has been done, which has given an analytical solution for calculating the magnetic forces between cylindrical permanent magnets, and the magnetic force can be expressed as follows:

$$F = -8\pi K_d R^2 \int_0^{+\infty} \frac{J_1^2(q)}{q} \sinh(q\tau_1)\sinh(q\tau_2)\, e^{-q\zeta} dq \tag{23}$$

where K_d is the magnetostatic energy constant, and other magnetic or geometric parameters can be referred to Figure 8.

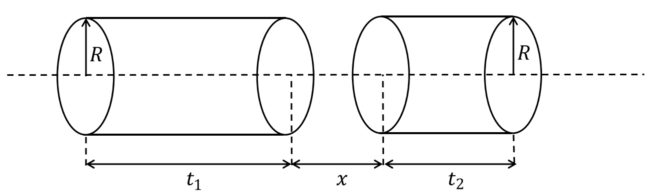

FIGURE 8. A scheme of the two interacting cylindrical permanent magnets with a common axis. Reproduced from [101].

When two magnets are identical and at a coaxial condition ($t_1 = t_2$ and $r = 0$), a new form of the equation can be obtained as shown in Equation (20). However, this simplified form can only provide a good force approximation when the distance between magnets is much larger than their length and diameters, and its accuracy will drop significantly if the distance gets smaller [102].

$$F = -\frac{1}{2}\pi K_d R^4 \left[\frac{1}{x^2} + \frac{1}{(x+2t)^2} - \frac{2}{(x+t)^2}\right] \tag{24}$$

Overall, by leveraging integral equations and advanced numerical techniques, this method provides a clear and precise way to determine the resulting forces. Integral Method is normally applied when the magnetic field distribution is complex and difficult to solve analytically, and it is widely used in the design and analysis of electromagnetic devices and permanent magnets. Improved versions of Integral Method often involve advanced numerical techniques and optimizations to enhance accuracy and efficiency. For instance, the use of Gaussian numerical integration and semi-analytical expressions can significantly improve the computational performance and accuracy of the method. A recent study has presented an integral definition method to solve the magnetic force between magnetic rings in permanent magnetic bearings [30]. Furthermore, Integral Method has been used to precisely evaluate the local magnetic force distribution within several magnetic systems, which demonstrates the effectiveness and computational efficiency of the method [103, 104]. This method provides precise force calculations by considering the entire magnetic field distribution, but again it sometimes brings intensive calculations.

8. BOUNDARY ELEMENT METHOD

BEM is a numerical computational technique always used to solve linear partial differential equations (PDEs) that have been reformulated as integral equations over the domain's boundary. In magnetostatics, BEM is particularly effective for calculating magnetic fields and forces in scenarios where the problem can be defined by boundary conditions, which reduces the problem dimensionality by one to lower computational complexity (3D problems become 2D, and 2D problems become 1D) [105, 106]. In magnetostatics, the governing equation for magnetic vector potential A in the absence of free currents is:

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{A}\right) = \mathbf{J}$$

$$(\text{It always simplifies to } \nabla^2\mathbf{A} = 0) \tag{25}$$

For scalar magnetic potential $\emptyset_m$ in regions with no current:

$$\nabla \cdot (\mu \nabla \emptyset_m) = 0 \tag{26}$$

Therefore, using Green's identity, the magnetic scalar potential $\emptyset_m$ at a point r can be expressed as:

$$\emptyset_m(\mathbf{r}) = \int \left[G(\mathbf{r},\mathbf{r}')\frac{\partial \emptyset_m}{\partial n}(\mathbf{r}') - \emptyset_m(\mathbf{r}')\frac{\boldsymbol{\partial}\mathbf{G}\left(\mathbf{r},\mathbf{r}'\right)}{\boldsymbol{\partial}\mathbf{n}}\right] d\Gamma \tag{27}$$

where $G(\mathbf{r},\mathbf{r}')\frac{\partial \emptyset_m}{\partial n}$ is the free-space Green's function, and $\frac{\partial}{\partial n}$ is the derivative normal to the boundary Γ. Once the magnetic vector potential is solved on the boundary, magnetic field and magnetic flux density are obtained. Then, Maxwell Tensor Method can be used to compute the force value as previously shown in Equation (16).

