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Comment on "Transfer Matrix Method for General Bianisotropic Layers": Correction of Explicit Formulations

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ABSTRACT: This comment addresses errors in the explicit expressions of the Ω matrix presented by Blankenship et al. (2025) [1]. We show that sign inconsistencies in the original formulation can lead to non-physical results, including the violation of energy conservation in passive lossless environments. We provide corrected formulas for the affected matrix entries and numerically verify that the corrected formulation restores physical consistency, with the total energy conserved to the displayed precision in the lossless benchmark.

1. INTRODUCTION

Recently, Blankenship et al. [1] proposed a transfer matrix method (TMM) for analyzing general bianisotropic layers, expanding upon foundational semi-analytical frameworks [3]. Their work aimed to provide a comprehensive, explicit formulation for the 4×4 system matrix Ω.

However, we have identified sign errors in the explicit formulas for Ω_{21}-Ω_{24} in Eqs. (9)–(12) of Ref. [1]. In general bianisotropic cases, where off-diagonal tensor elements are nonzero, these sign inconsistencies can lead to a significant violation of the law of conservation of energy (e.g., apparent amplification in passive media).

In this comment, we present the corrected explicit formulations for the affected Ω-matrix elements. We demonstrate through numerical validation that while the original formulation may appear correct in specific cases where certain tensor elements (e.g., μ_{xz}) are zero, it fails in the general case.

2. CORRECTED EXPLICIT FORMULATIONS

The spatial evolution of the transverse fields is governed by:

$$\frac{d}{d\tilde{z}} \begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} = \frac{-j}{\Omega_0} \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} \\ \Omega_{21} & \Omega_{22} & \Omega_{23} & \Omega_{24} \\ \Omega_{31} & \Omega_{32} & \Omega_{33} & \Omega_{34} \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ \tilde{H}_x \\ \tilde{H}_y \end{bmatrix} \quad (1)$$

where $\Omega_0 = \mu_{zz}\epsilon_{zz} - \zeta_{zz}\xi_{zz}$. The corrected expressions for the affected entries, corresponding to Eqs. (9)–(12) in [1], are:

$$\Omega_{21} = \epsilon_{zx}B - \zeta_{xx}\Omega_0 + (\zeta_{zx} + \tilde{k}_y)C \quad (2)$$

$$\Omega_{22} = \epsilon_{zy}B - \zeta_{xy}\Omega_0 + (\zeta_{zy} - \tilde{k}_x)C \quad (3)$$

$$\Omega_{23} = \mu_{zx}C - \mu_{xx}\Omega_0 + (\xi_{zx} - \tilde{k}_y)B \quad (4)$$

$$\Omega_{24} = \mu_{zy}C - \mu_{xy}\Omega_0 + (\xi_{zy} + \tilde{k}_x)B \quad (5)$$

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with intermediate variables defined as $A = \tilde{k}_y - \zeta_{xz}$, $B = -\mu_{xz}\zeta_{zz} - \mu_{zz}A$, and $C = \xi_{zz}A + \mu_{xz}\epsilon_{zz}$.

3. NUMERICAL VALIDATION

3.1. Benchmark with Rigorous Reference

We first checked consistency against the structure defined in [2]. In that case, $\mu_{xz} = 0$, so both the original and corrected formulations produce nearly identical results (difference on the order of 10^{-13}), which explains why the sign inconsistency can remain hidden in such special cases.

3.2. Violation of Energy Conservation

To illustrate the impact of the sign errors, we considered a general bianisotropic multilayer benchmark based on the second example of Ref. [1]. Specifically, we used the same source and geometry parameters, namely $\lambda_0 = 1\,\text{mm}$, $\theta = 29°$, $\phi = 79°$, $p_{TE} = 0.43 - 0.39j$, $p_{TM} = 1.00 + 0.17j$, and $L_1 = L_2 = \lambda_0/16$, but removed all loss terms to form a lossless counterpart for the energy-conservation test. The incident medium (reflection side) is vacuum ($\epsilon_r = 1$, $\mu_r = 1$). The lossless constitutive tensors were:

Layer 1:

$$[\epsilon_{r,1}] = \begin{bmatrix} 3.0 & 0.3 & -1.7 \\ 0.3 & 2.1 & -0.5 \\ -1.7 & -0.5 & 4.9 \end{bmatrix},$$

$$[\mu_{r,1}] = \begin{bmatrix} 2.9 & -0.5 & -0.2 \\ -0.5 & 1.3 & -0.6 \\ -0.2 & -0.6 & 2.8 \end{bmatrix},$$

$$[\zeta_{r,1}] = [\xi_{r,1}] = \begin{bmatrix} 3.2 & -0.2 & -0.5 \\ -0.2 & 2.2 & -0.4 \\ -0.5 & -0.4 & 3.6 \end{bmatrix}$$

TABLE 1. Total energy comparison, $R+T$, for the lossless counterpart of the second benchmark in [1].

Formulation	Total Energy ($R + T$)
Original Blankenship et al. [1]	147.067%
Corrected (this work)	100.000%

Layer 2:

$$[\epsilon_{r,2}] = \begin{bmatrix} 8.2 & 0.3 & -0.1 \\ 0.3 & 8.7 & -0.3 \\ -0.1 & -0.3 & 8.1 \end{bmatrix},$$

$$[\mu_{r,2}] = \begin{bmatrix} 4.8 & 2.3 & -0.3 \\ 2.3 & 8.0 & -2.4 \\ -0.3 & -2.4 & 3.2 \end{bmatrix},$$

$$[\zeta_{r,2}] = [\xi_{r,2}] = \begin{bmatrix} 9.0 & -1.3 & 1.3 \\ -1.3 & 5.7 & 0.2 \\ 1.3 & 0.2 & 7.3 \end{bmatrix}$$

The transmission substrate was also made lossless, with $\epsilon_{r,trn} = 2.14$ and $\mu_{r,trn} = 5.21$.

As shown in Table 1, the original explicit formulas lead to a substantial violation of energy conservation, exhibiting non-physical gain even in the lossless passive test. In contrast, the corrected formulas restore physical consistency, with the total energy conserved to the displayed precision.

4. CONCLUSION

The sign inconsistencies in the explicit formulas for Ω_{21}-Ω_{24} in Eqs. (9)–(12) of Ref. [1] can lead to non-physical results in general bianisotropic calculations. Correcting these expressions restores physical consistency in the tested cases and clarifies the explicit formulation without changing the broader BTMM framework.

REFERENCES

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