

Unbalance Vibration Compensation Control of Outer Rotor Coreless Bearingless Permanent Magnet Synchronous Motor Based on Optimized Secondary-Path Estimation VSS-FxLMS Algorithm

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ABSTRACT: Rotor mass eccentricity in an outer-rotor coreless bearingless permanent magnet synchronous motor (ORC-BPMSM) causes synchronous unbalance vibration, which degrades suspension stability and limits high-speed operation. To suppress this vibration, an optimized secondary-path estimation variable-step-size filtered-x least mean square (OSPE-VSS-FxLMS) feedforward compensation method is proposed. First, the rotor unbalance vibration mechanism is derived from the suspension force model. Then, a hyperbolic tangent–exponential (tanh-exp) function is introduced to adaptively regulate the FxLMS step size, improving the trade-off between convergence speed and steady-state error. Meanwhile, a normalized least mean square (NLMS)-based online secondary-path estimation strategy is developed to reduce gradient deviation caused by secondary-path mismatch. Simulations and experiments are conducted under constant-speed and acceleration conditions. Compared with the uncompensated condition at 3000 r/min, the displacement vibration amplitudes in the x - and y -directions are both reduced to 11 μm , corresponding to a reduction of about 69.4%. The effectiveness of the proposed unbalance vibration compensation method is confirmed by both simulated and experimental results.

1. INTRODUCTION

With the rapid growth of renewable-energy-based power systems, increasing demand is imposed on power grids for energy storage technologies with fast response, high efficiency, and long service life. Due to millisecond-level charge-discharge response, high power density, long cycle life, and superior energy conversion efficiency, flywheel energy storage has attracted increasing attention as a promising solution for future energy storage systems. In a flywheel energy storage system, the motor is regarded as the core energy-conversion component, and the system's energy-exchange efficiency and high-speed operating capability are directly determined by its performance. Conventional permanent magnet motors rely on mechanical bearings, resulting in issues such as high friction loss and limited reliability, which limits their applicability in high-speed and long-service-life operating conditions. Therefore, a more efficient outer-rotor coreless bearingless permanent magnet synchronous motor (ORC-BPMSM) is proposed to overcome limitations [1–3].

The ORC-BPMSM is constructed based on a bearingless permanent magnet synchronous motor, removing the stator iron core and adopting an outer-rotor configuration. It features frictionless operation, large rotational inertia, high power density, and low iron loss during suspended high-speed operation, making it particularly suitable for flywheel energy storage systems.

However, in practical manufacturing, the rotor's mass center does not coincide with its geometric center due to material properties, machining accuracy, assembly errors, and other factors. This causes unbalanced vibration during operation [4]. Unbalance vibration is further exacerbated by a larger mass distribution radius and moment of inertia of the outer rotor structure. Therefore, rotor unbalance vibration suppression is necessary to improve the suspension stability of the ORC-BPMSM.

Rotor unbalance vibration compensation control is commonly realized using feedforward compensation. Based on prior information about the disturbance or reference input, this method generates a compensation signal in advance to counteract the disturbance before it affects the system. Since vibration caused by rotor mass unbalance is periodic, synchronous vibration, feedforward compensation is suited for this type of disturbance and can achieve faster convergence and higher compensation accuracy [5]. In [6], an online unbalance estimation based on immersion and invariance theory is used for adaptive compensation. In [7], a control method based on the total least squares adaptive filtering algorithm is proposed, which achieves good vibration suppression and robustness under different operating conditions. In [8], a sliding-mode-observer-based method is proposed to estimate rotor unbalance and suppress the resulting vibration force and torque. These existing studies validate the feasibility and effectiveness of various control strategies for rotor unbalanced vibration suppression.

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Because the compensation signal passes through multiple dynamic links, amplitude attenuation, frequency response, and phase lag characteristics introduced during transmission reduce consistency between the compensation signal and the actual vibration response. To address the above problem, the filtered-x least mean square (FxLMS) algorithm is adopted [9]. FxLMS has also been combined with other control strategies to further improve the active vibration control performance [10]. By introducing an estimated secondary path into the weight update process, the reference signal is prefiltered to compensate for the influence of control-path dynamics on gradient estimation, enabling the FxLMS algorithm to approximate the true gradient direction more accurately. The step-size factor has a significant influence on convergence speed and steady-state error. Therefore, various improved strategies have been proposed to balance trade-off between them. In [11], the inverse hyperbolic sine function is adopted to establish nonlinear mapping between the error and step size, enabling rapid adjustment of the step size with error variation. In [12], an iterative seeking method is proposed to improve both the convergence speed and unbalance identification accuracy. In [13], the step size is adjusted according to the estimated signal power and accumulated instantaneous squared error, enabling adaptive step-size selection and improved estimation performance with low computational complexity. However, these methods have limited regulation capability in different error ranges and different operating conditions.

With the introduction of secondary-path estimation, the deviation of the prefiltered reference signal in the FxLMS algorithm from the true gradient direction under secondary-path mismatch should also be taken into consideration. To address this problem, in [14], the method of secondary-path identification combined with minimum-phase decomposition and inverse filtering frequency equalization is adopted to reduce the impact of secondary-path mismatch error on algorithm updates. In [15], the real-coded genetic algorithm is employed to eliminate the dependence on secondary-path identification and avoid secondary-path modeling error. However, it exhibits strong parameter sensitivity and tends to suffer from slow convergence or local optima under sudden changes in the secondary path. In [16], an auxiliary filter is introduced for an online adaptive secondary-path estimation model, enabling the coordinated online update of the secondary path and the controller.