In addition to the dimensionality reduction advantage, BEM naturally accommodates open-boundary problems without the need for artificial domain truncation. It is particularly effective when material properties remain uniform within each subdomain, which makes it a great solution for problems involving permanent magnets and linear magnetic materials. However, one significant drawback is the generation of dense system matrices, in contrast to sparse matrices of FEM, which results in increased memory requirements and computational load for large-scale applications. Moreover, BEM faces challenges when being applied to problems involving nonlinear magnetic behavior, such as saturation or hysteresis, often requiring hybrid formulations to overcome these limitations. The application of BEM in magnetic force computation has been widely explored in literature. For example, a 2022 study demonstrated the method's efficiency in electrostatic force calculations [107], while another recent investigation used a hybrid BEM to improve the accuracy and computational performance of force estimations in magnetically coupled systems [108]. In addition, in 2020, a novel Current Sheet Element (CSE) was introduced within the BEM framework to calculate magnetic forces and torques. This approach eliminates the need for volume meshing by modeling current distributions directly on surfaces, leading to more efficient and scalable simulations. The geometrical configuration used for field calculation with the CSE is illustrated in Figure 9.

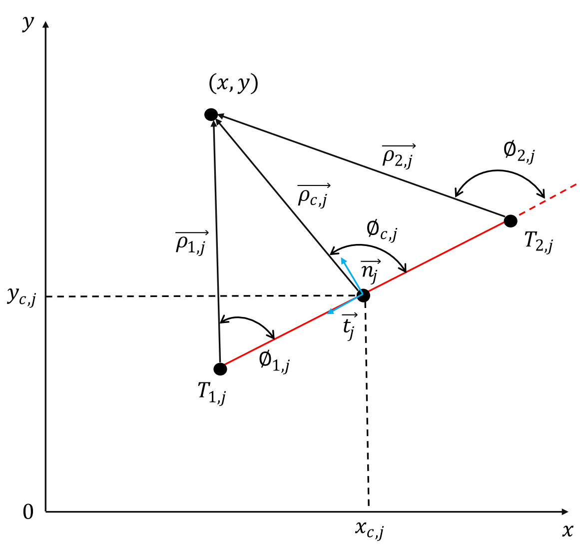

FIGURE 9. A simplified illustration of the geometrical parameters for the magnetic field calculation of a CSE. Reproduced from [109].

The magnetic field of the jth CSE is written as:

$$\delta\overrightarrow{B_J} = \frac{\mu_0 K_{Z,j}}{2\pi}\left(\overrightarrow{t_j}\,(\emptyset_{2,j}-\emptyset_{1,j})\right)\operatorname{sgn}\left(\overrightarrow{n_j}\cdot\overrightarrow{\rho_{c,j}}\right) + \overrightarrow{n_j}\,\text{In}\,\frac{\rho_{1,j}}{\rho_{2,j}} \quad (28)$$

where $K_{Z,j}$ is the current density, and $\overrightarrow{t_j}$ and $\overrightarrow{n_j}$ are unit vectors in different directions. Other parameters can be referred to Figure 9 [109]. Once the magnetic field is solved by Equation (28), the force can be calculated through Maxwell Stress Tensor Method. Therefore, BEM stands out as an efficient and accurate technique for magnetic force calculations in open-boundary and piecewise homogeneous problems. It is especially helpful for computing magnetic forces, torques, and field distributions when the problem domain is relatively large, infinite, or involves complex boundary geometries. In practical applications, BEM has been used in the analysis of magnetic shielding, eddy current problems, and contactless energy transfer systems, where the unbounded nature of the domain makes volume-based methods less efficient. It has also been applied to the study of magnetic levitation configurations and linear motors, where accurate evaluation of leakage fields in open space is crucial for predicting performance. These examples demonstrate that while BEM is less commonly used than FEM, it provides distinct advantages in applications where infinite or semi-infinite domains and boundary-focused accuracy are primarily important.

8.1. Complexity

Dipole Method is relatively simple, modeling magnetic sources as dipoles, which makes it straightforward but limited to small-scale systems. Filament Method is more complex, requiring detailed modeling of current paths, which can be challenging for intricate geometries. Finite Element Method (FEM) is highly complex, involving the discretization of the entire domain into small elements and solving Maxwell's equations numerically. Energy Method is moderately complex, as it involves calculating energy variations, which can be intricate for complex systems. Maxwell Tensor Method requires detailed field calculations at boundaries, adding to its complexity. Integral Method involves solving integral equations, which can be computationally intensive and complex. Boundary Element Method (BEM) reduces problem dimensionality but still requires solving complex integral equations on boundaries. Table 1 highlights the advantages, limitations, and applications of all these seven discussed methods.

