

An Abbe-Hopkins Unified Formulation of Optical Imaging for Efficient Cross-Model Verification in Computational Lithography

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ABSTRACT: Accurate simulation of partially coherent imaging is crucial for computational lithography, with Abbe and Hopkins as the two main formulations being used. Although the two methods are equivalent in theory, practical simulators making independent choices between Abbe and Hopkins could hardly produce consistent results that match the desired accuracy owing to the inherently different ways of numerically representing, discretizing, and truncating the illumination source and lens pupil function, etc. Moreover, classical Hopkins models require prior construction and/or eigen decomposition of the high-dimensional transmission cross coefficient (TCC), the prohibitive costs of which hinder timely model verification. To address these challenges, we developed a unified Abbe-Hopkins formulation in conjunction with a TCC-free Hopkins pointwise sampler for efficient cross-model validation. Our formulation supports both Abbe and Hopkins modeling in a single unified framework, with the two simulation modes using exactly the same numerical representations of the illumination source and projection lens. Cross-model verification for randomly sampled points is performed efficiently by evaluating the Hopkins quadratic form through a fast Fourier transform of an image and a few pointwise multiplications between images, without ever explicitly constructing a TCC and eigen-analyzing it. Numerical tests show that the Abbe and Hopkins results agree up to the machine precision level.

1. INTRODUCTION

As integrated circuit (IC) manufacturing advances toward the 5 nm technology node and beyond, optical lithography is pushed increasingly deeper below the diffraction limit, and the available process window continues to shrink [1–3]. Under these conditions, computational lithography (CL) has become essential for large-scale IC production, supporting process optimization, yield enhancement, and cost reduction [4–6]. Important CL applications, including optical proximity correction (OPC), source-mask optimization (SMO), process-window enhancement, and in-line process monitoring, critically rely on accurate and efficient simulations of partially coherent optical imaging. More broadly, these simulation and verification tasks are closely related to wave-based computational imaging, where reliable forward models are essential for inverse source recovery, inverse scattering, and super-resolution analysis [7–10].

From a physical standpoint, as illustrated in Fig. 1(a), a projection lithography system comprises a light source and condenser lens to produce a partially coherent light field, a photomask to spatially modulate the light field, and a projection lens to transfer the modulated light onto a wafer, forming an optical image whose intensity governs pattern generation [11, 12].

The theoretical description of partially coherent imaging is fundamentally based on the Abbe formulation and the equivalent Hopkins model [13–16], as a special case of the all-encompassing second-order coherence theory [17, 18].

Over the years, a number of alternative formulations have been proposed and investigated [19–22], but none seem able to surpass either Abbe in terms of model simplicity and physical intuitiveness or Hopkins in terms of computational efficiency. As such, Abbe and Hopkins remain the predominant formulations for high-performance industrial simulations, although each method has its own challenges.

The Abbe formulation adopts the Köhler model for illumination and treats an extended source as a collection of mutually incoherent point emitters, computes the coherent field amplitude on the image plane for each source point, and sums all the contributions to obtain the final intensity field. Near the numerical-aperture cutoff, even a small shift in the source position could lead to a pronounced change in the mask-pupil spectrum overlap and induce a significant image variation. Therefore, it requires exceedingly fine sampling over the source-point coordinate space, which makes full-chip Abbe imaging prohibitively expensive.

The Hopkins formulation exchanges the order of integration and lumps the illumination source and projection lens into a transmission cross-coefficient (TCC) [14–16]. The method enables highly efficient numerical simulations by decomposing

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the TCC into eigenmodes, called Hopkins kernels, and using a small number of Hopkins kernels to approximate the system [23, 24]. However, handling high-dimensional TCC traditionally incurs steep costs of computer memory and runtime, which rise sharply when the number of sampling points needs to increase in the space or frequency domain, or the dimension of the optical fields must be enlarged to capture the vectorial nature of light and cope with the illumination angle-dependent response of a thick photomask. Recently, a collaboration led by one of the present authors employed the Lanczos algorithm [25, 26] to extract the leading eigenmodes of a vectorial and thick-mask imaging system without explicitly forming a high-dimensional TCC [27]. However, the Lanczos process entails a heavy computational burden when a large number of Hopkins kernels is required for high accuracy or when the process needs to be repeated frequently in certain applications.

Although, in theory, the Abbe and Hopkins models are equivalent, in practice, they are mostly implemented mutually exclusively among industrial simulators, adopting inherently different numerical strategies to represent, discretize, and truncate the illumination source and lens pupil function, etc. This causes systematic discrepancies among software tools, especially between Abbe- and Hopkins-based simulators, which are difficult to debug and are hardly diminished by sampling more densely or keeping more kernels. Indeed, the CL industry has KLA's PROLITH, Synopsys' S-Litho, and Panoramic's HyperLith using the Abbe formulation, and all OPC tools, including Synopsys' Proteus, Siemens' Calibre, and ASML's Tachyon based on the Hopkins method. It has been a long-standing challenge in the CL industry to have different lithography simulators producing reasonably consistent results, for example, with a relative error down to the 10^{-3} level.