In this paper, an OSPE-VSS-FxLMS feedforward compensation method is proposed to suppress rotor unbalance vibration in the ORC-BPMSM. A tanh-exp function is used to adaptively regulate the FxLMS step size, and an NLMS-based online secondary-path estimation method is introduced to reduce the influence of secondary-path mismatch.

The main contributions are as follows:

1. The unbalance vibration mechanism of the ORC-BPMSM is analyzed based on the suspension force model, and the synchronous vibration characteristics caused by rotor mass eccentricity are derived.
2. A tanh-exp variable step-size FxLMS algorithm is proposed to improve the trade-off between convergence speed and steady-state error. An optimized secondary-path esti-

mation strategy based on NLMS is developed to enhance the accuracy of the filtered reference signal and the convergence stability of the FxLMS algorithm.

3. The effectiveness of the proposed method is verified by simulated and experimental results under constant-speed and acceleration conditions.

2. PRINCIPLE OF VIBRATION COMPENSATION

2.1. Suspension Force Modeling of the ORC-BPMSM

In the ORC-BPMSM, the iron-core stator is replaced by a core-less structure made of non-magnetic materials to reduce torque ripple and eddy-current loss. Surface-mounted permanent magnets are arranged on the inner side of the outer rotor and have three pole pairs. The stator has 36 slots, each of which accommodates two sets of windings, namely a suspension-force winding with two pole pairs and a torque winding with three pole pairs. In this configuration, the difference in pole-pair number between the torque winding P_M and the suspension-force winding P_B is 1, that is, $P_M = P_B \pm 1$. When electrical angular frequencies of the suspension force winding and torque winding are identical, the basic condition for rotor suspension of the ORC-BPMSM is satisfied [17]. Based on the Maxwell tensor method, the rotor suspension force is analyzed in terms of the air-gap flux density, and the radial suspension force is derived as follows:

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix} = (k_M - k_L) \begin{bmatrix} \psi_{Md} & \psi_{Mq} \\ \psi_{Mq} & -\psi_{Md} \end{bmatrix} \begin{bmatrix} i_{Bd} \\ i_{Bq} \end{bmatrix} + k_C \begin{bmatrix} x \\ y \end{bmatrix} \quad (1)$$

where $k_M = \frac{\pi P_M P_B L_B}{8lr\mu_0 N_M N_B}$, $k_L = \frac{n P_M N_B}{4r P_B N_M}$, $k_C = \frac{\pi P_M^2 \psi_M^2}{8lr\mu_0 N_M^2 \delta_0}$, ψ_{Md} and ψ_{Mq} are d - q axis components of the torque air-gap flux linkage; i_{Bd} and i_{Bq} are d - q axis components of suspension force winding currents; x and y are displacements in the stationary coordinate system, respectively; k_M is the Maxwell force constant; k_L is the Lorentz force constant; k_C is the offset constant; μ_0 is the vacuum permeability; r and l are the rotor's radius and length, respectively; δ_0 is the air-gap width when the rotor is in the equilibrium position; n is the number of motor phases; N_B and N_M are the number of turns of suspension force windings and torque windings, respectively; L_B is the self-inductance of suspension force windings.

2.2. Mechanism of Rotor Unbalance Vibration

Due to machining error, the rotor's center of mass deviates from its geometric center. This causes the permanent magnets to deviate from the geometric center, thereby disrupting the uniformity of the air-gap magnetic field inside the motor. Under a nonuniform magnetic field, an unbalanced magnetic pull is generated. Because the magnetic field intensity is higher in the eccentric direction, the magnetic pull in the same direction increases accordingly, resulting in a significant increase in rotor vibration amplitude in that direction. A brief schematic diagram of the rotor center of mass offset is shown in Fig. 1. The offset distance between the geometric center O and the mass center O' is denoted by ε . The angle between the eccentricity vector and the d -axis of the rotating coordinate system is

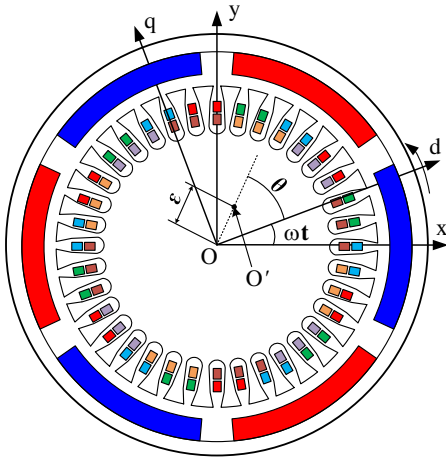


FIGURE 1. Rotor center of mass offset diagram.

denoted as θ . When the rotor rotates with mechanical angular velocity ω , the angular position of this vector with respect to the x -axis in the stationary coordinate system becomes $\omega t + \theta$.

When rotor unbalance vibration occurs, the periodic unbalance forces generated in the x - and y -directions are denoted as F_{ux} and F_{uy} , and are expressed as:

$$\begin{cases} F_{ux}(t) = m\varepsilon\omega^2 \cos(\omega t + \theta) \\ F_{uy}(t) = m\varepsilon\omega^2 \sin(\omega t + \theta) \end{cases} \quad (2)$$

where m is the rotor mass; ε is the distance between the mass center and geometric center; ω is the mechanical angular velocity [18]. The rotor's equation of motion can be written as:

$$\begin{cases} m\ddot{x} + c\dot{x} + kx = F_x - f_{mx} + F_{ux}(t) \\ m\ddot{y} + c\dot{y} + ky = F_y - f_{my} + F_{uy}(t) \end{cases} \quad (3)$$

where c and k are the equivalent damping and equivalent stiffness coefficients, respectively; f_{mx} and f_{my} are the resultant forces of gravity and other forces acting on the rotor in the x - and y -directions, respectively.

