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Approximating Processing Delays in High Energy Laser Directed Energy System Performance Prediction

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ABSTRACT: This study addresses an issue with high-energy laser directed energy weapon performance assessment when applied to the problem of countering swarms of uncrewed aerial systems (UAS). Queueing theory provides a suitable modelling framework for the performance assessment of such systems, as a single server queue can process only one threat at a time, based on the order in which threats arrive at the theatre of operation. Consequently, this introduces delays into the processing of sequences of threats. Delays in such queues typically have time-dependent service times, due to the target's movement. This results in considerable complexity in terms of producing performance predictions through stochastic models. In recent applications of queueing theory to directed energy systems, an ad hoc approximation has been used to estimate the delays that threats experience while waiting for service. This approach involves approximating the processing delay of a given threat by a constant value. In particular, it has been estimated by measuring the delay as a product of the expected service time and the number of threats present less one. Such an approximation can result in severely reduced and inaccurate performance predictions. In the current study, the mean delay will be used instead, and improvement on the aforementioned approximation will be demonstrated through explicit examples of swarm UAS defeat.

1. INTRODUCTION

The analysis reported in this study has resulted from the complexity of evaluating performance prediction probabilities associated with weapon systems used for counter uncrewed aerial system (UAS) applications. In particular, much interest has been devoted to the utility of high-energy laser (HEL) directed energy weapons (DEWs) [1] for such purposes [2–7]. The application of queueing theory to performance prediction modelling of HEL DEW systems was introduced in [8] and further refined in [9]. Modelling the arrival of such threats into the operational environment has been through a renewal process, which is equivalent to sensor system-based detection times [9, 10]. A HEL DEW will process one threat at a time, so that as threats are processed sequentially, their relative positions will change over time. This results in a time-dependent service time, and although one may apply expressions from standard queueing theory to determine the waiting time for threats to be processed, they are difficult to evaluate in practice. This is because the delays are defined through a recursive relationship, coupling the current delay of a threat with the previous delay associated with the last engaged threat and its elimination time. The latter is also determined through the system's ability to neutralise the target over an observational window dependent on its arrival time and associated delay. In the study [8], the delays are evaluated using Monte Carlo approximations, but this proved to be computationally expensive. Hence, in order to reduce modelling complexity, approximations for the delay have been examined, with a specific focus on deterministic estimates for it. In particular, it has been suggested that one may use an upper bound on it, based on an assumed average dwell time. Specifically, any one threat will have to wait a total of this average times the number of preceding threats, under the assumption that threats are processed sequentially. As pointed out in the discussion in [11, 12], this service time may be estimated to be around three seconds for a HEL thermal effect. Hence, if there are a total of N threats, the delay can be approximated by $3(N-1)$. This approximation has been applied in [13], but as the number of threats increases, this delay becomes quite large and results in significantly reduced performance results. Hence, it is useful to examine whether a better approximation for a queueing delay can be constructed. Therefore, this paper will introduce the idea of estimating the delay through its mean. This requires one to develop suitable expressions for it, which will be the focus in the next section. Once this is done, examples will be produced to demonstrate the improved performance predictions using this new approach. As a benchmark on performance, the case that there is no delay will be used to assess improved results compared to the current way in which delays are estimated.

2. DELAY APPROXIMATION

The relevant queue type is a single server with Markovian arrivals and a time-dependent service time. Suppose that a series of N threats are in the theatre of operation, which arrive according to a renewal process $\{S_n, n \in \{1, 2, \ldots, N\}\}$ where $S_n = X_1 + X_2 + \cdots + X_n$ and each X_j is non-negative and an independent and identically distributed interarrival time. The service time of threat j is denoted Y_j and the sequence of delays is $\{D_n, n \in \{1, 2, \ldots, N\}\}$, where $D_1 = 0$ because the

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first threat to arrive will not experience a waiting time. Then, the delays are given by the recursive Lindley expression [14]

$$D_{n+1}|\{D_n = d\} = \max\{0, Y_n + d - X_{n+1}\}, \tag{1}$$

where the dependency of the current delay on the previous one is emphasized through the conditional expression. This result implies that the delays are stochastic in nature. It is possible to express the distribution function of any delay in terms of a joint distribution function of the service times and interarrival times of all threats up to the current one [15]. This is not an easy expression to evaluate when the service times are time-dependent. Hence, it is useful to examine whether an alternative expression can be produced for delays. In particular, the focus will be on deriving an expression for the mean delay and using this as an estimate of the delay in performance predictions. Consequently, the approach in this investigation is to apply a deterministic approximation to the series of delays to avoid complexity in evaluating sequential probabilities of defeat of UAS. This approximation will be valid when there is a small variation in the delay's variance. In the case where a series of homogeneous threats are converging onto the HEL DEW's position, travelling at the same speed and with the same disruption threshold, one would expect that delays will be somewhat of the same order with little variation. This will be the assumed structure of the swarm of UAS in the current paper. Further work is required to establish a more rigorous understanding of the validity of approximating the delay by its mean value.

