(Received 26 February 2026, Accepted 27 March 2026, Scheduled 13 April 2026)

Formation of Multiple Electromagnetic Field Minima at Prescribed Locations

Denis Iuzvik[1, 2, *] and Maksim Stepanov[1, 2]

[1]Novosibirsk State Technical University, Karl Marx Av. 20, Novosibirsk 630073, Russian Federation
[2]Department of Radio Receiving and Radio Transmitting Devices at Novosibirsk State Technical University, Russian Federation

ABSTRACT: This study presents a method for generating multiple electromagnetic field minima in specified spatial regions. The relationship between the complex amplitudes of signals at receiving points and those of radiated signals is described using an S-parameter matrix. It is shown that the determination of the complex amplitudes of the radiated signals can be reduced to solving an underdetermined system of linear algebraic equations, which may be unstable because such a system can admit either infinitely many solutions or no solution. To address this issue, an optimization problem is formulated based on minimizing the squared error between the required and obtained electric field distributions. Its solution leads to a new system of linear algebraic equations, to which Tikhonov regularization is applied to ensure the stability and uniqueness of the solution. The proposed approach is validated by mathematical modeling for three electric field configurations, with the complex amplitudes of the radiated signals determined for each configuration. The modeling results confirm the correctness of the theoretical conclusions.

1. INTRODUCTION

Focusing the electromagnetic field on the required points in space is one of the ways to increase the signal-to-noise ratio (SNR) at the aperture of the receiving antenna of the subscriber device. Such an increase in the SNR allows for the use of high-order quadrature amplitude modulation (QAM), for example, 4096-QAM. The use of high-order QAM modulation allows for an increase in the information transfer rate in such wireless networks [1, 2]. In [3–5], the issues related to focusing linear antenna arrays over a finite distance were considered, and the effects of shifting the maximum of the electric field strength amplitude towards the antenna array relative to the focal point were studied.

Antenna array synthesis has traditionally been regarded as one of the fundamental problems of electrodynamics and antenna theory. The studies [6, 7] laid the foundation for analytical control of the radiation pattern, including null placement and the trade-off between main-lobe width and sidelobe level. Subsequently, the development of adaptive spatial filtering led to methods such as [8, 9], where interference suppression is achieved through the optimal selection of complex weights. The next important stage was the reformulation of synthesis as weighted least-squares and convex optimization problems, which made it possible to account for arbitrary array geometry, element directivity, as well as constraints not only in the far field but also in the near field [10, 11].

In recent years, interest in near-field synthesis has grown significantly due to applications in radio frequency identification (RFID), medical hyperthermia, wireless power transfer, radar, and emerging 5G/6G systems. Early studies on near-field phased arrays already showed that the problem can be naturally formulated as field control within a finite volume with minimization of the mean-square error. Modern reviews note that near-field focusing methods generally fall into two broad classes: conjugate-phase methods, which provide simple focusing at a prescribed point, and optimization-based methods, which enable the synthesis of more complex spatial field distributions, including contour-shaped and multi-point focal patterns [12–14].

For the near field, studies in which synthesis is formulated not in terms of an angular radiation pattern, but rather through field requirements at a set of spatial points, are of particular importance. Thus, [15] proposed an efficient synthesis method for planar antenna arrays to produce prescribed field contours in the near-field region. Later works introduced formulations based on electromagnetic inner products, making it possible to obtain closed-form solutions for various optimality criteria, as well as methods combining adaptive spatial discretization with convex optimization to improve focusing accuracy. These results show that the transition from "focusing at a single point" to "synthesizing a prescribed field over a region" has become a persistent trend in the modern literature [16, 17].

In recent years, increasing attention in antenna array research has been devoted not only to the synthesis of prescribed spatial field distributions, but also to inverse problems concerned with retrieving array parameters from a limited set of measurements. Of particular interest are approaches that reduce the number of required measurements by exploiting the ideas of sparse representations and compressed sensing (CS). In this context, the study [18] constitutes a significant contribution: the authors proposed a method for diagnosing array-element failures based solely on phaseless field measurements, while minimizing the

* Corresponding author: Denis Iuzvik (zetanicestar@gmail.com).

number of measurements through the use of compressed sensing theory. Importantly, the proposed procedure is applicable to both far-field and near-field regions and, according to the authors, represents the first CS-based technique capable of performing diagnostics using only phaseless measurements acquired exclusively in the near-field region.

In [19], the issues related to the formation of several maxima of the electric field strength amplitude in the required areas of space were considered. The formation of such maxima can be justified by the fact that the locations of subscriber devices may be known in advance. This is especially relevant when providing communication between smart home and Internet of Things devices, whose locations are usually static. This allows for communication between these devices while minimizing interference with other devices. However, this approach has a downside: for example, medical facilities house equipment that may be sensitive to excessive electromagnetic radiation from wireless telecommunications networks. Moreover, it is difficult to predict the locations of subscriber devices in such locations. Therefore, it is more practical to create minimum electric field amplitudes at the locations of sensitive piece of equipment and ensure acceptable signal levels elsewhere. Forming a single minimum electric field amplitude is possible through antenna array phasing. In practice, there may be more than one electromagnetically sensitive equipment in a room, requiring the creation of multiple zones within the space, where minimum electric field amplitudes are maintained. **The scientific novelty** of this work lies in the fact that previous studies have not considered methods for forming multiple zones of minimum electromagnetic field while simultaneously ensuring the required amplitude level of the electric field intensity in the remaining regions of space.