The rotor displacement response caused by unbalance excitation force F_u is given by:

$$\begin{cases} x_a = A \cos(\omega t + \eta - \gamma) \\ y_a = A \sin(\omega t + \eta - \gamma) \end{cases} \quad (4)$$

where A is the amplitude of the unbalance-induced displacement component, and γ is the phase delay of the displacement response with respect to the unbalance excitation force.

The expressions for A and γ can be written as:

$$\begin{cases} A = \frac{m\varepsilon\omega^2}{\sqrt{(k-m\omega^2)^2 + (c\omega)^2}} \\ \gamma = \arctan\left(\frac{c\omega}{k-m\omega^2}\right) \end{cases} \quad (5)$$

According to (2) and (4), the unbalance vibration caused by rotor eccentricity generates the periodic unbalance force and unbalance displacement synchronous with the mechanical angular velocity. Therefore, if a compensation signal can be generated before the vibration signal enters the controller so as to cancel the periodic vibration component, unbalance vibration can be compensated.

2.3. Principle of the FxLMS Algorithm Used in the Vibration Compensation Control

The FxLMS algorithm is a stochastic gradient adaptive algorithm based on the least mean square criterion. It introduces a secondary-path estimation to prefilter the reference signal, thereby correcting the influence of control-channel dynamics on gradient estimation. By minimizing the mean square value of the rotor displacement error signal, the amplitude and phase of the compensation signal are adaptively adjusted to effectively suppress synchronous unbalance vibration at the rotational frequency.

Figure 2 is the structural block diagram of the FxLMS algorithm. The reference input signals for the filter are $x_1(n)$ and $x_2(n)$, and the compensation control signal $u(n)$ is generated by online adjustment of the filter weights using an adaptive algorithm. The compensation signal is applied to the rotor system through the secondary path $S(z)$, and the resulting displacement response $y(n)$ is compared with the original unbalance vibration $d(n)$ to form the error signal $e(n)$. Meanwhile, a secondary-path estimate $\hat{S}(z)$ is introduced to filter the reference signal. The filtered signal $\hat{y}(n)$ is used for weight updating, achieving effective suppression of synchronous rotor unbalance vibration. The FxLMS algorithm uses the minimum mean square error criterion to update the weight coefficients of the adaptive filter. A finite impulse response (FIR) filter is adopted, and orthogonal sine and cosine signals are selected as reference inputs for synchronous rotor unbalance vibration.

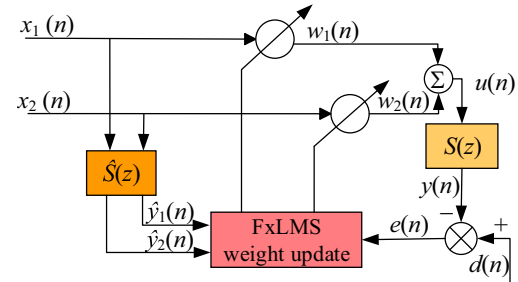


FIGURE 2. Structure block diagram of the FxLMS algorithm.

The expressions of two reference input signals are:

$$\begin{cases} x_1(n) = A_1 \sin(n\omega T) \\ x_2(n) = A_2 \cos(n\omega T) \end{cases} \quad (6)$$

where A_1 and A_2 are the amplitudes of two reference input signals, respectively; n is the number of iterations; T is the sampling period.

The adaptive filter adopts a dual-weight structure corresponding to in-phase and quadrature input signals, and the compensation control signal $u(n)$ is expressed as:

$$u(n) = w_1(n)x_1(n) + w_2(n)x_2(n) \quad (7)$$

where $x_1(n)$ and $x_2(n)$ are reference input signals. $w_1(n)$ and $w_2(n)$ are adaptive filter weights corresponding to sine and cosine reference inputs, respectively.

The error signal $e(n)$ is:

$$e(n) = d(n) - y(n) \quad (8)$$

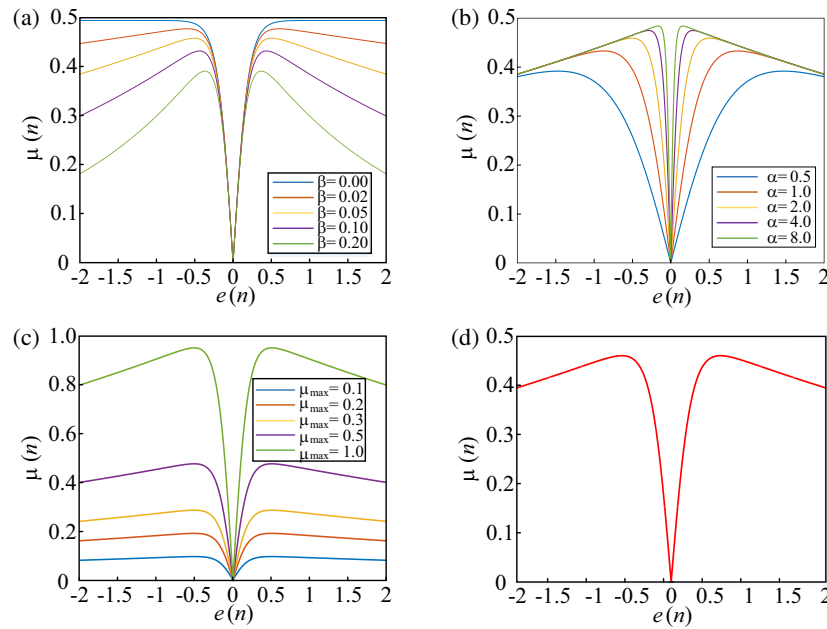


FIGURE 3. Relationship between $\mu(n)$ and $e(n)$ under different parameter settings.

where $d(n)$ is the original unbalance vibration disturbance, and $y(n)$ is the output of the secondary path.