At the same time, there is an ever-growing demand for Abbe-Hopkins cross-model validation, as an indispensable means for verifying model formulation, algorithm design, and code implementation, as well as quantifying series truncation errors, while the industry is developing Hopkins-styled models of increasing complexity, to deal with the so-called Mask3D effects and their dependence on off-axis illumination [28–30]. From a broader perspective, the importance of reliable forward modeling is also emphasized in wave-imaging and inverse-scattering studies, where model accuracy directly affects inverse reconstruction, source recovery, and super-resolution interpretation [7–10]. A lack of cross-model validation capability hinders the development of advanced CL, as it can be difficult to determine the root cause of an incorrect or inaccurate simulation, whether it is due to an inadequacy in model formulation, an error in algorithm design, or a bug in code implementation. Furthermore, in applications such as SMO, which require fast TCC decomposition and Hopkins simulation, while the optical system is frequently modified and updated, it is advantageous to have a Hopkins model strictly equivalent to an Abbe model and constantly validated by the latter.

In this study, we present a unified Abbe-Hopkins numerical framework, with the Abbe and Hopkins models derived from the same fundamental equation and implemented using the same numerical representations for all physical quantities

involved, thereby establishing a strict equivalence between the two formulations both physically and computationally. To facilitate efficient Abbe-Hopkins cross-model validation, we further develop a TCC-free Hopkins pointwise sampler (TF-Hopkins-PS), which computes the optical intensity at an arbitrary location on the image plane without constructing or eigen-decomposing a full TCC, but using efficient pointwise multiplication between images and fast Fourier transform (FFT) of images, while producing results in agreement with Abbe up to the level of machine precision. The unification of Abbe and Hopkins modeling and the invention of TF-Hopkins-PS enable a fast workflow of model validation by sampling image points on demand, in which the model formulation and numerical accuracy of Hopkins imaging can be verified quickly via focused inspection at critical layout features to provide a reliable basis for subsequent TCC construction, model decomposition, and heavy-duty Hopkins simulations.

2. UNIFIED ABBE-HOPKINS IMAGING THEORY

The following mathematical derivation is presented in terms of scalar waves to simplify the presentation. The same formulation can be extended to a full-vector theory without much difficulty, provided that care is taken to avoid changing the left-right order of physical quantities unwittingly, and to enforce a change of order intentionally via matrix vectorization.

2.1. Unified Abbe-Hopkins Framework

Abbe and Hopkins imaging are often presented separately as alternative formulations for partially coherent imaging. In this study, they are rigorously formulated as two equivalent representations of the same partially coherent imaging process. The purpose of the unified framework is to establish physical and numerical consistency simultaneously between the Abbe and Hopkins methods, ensuring their strict equivalence from theory to computation.

We began with a discrete representation of an extended illumination source. In the frequency domain, the illumination is decomposed into a finite set of point sources indexed by $\sigma = (\sigma_x, \sigma_y)$, each associated with an intensity $I_0(\sigma)$ and generating a plane wave illuminating the photomask. This source representation is fixed *a priori* and is used identically for Abbe and Hopkins.

The projection lens, as a linear shift-invariant system, is equivalently described by a point spread function (PSF) $P(\cdot)$ in space or its Fourier transform, called the pupil function $\tilde{P} \triangleq \mathcal{F}\{P\}$, where $\mathcal{F}$ denotes the Fourier transform. Traditionally, Abbe imaging is performed by filtering the photomask spectrum with a pupil function of finite support, whereas Hopkins modeling tends to use a PSF with a limited spatial extent, called the optical ambit or optical radius. To unify the two models in the same framework, either Abbe must admit a pupil function of enlarged support, or Hopkins needs to take a PSF of a larger radius. Ideally, the projection lens is best modeled as a linear system subject to domain limiting in both space and spatial frequency, as a straightforward generalization of the well-