Towards this objective, it is assumed that the renewal process is Poisson with exponentially distributed interarrival times with parameter ρ. The justification for such a choice has been based upon the observation that a Poisson process is a model for rare events, and so reflects a scenario where UAS appearance in the theatre of operation is more difficult to detect [10, 16]. Then note that one may write $\max\{0, Y_n + d - X_{n+1}\} = (Y_n + d - X_{n+1})\mathbb{I}[Y_n + d - X_{n+1} \geq 0]$, where $\mathbb{I}[z]$ is the indicator function, taking unity if condition z holds and is zero otherwise. Therefore by conditioning on the interarrival time X_{n+1}, one may show that

$$\begin{aligned}&\mathbb{P}(D_{n+1} > x|D_n = d)\\ &= \int_0^\infty \mathbb{P}(D_{n+1} > x|D_n = d, X_{n+1} = w) f_{X_{n+1}}(w)dw\\ &= \int_0^\infty \rho e^{-\rho w}\mathbb{P}\left((Y_n + d - w)\mathbb{I}[Y_n + d - w \geq 0]|D_n = d\right)dw,\end{aligned} \tag{2}$$

where $\mathbb{P}(\cdot)$ denotes the probability, and $f_{X_{n+1}}(w) = \rho e^{-\rho w}$ is the density of X_{n+1}. Since for any random variable X and event A, one may show that $\mathbb{P}(X\mathbb{I}[A]) = \mathbb{P}(X|A)\mathbb{P}(A)$, it can be demonstrated that the probability within the integral in (2) is reduced to

$$\begin{aligned}&\mathbb{P}(Y_n + d - w \geq x|Y_n \geq w - d, D_n = d) \times\\ &\mathbb{P}(Y_n \geq w - d|D_n = d).\end{aligned} \tag{3}$$

Observe that by expanding out the conditional probabilities in (3), this expression becomes

$$\begin{aligned}&\frac{\mathbb{P}(Y_n + d - w \geq x, Y_n \geq w - d, D_n = d)}{\mathbb{P}(Y_n \geq w - d, D_n = d)} \times\\ &\frac{\mathbb{P}(Y_n \geq w - d, D_n = d)}{\mathbb{P}(D_n = d)}\\ &= \mathbb{P}(Y_n \geq x + w - d|D_n = d),\end{aligned} \tag{4}$$

and by applying (4) to (2) the latter is

$$\begin{aligned}&\mathbb{P}(D_{n+1} > x|D_n = d)\\ &= \int_0^\infty \rho e^{-\rho w}\mathbb{P}(Y_n \geq x + w - d|D_n = d)dw.\end{aligned} \tag{5}$$

By applying complementation to expression (5), it is immediate that the conditional distribution function of the delay D_{n+1} is given by

$$\begin{aligned}&\mathbb{P}(D_{n+1} \leq x|D_n = d)\\ &= \int_0^\infty \rho e^{-\rho w} F_{Y_n|D_n=d}(x + w - d)dw,\end{aligned} \tag{6}$$

where $F_{Y_n|D_n=d}$ is the conditional distribution function of Y_n given $D_n = d$. One may now differentiate (6) with respect to x to yield the density

$$f_{D_{n+1}|D_n=d}(x) = \int_0^\infty \rho e^{-\rho w} f_{Y_n|D_n=d}(x + w - d)dw, \tag{7}$$

since the derivative of a distribution function is the corresponding probability density function. Therefore, by definition, the conditional mean (denoted $\mathbb{E}(\cdot)$) of the delay D_{n+1} is given by

$$\mathbb{E}[D_{n+1}|D_n = d] = \int_0^\infty x f_{D_{n+1}|D_n=d}(x)dx, \tag{8}$$

and to extract the unconditional mean of the delay D_n one may utilise the fact that

$$\mathbb{E}[D_{n+1}] = \mathbb{E}[\mathbb{E}[D_{n+1}|D_n]] \tag{9}$$

see [17]. In order to evaluate expression (9), one may utilise (8) and evaluate it as a function of d and then average the results. This requires a specific service time distribution to be introduced, which is the focus of the following section.