The weight coefficients are obtained by iterative updating using the least mean square algorithm, and the corresponding update equation can be expressed as:

$$\begin{cases} \mathbf{w}(n+1) = \mathbf{w}(n) - \mu(n)e(n)\hat{\mathbf{y}}(n) \\ \hat{\mathbf{y}}(n) = [\hat{y}(n), \hat{y}(n-1), \dots, \hat{y}(n-L_s+1)]^T \end{cases} \quad (9)$$

where $\mu(n)$ is the step-size function; $\hat{\mathbf{y}}(n)$ is the filtered reference vector; L_s is the order of the secondary path, i.e., the length of the discrete impulse response from the control input to sensor output.

In the conventional FxLMS algorithm, the step-size function $\mu(n)$ is a constant value. A larger $\mu(n)$ results in faster convergence but also increases the steady-state error. If $\mu(n)$ exceeds the allowable range, the algorithm diverges. Conversely, a smaller $\mu(n)$ reduces the steady-state error but slows down convergence.

3. DESIGN OF UNBALANCE VIBRATION COMPENSATION CONTROLLER

In practical applications, the fixed step size of the conventional FxLMS algorithm and dynamic characteristics of secondary-path estimation degrade the overall compensation performance. Therefore, a variable step-size FxLMS algorithm based on optimized secondary-path estimation is proposed. The algorithm is optimized in terms of the step-size function and secondary-path estimation, respectively, and its compensation performance is verified.

3.1. FxLMS Algorithm Based on the tanh-exp Function

As discussed in Section 2.3, the fixed step-size function $\mu(n)$ introduces certain limitations. A large step size leads to poor

algorithm accuracy, and a small one leads to slow convergence. To alleviate the trade-off between convergence speed and steady-state error, the variable step-size FxLMS (VSS-FxLMS) algorithm is widely adopted, in which the step size is dynamically adjusted by a nonlinear function.

To improve the algorithm's adaptive performance over different error intervals, a tanh-exp variable step-size function is employed. The function employs a hyperbolic tangent function to achieve a rapid increase in step size within the medium-error interval, thereby improving the algorithm's convergence speed. Meanwhile, an exponential decay term is introduced to suppress the step size in the large-error interval, preventing gradient divergence and enhancing system stability.

Since the rotor vibration error is symmetric about zero, the step-size function is designed as an even function of the error magnitude and is expressed as follows:

$$\mu(n) = \mu_{\max} \tanh(\alpha |e(n)|) \exp(-\beta |e(n)|) \quad (10)$$

where $\mu_{\max}$ is the maximum step-size upper bound; α is the tangent growth-rate coefficient; β is the exponential decay coefficient.

By adjusting these parameters, the shape of the step-size function curve can be precisely controlled. Effects of the three parameters on the step curve are shown in Fig. 3(a), Fig. 3(b), and Fig. 3(c).

For the FxLMS algorithm, the reference signal used for weight updating must first be prefiltered through the estimated secondary path. Therefore, its convergence behavior is mainly determined by the autocorrelation matrix of the filtered reference vector. According to the mean-square convergence condition of LMS-type adaptive algorithms, the variable step size should satisfy the following requirement:

$$0 < \mu(n) < \frac{2}{\lambda_{\max}(\mathbf{R}_{\hat{x}})} \quad (11)$$

where $\mathbf{R}_{\hat{x}}$ is the autocorrelation matrix of the filtered reference vector, and $\lambda_{\max}$ is the maximum eigenvalue. Since $\mu(n) < \mu_{\max}$, a conservative upper bound can be selected as $\mu_{\max}$:

$$\mu_{\max} = \eta \frac{2}{\lambda_{\max}(\mathbf{R}_{\hat{x}})} \quad 0 < \eta < 1 \quad (12)$$

where η is used to prevent system instability caused by secondary-path estimation errors and transient variations in operating conditions. In practical applications, it can be estimated from the filtered reference signal within a sliding window and selected within a conservative range to ensure stable weight updating.

The α and β determine growth and decay characteristics of the step-size curve, respectively. By setting the derivative of this function with respect to $|e(n)|$ to zero, the peak position can be obtained.

$$e_p = \frac{1}{2\alpha} a \sinh\left(\frac{2\alpha}{\beta}\right) \quad (13)$$

where e_p is the error magnitude corresponding to the maximum step size.

The α and β should be selected such that the maximum step size is located in the medium-error region, where rapid convergence is required. In the small-error region, the step size should remain sufficiently small to reduce steady-state misadjustment. In the large-error region, the exponential decay term should dominate to suppress excessive weight updating caused by transient disturbances or secondary-path mismatch.

The parameter selection procedure is as follows: first, $\mu_{\max}$ is determined according to the convergence bound; then, e_p is set based on the typical transient displacement error; finally, α and β are adjusted to satisfy the required peak position and decay rate.