8.2. Accuracy

Dipole Method provides good accuracy for small-scale systems with discrete magnetic moments but loses accuracy for larger systems. Filament Method is accurate for systems with well-defined current paths but less so for thick conductors or complex geometries. FEM offers high accuracy for complex geometries and material properties, which makes it one of the most precise methods. Energy Method can be accurate if the energy landscape is well understood, but this is often challenging. Maxwell Tensor Method is accurate for systems with well-defined boundaries but requires precise field calculations. Integral Method can be very accurate for systems with known boundary conditions. BEM is accurate for problems with homogeneous materials and well-defined boundaries but struggles with internal inhomogeneities. Across all these methods, accuracy can be significantly affected by magnetic leakage, as leakage fields alter the actual flux distribution compared with idealized assumptions. Methods that rely on simplified field representations, such as dipole or Filament Method, often underestimate or overestimate forces when leakage is significant. In contrast, numerical approaches like FEM, BEM, and Integral Method can explicitly model leakage paths, allowing for better agreement with experimental results in systems where fringing or stray fields are non-negligible.

TABLE 1. Detailed summary of seven magnetic force calculation methods.

Method	Advantages	Limitations	Applications
Dipole Method	Simple and effective for small-scale systems with discrete magnetic moments. Easy to implement and computationally efficient for small systems.	Accuracy decreases for larger systems or continuous distributions. Limited applicability to complex geometries.	Small-scale magnetic systems, magnetic particle analysis.
Filament Method	Effective for systems with well-defined current paths like coils and solenoids. Provides accurate force calculations for thin conductors.	Assumes thin conductors, computationally intensive for systems with many filaments. Less accurate for thick conductors or complex geometries.	Coils, solenoids, and electrodynamic levitation systems.
FEM	Highly versatile and accurate for complex geometries and material properties. Can handle inhomogeneous materials and non-linearities.	Requires significant computational resources and complex mesh generation. Time-consuming setup and refinement process.	Complex electromagnetic systems, structural analysis, heat transfer.
Energy Method	Useful for systems where energy variations are easier to compute than forces. Provides a way to derive forces from energy principles.	Accurate energy calculations can be challenging for complex systems. Requires precise knowledge of the system's energy landscape.	Systems with well-understood energy landscapes, magnetic actuators.
Maxwell Tensor Method	Direct link between field quantities and mechanical forces; effective for well-defined boundaries. Provides a comprehensive understanding of force distribution.	Requires accurate field calculations at boundaries; computationally demanding for irregular geometries. Integration over complex surfaces can be challenging.	Systems with clear boundary definitions, electromagnetic field analysis.
Integral Method	Solves integral equations effectively for known boundary conditions. Can handle infinite domains and provide accurate results for specific applications.	Computationally intensive; requires accurate boundary conditions and material properties. Complex mathematical formulations.	Systems with known boundary conditions, infinite domain problems.
BEM	Reduces problem dimensionality, efficient for large or infinite domains. Focuses on boundaries, reducing the number of unknowns.	Limited to homogeneous materials and well-defined boundaries. Not suitable for systems with significant internal inhomogeneities or nonlinear materials.	Large or infinite domain problems, homogeneous material systems.

8.3. Computational Efficiency

Dipole Method is computationally efficient for small systems but scales poorly with system size. Filament Method becomes computationally intensive for systems with many filaments. FEM requires significant computational resources, especially for large or detailed models. Energy Method is moderately efficient but depends on the complexity of energy calculations. Maxwell Tensor Method can be computationally demanding due to the need for detailed field calculations. Integral Method is computationally intensive due to the complexity of solving integral equations. BEM is computationally efficient for large or infinite domains but limited by the complexity of boundary conditions. When magnetic leakage is explicitly modelled, computational efficiency can be further impacted, as capturing leakage fields often requires extending the simulation domain, refining the mesh in air regions, and accounting for additional boundary conditions. Methods like FEM and BEM, while being capable of accurately representing leakage paths, experience increased computational loads in such cases, whereas simplified methods that neglect leakage maintain higher efficiency at the expense of accuracy.

8.4. Applicability

Dipole Method is most appropriate for small-scale systems characterized by discrete magnetic moments but is less applicable in designs where magnetic leakage plays a significant role, as leakage fields cannot be readily captured in its simplified formulation. Filament Method is well suited for configurations with clearly defined current paths, such as coils and solenoids, although its applicability decreases in systems with substantial fringing fields or open magnetic circuits. FEM offers high versatility, accommodating complex geometries, nonlinear or anisotropic materials, and explicit modelling of leakage paths, making it suitable for accurately representing real-world devices. Energy Method is advantageous when evaluating energy variations is more tractable than directly computing magnetic forces, but it requires a well-characterized magnetic field distribution, which can be complicated by leakage. Maxwell Stress Tensor Method is particularly effective for systems with well-defined boundaries, yet its performance depends on precise field evaluation in regions where leakage may occur. Integral Method is suitable for problems with known boundary conditions and can capture leakage effects when the domain is properly defined. BEM is ideal for large or infinite domains with homogeneous material properties, and it can handle leakage fields extending into unbounded regions while reducing computational complexity.