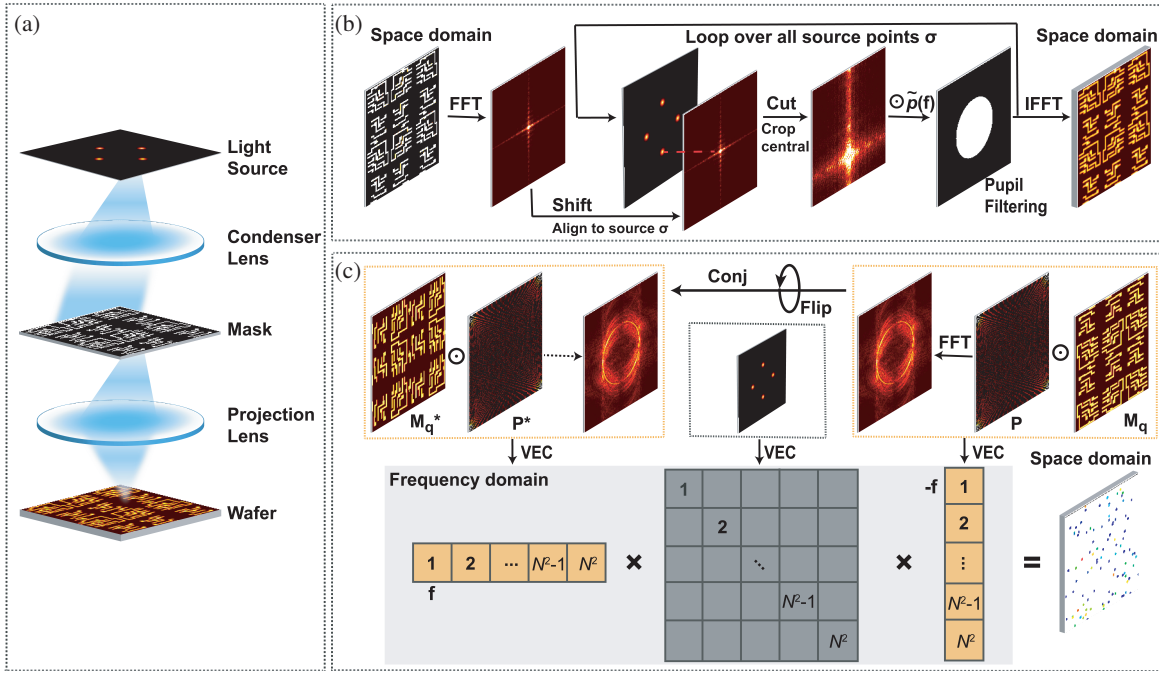


FIGURE 1. Partially coherent imaging system and comparison of Abbe and TF-Hopkins-PS workflows. (a) Schematic of a partially coherent imaging system. (b) Abbe imaging workflow based on frequency shifting, pupil filtering, and inverse Fourier transform. (c) Proposed TF-Hopkins-PS based on pointwise multiplications and the Fourier transform without explicit TCC construction.

known Slepian-Landau-Widom model of time-frequency channels [31–33].

In the Abbe formulation, each point source at a spatial frequency σ generates a coherent plane wave $\sqrt{I_0(\sigma)}e^{i\sigma \cdot \mathbf{r}}$ that illuminates the photomask. According to the theory of Fourier optics [34], the spectrum of the mask-modulated field $M^\sigma(\mathbf{r}) \triangleq e^{i\sigma \cdot \mathbf{r}}M(\mathbf{r})$ is filtered by the pupil function $\tilde{P}$ and produces an image field $E^\sigma(\mathbf{q})$ on the wafer,

$$E^\sigma(\mathbf{q}) = \sqrt{I_0(\sigma)}\mathcal{F}^{-1}[\mathcal{F}\{P(\mathbf{r})\} \times \mathcal{F}\{M^\sigma(\mathbf{r})\}], \quad (1)$$

which is equivalent to a convolution in the spatial domain,

$$E^\sigma(\mathbf{q}) = \sqrt{I_0(\sigma)} \int_{\mathbf{r}_1} M^\sigma(\mathbf{q} - \mathbf{r}_1)P(\mathbf{r}_1)d\mathbf{r}_1, \quad (2)$$

where $\mathbf{r} = (x, y)$ and $\mathbf{q} = (x', y')$ denote the mask (object) and wafer (image) plane coordinates, respectively, and $M(\mathbf{r})$ is a mask transmittance function representing the spatial light modulation by the photomask. The partially coherent image intensity is then obtained by incoherently summing the coherent intensities over all the illumination components:

$$I(\mathbf{q}) = \sum_{\sigma} |E^\sigma(\mathbf{q})|^2. \quad (3)$$

The numerical implementation of this Abbe formulation follows the workflow shown in Fig. 1(b): for each source point, the mask spectrum is shifted, aligned to the common frequency grid, filtered by the pupil function, and transformed back to the spatial domain by an inverse Fourier transform. By expanding

Eq. (3) and interchanging the summation over σ with the spatial integrations, we obtain the Hopkins formula [14, 15] in the form of a spatial “quadratic convolution,”

$$I(\mathbf{q}) = \int_{\mathbf{r}_1} \int_{\mathbf{r}_2} M^*(\mathbf{q} - \mathbf{r}_2)P^*(\mathbf{r}_2) \times J(\mathbf{r}_1, \mathbf{r}_2)P(\mathbf{r}_1)M(\mathbf{q} - \mathbf{r}_1)d\mathbf{r}_1d\mathbf{r}_2, \quad (4)$$

with $J(\mathbf{r}_1, \mathbf{r}_2)$ being the mutual intensity [15, 17] immediately before the mask plane.