The parameters are selected according to the convergence bound of the filtered reference signal and the desired peak position of the tanh-exp function. The final selected parameters are $\mu_{\max} = 0.5$, $\alpha = 2.0$, $\beta = 0.05$, and the corresponding waveform is shown in Fig. 3(d). This enables the overall step-size function to maintain a near-flat characteristic in the small-error interval, compensating for small-amplitude fluctuations, while preserving a sufficient step-size magnitude in the large-error interval to achieve stable suppression of unbalance excitation.

3.2. Optimized Secondary-Path Estimation

Although introducing a variable step-size strategy can improve convergence characteristics of the FxLMS algorithm to a certain extent, effectiveness still depends on accurately estimating the secondary path.

The secondary path is influenced by multiple factors, including dynamic characteristics of the actuator and controlled plant, power amplification and drive stages, and variations in operating conditions. Its dynamic characteristics can be hardly regarded as time-invariant. When the fixed secondary-path estimation is employed, any mismatch between the model and the actual secondary-path characteristics caused by variations in operating conditions introduces modeling error, which in

turn biases the gradient estimation in the FxLMS algorithm and thereby degrades its convergence characteristics and steady-state accuracy.

To improve the adaptability of the secondary-path estimation under different operating conditions, an NLMS-based online secondary-path estimation is employed to update the secondary-path estimation $\hat{S}(z)$ in real time. By normalizing the input-signal energy, the influence of input-amplitude variations on the parameter updating process can be effectively reduced, thereby improving the stability of the secondary-path estimation. The expression of NLMS is:

$$\hat{\mathbf{S}}(z, n+1) = \hat{\mathbf{S}}(z, n) - \mu_s \frac{e_s(n) \mathbf{u}_i(n)}{\|\mathbf{u}_i(n)\|^2 + \delta} \quad (14)$$

where $\hat{\mathbf{S}}(z, n)$ and $\hat{\mathbf{S}}(z, n+1)$ are the secondary-path estimations before and after updating, respectively; μ_s is the step size for secondary-path estimation; $e_s(n)$ is the error for secondary-path estimation; $\mathbf{u}_i(n)$ is the input vector for identification; δ is the regularization parameter.

In the proposed algorithm, $e_s(n)$ and $\mathbf{u}_i(n)$ are expressed as:

$$\begin{cases} e_s(n) = \hat{y}(n) - y(n) \\ \mathbf{u}_i(n) = [u(n), u(n-1), \dots, u(n-L_s+1)]^T \\ \hat{y}(n) = \hat{\mathbf{S}}(z) \mathbf{u}_i(n) \\ y(n) = \mathbf{S}(z) \mathbf{u}_i(n) \end{cases} \quad (15)$$

Initial parameters of the secondary-path estimation need not exactly coincide with those of the true secondary path. What is essential is that the filtered signal generated by $\hat{S}(z)$ preserves the correct gradient direction. Compared with magnitude estimation accuracy, delay and phase characteristics of the secondary-path estimation are more critical to the convergence stability of the FxLMS algorithm. Therefore, a secondary-path initialization method based on a prior physical model is adopted to ensure the rationality of the dynamic and phase characteristics of the compensation path in the first instance. The prior physical model $S(s)$ is expressed as:

$$S(s) = \frac{K_S e^{-\tau_d s}}{(T_a s + 1)(ms^2 + cs + k)} \quad (16)$$

where K_S is the sensor gain; τ_d is the total delay; T_a is the equivalent time constant of the actuation stage.

For online secondary-path estimation, the NLMS update step size should satisfy the standard convergence condition for normalized adaptive filters, i.e., $0 < \mu_s < 2$. In the proposed compensation system, the secondary-path estimate and the FxLMS controller weights are updated simultaneously. If both update processes operate on the same time scale, variations in the secondary-path model may distort the filtered reference signal and amplify gradient-estimation errors. A two-time-scale update strategy is adopted, and the secondary-path update step size is set as:

$$\mu_s \leq \rho \bar{\mu} \quad 0.01 \leq \rho \leq 0.1 \quad (17)$$

where $\bar{\mu}$ is the average value of the FxLMS variable step size within the sliding window, and ρ is the time-scale separation coefficient.

This condition ensures that the secondary-path estimate varies more slowly than adaptive controller weights, allowing the FxLMS weight update to be performed based on a relatively stable secondary-path model.

The functional block diagram of the optimized algorithm is shown in Fig. 4. To comprehensively evaluate the performance of the proposed method, the optimized algorithm is compared with the conventional FxLMS algorithm and the variable step-size FxLMS algorithm from two aspects, namely the energy characteristics of the error signal and the actual displacement amplitude.

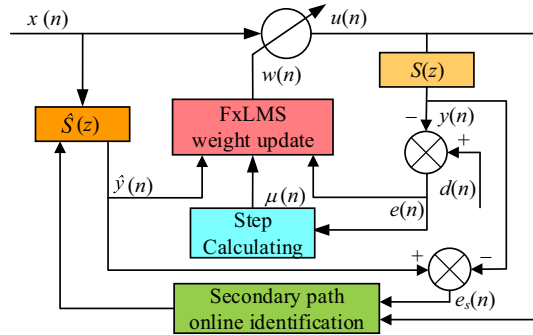


FIGURE 4. Block diagram of the OSPE-VSS-FxLMS algorithm.