This determines the **purpose of this work**: to substantiate the possibility of forming several minima of the amplitude of the electric field strength in given areas of space.

2. MATHEMATICAL MODEL OF THE SYSTEM

In [19], a method was proposed for representing an antenna system as a microwave multipole, linking the complex amplitudes of the radiated signals and the complex amplitudes at the receiving points through an S-parameter matrix. We will use this method as the basis for the mathematical model and briefly recall it [19].

The essence of the method is that N radiating antennas are located in space. M receiving points are randomly located. An example of the arrangement of radiating antennas and receiving points is shown in Figure 1. In the figure, the radiating antennas are located along the perimeter of the area (along the walls of the room), and the receiving points are located within this area. At each receiving point, the required values of the electric field strength amplitude are specified. Next, the complex transmission coefficients between each radiating antenna and each receiving point are determined. By solving a system of linear algebraic equations (SLAE), the complex amplitudes of the signals that must be applied to each radiating antenna to obtain the required field distribution in space are determined. The effectiveness of the method was demonstrated in [19].

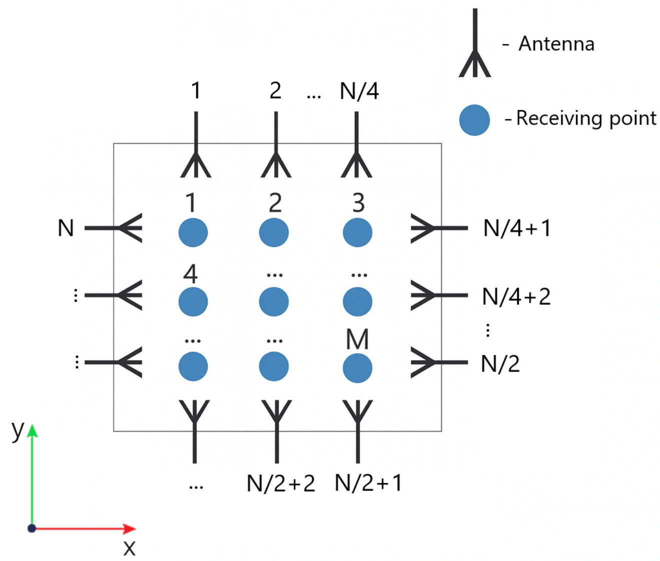

FIGURE 1. Radiating antenna-space system [19].

We briefly recall the method for controlling the electric field distribution in a given space, as demonstrated in [19]. The origin of the coordinates in the presented antenna-space system is located at the geometric center of the space. The "X" and "Y" coordinate axes are oriented parallel to the antenna arrays. The "Z" axis points upward.

Let us recall [19] the designations of the complex amplitudes of the radiated signals:

$$a_n = |a_n| \cdot e^{j \cdot \phi_n}, \tag{1}$$

where $|a_n|$ is the amplitude of the signal radiated from the n-th point; ϕ_n is the initial phase of the signal radiated from the n-th point; and j is the imaginary unit.

Complex amplitude at receiving points:

$$b_m = |b_m| \cdot e^{j \cdot \psi_m}, \tag{2}$$

where $|b_m|$ is the signal amplitude at the m-th point, and ψ_m is the signal phase at the m-th point.

The set of complex amplitudes of signals at the radiation points forms a column vector of the radiated signals $[A]$. The set of complex amplitudes of signals at the receiving points forms a column vector of the received signals $[B]$.

Because the system shown in Figure 1 is represented as a microwave multipole (see Figure 2) [19], the number of inputs to the multipole will be equal to $N + M$, where N is the number of radiating antennas, and M is the number of receiving points. Obviously, inputs 1 through N correspond to the radiating antennas, and inputs $N + 1$ through $N + M$ correspond to the receiving points.

Therefore, by specifying a column vector of the output signals of the multipole $[Y]$, it is possible to define a column vector of input signals $[X]$. They are connected by a scattering matrix (S-parameters — $[S]$):

$$[Y] = [S] \cdot [X], \tag{3}$$

We now set the boundary conditions. Considering that controlling the electric field distribution is required within the room volume, and not at its boundaries, the elements of the column

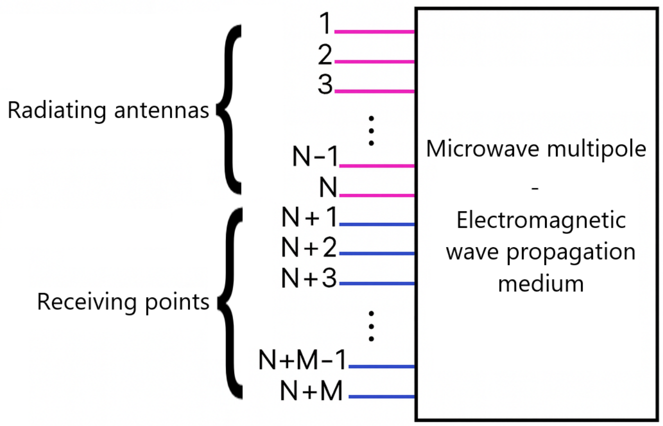

FIGURE 2. Representation of the radiating antenna-space system in the form of a microwave multipole [19].

