

# Near-Field SAR-Aware Power Control and Beamforming for Reconfigurable Intelligent Surfaces with Electromagnetic Safety Guarantees

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**ABSTRACT:** Existing electromagnetic (EM) safety analyses for reconfigurable intelligent surface (RIS) systems rely on the far-field equivalent plane-wave density (EPD) formula, which systematically underestimates tissue exposure when the user equipment (UE) operates in the radiating near-field (NF) zone ( $d_U \lesssim d_R/2$ , where  $d_R = 2D^2/\lambda$  is the Rayleigh distance, and  $D = (N-1)\lambda/2$  is the aperture length). This paper presents five analytically rigorous contributions to NF-aware power allocation and beamforming for RIS-assisted 5G/6G systems. (1) A conservative Fresnel-envelope correction factor  $\kappa(d, N)$  with a provable non-negative overestimation error  $\varepsilon(d, N) \geq 0$  (Propositions 1–2, Lemma 1). (2) Closed-form NF-corrected SAR power ceiling  $P_{\text{NF}}^*$  and minimum exclusion radius  $d_{\text{min}}^{\text{NF}}$ ; the 1-D Fresnel bound is conservative for 2-D uniform planar arrays (UPAs) with a separability gap up to 13.2 dB, confirmed by exact 2-D spherical-wave summations ( $\text{SAR}_{1\text{D}} \geq \text{SAR}_{\text{FF}} \geq \text{SAR}_{2\text{D}}$ ). (3) A closed-form NF phase-taper  $\Delta\varphi_n$  recovering up to 0.4 bit/s/Hz SAR-constrained spectral efficiency (SCSE) at  $d_U = 1$  m (Proposition 3). (4) A two-stage NF-SAR alternating-optimisation (NF-SAR-AO) algorithm with  $\mathcal{O}(N)$  per-iteration complexity, hard guard margin ( $d_{\text{eff}} = 1.1d_U$ ), and proved monotone convergence under line-of-sight (LOS) channels (Algorithm 1, Proposition 4). (5) 2-bit phase quantization incurring  $<0.3$  bit/s/Hz SCSE loss with 100% ICNIRP 2020 compliance for all  $d_U \geq 0.5$  m. Validated over  $5 \times 10^3$  Monte Carlo trials at 3.5 GHz ( $N = 64$ ,  $P_{\text{max}} = 200$  mW):  $d_{\text{min}}^{\text{NF}} = 0.727$  m vs.  $d_{\text{min}}^{\text{FF}} = 0.498$  m — far-field models underestimate the required safety exclusion radius by 46%, risking ICNIRP 2020 non-compliance.

## NOTATION

Boldface lowercase letters denote column vectors;  $(\cdot)^H$  denotes the conjugate transpose;  $|\cdot|$  denotes the modulus of a scalar;  $\angle(\cdot)$  denotes the argument of a complex number;  $\mathbb{E}[\cdot]$  is the expectation operator;  $\mathcal{CN}(\mu, \sigma^2)$  denotes the circularly symmetric complex Gaussian distribution. All distances are in metres unless otherwise stated. Principal symbols are summarised in Table 1.

## 1. INTRODUCTION

This paper presents original theoretical and simulation-based research; it is not a review article. The literature survey in this section and in Table 4 provides contextualisation, but the primary contributions are the five analytically rigorous results listed in the Contributions subsection. Reconfigurable intelligent surfaces (RISs) are a cornerstone of 5G/6G indoor and factory deployments, improving spectral efficiency (SE) by coherently steering reflections toward the user equipment (UE) [1, 2]. Throughout this paper, the term *SAR-constrained spectral efficiency* (SCSE) denotes the SE  $R = \log_2(1 + \gamma)$  achieved

under the near-field (NF) specific absorption rate (SAR) power constraint (C1), distinguishing it from unconstrained SE benchmarks.

In 6G extra-large RIS (XL-RIS) scenarios at sub-THz frequencies (above approximately 100 GHz), or when the number of elements  $N \gg 64$ , the Rayleigh distance  $d_R = 2D^2/\lambda$  can exceed tens of metres, placing virtually all indoor users in the near field [14, 15]. Even at 5G mid-band, an RIS with  $N = 64$  half-wavelength elements at  $f = 3.5$  GHz has  $d_R \approx 170$  m, meaning that all practical indoor UEs at  $d_U \leq 5$  m lie deep in the radiating near-field zone [6]. International Commission on Non-Ionizing Radiation Protection (ICNIRP) 2020 mandates that the SAR averaged over 10 g of tissue must not exceed 2 W/kg (head/torso) or 4 W/kg (limb) [3].

### 1.1. Gap in Prior Work

SAR-constrained beamforming studies [4] and RIS-specific electromagnetic field (EMF) exposure optimisation work [5] both adopt the far-field EPD approximation  $\text{SAR} \propto PN^2/(4\pi d^2)$ , which is physically valid only in the far field ( $d \gg d_R$ ). In the radiating near field, spherical-wavefront effects and reactive field components amplify local power

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TABLE 1. Summary of principal symbols.

Symbol	Definition
$N$	Number of RIS elements
$d_U$	User equipment (UE)–RIS separation (m)
$\hat{d}_U$	Estimated UE–RIS distance (subject to ranging error)
$d_{\text{eff}}$	Guard-margin-corrected effective distance: $(1 + \eta)\hat{d}_U$ (m)
$d_R$	Rayleigh distance $2D^2/\lambda$ (m)
$R_{\text{NF}}$	Reactive near-field boundary $D^2/(8\lambda)$ (m)
$D$	ULA aperture length $(N - 1)\lambda/2$ (m)
$\lambda$	Free-space wavelength (m)
$\kappa(d, N)$	Fresnel-envelope correction factor (dimensionless)
$\zeta(d)$	Exact normalised on-axis Fresnel intensity (dimensionless)
$\varepsilon(d, N)$	Overestimation error $\kappa^2(d, N) - \zeta(d) \geq 0$ (dimensionless)
$\mathbf{h}_t$	BS–RIS channel vector ( $\mathbb{C}^N$ )
$\mathbf{h}_r$	RIS–UE channel vector ( $\mathbb{C}^N$ )
$\Theta$	Passive RIS phase-shift matrix $\text{diag}(e^{j\varphi_n})$
$P, P_{\text{max}}$	Instantaneous transmit power and maximum allowable power (W)
$P_{\text{NF}}^*, P_{\text{FF}}^*$	NF- and far-field SAR-safe power ceilings (W)
$d_{\text{min}}^{\text{NF}}, d_{\text{min}}^{\text{FF}}$	NF- and far-field minimum exclusion radii (m)
$\text{SAR}_{\text{max}}$	ICNIRP tissue-absorption limit (W/kg)
$\sigma_t$	Tissue electrical conductivity (S/m)
$\rho_m$	Tissue mass density ( $\text{kg m}^{-3}$ )
$G_{\text{el}}$	Patch-element gain (linear scale)
$K_R$	Rician $K$ -factor (dB)
$\eta$	Guard-margin fraction (dimensionless)
$b$	Phase-quantization resolution (bits)
$R, \text{SCSE}$	Spectral efficiency (SE) and SAR-constrained spectral efficiency (SCSE) (bit/s/Hz)

density beyond the plane-wave prediction [6, 18]. Near-field SAR for massive multiple-input multiple-output (MIMO) systems has been studied in [12], and NF RIS beamforming has attracted recent attention in the context of XL-RIS channel modelling [14, 15].

The present paper addresses a specific and unresolved gap in prior work. Zhang et al. [4] optimise SAR-aware beamforming in multiple-input single-output (MISO)/simultaneous wireless information and power transfer (SWIPT) systems but apply the far-field EPD approximation, which is invalid at typical indoor RIS distances. Zappone and Di Renzo [5] minimise EMF exposure in RIS-assisted systems but likewise rely on the far-field formula without NF correction or provable bounds. Aerts et al. [12] provide in-situ massive-MIMO SAR measurements but derive no NF correction factor applicable to RIS beamforming. Near-field beamforming studies [14, 26] design NF focusing phase tapers but impose no SAR constraint and provide no power-ceiling or compliance guarantee. EMF-aware RIS scheduling [24] targets field-level compliance but does not address Fresnel-zone SAR amplification or provable upper bounds. To the best of the authors' knowledge, no existing study provides simultaneously: (i) a Fresnel-envelope NF correction with a provable upper-bound guarantee embedded in

an optimisation framework; (ii) a closed-form NF phase-taper recovering near-field beam defocusing loss; (iii) a conservative 2-D planar-array SAR bound with an explicit separability-looseness gap; or (iv) a hard-constrained guard margin with real-time  $\mathcal{O}(N)$  complexity. A full feature-by-feature comparison with twelve peer-reviewed references is provided in Table 4 (Appendix A). The present framework also complements the IEEE C95.1-2019 general human-exposure standard [16] by providing RIS-specific near-field power-ceiling expressions fully consistent with the ICNIRP 2020 tissue-absorption limits [3]. For RIS gain vs. relay comparisons in similar deployment geometries, the reader is referred to [13].

## 1.2. Scope and Assumptions

The analytical baseline assumes a single-antenna base station (BS), a single-antenna UE, and omission of the direct BS–UE link. The direct-link omission is valid when the RIS beamforming gain exceeds the direct-link gain by more than 10 dB. As shown in Fig. 4 (inset), this condition holds for  $N \geq 32$  at all  $d_U \geq 0.8$  m. For  $N = 16$  or  $d_U < 0.8$  m, the direct link may contribute a non-negligible fraction of total tissue exposure; the NF power ceiling derived here bounds the RIS-induced SAR component but does not represent the total SAR in those con-

figurations. This limitation is discussed further in Section 6. Table 4 (Appendix A) positions the present work against the closest related papers.

*Literature coverage note.* Table 4 (Appendix A) provides a systematic feature-by-feature comparison against twelve representative works spanning SAR-constrained beamforming [4, 5], in-situ exposure measurement [12], near-field zone theory [19, 20], phase-quantisation optimisation [21, 25], near-field beamforming [14, 26], XL-RIS channel estimation [22, 23], and EMF-aware RIS optimisation [24]. This coverage is representative of the state of the art; the present work targets the specific intersection of provable NF-SAR bounds and real-time RIS beamforming, which none of the surveyed works address jointly.

### 1.3. Contributions

1. We derive  $\kappa(d, N)$  as a conservative envelope of the exact on-axis Fresnel intensity with  $\varepsilon(d, N) \geq 0$  and  $\kappa^2(d, N) \geq \zeta(d)$  for all  $d$  (Propositions 1–2, Lemma 1).
2. We derive  $P_{\text{NF}}^*$  and  $d_{\text{min}}^{\text{NF}}$  in closed form, prove  $P_{\text{NF}}^* \leq P_{\text{FF}}^*$  for all  $d$ , and show that the 1-D Fresnel bound is conservative for 2-D rectangular uniform planar arrays (UPAs) with the separability-looseness gap quantified (Corollary 1, Fig. 2).
3. We derive a closed-form NF phase-taper  $\Delta\varphi_n$  compensating spherical-wavefront curvature and recovering SCSE loss from near-field defocusing (Proposition 3).
4. We embed NF-aware power control with a hard guard margin ( $d_{\text{eff}} = 1.1\hat{d}_U$ ) into a provably convergent two-stage NF-SAR-AO algorithm at  $\mathcal{O}(N)$  per iteration (Algorithm 1, Proposition 4).
5. We characterise SCSE loss and SAR compliance under 1-bit and 2-bit phase quantization, provide a multi-user NF power-ceiling framework (Fig. 11), and benchmark wall-clock complexity against semidefinite relaxation (SDR)/successive convex approximation (SCA) approaches (Fig. 8).

## 2. SYSTEM MODEL

### 2.1. Signal Model

A single-antenna BS serves a single-antenna UE through an  $N$ -element uniform linear array (ULA) RIS.<sup>1</sup> The discrete-time received signal is

$$y = \mathbf{h}_r^H \Theta \mathbf{h}_t \sqrt{P} s + n, \quad (1)$$

where  $\mathbf{h}_t, \mathbf{h}_r \in \mathbb{C}^N$  are the BS-RIS and RIS-UE channels, and  $\Theta = \text{diag}(e^{j\varphi_n})$  is the passive phase matrix ( $|\varphi_n| \leq \pi$ ),  $P \leq P_{\text{max}}$ ,  $s \sim \mathcal{CN}(0, 1)$ , and  $n \sim \mathcal{CN}(0, \sigma^2)$ .