3. COUNTER UAS PERFORMANCE MODEL

To provide a specific example of the utility of (9), it is necessary to introduce expressions for the probability of defeat of a swarm of UAS. Hence, an alternative to the development in [13] is derived in this section. Consider the situation where a HEL DEW is facing a series of N UASs that are travelling on a direct intercept course to the DEW, such that each is travelling at the same speed but spatially dispersed. This assumption is adopted so that the swarm remains coherent throughout the scenario [18]. In addition to this, the UAS will be assumed to be NATO Class 1 so that their speeds are limited, and they will have a lower disruption threshold [19]. As discussed in [9], a directed energy system, processing a threat which appeared at time S_n and experiencing a delay D_n, will have a service time whose complementary distribution function is given by

$$\mathbb{P}(Y_n > t) = \mathbb{P}\left[\int_{S_n+D_n}^{S_n+D_n+t} I_n(w)dw < \mu_n\right], \tag{10}$$

where $I_n(w)$ is the power density or irradiance which the system applies to the target at time w, and μ_n is the threat vulnerability threshold. This threshold and the fluence within the probability in (10) are measured in Watts per square meter (W/m^2). Expression (10) states that the probability that this service time exceeds t seconds (s) is equivalent to the event that the fluence on the target is smaller than the disruption threshold at this time.

The analysis in [8] evaluated expressions, such as (10), by utilising (1) with (10) and employing Monte Carlo methods sequentially over the number of threats. In contrast to this, the study in [13] applied the approximation $D_n \approx 3(N-1)$ to facilitate the analysis. In the current investigation, the mean delay, given through (9), will be used to instead approximate D_n.

Based upon the simple irradiance used in [13], it will be supposed that

$$I_n(w) = \frac{\kappa_n}{\nu_n^2(T_n - w)^2}. \tag{11}$$

Here, κ_n is a constant, dependent on the HEL DEW characteristics, and the threat is assumed to be travelling on a linear path to the DEW, at a constant speed of ν_n metres per second (m/s) taking T_n s to reach its target if not disrupted.

By applying (11) to (10) and evaluating the integral, it can be shown that

$$\mathbb{P}(Y_n > t) = \mathbb{P}\left([S_n + D_n + t - T_n][S_n + D_n - T_n] > \kappa_n t/(\nu_n^2 \mu_n)\right). \tag{12}$$

This expression may be simplified further by considering the expression within the probability as a quadratic function of $S_n + D_n$ and determining conditions under which the inequality holds. Consequently, it may be shown that (12), when conditioned on the delay D_n, becomes

$$\begin{aligned}\mathbb{P}(Y_n \le t|D_n) &= 1 - \mathbb{P}(\{\phi_{n,1}(t) < S_n + D_n < \phi_{n,2}(t)\}\,|D_n)\\ &= F_{S_n}(\phi_{n,2}(t) - D_n) - F_{S_n}(\phi_{n,1}(t) - D_n),\end{aligned} \tag{13}$$

where

$$\phi_{n,j}(t) = T_n - \frac{t}{2} \pm \frac{1}{2}\sqrt{t^2 + 4\kappa_n t/(\nu_n^2 \mu_n)}, \tag{14}$$

with the positive taken in ± when $j = 2$ and negative for $j = 1$, and F_{S_n} is the distribution function of the arrival time S_n.

Since the threats are arriving through a renewal process and travelling at the same speed, the probability that the swarm is defeated by time t is equivalent to the probability that the arrival time, plus the delay and service time, of the last threat to arrive is smaller than t. Hence

$$\mathbb{P}(\text{Swarm defeated by time } t) = \mathbb{P}(S_N + D_N + Y_N < t). \tag{15}$$

By conditioning on the service time in (15), one may show that this can be expressed in the form

$$\mathbb{P}(\text{Swarm defeated by time } t) = \int_0^\infty \mathbb{I}[t - D_N \ge x] F_{S_N|D_N}(t - x - D_N) f_{Y_N}(x)dx. \tag{16}$$

Hence, one may use Monte Carlo simulation to evaluate (16) by sampling from Y_N. Since the arrival process is Poisson, it can be demonstrated that

$$F_{S_N|D_N}(t) = 1 - \sum_{i=0}^{N-1} \frac{(\rho t)^i}{i!} e^{-\rho t}, \tag{17}$$

where ρ is the arrival rate. It is worth noting that (17) is equivalent to a Poisson complementary distribution function, which is useful from a numerical evaluation perspective. One may sample from Y_N by applying the inverse distribution function method to (13) and evaluating an average based upon evaluations of $\mathbb{I}[t - D_N \ge x] F_{S_N|D_N}(t - x - D_N)$ [20].