$$J(\mathbf{r}_1, \mathbf{r}_2) \triangleq \langle E^*(\mathbf{r}_1)E(\mathbf{r}_2) \rangle = \sum_{\sigma} I_0(\sigma)e^{-i\sigma \cdot \mathbf{r}_1 + i\sigma \cdot \mathbf{r}_2}, \quad (5)$$

where $\mathbf{r}_1$ and $\mathbf{r}_2$ are two spatial coordinates on the mask plane; $\langle \cdot \rangle$ denotes the statistical averaging over the source distribution; and $(\cdot)^*$ represents complex conjugation (or Hermitian conjugation for vector fields). Numerically, the mutual intensity is sampled at the same discrete source points used in the Abbe formulation.

Conventional Hopkins methods typically proceed by explicitly constructing the TCC

$$\text{TCC}(\mathbf{r}_1, \mathbf{r}_2) \triangleq P^*(\mathbf{r}_2)J(\mathbf{r}_1, \mathbf{r}_2)P(\mathbf{r}_1), \quad (6)$$

and discretizing it as a large and unwieldy $N^2 \times N^2$ TCC matrix, with N being the sampling size per spatial dimension and N^2 being the number of samples for each two-dimensional spatial coordinate $\mathbf{r}_1$ or $\mathbf{r}_2$. Although it is formally exact, this procedure entails substantial memory and computational overhead, particularly for large-scale simulations [27]. Moreover, ensuring strict numerical consistency with Abbe imaging often requires expanding the TCC into a large number (on the order

of thousands or more) of Hopkins kernels, whose generation, storage, and subsequent use in simulations further increase the computational burden.

The following section shows that, based on the unified Abbe-Hopkins framework, the Hopkins image intensity at any location can be evaluated directly by a fast algorithm without the explicit construction of the cumbersome TCC. In practice, a chosen set of representative image points may be prescribed, and the image intensities from the Abbe and Hopkins models are compared at the prescribed image points. The agreement between the results then provides a rapid validation of the Hopkins formulation, enabling potential errors to be uncovered and fixed before launching the computation-intensive tasks of the Hopkins model decomposition and full-chip simulations.

2.2. TCC-Free Hopkins Pointwise Sampler

For any fixed image point $\mathbf{q}$, each spatial convolution in Eq. (4) can be evaluated by taking the zero-frequency component of the FFT of pointwise multiplication [34]:

$$\int_{\mathbf{r}_1} P(\mathbf{r}_1) M(\mathbf{q} - \mathbf{r}_1) d\mathbf{r}_1 = \mathcal{F}\{P \odot M_{\mathbf{q}}\}(0, 0), \quad (7)$$

$$\int_{\mathbf{r}_2} M^*(\mathbf{q} - \mathbf{r}_2) P^*(\mathbf{r}_2) d\mathbf{r}_2 = \mathcal{F}\{M_{\mathbf{q}}^* \odot P^*\}(0, 0), \quad (8)$$

where $M_{\mathbf{q}}(\mathbf{r}) \triangleq M(\mathbf{q} - \mathbf{r})$, $\forall \mathbf{q} \in \mathbb{R}^2$, $\forall \mathbf{r} \in \mathbb{R}^2$, $\odot$ denotes the pointwise multiplication between two multi-component quantities, such as tuples, vectors, or matrices, and $\mathcal{F}\{\cdot\}$ represents the corresponding Fourier transform.

To perform computations in the frequency domain, the mutual intensity function is best represented in the frequency space as

$$\mathcal{F}\left\{\sum_{\boldsymbol{\sigma}} I_0(\boldsymbol{\sigma}) e^{-i\boldsymbol{\sigma} \cdot \mathbf{r}_1 + i\boldsymbol{\sigma} \cdot \mathbf{r}_2}\right\} = \sum_{\boldsymbol{\sigma}} I_0(\boldsymbol{\sigma}) \delta(\mathbf{f} + \boldsymbol{\sigma}) \delta(\mathbf{g} - \boldsymbol{\sigma}), \quad (9)$$

where $\mathbf{f}$ and $\mathbf{g}$ are the spatial frequency variables conjugate to $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively. In the discrete setting, $\mathcal{F}\{P \odot M_{\mathbf{q}}\}$ is vectorized by column-wise stacking, such that

$$\mathcal{F}\{P \odot M_{\mathbf{q}}\} \triangleq [A_1(\mathbf{f}_1) A_1(\mathbf{f}_2) \dots A_1(\mathbf{f}_{N^2})]^T, \quad (10)$$