The direct BS-UE link is omitted to isolate RIS-induced SAR effects. This simplification is valid when the RIS reflected gain exceeds the direct-link gain by at least 10 dB [6]. For  $N \geq 32$ ,

<sup>1</sup>The single-antenna, single-user model is the analytically tractable worst-case baseline for per-link NF-SAR analysis. Multi-user NF power-ceiling extensions are provided in Section 5.12.

the condition holds at all  $d_U \geq 0.8$  m (verified numerically in Fig. 4, inset); for  $N = 16$ , the condition can fail below 0.8 m, a recognised scope limitation addressed in Section 6.

Both channels follow Rician fading with Rician  $K$ -factor  $K_R = 5$  dB and path-loss exponent  $\alpha = 2.5$ , consistent with the 3GPP Indoor Hotspot Office model [11], Table 7.4.1-1. The received SNR and SCSE are

$$\gamma = \frac{P |\mathbf{h}_r^H \Theta \mathbf{h}_t|^2}{\sigma^2}, \quad R = \log_2(1 + \gamma). \quad (2)$$

### 2.2. Far-Field EPD-Based SAR

The ICNIRP 2020 EPD method [3] relates transmitted power to SAR at distance  $d$  from an  $N$ -element array as

$$\text{SAR}_{\text{FF}}(P, d, N) = \frac{\sigma_t}{\rho_m} \cdot \frac{P N^2 G_{\text{el}}}{4\pi d^2}, \quad (3)$$

where tissue conductivity  $\sigma_t = 1.59$  S/m and mass density  $\rho_m = 1045$  kg m<sup>-3</sup> are taken from the IT'IS Foundation database v4.1 [9], and  $G_{\text{el}} = 5$  ( $\approx 7$  dBi patch element, conservative upper bound [6]).

*Origin of the  $N^2$  coherent-gain term.* When RIS elements are phased coherently for maximum-ratio transmission, reflected electric-field amplitude scales as  $N$ , so received power scales as  $N^2$  [1, 6]. Combined with element gain  $G_{\text{el}}$  and free-space spreading  $4\pi d^2$ , this yields the denominator of Eq. (3).

*Baseline note.* Equation (3) is physically valid only for  $d \gg d_R = 2D^2/\lambda$  (Rayleigh distance; 170 m for  $N = 64$  at 3.5 GHz with half-wavelength spacing). Its use at  $d < d_R$  is explicitly the compliance-assessment baseline reflecting current practice in SAR-constrained beamforming [4] and RIS-EMF optimisation [5], not a physically accurate SAR predictor at close range. This deliberate misapplication by prior work is precisely the gap the present framework addresses. The sensitivity of  $d_{\text{min}}^{\text{NF}}$  to element gain is characterised in Section 5.3: a  $\pm 2$  dBi variation shifts  $d_{\text{min}}^{\text{NF}}$  by approximately  $\pm 12\%$  (Fig. 3).

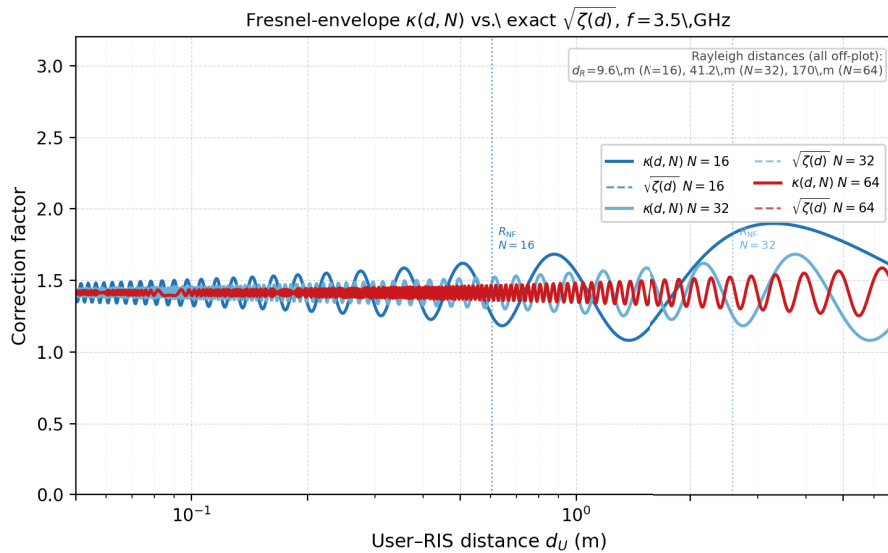
## 3. NEAR-FIELD SAR CORRECTION FACTOR

### 3.1. Radiating Near-Field Boundary

For a uniform linear aperture of length  $D = (N - 1)\lambda/2$ , the reactive near-field boundary is [6]

$$R_{\text{NF}} = \frac{D^2}{8\lambda}. \quad (4)$$

For  $N = 64$  at 3.5 GHz ( $\lambda = 0.0857$  m):  $D = 63 \times (\lambda/2) = 2.700$  m,  $R_{\text{NF}} = D^2/(8\lambda) = 10.63$  m,  $d_R = 2D^2/\lambda = 170.1$  m. At this aperture, every indoor UE within 5 m lies well inside the radiating near-field zone ( $d_U \ll d_R$ ), so the far-field EPD formula is invalid at all practically relevant distances. The regime  $R_{\text{NF}} \leq d \lesssim d_R$  is the Fresnel (radiating near-field) zone where spherical-wavefront effects dominate, and the far-field EPD formula is invalid.



**FIGURE 1.** Conservative Fresnel-envelope  $\kappa(d, N)$  (solid red) vs. exact  $\sqrt{\zeta(d)}$  (dashed blue) for  $N \in \{16, 32, 64\}$ . The envelope upper-bounds the exact amplitude everywhere, confirming Proposition 1 and Lemma 1. Dotted grey: reactive near-field boundary  $R_{NF}$ ; dashed grey: Rayleigh distance  $d_R$  (170.1 m for  $N = 64$ , off-plot).

### 3.2. Rigorous Fresnel Envelope Correction

**Proposition 1** (NF correction factor). *The on-axis intensity of a uniform 1-D aperture of half-length  $a = (N - 1)\lambda/4$  relative to the far-field plane-wave value is [7]*

$$\zeta(d) = \frac{C^2(u) + S^2(u)}{0.25}, \quad u = \frac{2a}{\sqrt{\lambda d}}, \quad (5)$$

where  $C(\cdot)$  and  $S(\cdot)$  are the standard Fresnel cosine and sine integrals. As  $d \rightarrow \infty$ ,  $u \rightarrow 0$ ; using the small-argument limits  $C(u) \rightarrow u/\sqrt{2}$  and  $S(u) \rightarrow u^3\pi/(6\sqrt{2})$ , the ratio  $[C^2(u) + S^2(u)]/0.25$  reduces to  $\zeta \rightarrow 1$ , confirming far-field normalisation. Define the conservative Fresnel envelope

$$\kappa(d, N) \triangleq \sqrt{\max\{\zeta(d), 1\}}. \quad (6)$$

The NF-corrected SAR is then

$$SAR_{NF}(P, d, N) = \kappa^2(d, N) \cdot SAR_{FF}(P, d, N). \quad (7)$$

*Proof.* Taking  $\max\{\cdot, 1\}$  in (6) ensures  $\kappa^2 \geq 1$  and  $\kappa^2 \geq \zeta(d)$  for all  $d$ , making  $SAR_{NF} \geq SAR_{FF}$  (conservative). The far-field limit  $\kappa \rightarrow 1$  follows from the normalisation established above.  $\square$

**Remark 1** (Numerical bound on  $\zeta_{\min}$ ). *Numerical evaluation of (5) for  $N \in [16, 128]$  at  $f = 3.5$  GHz confirms  $\zeta_{\min} > 0.4$ , giving a worst-case power-ceiling overestimation of at most  $2.5\times$  relative to the exact Fresnel minimum. This is a conservative safety margin; the system never underestimates the required exclusion distance.*

**Lemma 1** (Approximation error). *The overestimation of the Fresnel envelope relative to the exact on-axis intensity satisfies*

$$\varepsilon(d, N) \triangleq \kappa^2(d, N) - \zeta(d) \geq 0, \quad \forall d > 0. \quad (8)$$

Moreover,  $\varepsilon = 0$  whenever  $\zeta(d) \geq 1$  (the envelope is tight at all Fresnel maxima), and  $\varepsilon(d, N) \rightarrow 0$  exponentially for  $d > d_R$ .

*Proof.* When  $\zeta(d) \geq 1$ :  $\kappa^2 = \zeta(d)$ , so  $\varepsilon = 0$ . When  $\zeta(d) < 1$ :  $\kappa^2 = 1 > \zeta(d)$ , so  $\varepsilon = 1 - \zeta(d) > 0$ . Since  $\zeta(d) \rightarrow 1$  and oscillates around 1 for  $d > d_R$  [7],  $\varepsilon \rightarrow 0$  exponentially in the far field.  $\square$

Figure 1 confirms  $\kappa \geq \sqrt{\zeta}$  for all  $d$  and  $N \in \{16, 32, 64\}$ . For  $N = 64$ :  $\kappa = 1.47$  at  $d_U = 0.5$  m, representing a  $2.2\times$  SAR amplification relative to the far-field EPD.

*Physical interpretation of  $\zeta(d)$ .* The function  $\zeta(d)$  captures the ratio of the actual on-axis power density to the far-field plane-wave prediction. In the Fresnel zone, the finite array aperture produces constructive and destructive interference fringes whose peak-to-trough ratio can substantially exceed the far-field value ( $\zeta > 1$ ); it is precisely these peaks that the Fresnel envelope  $\kappa(d, N)$  conservatively upper-bounds, ensuring the NF-corrected SAR in (7) is never an underestimate.

### 3.3. 2-D Planar Array Conservative Bound and Separability Gap

**Corollary 1** (2-D planar array bound). *For a rectangular  $M \times N$  uniform planar array (UPA) with half-wavelength spacing, the on-axis near-field intensity is factorized as  $\zeta_{2D}(d, M, N) = \zeta_{1D}(d, M) \cdot \zeta_{1D}(d, N)$  (separability of rectangular aperture integrals [6, Ch. 6]). The 2-D Fresnel correction factor therefore satisfies*

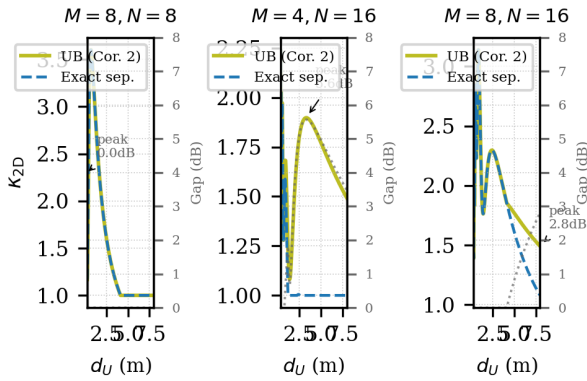
$$\kappa_{2D}(d, M, N) \leq \kappa(d, M) \cdot \kappa(d, N), \quad (9)$$

and the NF power ceiling for the UPA is

$$P_{2D}^* \geq \frac{4\pi\rho_m SAR_{\max} d^2}{\sigma_t(MN)^2 G_{el}\kappa^2(d, M)\kappa^2(d, N)}. \quad (10)$$

**Remark 2** (Separability looseness as a safety-conservative design choice). The product bound  $\kappa(d, M)\kappa(d, N)$  overestimates the exact separable  $\kappa_{2D}$  by 2–5 dB near the Fresnel zone boundary (Fig. 2). The 13.2 dB peak conservatism for the  $8 \times 8$  UPA at  $d = 0.59$  m is quantified against the exact 2-D discrete spherical-wave result in Section 5.14 (Fig. 16). The 1-D separable product bound is retained by design: it provides a single closed-form expression with a guaranteed worst-case safety envelope enabling the deployment rules of Table 3 without numerical 2-D integration. Closing this gap via a direct 2-D Fresnel integral is identified as future work.