vector $[Y]$ from 0 to N will be equal to zero. Elements $N+1$ to $N+M$ determine the complex amplitudes of the signals (b_m) at the given receiving points M.

$$[Y] = [y_1 = 0; \quad \ldots; \quad y_N = 0; \quad y_{N+1} = b_1; \quad \ldots; \quad y_{N+M} = b_M]^T. \tag{4}$$

It is also assumed that in the input signal (influence) column vector $[X]$, the elements from $N+1$ to $N+M$ are equal to zero. A simplification is that there is no reflection at the points under consideration.

$$[X] = [x_1 = a_1; \quad \ldots; \quad x_N = a_N; \quad x_{N+1} = 0; \quad \ldots; \quad x_{N+M} = 0]^T. \tag{5}$$

The S-parameter matrix of the multipole has the following form [19]:

$$[S] = \begin{bmatrix} s_{1,1} & .. & s_{1,N} & s_{1,N+1} & .. & s_{1,N+M} \\ .. & .. & .. & .. & .. & .. \\ s_{N,1} & .. & s_{N,N} & s_{N,N+1} & .. & s_{N,N+M} \\ s_{N+1,1} & .. & s_{N+1,N} & s_{N+1,N+1} & .. & s_{N+1,N+M} \\ .. & .. & .. & .. & .. & .. \\ s_{N+M,1} & .. & s_{N+M,N} & s_{N+M,N+1} & .. & s_{N+M,N+M} \end{bmatrix}. \tag{6}$$

The diagonal elements of the matrix are the reflection coefficients from the corresponding multipole input. The off-diagonal elements of the matrix are the transmission coefficients between the corresponding multipole inputs.

In free space, the transmission coefficients are determined using the following expression [19]:

$$s_{i,k} = F_{i,k} \cdot F_{k,i} \cdot r_{i,k}^{-q} \cdot \exp\left(-j \cdot \beta \cdot r_{i,k}\right), \tag{7}$$

where $r_{i,k}$ is the distance from the point corresponding to the i-th input of the multipole network to the point corresponding to the k-th input of the multipole network; $\beta = \frac{2\cdot\pi}{\lambda}$ is the phase coefficient; $F_{i,k}$ are the elements of the matrix containing the field gain coefficients of the antenna installed at the point corresponding to input k in the direction of the point corresponding to input i; and the factor $r_{i,k}^{-q}$ determines the decrease in the amplitude of the electromagnetic wave strength during propagation from the point corresponding to the input of the multipole i to the point corresponding to the input of the multipole k. For the electromagnetic field in the near zone $q = 2$, in the far zone — $q = 1$.

The distance between the inputs of a multipole is determined using the expression [19]:

$$r_{i,k} = \sqrt{\left(x_i - x_k\right)^2 + \left(y_i - y_k\right)^2 + \left(z_i - z_k\right)^2}, \tag{8}$$

where (x_i, y_i, z_i) are the Cartesian coordinates of the i-th point.

The set of multipole inputs, which are radiating antennas, can be analyzed as an antenna array. Accordingly, when calculating the transmission coefficients using Equation (7), it is necessary to determine the zone of the antenna array field, in which the multipole input for which the transmission coefficient is being calculated is located to correctly select the value of q. To determine the boundaries of the near zone, Equation (9) [20] is used:

$$r_{nf} \leq \frac{2 \cdot D^2}{\lambda}, \tag{9}$$

where D is the aperture size of the antenna array.

Thus, we can write the condition for determining the value of q:

$$q = \begin{cases} 2, & r_{i,k} \leq \frac{2\cdot D^2}{\lambda} \\ 1, & r_{i,k} > \frac{2\cdot D^2}{\lambda} \end{cases} \tag{10}$$

Equations (9) and (10) are used as engineering estimates of the boundary between the near-field and far-field radiation regions, depending on the aperture size of the antenna array [20]. Clearly, in the general case, the transition from the near-field region to the far-field region is not abrupt but occurs through an intermediate zone. Therefore, the formula used does not define a strict physical boundary, but rather makes it possible to estimate the regime in which the antenna configuration under consideration operates — near field or far field — and, accordingly, to select the value of the parameter q. A more accurate determination of the zone boundaries and analysis of the field structure for a specific antenna-array geometry require a full-wave electromagnetic simulation.

In the work [19], it is shown that the matrix of S-parameters (6) of a multipole can be divided into submatrices:

$$[S] = \begin{bmatrix} [S_{TT}] & [S_{TR}] \\ [S_{RT}] & [S_{RR}] \end{bmatrix}, \tag{11}$$

where $[S_{TT}]$ is a submatrix that determines the transmission coefficients from one radiation point to another radiation point (submatrix of mutual transmission coefficients), with dimensions $[N$ rows; N columns$]$; $[S_{TR}]$ is a submatrix that defines the transmission coefficients from the receiving point to the transmitting point (submatrix of inverse transmission coefficients), with dimensions $[N$ rows; M columns$]$; $[S_{RT}]$ is a submatrix that defines the transmission coefficients from the emission point to the receiving point (submatrix of direct transmission coefficients), with dimensions $[M$ rows; N columns$]$;