where the two-dimensional $N \times N$ frequency grid is flattened into an N^2 -dimensional vector using a fixed column-wise indexing rule. Specifically, for $\mathbf{f}_j = (f_x[p], f_y[\ell])$, the vector index is defined as $j = p + (\ell - 1)N$, with $p, \ell = 1, \dots, N$. This same ordering is used for the source weights, pupil samples, shifted mask spectrum, and Hopkins quadratic form, so that each vector entry corresponds to a unique frequency sample on the shared grid. With $(\cdot)^T$ denoting vector transposition, its Hermitian conjugate reads

$$\begin{aligned} \mathcal{F}\{M_{\mathbf{q}}^* \odot P^*\} &\triangleq [A_2(\mathbf{g}_1) A_2(\mathbf{g}_2) \dots A_2(\mathbf{g}_{N^2})] \\ &= [A_1^*(-\mathbf{g}_1) A_1^*(-\mathbf{g}_2) \dots A_1^*(-\mathbf{g}_{N^2})], \end{aligned} \quad (11)$$

where $A_1(\mathbf{f}_j)$ and $A_2(\mathbf{g}_j)$ denote complex amplitudes at the j -th frequency coordinate. The same column-wise vectorization

is used in Eqs. (10) and (11), so each frequency sample is paired with its corresponding source weight in Eq. (12). With this representation, the mutual intensity becomes diagonal in the frequency domain, and the Hopkins image intensity at a single image point $\mathbf{q}$ can be written as

$$I(\mathbf{q}) = [A_2(\mathbf{f}_1), A_2(\mathbf{f}_2), \dots, A_2(\mathbf{f}_{N^2})] \begin{bmatrix} I_0(\mathbf{f}_1) & & 0 \\ & I_0(\mathbf{f}_2) & \ddots \\ & & \ddots & 0 \\ 0 & \dots & 0 & I_0(\mathbf{f}_{N^2}) \end{bmatrix} \begin{bmatrix} A_1(-\mathbf{f}_1) \\ A_1(-\mathbf{f}_2) \\ \vdots \\ A_1(-\mathbf{f}_{N^2}) \end{bmatrix}, \quad (12)$$

This immediately gives rise to a highly efficient TF-Hopkins-PS. As illustrated in Fig. 1(c), for any fixed image point $\mathbf{q}$, we compute the spectrum of $P \odot M_{\mathbf{q}}$ using a single FFT, prepare its complex conjugate counterpart, and then compute the perfect-squared quadratic form of Eq. (12) using only pointwise multiplications. Such a TF-Hopkins-PS provides a means for sampling Hopkins image intensities efficiently without constructing a high-dimensional TCC or eigen-analyzing it. Interestingly, TF-Hopkins-PS is amenable to a PSF $P(\cdot)$ of unlimited support, which is another advantage of its TCC-freeness.

3. NUMERICAL TESTS

3.1. Simulation Setup and Data Preparation

For a unified platform supporting both full-field Abbe imaging and the proposed TF-Hopkins-PS, the source, photomask, and pupil were all discretized on the same $N \times N$ grid with $N = 401$, and shared a common coordinate system in the frequency domain. Our test uses a scalar, thin-mask, quasi-monochromatic extreme ultraviolet (EUV) lithography setting with wavelength $\lambda = 13.5$ nm and numerical aperture $\text{NA} = 0.33$, corresponding to a diffraction-limited imaging resolution $\lambda/(2 \times \text{NA}) \approx 20.45$ nm. The imaging configuration employs the standard Köhler illumination, in which different source points are assumed to be spatially mutually incoherent, whereas the light field emitted from any fixed source point produces a temporally coherent plane wave. The key modeling assumptions and the numerical elements kept identical in the Abbe and TF-Hopkins-PS modes are summarized in Table 1. These settings are fixed before the simulation and are shared by the two formulations.

The illumination source had a quadrupole configuration [35], as shown in Fig. 2(a). In this configuration, the source was described by its angular intensity distribution mapped to the pupil coordinates. Let $f_{\text{cut}} = \text{NA}/\lambda$ denote the spatial cut-off frequency and define the normalized coordinates $(u, v) = (f_x/f_{\text{cut}}, f_y/f_{\text{cut}})$. In numerical simulations over a finite window $L_x \times L_y$ (here $L_x = L_y = 6 \mu\text{m}$), the corresponding normalized spatial frequencies are

$$\begin{aligned} u[k] &= \frac{k - (N + 1)/2}{f_{\text{cut}} L_x}, \\ v[\ell] &= \frac{\ell - (N + 1)/2}{f_{\text{cut}} L_y}, \end{aligned} \quad k, \ell = 1, \dots, N. \quad (13)$$

TABLE 1. Shared assumptions and numerical settings.