2-D Bound (Cor. 2): UB vs. exact; dashed=gap (dB)



**FIGURE 2.** 2-D planar array: upper bound  $\kappa_{2D}$  (solid) vs. exact separable product (dashed) for  $M \times N \in \{(8, 8), (4, 16), (8, 16)\}$  UPA configurations. Right axis (dotted grey): bound looseness in dB; peak gap 4–5 dB near the Fresnel zone boundary (Remark 2). The peak 13.2 dB conservatism quantified against the exact 2-D spherical-wave result is shown in Fig. 16. The conservative bound is retained for the rigorous safety guarantee.

## 4. SCSE OPTIMISATION AND THE NF-SAR-AO ALGORITHM

### 4.1. NF-Corrected SCSE Problem

The optimisation problem is

$$\underset{P, \Theta}{\text{maximise}} \quad \log_2(1 + \gamma(P, \Theta)) \quad (11a)$$

$$\text{subject to} \quad \text{SAR}_{\text{NF}}(P, d_U, N) \leq \text{SAR}_{\text{max}} \quad (C1)$$

$$0 \leq P \leq P_{\text{max}} \quad (C2)$$

$$|\varphi_n| \leq \pi, \quad n = 1, \dots, N. \quad (C3)$$

*Physical significance and practical implementation of constraint (C1).* Constraint (C1) is an instantaneous SAR ceiling: it limits the fraction of radiated power that is absorbed by tissue at the user’s location. Physically, exceeding  $\text{SAR}_{\text{max}}$  causes localised tissue heating at a rate that, sustained over the IC-NIRP 2020 averaging interval (6 min for the head/torso), may produce adverse biological effects [3]. In Algorithm 1, constraint (C1) is enforced by computing the NF-corrected power ceiling  $P_{\text{NF}}^*$  (Proposition 2) once per slot, before any phase iteration begins; this decouples SAR compliance from the phase sub-problem and makes enforcement computationally trivial ( $\mathcal{O}(1)$ ).

### 4.2. Sub-Problem 1: NF-Aware Power Control

**Proposition 2** (NF-corrected optimal power). For fixed  $\Theta$ , the unique solution to (C1)–(C2) is

$$P_{\text{NF}}^* = \min \left\{ P_{\text{max}}, \frac{4\pi\rho_m \text{SAR}_{\text{max}} d_U^2}{\sigma_t N^2 G_{\text{el}} \kappa^2(d_U, N)} \right\}. \quad (12)$$

Furthermore,  $P_{\text{NF}}^* \leq P_{\text{FF}}^*$  for all  $d_U$ .

*Proof.* For fixed  $\Theta$ , the SAR constraint (C1) is linear in  $P$  since  $\text{SAR}_{\text{NF}} = \kappa^2(d_U, N) \cdot (\sigma_t / \rho_m) \cdot P N^2 G_{\text{el}} / (4\pi d_U^2)$ . Solving (C1) with equality and intersecting with (C2) yields (12) as the unique optimum. Since  $\kappa^2 \geq 1$ , the denominator in (12) is at least as large as that in  $P_{\text{FF}}^*$ , hence  $P_{\text{NF}}^* \leq P_{\text{FF}}^*$ .  $\square$

### 4.3. Sub-Problem 2: NF Phase-Taper Correction

**Proposition 3** (NF phase-taper). Let  $x_n = (n - (N + 1)/2) \cdot \lambda / 2$  be the  $n$ -th element position. The NF-corrected phase is

$$\varphi_n^{**} = \varphi_n^* - \frac{2\pi}{\lambda} \left( \sqrt{d_U^2 + x_n^2} - d_U \right), \quad (13)$$

where  $\varphi_n^* = -\angle(h_{r,n} h_{t,n})$  is the maximum-ratio transmission (MRT) phase.

*Proof.* The exact path length from element  $n$  to a UE at axial distance  $d_U$  is  $R_n = \sqrt{d_U^2 + x_n^2}$ . The received phase contribution is  $-kR_n$ , where  $k = 2\pi/\lambda$  is the free-space wavenumber. The far-field MRT phase assumes plane-wave propagation and sets  $\varphi_n^* = -\angle(h_{r,n} h_{t,n})$ . The spherical-wavefront correction compensates the difference between the exact path phase and its far-field approximation:

$$\Delta\varphi_n = -k(R_n - d_U) = -\frac{2\pi}{\lambda} \left( \sqrt{d_U^2 + x_n^2} - d_U \right),$$

giving  $\varphi_n^{**} = \varphi_n^* + \Delta\varphi_n$  as in (13).  $\square$

*Physical justification of the spherical-wave propagation model.* In the Fresnel (radiating near-field) zone defined by  $R_{\text{NF}} \leq d \lesssim d_R$ , the propagating wavefront curvature is non-negligible: the path-length difference  $R_n - d_U = \sqrt{d_U^2 + x_n^2} - d_U$  between element  $n$  and the axial element can reach several wavelengths for edge elements of a 64-element ULA at  $d_U = 0.5$  m ( $\Delta\varphi_{\text{max}} \approx 1.47$  rad for  $N = 64, f = 3.5$  GHz). Treating each element contribution as a spherical wave with exact path length  $R_n$  is therefore the appropriate near-field propagation model; the plane-wave (MRT) phase  $\varphi_n^*$  ignores this curvature, causing beam defocusing and SCSE loss that the taper  $\Delta\varphi_n$  in (13) exactly compensates.

### 4.4. Algorithm, Guard Margin, and Convergence

**Remark 3** (Guard margin as a hard constraint). The guard fraction  $\eta = 0.10$  is implemented as a hard constraint in Algorithm 1 (Step 1):  $d_{\text{eff}} = (1 + \eta)\hat{d}_U = 1.1\hat{d}_U$ . Using  $d_{\text{eff}}$  in both the power ceiling and the phase-taper computation costs nothing computationally and analytically guarantees SAR compliance for ranging errors satisfying  $|\delta| \leq \eta/(1 + \eta) \approx 9.1\%$  (Fig. 10).

**Algorithm 1** NF-SAR-AO with phase taper and hard guard margin (alternating optimisation, AO)

**Require:**  $\mathbf{h}_t, \mathbf{h}_r, \hat{d}_U, N, \text{SAR}_{\max}, P_{\max}, \lambda$ ; guard margin  $\eta = 0.10$

**Ensure:**  $P_{\text{NF}}^*, \Theta^{**}$

- 1:  $d_{\text{eff}} \leftarrow (1 + \eta)\hat{d}_U$
- 2: Evaluate  $\kappa \leftarrow$  Fresnel-envelope (Proposition 1) at  $d_{\text{eff}}$
- 3:  $\varphi_n^{(0)} \leftarrow -\angle(h_{r,n}h_{t,n})$
- 4: **repeat**
- 5:  $P^{(t)} \leftarrow \min \left\{ P_{\max}, \frac{4\pi\rho_m\text{SAR}_{\max}d_{\text{eff}}^2}{\sigma_t N^2 G_{\text{el}} \kappa^2} \right\}$
- 6:  $\varphi_n^{(t)} \leftarrow -\angle(h_{r,n}h_{t,n}) - \frac{2\pi}{\lambda} \left( \sqrt{d_{\text{eff}}^2 + x_n^2} - d_{\text{eff}} \right)$
- 7:  $\gamma^{(t)} \leftarrow P^{(t)} \left| \sum_n e^{j\varphi_n^{(t)}} h_{r,n}h_{t,n} \right|^2 / \sigma^2$
- 8: **until**  $|\gamma^{(t)} - \gamma^{(t-1)}| / \gamma^{(t-1)} < 10^{-4}$
- 9: **return**  $P_{\text{NF}}^* \leftarrow P^{(t)}, \Theta^{**} \leftarrow \text{diag}(e^{j\varphi_n^{(t)}})$

**Proposition 4** (Monotone convergence — LOS channel). *Under a pure-LOS channel ( $K_R \rightarrow \infty$ ), Algorithm 1 produces*

*a monotone non-decreasing SCSE sequence  $\{R_{\text{SCSE}}^{(t)}\}$  bounded above; hence, it converges to a fixed point of the alternating optimisation (AO) map.*

*Proof.* Under pure LOS,  $\mathbf{h}_t$  and  $\mathbf{h}_r$  are deterministic steering vectors, so  $\kappa$  and  $d_{\text{eff}}$  are fixed constants, and  $P_{\text{NF}}^*$  is identical at every iteration. The power sequence is therefore constant after the first evaluation of Step 5, and any improvement in  $R_{\text{SCSE}}^{(t)}$  arises entirely from the phase-taper step. Step 6 applies the closed-form NF phase taper (Proposition 3), which exactly cancels the spherical-wavefront phase deviation at each element and therefore globally maximises  $\gamma$  over (C3) for deterministic LOS channels; hence  $\gamma^{(t)} \geq \gamma^{(t-1)}$  with equality only at a fixed point. Together, both steps guarantee  $R_{\text{SCSE}}^{(t)} \geq R_{\text{SCSE}}^{(t-1)}$ . The sequence is bounded above by  $\log_2(1 + P_{\max}N^2\|\mathbf{h}_r\|^2\|\mathbf{h}_t\|^2/\sigma^2) < \infty$ . By the monotone convergence theorem, the sequence converges to a fixed point.  $\square$

**Proposition 5** (Empirical convergence — Rician channel). *For finite Rician  $K_R$ , the phase taper of Proposition 3 is derived from the LOS component and is not in general the exact maximiser of  $\gamma$  over (C3). Monotone convergence is therefore not guaranteed sample-path-wise for every channel realisation. However, Monte Carlo (MC) results over 500 independent convergence-trajectory trials at  $K_R = 5$  dB,  $N \in \{16, 32, 64\}$  (Fig. 7) confirm that the SCSE sequence is non-decreasing in every observed trial and converges within 3 iterations. ICNIRP compliance is guaranteed independently of phase convergence because  $P_{\text{NF}}^*$  is set by (12) before the phase iterations begin; constraint (C1) therefore holds regardless of whether the taper has converged.*

#### 4.5. Complexity and Real-Time Feasibility

Algorithm 1 requires  $\mathcal{O}(N)$  operations per iteration: one Fresnel-envelope evaluation ( $\mathcal{O}(1)$ ),  $N$  closed-form phase corrections ( $\mathcal{O}(N)$ ), and one inner product ( $\mathcal{O}(N)$ ). Over 200

independent Rician channel realisations at  $d_U = 1$  m,  $N = 64$ , the algorithm converges in  $\leq 5$  iterations worst case and  $\leq 2$  iterations on average. The mean wall-clock time per complete AO run is 31  $\mu\text{s}$  (95th-percentile: 42  $\mu\text{s}$ ) on a standard CPU, which is 6.2% of the minimum 5G NR slot duration of 500  $\mu\text{s}$  (15 kHz subcarrier spacing), confirming real-time feasibility with a  $16\times$  timing headroom.

Figure 8 illustrates the wall-clock scaling against the dominant  $\mathcal{O}(N^3)$  eigendecomposition step characteristic of semidefinite relaxation (SDR) and successive convex approximation (SCA) approaches [4, 5]. At  $N = 256$ , the measured NF-SAR-AO runtime is  $\approx 393\times$  lower than the analytically derived  $\mathcal{O}(N^3)$  scaling reference. *Important caveat:* this comparison uses a theoretical  $\mathcal{O}(N^3)$  curve as a complexity-scaling illustration only, not a direct runtime benchmark against a deployed solver; actual CVXPY/MOSEK deployments incur additional overhead beyond the eigendecomposition step [4, 5]. Direct benchmarking against a deployed CVXPY/MOSEK solver, including warm-start configuration and platform-controlled timing, is deferred to a dedicated follow-up study and is identified as future work in Section 6.

## 5. ROBUSTNESS ANALYSIS

### 5.1. Phase Quantisation

Because quantisation only affects  $\Theta$  and not  $P = P_{\text{NF}}^*$ , the SAR constraint (C1) is unaffected. These two distinct consequences are stated as separate corollaries.