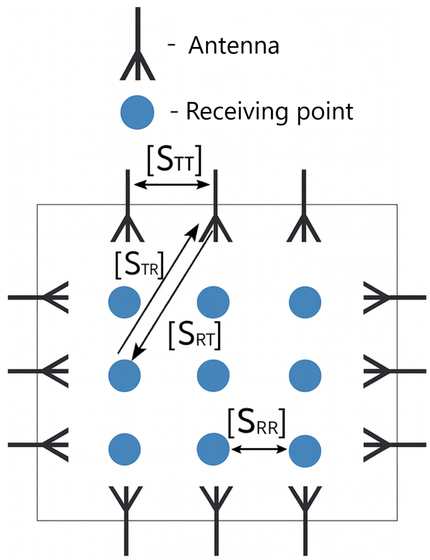

FIGURE 3. Schematic representation of the role of submatrices [19].

$[S_{RR}]$ is a submatrix that determines the transmission coefficients from a receiving point to another receiving point (submatrix of mutual transmission coefficients), with dimensions [M rows; M columns].

Figure 3 [19] shows a schematic representation of the roles of the submatrices.

In the ideal case, when all receiving and transmitting antennas are matched, and there is no mutual influence between the transmitting points and receiving points, (3) can be expressed as:

$$[B] = [S_{RT}] \cdot [A], \tag{12}$$

Then, the number of equations will be reduced from $N + M$ to M.

2.1. Formation of Multiple Electromagnetic Field Minima

If the condition $b_m = 0$ is specified for the corresponding receiving point, the minimum electric field strength will not be a single occurrence but rather a repeating one, extending to many subsequent points in space. A single occurrence of the minimum electric field amplitude is ensured by introducing support points into the problem, which define the electric field amplitude at points in space that do not correspond to the minimum points. Accordingly, the column vector of received signals $[B]$ must be specified such that at the receiving points, where the minimum is required, the value of the $b_m = 0$ element is zero, and at the specified support point, the value is greater than zero, i.e., $b_m > 0$.

In [19], the problem of forming the maxima of the electric field strength amplitude in the required spatial regions was solved. In this study, the system of Equation (12) was solved using the Gaussian method; however, when additional support points are introduced into the system, instability of the solution may arise [21–23]. In addition, if the number of radiating antennas N is greater than the number of receiving points M in the system, the system will be underdetermined. Such an underdetermined system may have an infinite number of solutions or none at all. Therefore, to solve system (12), we introduce a criterion formulation of the problem: finding the column vector $[A]$ that minimizes the quadratic error of the approximation of the required field distribution.

To control the suppression depth at the given minimum points, it is necessary to introduce additional weights w_m in the form of a diagonal matrix:

$$[W] = \text{diag}(w_1, w_2, \ldots, w_M), \tag{13}$$

The higher the value of w_m is, the higher the priority is to ensure the conditions $b_m \approx 0$ and $b_m \approx 1$ depending on which are necessary to ensure the maximum or minimum amplitude of the electric field strength at the required receiving point.

To determine the column vector $[A]$, we need to solve the following problem:

$$\text{M}([A]) = \|[W] \cdot ([S_{RT}] \cdot [A] - [B])\|_2^2\,, \tag{14}$$

where $\| \ \|_2$ — L2-norm: $\|x\|_2 = \sqrt{x_1^2 + \ldots x_n^2}$.

In fact, if we introduce the mathematical definition of the error for the introduced system, we obtain:

$$[\varepsilon] = [S_{RT}][A] - [B], \tag{15}$$

in an ideal case $[\varepsilon] = 0$.

So, in fact, we have:

$$\text{M}([A]) = \|[W] \cdot [\varepsilon]\|_2^2\,, \tag{16}$$

Thus, (14) and (16) are the criteria for error minimization.

The introduction of additional support points into the system can lead to instability of the solutions [21–23]. Therefore, the Tikhonov regularization method [24] is introduced into the system (14). The introduction of Tikhonov regularization, for $\mu > 0$, allows us to obtain the uniqueness of the solution. Then, Equation (14) takes the following form:

$$\text{M}([A]) = \|[W] \cdot ([S_{RT}] \cdot [A] - [B])\|_2^2 + \mu\,\|[A]\|_2^2\,, \tag{17}$$

where μ — Tikhonov regularization coefficient.

We expand the norm through Hermitian conjugation. It is known [21–23] that $\|x\|_2^2 = x^H \cdot x$, where H is a Hermitian conjugation, that is, the operation of transposition and complex conjugation.