Item	Shared assumption/setting
Imaging model	Scalar EUV projection imaging; Köhler illumination; $\lambda = 13.5$ nm; NA = 0.33.
Coherence assumption	Mutually incoherent across source points; coherent per source point.
Mask model	Infinitely thin scalar transmission mask; no thick-mask or Mask3D effects.
Source discretization	Same quadrupole source, weights, support, and $N \times N$ spectral grid.
Pupil sampling	Same circular pupil, cutoff frequency, sampling coordinates, and Fourier-transform convention.
Mask and window	Same $L_x \times L_y$ window, GDS rasterization, supersampling, box filtering, and spectrum cropping.
Indexing convention	Same object/image coordinates, frequency grid, and column-wise vectorization order.

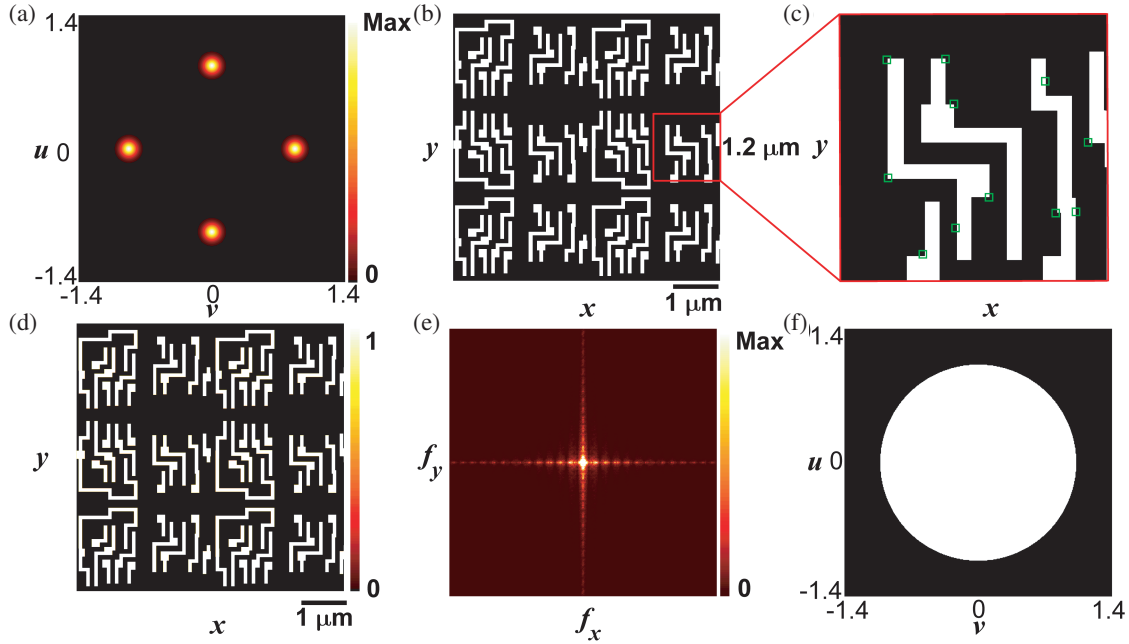


FIGURE 2. Source configuration, photomask construction, and pupil modeling used in the numerical simulations. (a) Quadrupole illumination source distribution. (b) Supersampled binary photomask rasterized from a GDS layout within an $L_x \times L_y$ window. (c) Enlarged view of the region in (b), and green boxes mark the sampling pixels. (d) Working-resolution photomask obtained by 2D box filtering. (e) Amplitude spectrum of the photomask after the Fourier transform. (f) Circular pupil function.

The quadrupole source was implemented on the (u, v) spectral grid as four Gaussian lobes centered at $(c_{x,m}, c_{y,m}) \in \{(\pm\rho_0, 0), (0, \pm\rho_0)\}$ with $\rho_0 = 0.85$,

$$\tilde{S}(\mathbf{f}) \triangleq \tilde{S}[k, \ell] = \text{const} \times \sum_{m=1}^4 \exp\left(-\frac{(u[k] - c_{x,m})^2 + (v[\ell] - c_{y,m})^2}{2f_\sigma^2}\right), \quad (14)$$

with $f_\sigma = 0.05$. The source weights are clipped to zero outside the unit disk $u[k]^2 + v[\ell]^2 \leq 1$ and normalized such that $\sum_{k,\ell} S[k, \ell] = 1$.

The photomask pattern was imported from a GDS file for a real IC product. The mask was modeled as an infinitely thin spatial light modulator, and no thick-mask propagation or Mask3D electromagnetic response is included in this validation experiment. An $L_x \times L_y$ region of interest was extracted

and rasterized into a binary transmission map on a $4L \times 4L$ over-sampled grid with $L = 2001$, as shown in Fig. 2(b). The over-sampled photomask pattern is then downsampled to an $L \times L$ gray-scale photomask transmittance function $M(\mathbf{r})$, as shown in Figs. 2(c), (d), using a 2D box filter for area averaging [36, 37]. An FFT was applied to obtain the photomask spectrum $\tilde{M}(\mathbf{f})$, as shown in Fig. 2(e), followed by spectrum cropping to the $N \times N$ frequency grid.