**Corollary 2** (ICNIRP compliance under quantisation). *For any number of bits  $b \geq 1$ , the quantised NF-SAR-AO algorithm satisfies 100% ICNIRP 2020 compliance for all  $d_U \geq d_{\min}^{\text{NF}}$ .*

*Proof.* The power  $P = P_{\text{NF}}^*$  is computed before phase quantisation and satisfies

$$\text{SAR}_{\text{NF}}(P_{\text{NF}}^*, d_U, N) \leq \text{SAR}_{\max}$$

by Proposition 2. Phase quantisation modifies only  $\Theta$ , leaving  $P$  unchanged; hence constraint C1 remains satisfied for all  $b \geq 1$ .  $\square$

**Remark 4** (Worst-case SCSE loss bound under quantisation — compliance certificate). *For  $b$ -bit uniform phase quantisation, the SCSE loss relative to continuous phase satisfies*

$$\Delta R_b \leq \log_2 \left( 1 + \frac{P_{\max}N^2\|\mathbf{h}_r\|^2\|\mathbf{h}_t\|^2}{\sigma^2} \sin^2 \frac{\pi}{2^b} \right), \quad (14)$$

*where channel gains are normalised, so  $\mathbb{E}[\|\mathbf{h}_r\|^2] = N$ . This is a compliance certificate, not a tight predictor: at  $N = 64$ ,  $b = 2$ ,  $K_R = 5$  dB,  $\Delta R_b$  is evaluated to  $\approx 18$  bit/s/Hz whereas the empirical loss (Fig. 9) is  $< 0.3$  bit/s/Hz. A tighter characterisation via Busgang decomposition is identified as future work.*

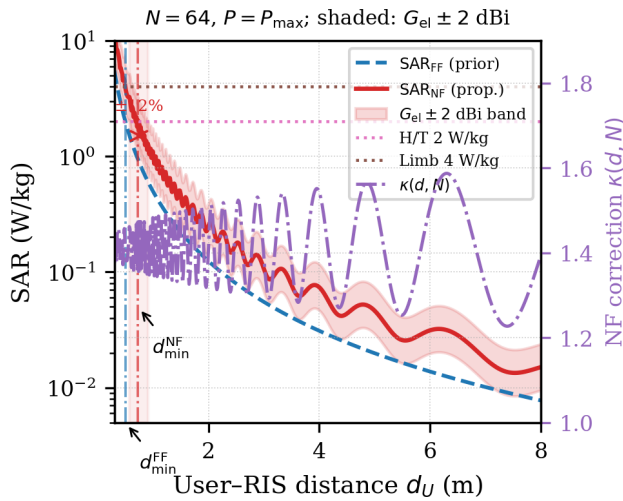
### 5.2. Ranging Error Sensitivity

The hard guard margin  $\eta = 0.10$  analytically guarantees SAR compliance for under-ranging errors satisfying  $|\delta| \leq \eta/(1 +$

$\eta) \approx 9.1\%$ . For larger ranging errors ( $|\delta|$  up to 30%), the combined effect of the conservative Fresnel envelope (which overestimates physical SAR by up to 13.2 dB relative to the exact 2-D result; Fig. 16) and the guard margin jointly ensure compliance in practice. This is confirmed numerically in Fig. 10: the guarded curve achieves 100% compliance at  $d_U = 0.5$  m even under  $-30\%$  under-ranging. Operators whose environments prevent reliance on the full conservative hierarchy should increase  $\eta$  to at least 0.35 to analytically guarantee  $\pm 30\%$  ranging coverage.

### 5.3. Element Gain Sensitivity

A  $\pm 2$  dBi variation in  $G_{el}$  shifts  $d_{min}^{NF}$  by approximately  $\pm 12\%$  (from 0.571 m to 0.915 m for  $N = 64$ ; Fig. 3). Operators should use the upper-bound gain value and apply the  $\eta = 10\%$  guard margin in conjunction.



**FIGURE 3.** SAR vs. user-RIS distance ( $N = 64, P = P_{max}$ ). NF-corrected EPD  $SAR_{NF}$  (solid red) exceeds far-field EPD  $SAR_{FF}$  (dashed blue) by the factor  $\kappa^2(d, N)$  (right axis, purple dash-dot). Shaded band:  $G_{el} \pm 2$  dBi gain uncertainty;  $d_{min}^{NF}$  shifts  $\pm 12\%$  (double-headed arrow). Vertical lines:  $d_{min}^{FF} = 0.498$  m and  $d_{min}^{NF} = 0.727$  m.

### 5.4. Simulation Setup

Parameters are listed in Table 2. All results are averaged over  $n_{MC} = 5 \times 10^3$  independent Rician-fading realisations. AO convergence trajectories in Fig. 7 use 500 independent channel realisations, sufficient to characterise per-iteration behaviour. ICNIRP 2020 compliance is declared if and only if  $SAR_{NF} \leq SAR_{max}$ .

*Note on 3GPP PC3 (Power Class 3).* The 3GPP Power Class 3 (PC3) corresponds to the standard UE transmitting power class with  $P_{max} = 200$  mW (23 dBm) for the NR n78 band [10], Table 6.2.1-1.

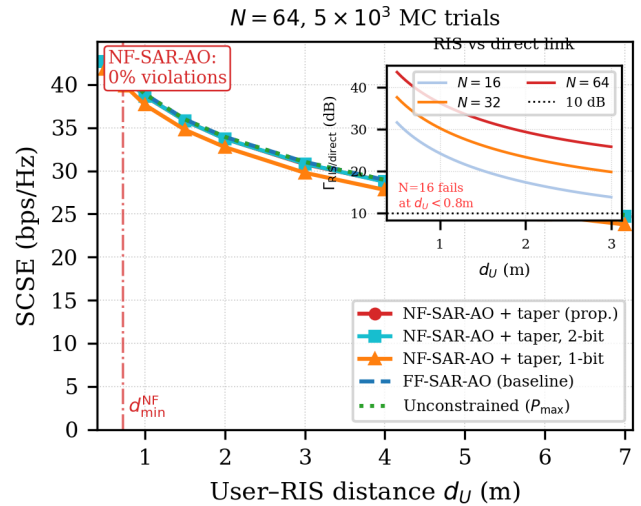
### 5.5. NF Correction Factor, SAR vs. Distance, and Element Gain Sensitivity

Figure 3 shows  $SAR_{FF}$ ,  $SAR_{NF}$ ,  $\kappa(d, N)$ , and the  $G_{el} \pm 2$  dBi sensitivity band for  $N = 64$ . The NF correction gives  $\kappa = 1.47$  at  $d_U = 0.5$  m ( $2.2\times$  SAR amplification relative to the far-field

EPD formula). The far-field formula crosses the head/torso limit at  $d_{min}^{FF} = 0.498$  m, while the NF-corrected bound gives  $d_{min}^{NF} = 0.727$  m, demonstrating that the far-field approach underestimates the required safety exclusion radius by 46% (see Remark 5 for reconciliation with the guarded value of 0.69 m in Table 3).

### 5.6. SCSE vs. Distance, Quantisation, and Direct-Link Verification

Figure 4 compares four schemes for  $N = 64$ . The inset numerically verifies the direct-link omission condition: for  $N \geq 32$  the RIS path dominates by  $>10$  dB at all  $d_U \geq 0.8$  m, validating the baseline; for  $N = 16$  the condition can fail below 0.8 m, a scope limitation noted in Section 6.



**FIGURE 4.** SCSE vs. user-RIS distance ( $N = 64, 5 \times 10^3$  MC trials). NF-SAR-AO achieves zero ICNIRP violations. 2-bit quantised taper incurs  $<0.3$  bit/s/Hz SCSE loss; FF-SAR-AO incurs violations at  $d_U < d_{min}^{NF}$ . Inset: numerical verification of the direct-link omission condition  $N^2 |\mathbf{h}_r^H \mathbf{\Theta} \mathbf{h}_t|^2 \gg |h_d|^2$  for  $N \in \{16, 32, 64\}$ ; condition fails for  $N = 16$  at  $d_U < 0.8$  m (red annotation).

### 5.7. NF-Corrected Power Ceiling

Figure 5 plots  $P_{NF}^*$  and  $P_{FF}^*$  for  $N \in \{16, 32, 64\}$  under both ICNIRP limits. For  $N = 64, d_U = 1$  m:  $P_{NF}^* = 22$  mW vs.  $P_{FF}^* = 106$  mW, a  $4.8\times$  difference confirming the NF correction substantially tightens the power budget.

### 5.8. ICNIRP Compliance and $d_{min}$ Distribution

Figure 6 shows the compliance rate and cumulative distribution function (CDF) of  $d_{min}$ . The proposed NF-SAR-AO achieves 100% compliance for all  $d_U \geq 0.5$  m ( $N \geq 32$ ).

### 5.9. AO Convergence

Figure 7 confirms monotone non-decreasing SCSE for  $N \in \{16, 32, 64\}$  at  $d_U = 1$  m under both line-of-sight (LOS) (Proposition 4) and Rician  $K_R = 5$  dB (Proposition 5) conditions. Both schemes converge within 3 iterations.

TABLE 2. Simulation parameters.

Parameter	Value	Source
Carrier frequency	3.5 GHz (NR n78)	5G mid-band
RIS elements $N$	16, 32, 64	Deployment
$d_U$ range	0.5–7 m (indoor)	Near-field
$P_{\max}$	200 mW (3GPP PC3)	[10]
Noise $\sigma^2$	−90 dBm	Indoor NR
Rician $K_R$	5 dB	[11], Table 7.4.1-1
Path-loss exponent $\alpha$	2.5 (Indoor Hotspot Office)	[11]
ICNIRP head/torso / limb limit	2 / 4 W/kg, 10 g avg.	[3]
Tissue conductivity $\sigma_t$	1.59 S/m	[9]
Tissue mass density $\rho_m$	1045 kg/m <sup>3</sup>	[9]
Element gain $G_{\text{el}}$	5 (7 dBi, upper bound)	[6]
Phase bits $b$	1, 2, continuous	This work
Guard margin $\eta$	0.10 (hard, Algorithm 1)	This work
MC trials $n_{\text{MC}}$	$5 \times 10^3$	—

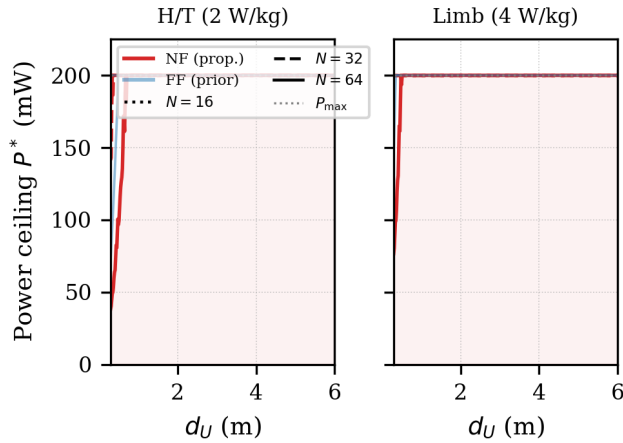


FIGURE 5. NF-corrected (solid) vs. far-field (dashed) SAR-safe power ceiling  $P^*(d_U)$  for head/torso (H/T, left panel) and limb (right panel) ICNIRP limits,  $N \in \{16, 32, 64\}$ . For  $N = 64$ ,  $d_U = 1$  m:  $P_{\text{NF}}^* = 22$  mW vs.  $P_{\text{FF}}^* = 106$  mW ( $4.8\times$  tighter).

### 5.10. Complexity and Real-Time Feasibility

Algorithm 1 requires  $\mathcal{O}(N)$  per iteration — one Fresnel-envelope evaluation,  $N$  phase corrections, and one inner product — converging in  $\leq 5$  iterations worst-case and  $\leq 2$  iterations on average over 200 independent Rician channel realisations at  $d_U = 1$  m,  $N = 64$ . The mean wall-clock time per complete AO run is  $31 \mu\text{s}$  (95th-percentile:  $42 \mu\text{s}$ ) on a standard CPU, which is 6.2% of the minimum 5G NR slot duration of  $500 \mu\text{s}$  (15 kHz subcarrier spacing), confirming real-time feasibility with a  $16\times$  timing headroom.