8.5. Mathematical Properties

Dipole Method uses simple mathematical models based on dipole interactions. Filament Method involves detailed mathematical modeling of current paths. FEM uses numerical solutions to Maxwell's equations, involving complex mathematical formulations. Energy Method relies on energy principles and variations, requiring precise mathematical descriptions of the energy landscape. Maxwell Tensor Method uses Maxwell stress tensor, involving detailed mathematical integration over surfaces. Integral Method involves solving integral equations derived from Maxwell's equations, requiring advanced mathematical techniques. BEM reduces problem dimensionality by focusing on boundaries, involving complex integral equations on these surfaces.

9. POTENTIAL SOLUTIONS AND RESEARCH DIRECTION

Based on the previously discussed limitations and challenges of each numerical approach to magnetic force calculation, this section introduces a range of potential solutions aimed at addressing these methodological constraints. These solutions are designed not only to mitigate current shortcomings but also to enhance the overall accuracy, computational efficiency, and applicability of the methods across various scenarios. In parallel, several future research directions are proposed, offering pathways for continued advancement and innovation in the field. Given that each method presents its own unique set of challenges, targeted improvements can be achieved through the application of advanced numerical techniques and the development of hybrid modeling approaches. In particular, future efforts should concentrate on improving the performance of these methods when they are applied to systems with complex geometries, nonlinear material behaviors, and dynamic operating conditions. By leveraging ongoing advancements in computational power and algorithm design, researchers can significantly expand the capabilities and practical use of magnetic force calculation techniques, ultimately enabling more accurate and robust analyses across a wider range of engineering applications.

9.1. Dipole Method

To improve the accuracy of the Dipole Method for larger systems, hybrid techniques that combine basic dipole approximations with higher-order multipole expansions may be adapted. This approach allows for a more accurate representation of magnetic interactions while retaining computational efficiency. Additionally, machine learning algorithms can be used to predict dipole interactions in complex systems, potentially accelerating computations and increasing accuracy. Another promising solution is the development of adaptive algorithms that dynamically adjust the level of approximation based on system size and complexity, which help optimise performance across varying scales.

Future research should emphasize the refinement of such adaptive algorithms to enhance scalability and robustness. At the nanoscale, incorporating quantum mechanical effects could offer deeper insights into magnetic interactions that classical dipole models fail to capture. Moreover, the exploration of advanced materials with unique magnetic properties may further extend the applicability of the Dipole Method across emerging domains such as spintronics and nanoscale sensing.

9.2. Filament Method

Enhancing the computational efficiency of Filament Method can be achieved by optimizing numerical integration techniques used to calculate magnetic forces between current-carrying filaments. The application of parallel computing and GPU acceleration has the potential to significantly reduce computation times in systems comprising numerous filaments. Moreover, incorporating detailed physical models that consider conductor thickness and complex geometries can improve both accuracy and applicability.

Future work should prioritize the development of these advanced models, particularly for systems where conductor shape significantly affects magnetic behaviours. Integrating real-time sensor data in practical applications — such as magnetic levitation systems or wireless power transfer — can improve the Filament Method's responsiveness and accuracy. Additionally, exploring novel conductor materials and structural designs could open new avenues for performance optimization.

9.3. FEM

To address the high computational demands of FEM, researchers can focus on the development of more efficient meshing strategies, including adaptive mesh refinement and intelligent domain decomposition. The use of advanced solvers supported by parallel processing and cloud computing platforms can further reduce simulation time and resource requirements. Hybridizing FEM with other numerical methods, such as BEM, may also lead to robust and flexible solutions capable of handling diverse system configurations.

Future research should aim to improve FEM's accuracy in modelling nonlinear materials and intricate boundary conditions. The integration of machine learning techniques to optimize mesh generation and solver performance could further elevate FEM's efficiency. Additionally, expanding its use to multi-physics environments involving thermal, structural, and electromagnetic interactions can broaden its impact across scientific and engineering disciplines.

9.4. Energy Method

Improving Energy Method involves the formulation of more accurate models for evaluating energy variations in systems with complex geometries and interactions. The application of optimisation algorithms to minimize the computational burden of energy-based calculations can enhance efficiency. Furthermore, coupling Energy Method with complementary approaches such as FEM can conduct more comprehensive and precise analyses, particularly for systems with nonuniform field distributions.