An ideal circular aperture is used for the pupil function.

$$\tilde{P}(\mathbf{f}) \triangleq \tilde{P}[k, \ell] = \begin{cases} 1, & u[k]^2 + v[\ell]^2 \leq 1, \\ 0, & \text{otherwise,} \end{cases} \quad (15)$$

as shown in Fig. 2(f), which was sampled on the same spectral grid as the source. For Abbe imaging, the pupil function $\tilde{P}(\mathbf{f})$ is used to directly filter the photomask spectrum $\tilde{M}(\mathbf{f})$, whereas in TF-Hopkins-PS, the PSF $P(\mathbf{r}) = \mathcal{F}^{-1}\{\tilde{P}(\mathbf{f})\}$ is a spatial

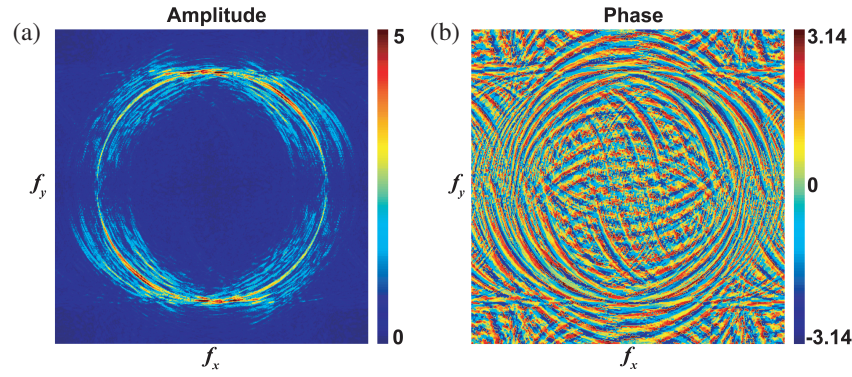


FIGURE 3. Frequency domain characteristics of the proposed TF-Hopkins-PS for a photomask pattern. (a) Amplitude of the Fourier transform of the spatial-domain pointwise multiplication between the shifted photomask M_q and the PSF P . (b) Corresponding phase distribution, exhibiting conjugate centro-symmetry between the $\pm f$ frequencies.

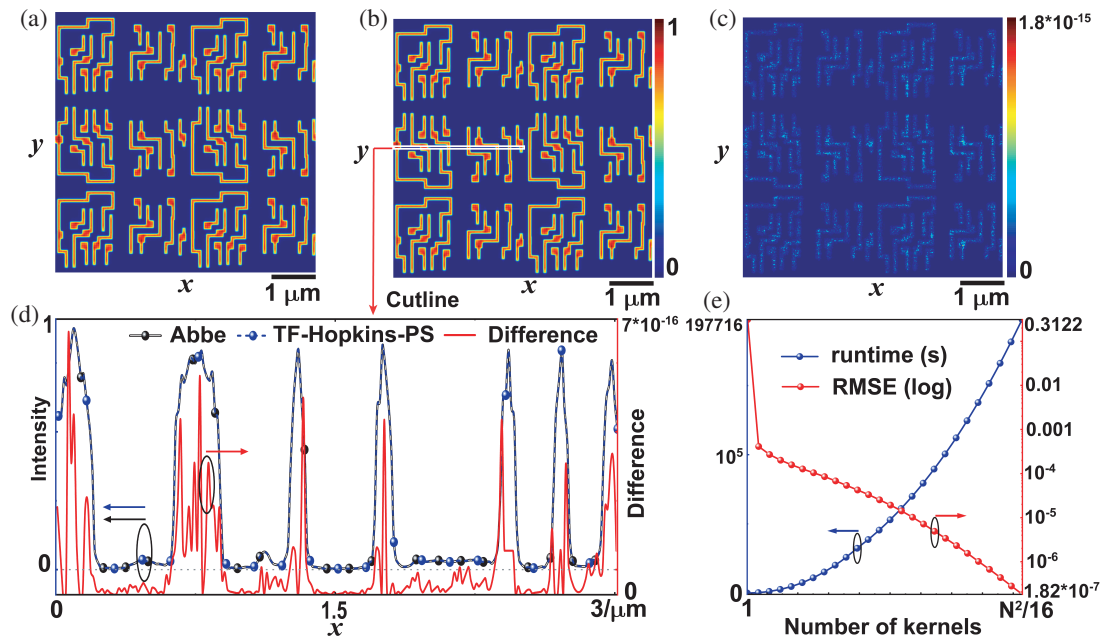


FIGURE 4. Full-field comparison between the Abbe and TF-Hopkins-PS images. (a) Abbe image I_A . (b) TF-Hopkins-PS image I_H . (c) Absolute difference between I_H and I_A . (d) Intensity profiles and their difference along the cutline indicated in (b), manifesting Abbe-Hopkins agreement up to the level of machine precision. (e) Effects of kernel truncation on runtime and RMSE for the conventional Hopkins method.

pointwise multiplier that modulates the photomask transmittance function.