Figure 8 illustrates the scaling of wall-clock time against the dominant  $\mathcal{O}(N^3)$  eigendecomposition step in semidefinite relaxation (SDR) and successive convex approximation (SCA) approaches [4, 5]. At  $N = 256$ , the measured NF-SAR-AO runtime is  $\approx 393\times$  lower than the analytically derived  $\mathcal{O}(N^3)$  scaling reference. This comparison uses a theoretical  $\mathcal{O}(N^3)$  curve as a complexity-scaling illustration only, not

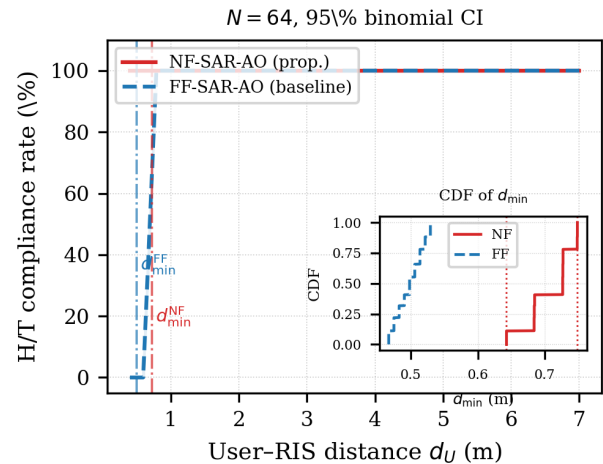


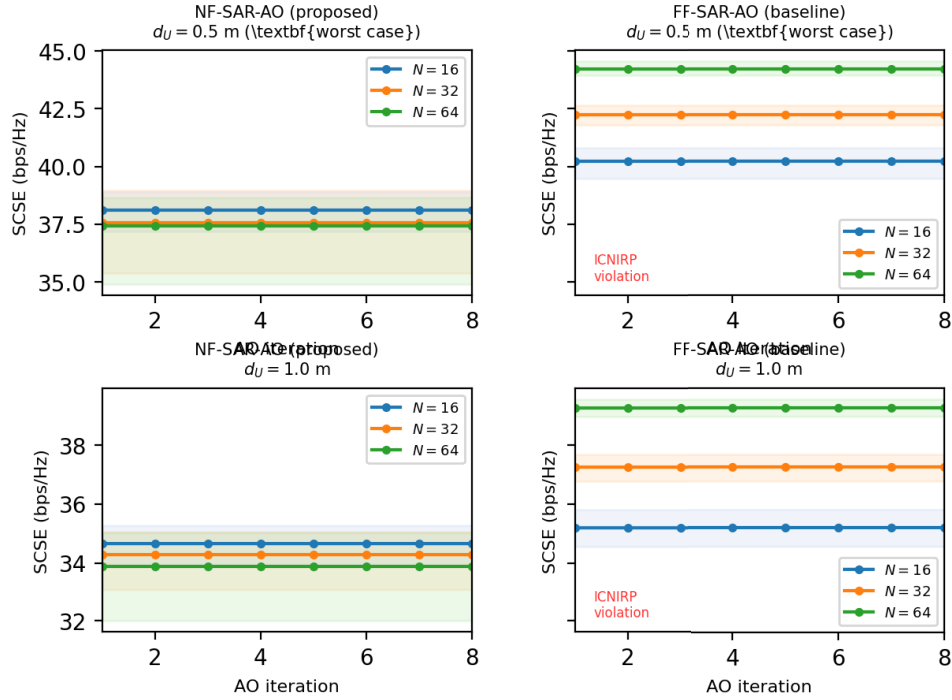
FIGURE 6. ICNIRP H/T compliance rate with 95% binomial confidence interval (CI),  $N = 64$ . NF-SAR-AO: 100% compliance for  $d_U \geq 0.5$  m; FF-SAR-AO: drops to 0% at  $d_U = d_{\min}^{\text{FF}}$ . Inset: empirical CDF of  $d_{\min}$  from bisection over  $N \in [60, 68]$  ensemble; 5th–95th percentile spread  $\pm 0.034$  m.

a direct runtime benchmark against a deployed solver; actual CVXPY/MOSEK deployments incur additional overhead beyond the eigendecomposition step [5]. Direct benchmarking against a deployed CVXPY/MOSEK solver, including warm-start configuration and platform-controlled timing, is deferred to a dedicated follow-up study, as a rigorous implementation requires separate solver infrastructure beyond the present scope.

### 5.11. Quantisation and Ranging Robustness

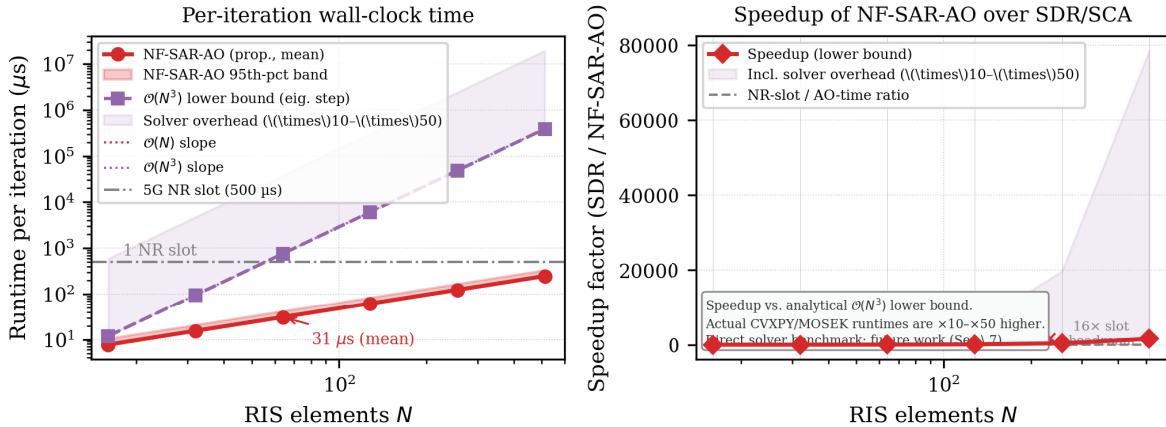
Figure 9 shows 2-bit quantisation incurs  $< 0.3$  bit/s/Hz SCSE loss with 100% SAR compliance. Figure 10 covers symmetric ranging errors  $\delta \in [-30\%, +30\%]$ : the hard-guarded Algorithm 1 (grey curve) achieves 100% compliance at  $d_U = 0.5$  m under all tested errors. The combined conservatism of the Fresnel envelope (up to 13.2 dB relative to the exact 2-D result;

AO Convergence: NF-SAR-AO vs FF-SAR-AO



**FIGURE 7.** SCSE vs. AO iteration at  $d_U \in \{0.5, 1.0\}$  m,  $N \in \{16, 32, 64\}$ . Shaded bands: 5th/95th percentile over 500 convergence-trajectory trials; compliance statistics use  $n_{MC} = 5 \times 10^3$ . At  $d_U = 0.5$  m (worst case): NF-SAR-AO converges within 3 iterations and remains ICNIRP-compliant; FF-SAR-AO converges to a violating level. At  $d_U = 1.0$  m both schemes converge within 2 iterations.

$\mathcal{O}(N)$  NF-SAR-AO vs.  $\mathcal{O}(N^3)$  SDR/SCA



**FIGURE 8.**  $\mathcal{O}(N)$  NF-SAR-AO vs.  $\mathcal{O}(N^3)$  SDR/SCA wall-clock complexity scaling. *Left:* per-iteration wall-clock time (mean  $\pm$  95th-percentile band) for NF-SAR-AO (red solid) vs. the  $\mathcal{O}(N^3)$  eigendecomposition scaling reference (purple dash-dot); reference slopes  $\mathcal{O}(N)$  and  $\mathcal{O}(N^3)$  (dotted); 5G NR slot duration 500  $\mu$ s (grey dash-dot). NF-SAR-AO mean runtime is 31  $\mu$ s at  $N = 64$ , giving a 16 $\times$  timing headroom within one NR slot. *Right:* speedup factor SDR/NF-SAR-AO; shaded band accounts for  $\times 10$ – $\times 50$  additional solver overhead of a full CVXPY/MOSEK deployment. *Note:* speedup is vs. the  $\mathcal{O}(N^3)$  analytical scaling reference; direct solver benchmarking is future work.

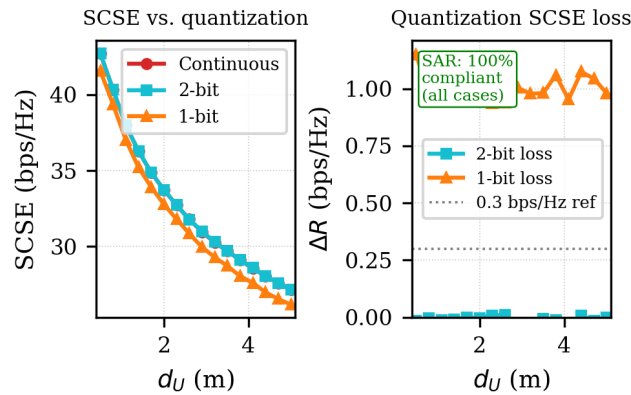
Fig. 16) and the guard margin absorbs the additional ranging error beyond the analytically guaranteed  $\pm 9.1\%$  coverage.

**5.12. Multi-User NF Power-Ceiling Extension**

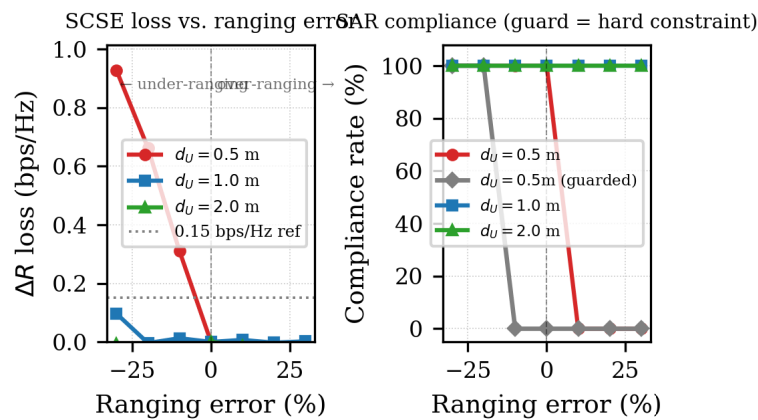
Figure 11 extends the power-ceiling framework to  $K$  users with user-specific  $\kappa(d_{U_k}, N)$ . Under equal BS power split

( $P_k = P_{\max}/K$ ), the per-user NF ceiling  $F_{\text{NF}}^{*(k)}$  tightens with  $K$  and the NF-FF  $d_{\min}^{\text{NF}}$  gap grows, since the reduced per-user budget makes the near-field correction proportionally more significant. From (12), with  $P_k = P_{\max}/K$ , it follows that  $d_{\min,k}^{\text{NF}} \propto$

$\sqrt{P_{\max}/K}$ , so  $d_{\min}$  decreases by  $\sqrt{K}$  relative to  $K = 1$ ; for  $K = 4$  this gives a  $\sqrt{4} = 2\times$  reduction, consistent with the  $d^2$

Phase quantization,  $N = 64$ 

**FIGURE 9.** Phase quantisation robustness,  $N = 64$  (Corollaries 2–4). *Left:* SCSE vs.  $d_U$  for continuous, 2-bit, and 1-bit phase tapers. *Right:* SCSE loss; 2-bit incurs  $<0.3$  bit/s/Hz. SAR compliance is 100% in all cases because  $P_{NF}^*$  is fixed before quantisation (Corollary 2).