The mean square error (MSE) is used as the evaluation metric, and the secondary-path estimation error is analyzed over the iteration index to assess the algorithms' convergence speed and steady-state error energy level during the iterative process, and the expression is given by:

$$\text{MSE}_{\text{dB}}(k) = 10 \log_{10} \left(\frac{1}{N_w} \sum_{i=k-N_w+1}^k e_s^2(i) \right) \quad (18)$$

where N_w is the length of the sliding statistical window.

To further reflect the physical significance of the control effect, a sliding root mean square (RMS) metric is introduced to evaluate the displacement error signal, characterizing the equivalent amplitude of the residual vibration, and the expression is given by:

$$\text{RMS}(n) = \sqrt{\frac{1}{N_w} \sum_{i=n-N_w+1}^n e^2(i)} \quad (19)$$

The conventional FxLMS, tanh-exp-based VSS-FxLMS, and the proposed OSPE-VSS-FxLMS algorithms are simulated and compared, and the results are shown in Table 1 and Fig. 5. The results show that VSS-FxLMS achieves faster convergence than conventional FxLMS by introducing the proposed tanh-exp variable step-size function. The proposed OSPE-VSS-FxLMS further improves convergence stability and reduces residual vibration by reducing the gradient deviation caused by secondary-path mismatch.

In summary, the adopted variable step-size strategy can adaptively adjust the step size based on the error energy signal, which improves the convergence speed and steady-state accuracy of the FxLMS algorithm. In addition, the online

TABLE 1. Performance comparison table of three algorithms.

	MSE (dB)	Iterations	RMS (μm)	Time (s)
Conventional FxLMS	-19.1	557	0.12	0.55
VSS-FxLMS	-22.5	356	0.09	0.35
OSPE-VSS-FxLMS	-25.5	178	0.05	0.18

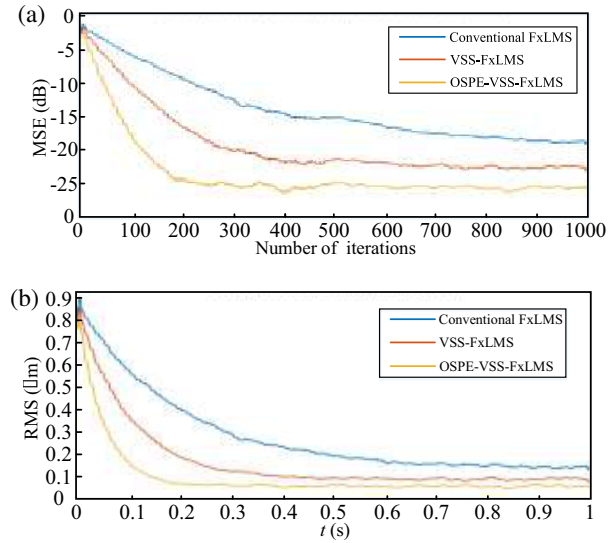


FIGURE 5. Performance comparison of the three algorithms. (a) MSE and (b) RMS.

secondary-path estimation further improves the accuracy of gradient estimation, enabling the algorithm to exhibit superior overall performance in terms of both convergence characteristics and physical vibration suppression.

4. SIMULATION TEST

In the preceding section, the convergence characteristics and steady-state performance of the proposed compensation strategy have been analyzed at the algorithm level, and its effectiveness has been verified. To provide a more intuitive demonstration of the compensation performance of the control method, a control system simulation model is established on the MATLAB/Simulink platform, and simulation and comparative analyses are conducted under medium speed (3000 r/min) and medium-to-high speed (5000 r/min) operating conditions.

The overall control block diagram of the ORC-BPMSM is constructed, as shown in Fig. 6. The torque control system is implemented using the maximum torque per ampere (MTPA) strategy, while the suspension force control system employs a dual closed-loop structure consisting of displacement and current loops. The unbalance vibration compensation system uses measured rotor displacement and two-phase currents in the rotating reference frame as inputs to generate unbalance compensation control force. The generated compensation force is superimposed on the radial suspension force and introduced into the feedforward compensation channel, thereby producing the required suspension force for unbalance vibration compensation control.