These settings provided a physically and numerically consistent set of input data for subsequent Abbe and Hopkins simulations and cross-model validation.

3.2. Numerical Results and Performance Analysis

Before comparing the final simulation results, we inspected key intermediate images to ensure the correct implementation of the TF-Hopkins-PS. Fig. 3 shows the spectrum of $P \odot M_q$ computed via a single pointwise multiplication followed by one FFT. Its complex conjugate counterpart is produced simply by array reversal and complex conjugation, saving another pointwise multiplication and FFT.

For cross-model validation, the Abbe model was activated to generate a full-field normalized intensity distribution as a reference, called the Abbe image I_A , as shown in Fig. 4(a). Then, TF-Hopkins-PS was invoked to “sample” all of the same grid points exhaustively to produce a Hopkins image I_H of the same size, as shown in Fig. 4(b). The difference between the simulation results was quantified using the maximum absolute error (MAE), defined as

$$\text{MAE} \triangleq \max_{i,j} |I_H(i,j) - I_A(i,j)|, \quad (16)$$

or a root-mean-square error (RMSE) defined as

$$\text{RMSE} \triangleq \sqrt{N^{-2} \sum_{i=1}^N \sum_{j=1}^N |I_H(i,j) - I_A(i,j)|^2}. \quad (17)$$

Our experiments found $\text{MAE} = 1.887 \times 10^{-15}$ and $\text{RMSE} = 1.451 \times 10^{-16}$, indicating nearly perfect numerical agreement at

the level of machine precision. The absolute error map and one-dimensional profiles on a representative cutline are shown in Figs. 4(c) and (d), respectively. It is quite reasonable to attribute the extremely small difference mainly to the rounding errors of the double-precision floating-point arithmetic.

The runtime was also measured on a workstation equipped with two Intel Xeon Gold 6330 CPUs (2.00 GHz) and 512 GB RAM. For comparison, the conventional Hopkins method is also performed, which is based on explicit TCC construction and Lanczos eigen-decomposition. As shown in Fig. 4(e), the TCC was assembled and decomposed to extract up to the leading $N^2/16$ Hopkins kernels for model reconstruction. Even under this extreme setting, the resulting intensity still bears a relative error on the order of 10^{-7} , whereas the average evaluation time skyrocketed to 1.230 s per pixel. In contrast, TF-Hopkins-PS requires only 0.017 s per pixel on the same platform, representing a more than 70-fold speedup. The speedup receives another drastic boost for applications that only require sampling a small portion of the image points. Such an advantage of efficient sampling stems from a fundamental difference in the numerical workflows: TF-Hopkins-PS involves only a Fourier transform and a few pointwise multiplications in the frequency domain, whereas the conventional Hopkins approach requires a large number of full-field convolutions between the Hopkins kernels and the photomask transmittance function.

Therefore, TF-Hopkins-PS affords a fast and reliable means for Abbe-Hopkins cross-model validation, achieving perfect numerical consistency between the two simulation methods up to the level of machine precision. At the software development stage, such rigorous verification of the Hopkins model provides useful information to guide algorithm design and code implementation. In many practical applications, such as SMO, a working Hopkins model is critical for subsequent TCC-based analyses and simulations.

4. CONCLUSION

A unified Abbe-Hopkins framework is established to facilitate perfect Abbe-Hopkins consistency and cross-model validation, with Abbe and Hopkins simulation results agreeing up to the level of machine precision. Such perfect consistency relies on a strict convention for the two models to use exactly the same numerical representations and discretization schemes for the illumination source, photomask, and projection lens. The spatial boundaries and spectral filters of the images must be carefully examined and meticulously aligned.

Based on the unified framework, a TF-Hopkins-PS was further developed for partially coherent imaging in the style of Hopkins, which can compute the intensity at any given point rapidly by avoiding the construction and decomposition of a TCC, but only invoking a few pointwise multiplications between images and a single image FFT. Fast model validation using TF-Hopkins-PS should help discover and fix model discrepancies in a timely manner to avoid unnecessary resource consumption caused by a faulty Hopkins formulation. The TF-Hopkins-PS can even serve as a stand-alone workhorse for applications that only require sparse image simulations at critical layout features.

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