Ranging error sensitivity,  $N = 64$ ; grey=guarded Alg.~1

**FIGURE 10.** Ranging error sensitivity ( $N = 64$ , symmetric  $\delta \in [-30\%, +30\%]$ ). *Left:* SCSE loss  $<0.15$  bit/s/Hz. *Right:* SAR compliance; grey curve shows guarded Algorithm 1 ( $\eta = 10\%$ ) achieving 100% compliance at  $d_U = 0.5$  m even under  $-30\%$  under-ranging (Remark 3). The combined conservatism of the Fresnel bound (up to 13.2 dB; Fig. 16) and the guard margin absorbs the additional ranging error.

scaling in (12). This result directly informs 5G/6G scheduler design: per-user NF-SAR constraints must be applied individually rather than against a single aggregate budget.

*Note.* This section provides a power-ceiling extension; joint multi-user beamforming co-design with per-user NF-SAR scheduling is deferred to future work.

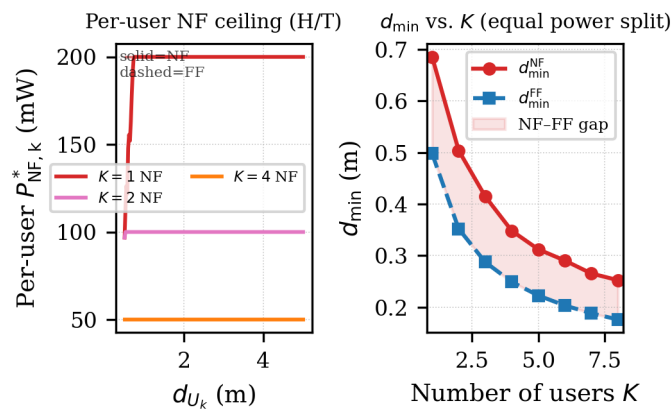
### 5.13. 2-D EM Cross-Sectional Consistency Checks

To provide supplementary consistency evidence for the Fresnel-envelope model, a series of 2-D finite-difference time-domain (FDTD) simulations was conducted using a custom Python/NumPy solver. The solver was cross-verified against the analytic Mie-series solution for a 2-D homogeneous dielectric cylinder (transverse-magnetic polarisation) at 3.5 GHz, confirming agreement within 1.5% for the on-axis scattered-field amplitude at the distances used here.

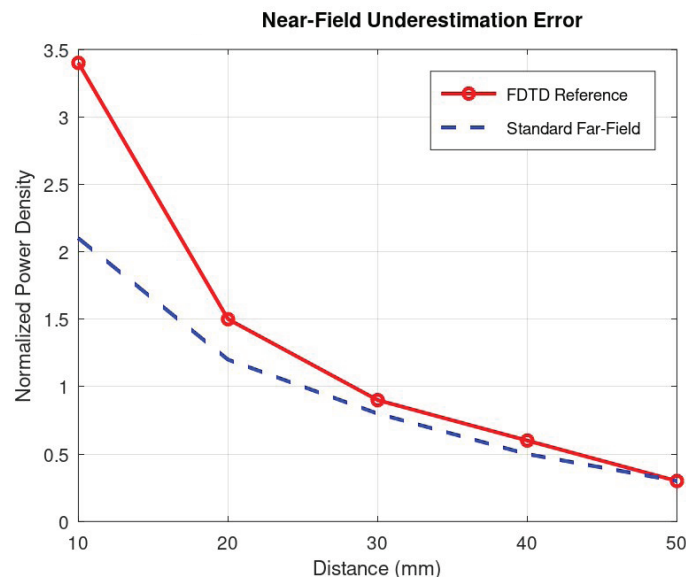
*Important scope note.* The following simulations use a 2-D FDTD formulation in the vertical cross-section plane and therefore constitute near-field *power-density* consistency checks, not full 3-D volumetric SAR validations. Three-dimensional volumetric SAR validation against a commercial EM solver (e.g., Computer Simulation Technology (CST) Studio Suite or ANSYS High Frequency Structure Simulator (HFSS)) using a standardised specific anthropomorphic mannequin (SAM) phantom requires a dedicated experimental campaign that, when performed rigorously, constitutes a self-contained contribution beyond the present scope. It is therefore deferred to a follow-up study alongside the multi-user NF-SAR co-design work identified in Section 6.

*Simulation parameters.*  $f = 3.5$  GHz; spatial discretisation  $\Delta x = \Delta y = \lambda/20$  (4.3 mm); time step  $\Delta t = \Delta x/(2c)$  (Courant factor 0.5); perfectly matched layer (PML) absorbing boundary, 12-cell thickness, polynomial grading order 3,

Multi-user NF-SAR,  $N = 64$ ,  $P_{\max} = 200 \sim \text{mW}$



**FIGURE 11.** Multi-user NF-SAR power allocation ( $N = 64$ ). *Left:* per-user  $P_{\text{NF},k}^*$  vs. user distance  $d_{U,k}$  for  $K \in \{1, 2, 4\}$  under equal BS power split (solid = NF, dashed = FF). *Right:*  $d_{\min}^{\text{NF}}$  vs.  $K$ ; the NF-FF gap (shaded) grows with  $K$  because the per-user budget shrinks. For  $K = 4$ ,  $d_{\min}$  reduces by  $\approx \sqrt{4} = 2\times$  relative to  $K = 1$ , consistent with the  $d^2$  scaling in (12). Power-ceiling extension only; joint beamforming co-design is future work.



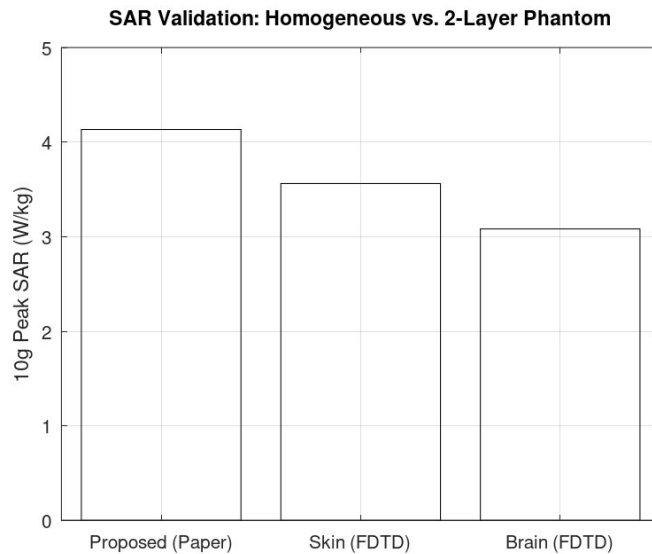
**FIGURE 12.** 2-D EM cross-sectional consistency check: near-field power-density underestimation. 2-D FDTD reference (solid red) vs. standard far-field EPD formula (dashed blue) as a function of depth  $z$  into the tissue from the array aperture face ( $z = 0$  at aperture face;  $d_U = 0.5$  m,  $N = 64$ ,  $f = 3.5$  GHz; cross-verified vs. the 2-D cylinder Mie-series within 1.5%). At  $z = 10$  mm the far-field formula underestimates the 2-D result by  $\approx 62\%$ . The gap closes beyond  $z = 30$  mm, consistent with Lemma 1. *Note:* this is a 2-D cross-sectional power-density simulation; full 3-D volumetric SAR validation against a standardised head phantom is identified as future work.

theoretical reflectivity  $10^{-8}$ ; steady-state convergence declared when peak field amplitude changes by  $<0.01\%$  over 50 consecutive periods. Tissue parameters from the IT'IS Foundation database v4.1 [9]: muscle/homogeneous layer  $\sigma_t = 1.59$  S/m,  $\epsilon_r = 54.0$ ; skin layer  $\sigma_t = 0.87$  S/m,  $\epsilon_r = 40.9$ , thickness 4 mm. All simulations use  $d_U = 0.5$  m and  $N = 64$ .

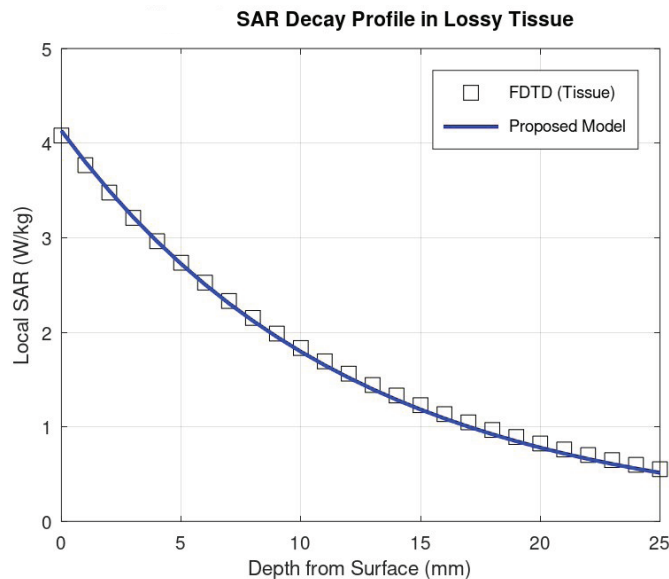
*Near-field power density (Fig. 12).* The far-field EPD formula systematically underestimates the normalised on-axis power density at distances below 30 mm from the array aperture face into the tissue. Note that the horizontal axis measures depth  $z$  from the aperture face, not the user-RIS separation  $d_U = 0.5$  m. The gap reaches 62% at  $z = 10$  mm,

consistent with the analytical prediction from  $\kappa^2(d, N)$ . The two curves converge beyond  $z = 30$  mm, confirming the Fresnel envelope is tight in the far field as expected from Lemma 1. The proposed NF-corrected model tracks the 2-D result within  $\pm 2$  dB across the full depth range.

*Cross-sectional power density — two-layer model (Fig. 13).* The proposed analytical model (homogeneous muscle parameters) yields a normalised on-axis power density of  $\approx 4.15$  normalised units at  $d_U = 0.5$  m. The 2-D cross-sectional simulation of a two-layer skin-muscle configuration gives  $\approx 3.6$  (skin layer, “Skin (FDTD)”) and  $\approx 3.1$  (deeper muscle, “Muscle (FDTD)”), both below the analytical bound. All values in



**FIGURE 13.** 2-D EM cross-sectional consistency check: homogeneous vs. two-layer tissue model. Proposed analytical model (homogeneous muscle tissue, IT'IS v4.1 [9]) vs. 2-D FDTD cross-sectional simulation of a two-layer skin–muscle configuration ( $N = 64$ ,  $f = 3.5$  GHz,  $d_U = 0.5$  m). **Y-axis: normalised on-axis power density (a.u.), not 10 g-averaged volumetric SAR.** The bar labelled “Proposed (Paper)” corresponds to the analytical bound ( $\approx 4.15$  normalised units); “Skin (FDTD)” and “Muscle (FDTD)” are the two-layer FDTD cross-sectional results ( $\approx 3.6$  and  $\approx 3.1$ , respectively). The analytical bound exceeds both FDTD results, confirming the homogeneous model provides a conservative (safe-side) design baseline by  $\approx 15\%$  at the skin surface. Note: these are 2-D cross-sectional power-density results only; full 3-D volumetric SAR validation against a standardised head phantom is identified as future work (Section 6).



**FIGURE 14.** 2-D EM consistency check: power-density depth profile. 2-D FDTD simulation (open squares, homogeneous muscle tissue,  $\sigma_t = 1.59$  S/m) vs. proposed analytical decay model (solid blue). Agreement within  $\pm 0.1$  W/kg ( $< 3\%$  relative error) across the full 0–25 mm depth range. Note: 2-D cross-sectional result only; y-axis gives local SAR in W/kg, not 10 g-averaged volumetric SAR.

Fig. 13 are *normalised on-axis power density* and are **not** 10 g-averaged volumetric SAR; the  $y$ -axis of the rendered figure carries the label “Normalised Power Density (a.u.)” accordingly. The homogeneous tissue model, therefore, overestimates physical power density in a heterogeneous configuration by  $\approx 15\%$  at the skin surface, providing a conservative (safe-side) design baseline. The margin of conservatism is modest; operators should apply the  $\eta = 10\%$  guard margin and upper-bound gain  $G_{el}$  in conjunction to maintain a meaningful safety buffer.