Then, the first part of expression (17) will take the form:

$$\|[W][\varepsilon]\|_2^2 = ([W][\varepsilon])^H([W][\varepsilon]) = [\varepsilon]^H[W]^H[W][\varepsilon] \tag{18}$$

Since the weight matrix $[W]$ is real and diagonal, we obtain:

$$[W]^H[W] = [W]^2 = \text{diag}(w_1^2, \ldots, w_M^2) \tag{19}$$

Then expression (17) will take the form:

$$\text{M}([A]) = ([S_{RT}] \cdot [A] - [B])^H \cdot [W]^2 \cdot ([S_{RT}] \cdot [A] - [B]) + \mu[A]^H[A] \tag{20}$$

Let's open the brackets in (20):

$$\begin{aligned}\text{M}([A]) = {} & [A]^H[S_{RT}]^H[W]^2[S_{RT}][A] - [A]^H[S_{RT}]^H[W]^2[B] \\ & -[B]^H[W]^2[S_{RT}][A] \\ & +[B]^H[W]^2[B] + \mu[A]^H[A]\end{aligned} \tag{21}$$

As stated in [25], it is necessary to find a complex column vector $[A]$ for which the expression $\text{M}([A])$ does not decrease with small changes in the real and imaginary parts; that is, a stationary point is found the global minimum of the function.

$$\frac{\partial \mathrm{M}([A])}{\partial [A]^*} = 0 \tag{22}$$

where $[A]^*$ — complex conjugate column vector $[A]$.

It is necessary to differentiate expression (21) according to (22).

As a result, we obtain the expression:

$$\frac{\partial \mathrm{M}([A])}{\partial [A]^*} = [S_{RT}]^H [W]^2 [S_{RT}][A] - [S_{RT}]^H [W]^2 [B] + \mu [A] \tag{23}$$

As a minimum condition, we write:

$$[S_{RT}]^H [W]^2 [S_{RT}][A] - [S_{RT}]^H [W]^2 [B] + \mu [A] = 0 \tag{24}$$

Then:

$$[S_{RT}]^H [W]^2 [S_{RT}][A] + \mu [A] = [S_{RT}]^H [W]^2 [B] \tag{25}$$

Let's transform (25):

$$\left([S_{RT}]^H [W]^2 [S_{RT}] + \mu [I]\right)[A] = [S_{RT}]^H [W]^2 [B] \tag{26}$$

where $[I]$ is the identity matrix.

Thus, by solving system (26), a column vector of radiated signals $[A]$ is determined to ensure the required distribution of the electric field in a given region of space.

From (26), we can express $[A]$:

$$[A] = \left([S_{RT}]^H [W]^2 [S_{RT}] + \mu [I]\right)^{-1} [S_{RT}]^H [W]^2 [B] \tag{27}$$

In addition, the feasibility of using sparse sets of control points is consistent with recent results on reduced near-field data acquisition. In [18], the authors studied the diagnosis of realistic arrays from a reduced number of phaseless near-field measurements and showed that reliable reconstruction can still be achieved when sparsity is properly exploited through compressed-sensing-based formulations. Although that study addresses array diagnosis rather than field synthesis, it supports the broader idea that sparsity-aware formulations and carefully selected spatial samples may still provide sufficient information for solving electromagnetic inverse and design problems. Therefore, these results further motivate the extension of the proposed approach to sparse array configurations and to sparse sets of minimum-field locations.

Let us test the obtained mathematical apparatus using mathematical modeling.

3. MATHEMATICAL SIMULATION

Consider an antenna system comprising 28 radiators located around the perimeter of a space. Seven antennas are located on each side of the space. This system is designed to operate with nine receiving points. The operating frequency is 2.5 GHz, and the distance between the radiators is half a wavelength, $\lambda/2$. The radiation pattern of a single radiator is assumed to be isotropic.

Figure 4 shows the required configurations of the electric field in space.

Table 1 presents the coordinates of the radiators.

TABLE 1. Coordinates of radiators.

No.	X, m	Y, m	Z, m
1	−0.1800	−0.2100	0
2	−0.1200	−0.2100	0
3	−0.0600	−0.2100	0
4	0	−0.2100	0
5	0.0600	−0.2100	0
6	0.1200	−0.2100	0
7	0.1800	−0.2100	0
8	0.2100	−0.1800	0
9	0.2100	−0.1200	0
10	0.2100	−0.0600	0
11	0.2100	0	0
12	0.2100	0.0600	0
13	0.2100	0.1200	0
14	0.2100	0.1800	0
15	0.1800	0.2100	0
16	0.1200	0.2100	0
17	0.0600	0.2100	0
18	0	0.2100	0
19	−0.0600	0.2100	0
20	−0.1200	0.2100	0
21	−0.1800	0.2100	0
22	−0.2100	0.1800	0
23	−0.2100	0.1200	0
24	−0.2100	0.0600	0
25	−0.2100	0	0
26	−0.2100	−0.0600	0
27	−0.2100	−0.1200	0
28	−0.2100	−0.1800	0

Table 2 lists the amplitudes and initial phases of the radiated signals, obtained using expression (27), to form the required distribution of the electric field in space.

Figure 5 shows the results of the mathematical modeling of the distribution of the electric field in space.

Table 3 presents the values of the electric field strength at the receiving points.

As can be seen from the results shown in Figure 5 and Table 3, the proposed method for determining the amplitudes and initial phases of signals allows for the minimum amplitude of the electric field strength to be ensured at several required points in space, and also for the difference between the minimum and maximum points to be 5.41 dB in the worst case and 17.71 dB in the best case, which indicates the potential possibility of applying the proposed method in practice.