Future directions may include extending the Energy Method to dynamic systems where temporal energy fluctuations are significant. Its integration into multi-physics simulations — accounting for coupled thermal, mechanical, and magnetic effects — can also enhance its relevance to real-world problems. Investigating novel energy storage mechanisms and the use of functional magnetic materials may further enrich its applicability.

9.5. Maxwell Tensor Method

The accuracy of Maxwell Tensor Method can be improved through the development of more refined algorithms for evaluating electromagnetic field quantities at boundaries. Advanced numerical integration techniques can contribute to better precision and reduced computational costs. Additionally, hybrid models that integrate the Maxwell Tensor Method with FEM or other spatially adaptive methods can provide more complete solutions for systems with complex structures.

Future research could focus on extending the applicability of the Maxwell Tensor Method to handle geometrically intricate and materially inhomogeneous domains. Incorporating real-time sensor feedback in practical applications — such as actuator systems or contactless sensors — may also enhance the method's performance. Moreover, studies involving advanced materials and unconventional boundary conditions could reveal new ways to optimise force estimation using this method.

9.6. Integral Method

To mitigate the computational intensity of the Integral Method, efforts should be directed toward developing more efficient solvers for integral equations, particularly those capable of parallel execution. High-performance computing techniques and algorithmic improvements, including fast multipole methods and adaptive quadrature, can significantly reduce computation time. Combining Integral Method with FEM or other localized numerical techniques can also help address challenges associated with domain complexity.

Research should explore the method's applicability to systems with complex boundary conditions and strongly nonlinear materials. The formulation of new integral representations that better capture physical behaviours in such environments could enhance both accuracy and efficiency. Furthermore, the use of advanced magnetic materials and multi-domain configurations may reveal new opportunities for applying the Integral Method in emerging applications.

9.7. BEM

Enhancing the efficiency of the boundary element method involves the development of advanced algorithms for solving boundary integral equations with improved convergence properties. Adaptive meshing, domain partitioning, and parallel computing techniques can greatly accelerate simulations, particularly for large-scale or open-domain problems. When being integrated with FEM or other volume-based methods, BEM can serve as a powerful tool in hybrid frameworks that leverage the strengths of each technique.

Future research should focus on extending BEM to better accommodate nonlinear and inhomogeneous material properties. Real-time data integration, especially in practical applications such as magnetic field mapping and system monitoring, could significantly improve accuracy and responsiveness. In addition, investigating novel formulations and leveraging emerging materials may unlock new use cases and further enhance the method's versatility.

10. CONCLUSION

This review has analysed seven advanced numerical approaches for magnetic force calculations, including Dipole Method, Filament Method, FEM, Energy Method, Maxwell Tensor Method, Integral Method, and BEM, with respect to their accuracy, computational efficiency, and applicability. The comparative findings highlight that Dipole Method is computationally efficient and suitable for small-scale or weakly coupled systems but lacks accuracy in large or complex configurations. Filament Method offers high accuracy for coils and solenoids, particularly in misaligned cases, though its efficiency decreases as the number of filaments increases. FEM achieves the highest accuracy and versatility for complex geometries, nonlinear materials, and anisotropic properties, but at the cost of high computational demand. Energy Method is moderately efficient and effective where energy variation is easier to compute than force, especially when being integrated with FEA for complex structures. Maxwell Tensor Method provides highly accurate local force and torque evaluations in systems with well-defined boundaries, though it requires dense field data and can be computationally intensive. Integral Method is robust for problems with infinite or semi-infinite domains but is computationally heavy due to integral equation complexity. BEM is most efficient for large or open-boundary problems, reducing dimensionality and computational effort, though it is less flexible in handling heterogeneous materials.

From an accuracy perspective, FEM and Maxwell Tensor Methods rank the highest, followed by Filament and Energy methods for structured geometries, while Dipole and Integral methods are more approximate. From a computational efficiency perspective, Dipole and BEM methods perform the best in their respective domains, while FEM demands a lot of resources. In terms of applicability, FEM and Maxwell Tensor methods are the most versatile across diverse electromagnetic systems, while dipole, filament, and BEM excel in specialised contexts.

In conclusion, these findings show that no single method is universally optimal, and the choice depends on the trade-off among accuracy, efficiency, and problem domain. Future progress will rely on hybrid methods that integrate complementary strengths, as well as advanced computational techniques such as parallelization and machine learning, to overcome existing limitations and further improve force calculation accuracy in complex electromagnetic systems.

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