Hence, (16) provides a simple metric to examine the impact of approximations of the delay on performance predictions. In addition to this, it provides a simpler mechanism than that in [13] to evaluate the mission success of the DEW in defeating the swarm of UAS under the assumed conditions. The next section will provide an illustration of these results.

4. ANALYSIS OF DELAY APPROXIMATIONS

As in [13], a Gaussian beam profile is assumed for the HEL DEW, with a simple model for irradiance, which results in $\kappa_n = \frac{P_T}{2\pi(\alpha_D^2 + \alpha_J^2)}$, where P_T is the laser peak power, taken to be 10 kW; $\alpha_D = \frac{2}{\pi} M^2 \frac{\lambda}{D}$ is the angular standard deviation of the Gaussian diffraction pattern; M^2 is the laser beam quality; λ is the wavelength; D is the laser aperture diameter; and α_J is the beam jitter. The specific parameters used are $\lambda = 1.045\,\mu\text{m}$, $D = 0.8\,\text{m}$, $M^2 = 1$ and $\alpha_J = 0.001$ radians. These choices imply that a small swarm of homogeneous UAS are converging on the HEL DEW's position. A common disruption threshold will be assumed and taken to be $\mu_n = 10^4$ W/m^2. As can be inferred from the discussions in [5] and [9], thresholds in the range from 10^2 to 10^6 W/m^2 are indicative of typical vulnerabilities in UAS. On the lower end of the spectrum, the threshold is appropriate when measuring the disruption of a UAS by minor damage. An example is when a propellor is hit by the incident HEL beam. Larger thresholds coincide with direct thermal damage, such as burning a hole through the UAS fuselage. The choice adopted in this study has been selected to represent a middle range of disruption. The arrival time parameter is selected to be $\rho = 1$ throughout. This parameter controls the appearance of the swarm in the theatre of operation. Choices of $\rho >> 1$ will imply that the swarm appears almost instantaneously or equivalently, each member is detected almost simultaneously. In situations where $\rho << 1$ the swarm appears spatially more sparsely distributed. A series of $N = 8$ UAS will be converging on the HEL's position under two speed cases, but with common $T_n = 100$ s. The two speed cases, denoted as

TABLE 1. Delay estimates of each threat for each of the two cases, estimated by (9). The average has been taken over 50 measurements of d, between 0 and 100.

Delay	D_1	D_2	D_3	D_4	D_5	D_6	D_7	D_8
Case 1	0	0.19508	0.31374	0.18384	0.45147	0.41966	0.61982	0.7106
Case 2	0	1.73204	1.92829	1.44795	0.56206	2.8209	4.26071	4.35636

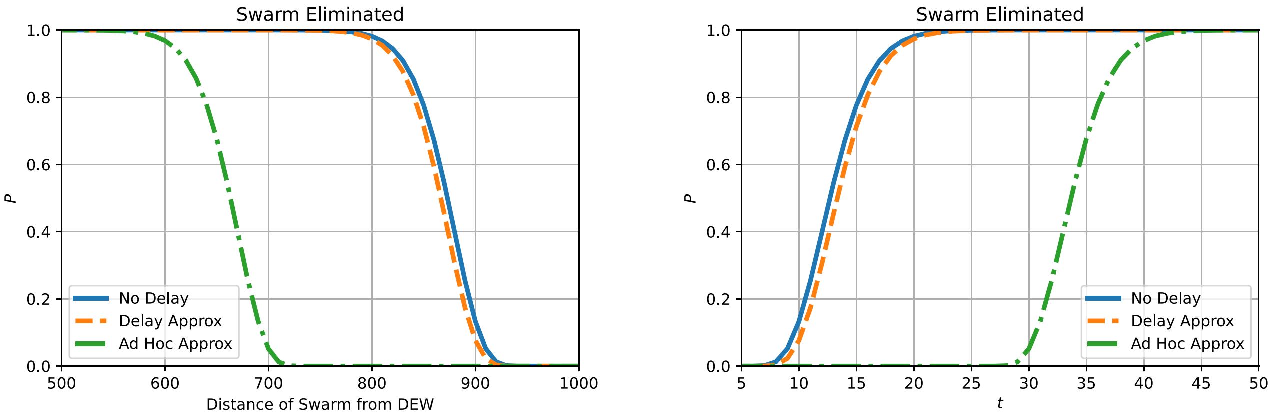

FIGURE 1. Plot of the probability of defeat of the swarm, under Case 1. The left subplot shows this as a function of the distance of the last threat to arrive, while the right subplot plots it as a function of time.