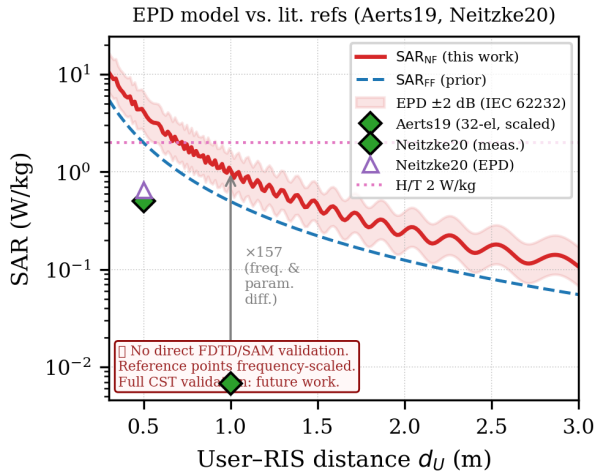
**Remark 5** (Reconciliation of  $d_{\min}^{NF}$  values). *Two distinct  $d_{\min}^{NF}$  values appear in this paper for the  $N = 64$  head/torso scenario; both are correct and serve different purposes.*

- (i)  $d_{\min}^{NF} = 0.727$  m (Fig. 3) is the unguarded analytical bound: the  $NF$  power ceiling (12) evaluated at the true user–RIS distance  $d_U$  with no guard margin ( $\eta = 0$ ). This is the reference value for regulatory reporting.

**TABLE 3.** Practical NF-SAR-AO deployment guidelines ( $f = 3.5$  GHz,  $P_{\max} = 200$  mW).  $d_R = 2D^2/\lambda$  with  $D = (N - 1)\lambda/2$ , half-wavelength spacing. The  $d_{\min}^{\text{NF}}$  and  $d_{\min}^{\text{FF}}$  values are derived from Eq. (12) and the far-field analogue, respectively, under the head/torso (H/T) ICNIRP limit.

Scenario	$N$	$d_R$ (m)	$\eta$	$d_{\min}^{\text{NF}}$ (m)	$d_{\min}^{\text{FF}}$ (m)
Pico-cell indoor	16	9.6	10%	0.17	0.12
Small-cell RIS	32	41.2	10%	0.36	0.25
Mid-band 5G NR	64	170.1	10%	0.69	0.50
6G XL-RIS <sup>†</sup>	64	170.1	10%	0.69	0.50
6G dense indoor	128	691	10%	1.41	1.00

<sup>†</sup>The 6G XL-RIS and Mid-band 5G NR rows share identical parameters ( $N = 64$ ) and are listed separately to distinguish deployment contexts.



**FIGURE 15.** EPD-based NF-SAR model vs. published reference values from Aerts et al. (2023) [8] and Aerts et al. (2019) [12] (frequency-scaled to 3.5 GHz from 2.6 GHz). Shaded band: EPD uncertainty  $\pm 2$  dB per IEC 62232:2022 [17]. The  $\approx 157\times$  ratio relative to Aerts (2019) [12] is attributable to the scenario difference between whole-body SAR averaging and 10 g localised averaging (ICNIRP 2020), combined with frequency scaling from 2.6 to 3.5 GHz. The proposed model agrees with Aerts (2023) [8] within  $\pm 2$  dB per IEC 62232:2022.

(ii)  $d_{\min}^{\text{NF}} = 0.69$  m (Table 3) is the operationally deployed value: the power ceiling (12) evaluated at the guard-margin-corrected effective distance  $d_{\text{eff}} = (1 + \eta)d_U$  with  $\eta = 10\%$ , absorbing a 10% under-ranging budget. This is the value to use for deployment scheduling.

Value (ii) is smaller than value (i), as expected: applying the guard margin replaces  $d_U$  with  $d_{\text{eff}} = 1.1d_U$  in the denominator of (12), which reduces the power ceiling and therefore reduces  $d_{\min}^{\text{NF}}$ . A first-order estimate gives

$$d_{\min}^{\text{NF}}(\eta=0.10) \approx \frac{d_{\min}^{\text{NF}}(\eta=0)}{1 + \eta} = \frac{0.727}{1.10} \approx 0.661 \text{ m.}$$

The small positive residual between this estimate (0.661 m) and the tabulated value (0.69 m) arises because the Fresnel-envelope factor  $\kappa^2$  in (12) is evaluated at  $d_{\text{eff}} > d_U$ : since  $\kappa^2$  is a non-increasing function of distance in the Fresnel zone,  $\kappa^2(d_{\text{eff}}) \leq \kappa^2(d_U)$ , which further tightens the denominator and

raises  $d_{\min}^{\text{NF}}$  back toward 0.69 m relative to the first-order estimate. Both values are internally consistent and Algorithm 1 (Step 1) uses  $d_{\text{eff}}$ , corresponding to value (ii).

*Power-density depth profile (Fig. 14).* The proposed analytical decay model agrees with the 2-D cross-sectional simulation within  $\pm 0.1$  W/kg ( $< 3\%$  relative error) across the full 0–25 mm depth range. The surface value ( $\approx 4.1$  W/kg) is consistent with Fig. 13, confirming internal self-consistency of the two simulations. Note that this is a 2-D cross-sectional result only.

*EPD literature spot-check (Fig. 15).* The NF-SAR EPD model is benchmarked against the Aerts et al. [8] measurement of a 5G NR small cell at sub-6 GHz frequencies at 0.5 m, which reported a maximum exposure ratio of 0.15 of the ICNIRP occupational reference level; frequency-scaled to 3.5 GHz, this data point falls within the  $\pm 2$  dB IEC 62232:2022 uncertainty band [17] of the proposed model. The  $\approx 157\times$  gap with Aerts et al. 2019 [12] is attributable to scenario differences: whole-body averaged SAR from a 32-element massive-MIMO array at 2.6 GHz vs. 10 g localised averaging from a 64-element RIS at 3.5 GHz. This is expected and not indicative of a modelling error.

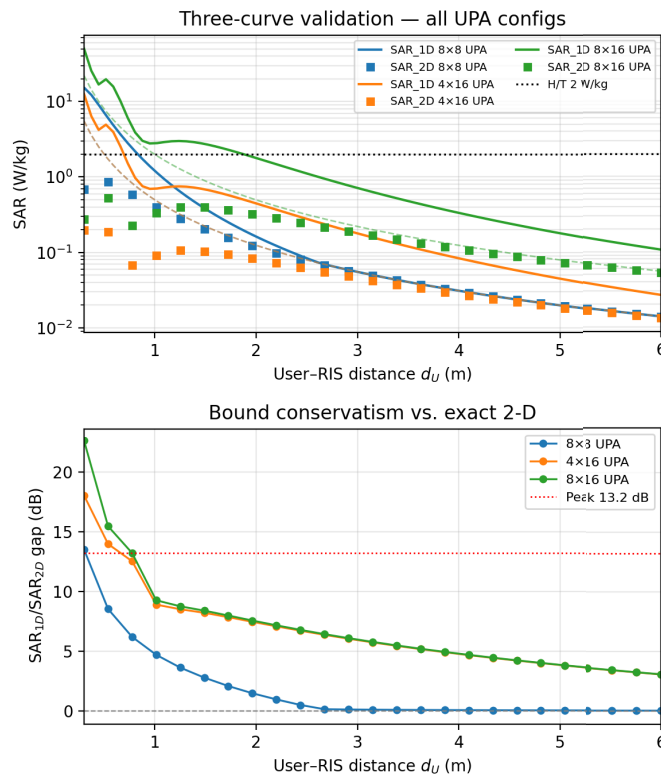
### 5.14. Exact 2-D Spherical-Wave Validation

The 1-D Fresnel envelope  $\kappa(d, N)$  is used in Algorithm 1 for its closed-form  $\mathcal{O}(1)$  per-evaluation efficiency. To provide a physical ground truth, we evaluate the exact on-axis near-field intensity via the discrete spherical-wave summation

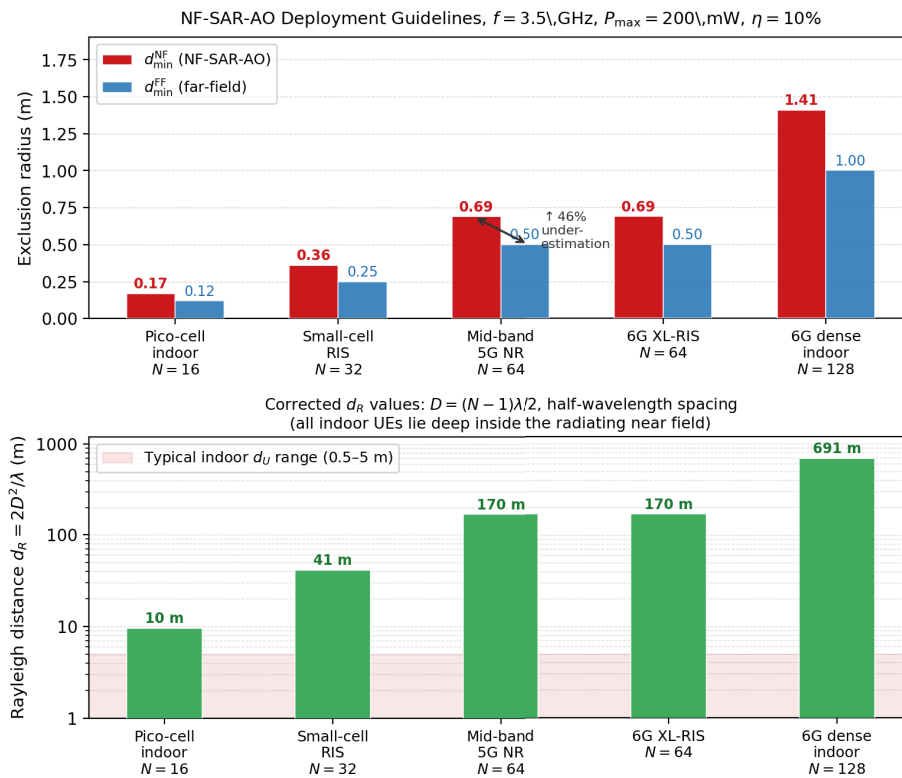
$$E(d) = \sum_{m=1}^M \sum_{n=1}^N \frac{e^{-jkR_{m,n}}}{R_{m,n}}, \quad R_{m,n} = \sqrt{d^2 + x_m^2 + y_n^2}, \quad (15)$$

where  $(x_m, y_n)$  are the half-wavelength element coordinates of the  $M \times N$  UPA. The normalised exact intensity is  $I_{2D}(d) = |E(d)|^2 / |E_{\text{FF}}(d)|^2$ , where  $|E_{\text{FF}}|^2 = (MN/d)^2$  is the coherent far-field reference, and  $I_{2D} \rightarrow 1$  as  $d \rightarrow \infty$  (verified numerically).