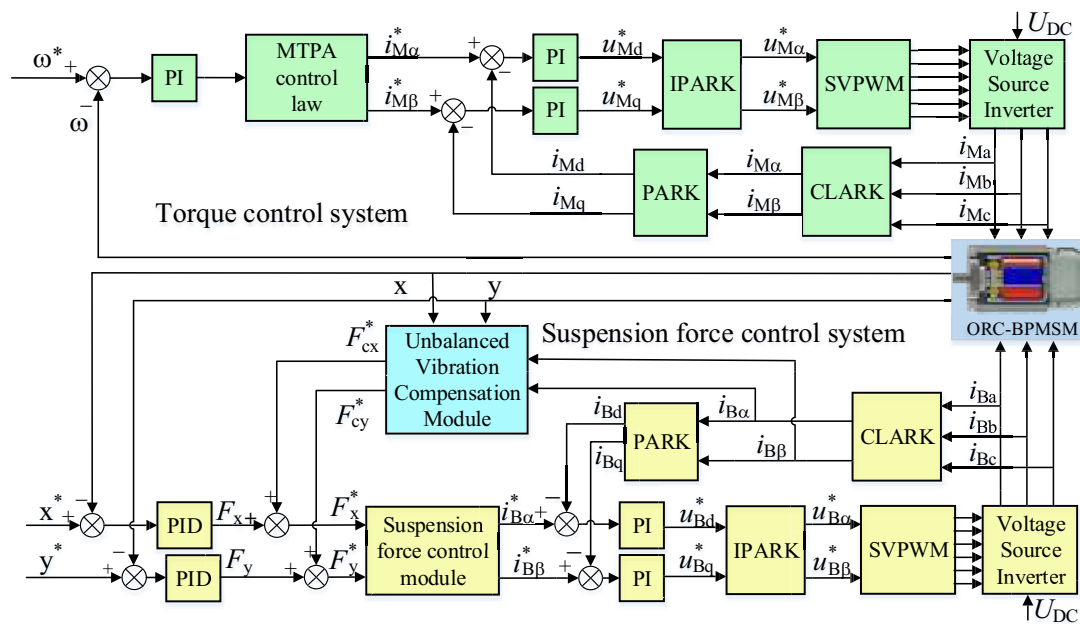


FIGURE 6. Block diagram of the ORC-BPMSM vibration compensation control.

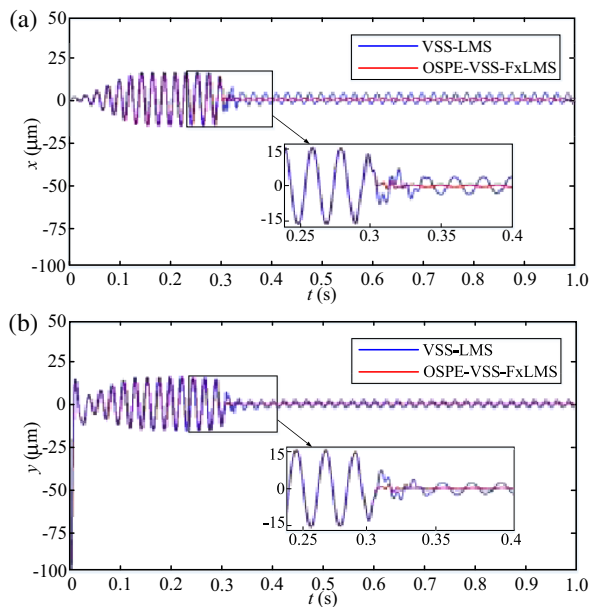


FIGURE 7. Diagram of rotor vibration compensation at 3000 r/min. (a) Radial displacement in the x direction and (b) radial displacement in the y direction.

Based on the control block diagram, simulation tests are conducted on the ORC-BPMSM under different rotational speeds. No compensation is applied during startup. At 0.3 s, the OSPE-VSS-FxLMS algorithm and the VSS-LMS algorithm are introduced, respectively, and the rotor displacements before and after compensation are comparatively analyzed, with the rotor's initial position $x = 0 \mu\text{m}$, $y = -100 \mu\text{m}$. The results are shown in Fig. 7 and Fig. 8.

Figure 7 shows rotor radial displacement waveforms in the x - and y -directions at 3000 r/min. As shown in Figs. 7(a) and 7(b), without compensation control at the initial stage, the rotor radial displacement exhibits periodic oscillations with large

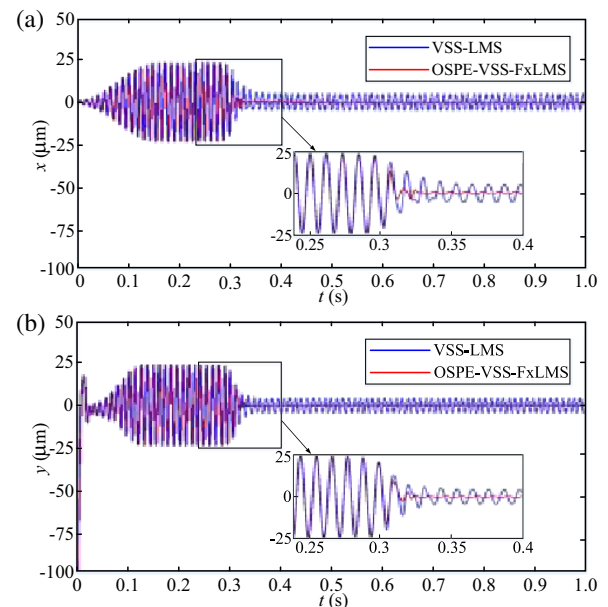


FIGURE 8. Diagram of rotor vibration compensation at 5000 r/min. (a) Radial displacement in the x direction and (b) radial displacement in the y direction..

amplitudes and slow attenuation due to rotor eccentricity. Vibration compensation control is introduced at 0.3 s. The results show that, compared with the VSS-LMS algorithm, the proposed method compensates rotor vibration more effectively, reducing the overall rotor vibration amplitude to approximately 7.9% of its original level. Meanwhile, the convergence speed is further improved, and the convergence time is shortened to about 0.33 s.

Figure 8 shows rotor radial displacement waveforms in the x - and y -directions at 5000 r/min. Compared with the low-speed condition, both the vibration frequency and amplitude of the rotor increase significantly under the medium-to-high-

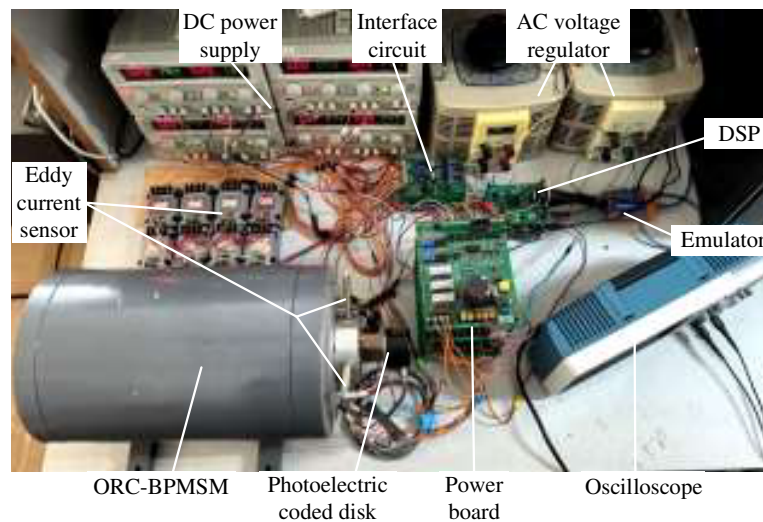


FIGURE 9. Experimental platform.