The difference in the normalized electric field amplitude values between the target values and the simulation results is also noteworthy. It's worth noting that the difference in the target values was 6 dB at best and 9.3 dB at worst. This difference is due to the generated electric field distribution and the fact that, for a given configuration of transmitters and receiving points,

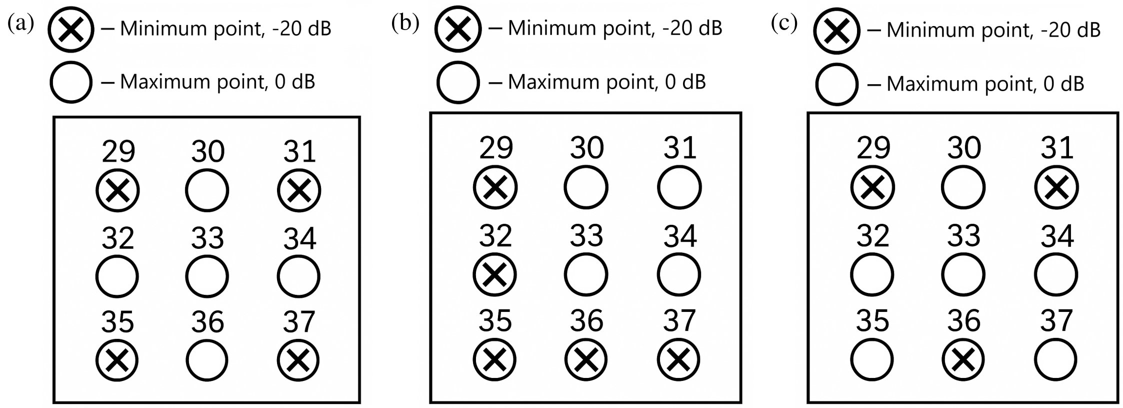

FIGURE 4. Required configurations of the electric field in space. (a) Minima at the corners of space, (b) anti-letter "L", (c) anti-letter "Y".

TABLE 2. Amplitudes and initial phases of radiated signals.

Antenna No.	Minima in the corners of space		Anti-letter "L"		Anti-letter "Y"	
	Amplitude, V/m	Phase, deg	Amplitude, V/m	Phase, deg	Amplitude, V/m	Phase, deg
1	6.5613	179.2672	6.6566	−166.4933	10.031	−142.3760
2	5.7318	−65.5734	4.8291	1.9463	3.6435	−42.1162
3	14.882	−23.9547	7.0462	7.5275	2.8778	−14.3132
4	16.138	−61.4117	8.4541	−39.8983	8.5155	−55.4727
5	14.882	−23.9548	6.6861	−13.7470	2.8778	−14.3134
6	5.7318	−65.5735	2.5934	−51.2732	3.6435	−42.1163
7	6.5613	179.2672	8.2750	−148.8535	10.031	−142.3760
8	6.5613	179.2673	5.7930	−154.6830	7.7753	−136.1498
9	5.7317	−65.5731	5.6489	−2.9597	7.2216	−19.9073
10	14.882	−23.9548	8.2436	−8.8473	4.2483	−5.5792
11	16.138	−61.4117	11.616	−43.7278	11.119	−49.4333
12	14.882	−23.9548	5.3178	−30.2047	6.8516	−29.6463
13	5.7318	−65.5733	5.5597	−47.5809	3.4566	−69.9606
14	6.5613	179.2673	11.147	−134.1174	9.1233	−139.6095
15	6.5613	179.2672	11.147	−134.1174	7.7455	−141.3899
16	5.7318	−65.5733	5.5596	−47.5810	3.6085	−42.3863
17	14.882	−23.9548	5.3178	−30.2048	4.9516	−28.1213
18	16.138	−61.4117	11.616	−43.7279	13.037	−45.9550
19	14.882	−23.9548	8.2436	−8.8472	4.9516	−28.1214
20	5.7318	−65.5733	5.6489	−2.9596	3.6085	−42.3863
21	6.5613	179.2673	5.7930	−154.6828	7.7455	−141.3900
22	6.5613	179.2672	8.2750	−148.8537	9.1233	−139.6095
23	5.7317	−65.5734	2.5933	−51.2730	3.4566	−69.9608
24	14.882	−23.9548	6.6861	−13.7471	6.8515	−29.6464
25	16.138	−61.4117	8.4542	−39.8981	11.119	−49.4333
26	14.882	−23.9547	7.0462	7.5277	4.2483	−5.5794
27	5.7318	−65.5732	4.8291	1.9465	7.2216	−19.9072
28	6.5613	179.2673	6.6565	−166.4931	7.7753	−136.1499

the task of generating the required electric field distribution in space inevitably involves a compromise. Some requirements are mutually limiting; therefore, the achievable solution is determined by the minimum possible value of the squared error between the target and resulting field distributions in space. Consequently, the calculated complex amplitudes of the radiated signals provide the best approximation to the target electric field amplitude values at the receiving points.