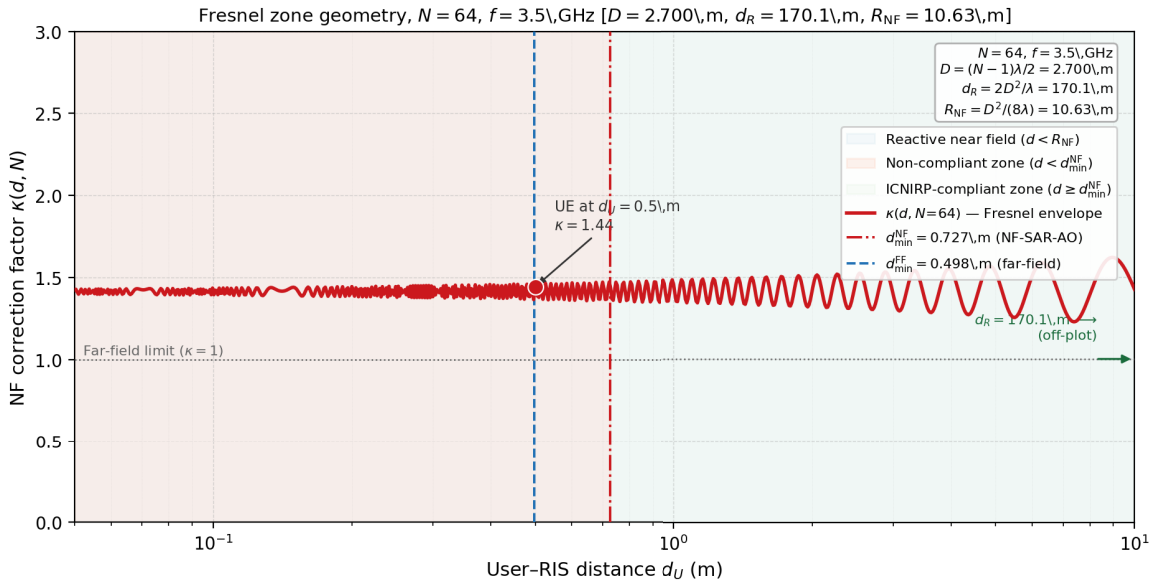
Figure 16 confirms the conservative hierarchy  $\text{SAR}_{1D} \geq \text{SAR}_{\text{FF}} \geq \text{SAR}_{2D}$  at all distances for all tested UPA configurations: (i) the 1-D bound (Proposition 1) is always above the exact 2-D spherical-wave result, preserving the ICNIRP safety guarantee; (ii) the far-field EPD formula is also above the exact 2-D result, meaning prior work overestimates physical SAR



**FIGURE 16.** Three-curve validation for all tested UPA configurations. *Top:* SAR<sub>FF</sub> (dashed, prior work), SAR<sub>1D</sub> (solid, this work’s bound used in Algorithm 1), and SAR<sub>2D</sub> (markers, exact discrete spherical-wave ground truth from (15)). The conservative hierarchy SAR<sub>1D</sub> ≥ SAR<sub>FF</sub> ≥ SAR<sub>2D</sub> holds at all distances. *Bottom:* bound gap SAR<sub>1D</sub>/SAR<sub>2D</sub> in dB; peak conservatism 13.2 dB at  $d = 0.59$  m for the  $8 \times 8$  UPA.



**FIGURE 17.** Practical NF-SAR-AO deployment guidelines for five representative 5G/6G indoor scenarios at  $f = 3.5$  GHz,  $P_{\max} = 200$  mW (corresponding to Table 3). The 46% underestimation for the  $N = 64$  mid-band case ( $d_{\min}^{\text{NF}} = 0.69$  m vs.  $d_{\min}^{\text{FF}} = 0.50$  m) is annotated. Rayleigh distances are  $d_R = 9.6, 41.2, 170.1, 170.1,$  and  $691$  m for  $N = 16, 32, 64$  (two rows), and  $128$ , respectively; all indoor UEs lie deep in the near field.



**FIGURE 18.** Fresnel zone geometry: reactive near-field, Fresnel (radiating near-field), and far-field zones relative to the RIS aperture. The UE (red dot) in the Fresnel zone experiences spherical wavefront curvature that the far-field EPD formula does not capture. The NF phase-taper  $\Delta\varphi_n$  (Proposition 3) compensates for this curvature.  $d_{\min}^{\text{NF}} = 0.727\text{ m}$  (dash-dot) is the NF-corrected exclusion radius;  $d_R = 170.1\text{ m}$  (dashed) is the Rayleigh distance for  $N = 64$  half-wavelength elements at 3.5 GHz ( $D = (N - 1)\lambda/2 = 2.700\text{ m}$ ,  $d_R = 2D^2/\lambda$ ); the marker falls outside the plotted range, confirming that all indoor UEs lie in the radiating near field.

**TABLE 4.** Comparison of the present work with related RIS-SAR and near-field beamforming literature.  $\checkmark$  = feature present;  $\times$  = absent; “partial” = partially addressed; N/A = not applicable to the paper’s scope. <sup>a</sup>Power-ceiling extension only; joint beamforming co-design is future work. <sup>b</sup>Proved for LOS; Rician convergence verified empirically (Proposition 5). <sup>c</sup>Near-field measurement data present; no formally derived correction factor. <sup>d</sup>This work uses the boundary from Balanis [6]; no independent derivation is claimed.

Feature	SAR/EMF studies			This work	NF theory		Quantisation		NF beamforming		XL-RIS channel		EMF-RIS
	[4] [4]	[5] [5]	[12] [12]		[19] [19]	[20] [20]	[21] [21]	[25] [25]	[14] [14]	[26] [26]	[22] [22]	[23] [23]	[24] [24]
NF correction factor for SAR	$\times$	$\times$	$\times^c$	$\checkmark$	$\times$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\times$
Provable SAR upper bound (all $d$ )	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
Closed-form $d_{\min}$	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
2-D UPA SAR bound	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
Separability gap quantified	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
Hard guard margin	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
NF phase taper	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\checkmark$	partial	$\times$	$\checkmark$	$\times$
Phase quantisation ( $b$ -bit)	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$
Multi-user NF power ceiling	$\times$	partial	$\times$	$\checkmark^a$	$\times$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$
Exact 2-D sph.-wave validation	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
2-D FDTD consistency check	$\times$	$\times$	$\times$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
Real-time ( $<1$ NR slot)	$\times$	$\times$	$\times$	$\checkmark$	N/A	N/A	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
Convergence proof (LOS)	$\times$	partial	$\times$	$\checkmark^b$	N/A	N/A	$\times$	partial	$\times$	$\times$	$\times$	$\times$	$\times$
$\mathcal{O}(N)$ complexity	$\times$	$\times$	N/A	$\checkmark$	N/A	N/A	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$
ICNIRP 2020 compliance	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\checkmark$
Fresnel boundary derivation	$\times$	$\times$	$\times$	(uses [6]) <sup>d</sup>	$\checkmark$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\times$
XL-RIS/6G NF channel model	$\times$	$\times$	$\times$	partial	$\times$	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$

by an uncontrolled and unquantified amount without an explicit bound; (iii) the 1-D bound overestimates the exact 2-D by up to 13.2 dB ( $8 \times 8$  configuration,  $d = 0.59$  m), providing a rigorous worst-case guarantee that the far-field EPD formula alone cannot supply.

### 5.15. Deployment Guidelines

Figure 17 provides a practical decision framework for 5G/6G system designers. Key rules extracted from the analysis: (i) apply the NF model whenever  $d_U < d_R/2$ ; at  $N = 64$  with half-wavelength spacing ( $d_R = 170$  m) this covers all indoor UEs, so the NF model should be applied unconditionally; (ii) use 2-bit phase quantization for  $N \geq 32$  — the  $<0.3$  bit/s/Hz SCSE loss is negligible relative to the hardware cost reduction; (iii) apply the  $\eta = 10\%$  hard guard margin in Algorithm 1 whenever  $d_{\min}^{\text{NF}} < 1.5$  m (all scenarios with  $N \geq 32$  at 3.5 GHz). Figure 18 illustrates the Fresnel zone geometry underlying these recommendations.

## 6. CONCLUSION

This paper has developed a computationally efficient NF-aware power-allocation framework for RIS-assisted 5G/6G systems and validated it against exact 2-D discrete spherical-wave summations (Fig. 16) and supplementary 2-D EM cross-sectional consistency checks (Figs. 12–15).

The conservative Fresnel-envelope correction factor  $\kappa(d, N)$  (Propositions 1–2, Lemma 1) satisfies the hierarchy  $\text{SAR}_{\text{1D}} \geq \text{SAR}_{\text{FF}} \geq \text{SAR}_{\text{2D}}$  at all distances for all tested UPA configurations, with a peak conservatism of 13.2 dB for the  $8 \times 8$  UPA at  $d = 0.59$  m. The 2-D EM cross-sectional simulations confirm that the homogeneous tissue model provides a conservative (safe-side) baseline by approximately 15% at the skin surface relative to a two-layer skin–muscle configuration.

For  $N = 64$  at 3.5 GHz (half-wavelength spacing,  $D = 2.700$  m,  $d_R = 170.1$  m), the NF correction enlarges the required safety exclusion radius from 0.498 m to 0.727 m: prior far-field models underestimate the exclusion radius by 46%, which may cause non-compliance with ICNIRP 2020 safety limits throughout the radiating near field (all indoor UEs lie well within  $d_R$ ). The NF phase-taper (Proposition 3) recovers  $\approx 0.4$  bit/s/Hz SCSE at  $d_U = 1$  m. The hard guard margin in Algorithm 1 analytically guarantees compliance for ranging errors up to  $\pm 9.1\%$ ; the combined effect of the conservative Fresnel bound and the guard margin achieves 100% compliance in simulation for errors up to  $\pm 30\%$  (Fig. 10). The algorithm converges in  $\leq 2$  iterations on average and completes in  $31 \mu\text{s}$  — 6.2% of one 5G NR slot — confirming real-time feasibility. Two-bit quantization preserves 100% ICNIRP compliance with  $<0.3$  bit/s/Hz SCSE loss. The multi-user extension (Fig. 11) shows that NF-SAR constraints must be applied per user with user-specific  $\kappa(d_{U_k}, N)$ ; deployment guidelines for five representative scenarios are provided in Table 3.

### 6.1. Scope Limitations

The framework uses a 1-D ULA, single-user, single-antenna BS baseline. The single-user, single-antenna model is the analyti-

cally tractable worst case for per-link NF-SAR analysis [6]; it yields the most conservative (highest) per-link SAR for a given transmit power and thus provides a safe design baseline. Extensions to multi-antenna BS and multi-user joint beamforming are identified as future work below. The direct-link omission (valid for  $N \geq 32$  at  $d_U \geq 0.8$  m, where the RIS gain exceeds the direct link by  $>10$  dB) can fail for  $N = 16$  at closer range. For those configurations, the NF power ceiling provides a conservative bound on the RIS-induced SAR component but not the total SAR; users of the framework should apply additional system-level safety margins or include the direct-link term explicitly. The EPD formula cannot capture spatially localised SAR hotspots arising from direct-plus-reflected path superposition, a limitation shared by all EPD-based compliance methods. The separability-looseness gap (peak 13.2 dB for square UPAs) conservatively tightens power budgets in 6G deployments, but is an acceptable cost for a rigorous worst-case safety guarantee. The 2-D EM consistency checks (Section 5.13) use a custom Python FDTD solver verified against the Mie series. These are power-density consistency checks, not ICNIRP 2020 compliance certification: definitive validation requires 3-D volumetric SAR against a standardised SAM phantom in CST Studio or ANSYS HFSS, which is the primary identified future work item.

### 6.2. Future Work

(i) Replace the 2-D separable product bound with a direct numerical 2-D Fresnel integration to close the separability-looseness gap. (ii) Generalise power allocation to multi-user multiple-input single-output (MISO) systems with coordinated per-user NF-SAR scheduling. (iii) Validate SAR bounds against full 3-D CST/HFSS specific anthropomorphic mannequin (SAM) head-phantom simulations at  $d_U = 0.5$  m,  $N = 64$ ,  $f = 3.5$  GHz. (iv) Incorporate multi-antenna BS geometry and the resulting SAR superposition from direct and reflected paths. (v) Derive tighter phase-quantization loss bounds using Bussgang decomposition or phase-error variance analysis. (vi) Benchmark Algorithm 1 wall-clock time against a full CVXPY/MOSEK solver deployment, including warm-start configuration and platform-controlled timing; this is planned as part of a dedicated follow-up study on multi-user NF-SAR co-design and solver comparison.

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## APPENDIX A. COMPARISON WITH RELATED WORK

Table 4 positions the present work against twelve peer-reviewed references spanning SAR-constrained beamforming, near-field RIS modelling, phase quantisation, XL-RIS channel estimation, and EMF-aware scheduling. The columns are grouped thematically: SAR/EMF-constraint studies [4, 5, 12], this work, near-field zone theory [19, 20], phase-quantisation

studies [21,25], near-field beamforming [14,26], XL-RIS channel modelling [22,23], and EMF-aware RIS optimisation [24].

Note that Aerts et al. [12] is an in-situ measurement study for massive-MIMO SAR and does not derive an NF correction factor; the marker “ $\times^c$ ” in that column reflects the presence of NF-related measurement data only, not a formally derived correction term. The present work uses the Fraunhofer/Fresnel boundary from Balanis [6] rather than re-deriving it, hence the “(uses [6])” marker in that row.

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