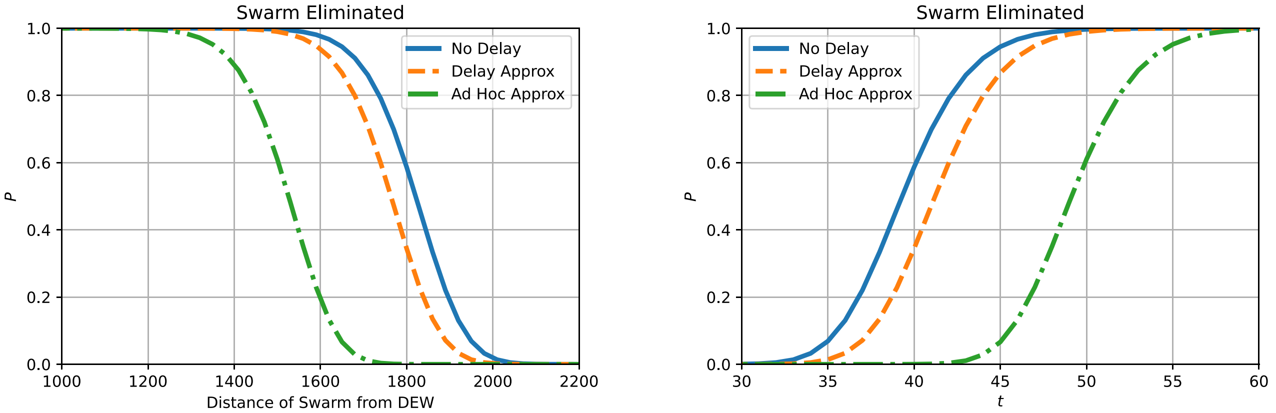

FIGURE 2. Case 2 performance predictions showing the impact of delay estimations.

Case 1 and Case 2, will be for common speeds of 10 and 30 m/s, respectively. These choices imply a small swarm of homogeneous UAS are convering on the HEL DEW's position, such that they take the same amount of time to reach their intended target from the point of appearance in the theatre of operation. The two speed options have been selected to reflect slow and fast UAS, to allow comparison of results.

Table 1 provides estimates of each expected delay based on evaluation of (9), using an average of 50 data points by varying d from 0 to T_N in steps of 2. Although this is a small sample on which to base the estimation of mean delay, it nonetheless provides a suitable approximation whose accuracy may be improved by increasing the sample size. One can observe that the mean delays increase with the UAS speed. Note that the approximation for the delay is given by 21 s based on the results in [13], which, compared to the results in Table 1, show that it is a severe overestimation of the maximum delay. To see the impact of this overestimation, Figures 1–2 provide examples of the probability of defeat (16) under three delay instances. Each figure plots three curves, one for the situation where there is no delay (to function as a benchmark), one using the approximation of 21 s, and the third selecting the maximum mean delay for each case in Table 1. For each data point, a total of 5000 Monte Carlo simulations have been used to evaluate (16).

The plots in Figure 1 are for Case 1, with the left subplot plotting the probability of swarm defeat as a function of the swarm's distance from the HEL DEW. This distance is measured as that of the last threat to arrive at the combat scene. The right subplot graphs the same probability but as a function of time. In each figure, the delay for the last threat to appear is 0.7106 s from Table 1. The curve labelled ad hoc delay refers to the application of the estimate of delay of 21 s. Figure 1 shows that the ad hoc approximation will result in a severely inaccurate estimate of the distance at which the swarm will be eliminated. This is also clarified in the right subplot in Figure 1, where it is clear that the delay inaccuracy can lead to the conclusion that the HEL DEW will not be able to eliminate the swarm until after 40 s. This can lead system designers to then consider increasing the laser power to eliminate the swarm more rapidly, which would be obviously erroneous based on the performance prediction result with maximal delay estimated based upon the approach in this study. Finally, Figure 2 shows the effect of delay approximations for Case 2. Similar conclusions can be drawn as in Case 1.

5. CONCLUSIONS

The purpose of this study has been to introduce a method through which delays may be estimated in a queueing process with time-dependent service times. The approach introduced in [13] used an approximation of delays through a constant based upon ad hoc analysis. In the current study, sequential delays have been estimated by the mean delay. The motivation for this has been problems associated with the over-estimation of the delays in HEL DEW performance prediction studies. The approach developed in this analysis was shown to produce better delay estimates than the ad hoc solution used in [13], and a series of performance predictions demonstrated the importance of using more accurate estimates of such delays. In addition to this, the paper introduced an improved way in which performance predictions can be quantified for the case of homogenous threats, improving the analysis in [13]. The study indicated the need for further investigations into the distribution of the delays and assessing the relative merits of applying approximations.

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