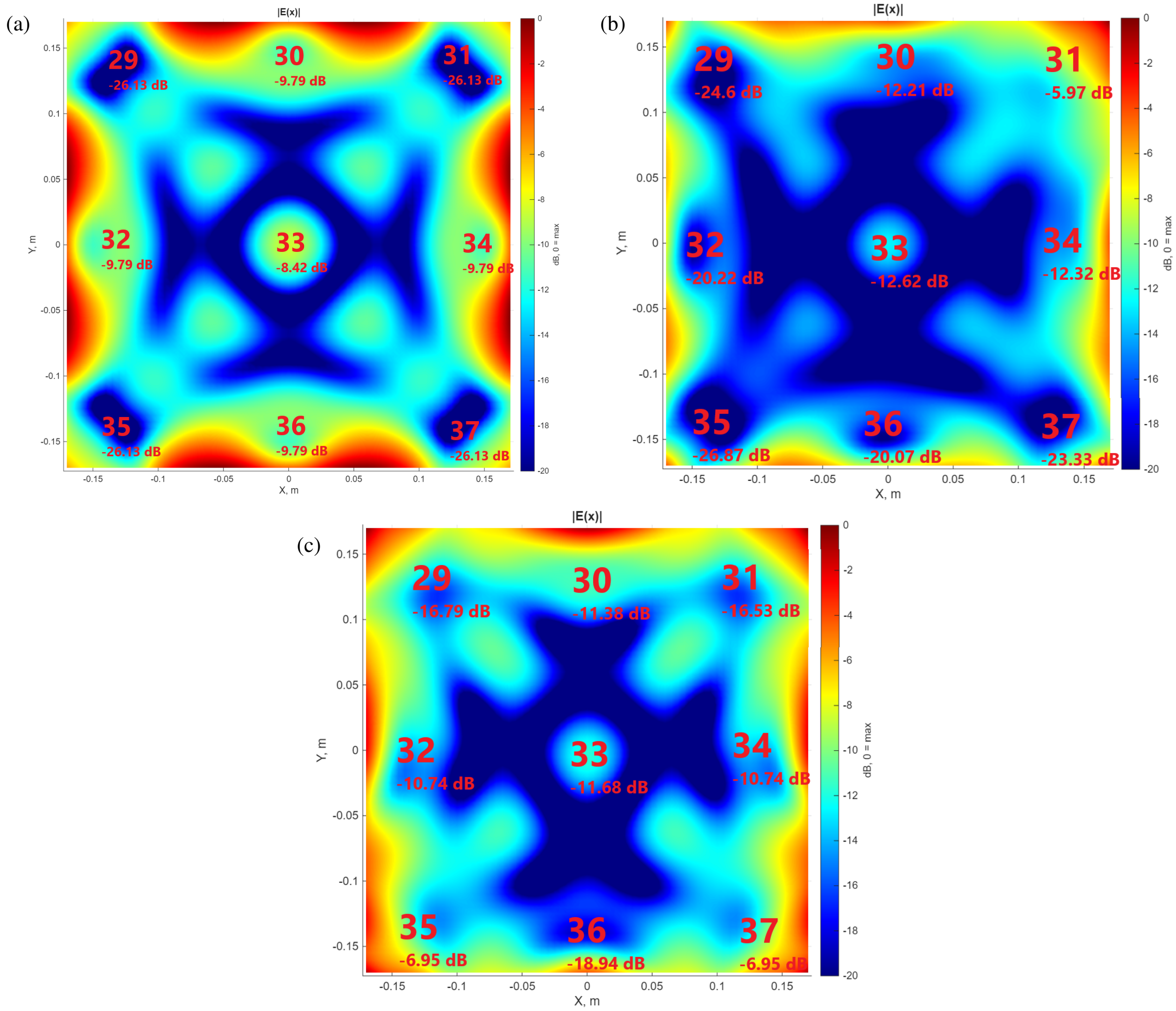

FIGURE 5. Electric field distributions. (a) Minima in the corners of space, (b) anti-letter 'L", (c) anti-letter "Y".

TABLE 3. Electric field strength values at receiving points.

Receiving point No.	Minima in the corners of space, dB	Anti-letter "L", dB	Anti-letter "Y", dB
29	**−26.13 (Minima point)**	**−24.6 (Minima point)**	**−16.79 (Minima point)**
30	−9.79	−12.21	−11.38
31	**−26.13 (Minima point)**	−5.97	**−16.53 (Minima point)**
32	−9.79	**−20.22 (Minima point)**	−10.74
33	−8.42	−12.62	−11.68
34	−9.79	−12.32	−10.74
35	**−26.13 (Minima point)**	**−26.87 (Minima point)**	−6.95
36	−9.79	**−20.07 (Minima point)**	**−18.94 (Minima point)**
37	**−26.13 (Minima point)**	**−23.33 (Minima point)**	−6.95

It is worth noting that no universal upper bound on N exists for the proposed method. In principle, the method remains applicable to arbitrary N, since it is based on solving a regularized linear system. The actual limitation is practical rather than theoretical and is determined by the conditioning of the system matrix, the available computational resources, and the number of effective spatial degrees of freedom relative to the imposed field constraints.

4. CONCLUSION

This study proposes a method for determining the complex amplitudes of radiated signals that can ensure several minima of the electric field strength amplitude in given spatial regions. The relationship between the complex amplitudes of radiated signals and the complex amplitudes at receiving points is described by S-parameter matrix. This study proposes calculating the complex amplitudes of radiated signals by solving a linear equation (SLAE). The resulting SLAE can be an underdetermined system, which can lead to an infinite number of solutions or instability. Therefore, the problem was reduced to minimizing the squared error between the required and obtained spatial distributions of the electric field. Tikhonov regularization was also introduced into the system to ensure the uniqueness of the solution to the SLAE. Solving the optimization problem with the introduced squared error measure yields a solution to the new SLAE.

Three electric field configurations were proposed, and the signal amplitudes and initial phases were calculated for each configuration. Mathematical modeling of the electric field distribution using the obtained signal amplitudes and initial phases demonstrated the viability of the proposed method.

REFERENCES

[1] Voudoukis, N., "Performance analysis, characteristics, and simulation of digital QAM," *European Journal of Electrical Engineering and Computer Science*, Vol. 1, No. 1, 2017.

[2] Svensson, A., "An introduction to adaptive QAM modulation schemes for known and predicted channels," *Proceedings of the IEEE*, Vol. 95, No. 12, 2322–2336, 2007.

[3] Hansen, R., "Focal region characteristics of focused array antennas," *IEEE Transactions on Antennas and Propagation*, Vol. 33, No. 12, 1328–1337, 1985.

[4] Iuzvik, D. and M. Stepanov, "Formation of the maximum amplitude of the electric field strength using two one-dimensional linear antenna arrays located orthogonal to each other," *Journal of Electromagnetic Waves and Applications*, Vol. 38, No. 18, 2106–2123, 2024.

[5] Iuzvik, D. and M. Stepanov, "Ensuring the maximum amplitude of the electric field strength in given coordinates using a linear antenna array focused on a finite distance," *Journal of Electromagnetic Waves and Applications*, Vol. 38, No. 4, 411–427, 2024.

[6] Schelkunoff, S. A., "A mathematical theory of linear arrays," *The Bell System Technical Journal*, Vol. 22, No. 1, 80–107, 1943.

[7] Hansen, R. C., *Phased Array Antennas*, John Wiley & Sons, 2009.

[8] Capon, J., "High-resolution frequency-wavenumber spectrum analysis," *Proceedings of the IEEE*, Vol. 57, No. 8, 1408–1418, 1969.

[9] Frost, O. L., "An algorithm for linearly constrained adaptive array processing," *Proceedings of the IEEE*, Vol. 60, No. 8, 926–935, 1972.

[10] Carlson, B. D. and D. Willner, "Antenna pattern synthesis using weighted least squares," *IEE Proceedings H (Microwaves, Antennas and Propagation)*, Vol. 139, No. 1, 11–16, 1992.

[11] Lebret, H. and S. Boyd, "Antenna array pattern synthesis via convex optimization," *IEEE Transactions on Signal Processing*, Vol. 45, No. 3, 526–532, 1997.

[12] Pino, M. R., R. G. Ayestarán, P. Nepa, and G. Manara, "An overview on synthesis techniques for near-field focused antennas," *Recent Wireless Power Transfer Technologies*, 2019.

[13] Ni, H., M. Anjum, D. Mishra, and A. Seneviratne, "Energy-efficient near-field beamforming: A review on practical channel models," *Energies*, Vol. 18, No. 11, 2966, 2025.

[14] Nepa, P. and A. Buffi, "Near-field-focused microwave antennas: Near-field shaping and implementation," *IEEE Antennas and Propagation Magazine*, Vol. 59, No. 3, 42–53, 2017.

[15] Chou, H.-T., N.-N. Wang, H.-H. Chou, and J.-H. Qiu, "An effective synthesis of planar array antennas for producing near-field contoured patterns," *IEEE Transactions on Antennas and Propagation*, Vol. 59, No. 9, 3224–3233, 2011.

[16] Buttazzoni, G., F. Babich, S. Pastore, F. Vatta, and M. Comisso, "Gaussian approach for the synthesis of periodic and aperiodic antenna arrays: Method review and design guidelines," *Sensors*, Vol. 21, No. 7, 2343, 2021.

[17] Yang, S. J. and Y. D. Kim, "An accurate near-field focusing of array antenna based on near-field active element pattern and infinitesimal dipole modeling," *IEEE Access*, Vol. 9, 143 771–143 781, 2021.

[18] Palmeri, R., G. M. Battaglia, A. F. Morabito, S. Costanzo, F. Venneri, and T. Isernia, "Fault diagnosis of realistic arrays from a reduced number of phaseless near-field measurements," *IEEE Transactions on Antennas and Propagation*, Vol. 71, No. 9, 7206–7219, 2023.

[19] Iuzvik, D. and M. Stepanov, "Focusing of the electromagnetic field in several given areas of space," *Progress In Electromagnetics Research M*, Vol. 113, 11–22, 2022.

[20] Skolnik, M. I., *Radar Handbook*, 3rd ed., McGraw-Hill, New York, NY, USA, 2012.

[21] Strang, G., *Linear Algebra and Its Applications*, 4th ed., Cengage Learning, 2006.

[22] Gantmakher, F. R., *The Theory of Matrices*, LitRes, Moscow, Russia, 2016 (in Russian).

[23] Ben-Israel, A. and T. N. E. Greville, *Generalized Inverses: Theory and Applications*, Springer, 2003.

[24] Tikhonov, A. N. and V. Y. Arsenin, *Solutions of Ill-posed Problems*, V. H. Winston/John Wiley & Sons, 1977.

[25] Brandwood, D. H., "A complex gradient operator and its application in adaptive array theory," *IEE Proceedings F (Communications, Radar and Signal Processing)*, Vol. 130, No. 1, 11–16, 1983.