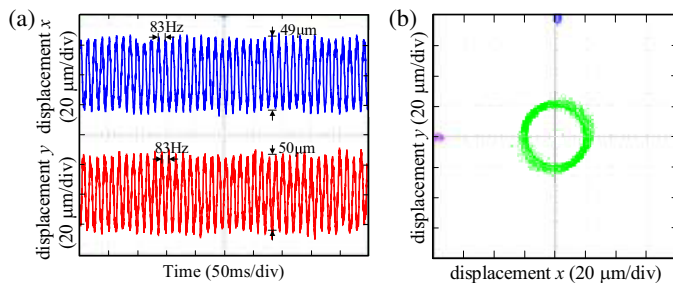


FIGURE 10. Diagrams of rotor displacement without compensation at 5000 r/min. (a) Waveform of displacement and (b) displacement trajectory.

speed condition. The overall trend remains consistent with that under the low-speed condition. When the proposed method is adopted, the rotor vibration amplitude is reduced to 27.8% of that with the VSS-LMS algorithm, while the convergence time is reduced by 46.9%. In addition, after the system reaches a steady state, the displacement fluctuation amplitude is further reduced, and the rotor suspension performance is improved. These results indicate that the proposed control method provides good convergence speed and steady-state accuracy over a range of operating speeds.

5. EXPERIMENTAL RESEARCH

To further verify the feasibility of the proposed vibration compensation control method, the ORC-BPMSM is used as an experimental prototype to construct the experimental platform. In this paper, the digital control system is based on the TMS320F28335 DSP to build the core hardware platform, and the ORC-BPMSM algorithm is compiled, debugged, and executed in the CCS11.2.0 integrated development environment. A visual human-machine interface is also developed based on VB6.0 for real-time monitoring of experimental data and for reading, display, and dynamic adjustment of control parameters. The experimental platform is shown in Fig. 9. Main parameters of the ORC-BPMSM are

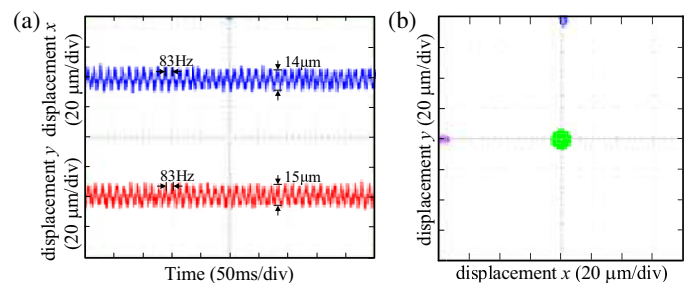


FIGURE 11. Diagrams of rotor displacement with OSPE-VSS-FxLMS at 5000 r/min. (a) Waveform of displacement and (b) displacement trajectory.

shown in Table 2. Experiments are conducted on the radial two-degree-of-freedom system for validation.

5.1. Vibration Compensation Experiment at Rated Speed

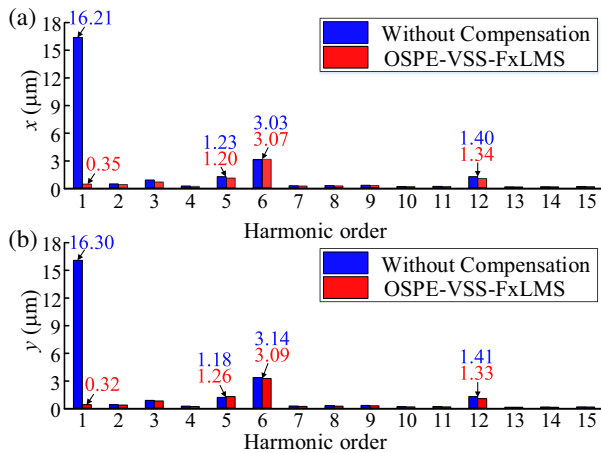
When the ORC-BPMSM speed is stabilized at 5000 r/min, rotor displacement waveforms in the x - and y -directions are sampled using eddy-current displacement sensors, and the results are shown in Fig. 10 and Fig. 11.

Rotor displacement waveforms without compensation are shown in Figs. 10(a) and 10(b). The vibration amplitudes of displacement in the x - and y -directions are approximately $49\ \mu\text{m}$ and $50\ \mu\text{m}$, respectively. In addition, the trajectory boundary exhibits slight dispersion and minor fluctuations under the influence of disturbances. When the OSPE-VSS-FxLMS algorithm is used for control, the displacement vibration amplitudes in the x - and y -directions are approximately reduced to $14\ \mu\text{m}$ and $15\ \mu\text{m}$. The experiment demonstrates that vibration signals are reduced to approximately 25% of the original level by the proposed method.

According to the fast Fourier transform (FFT) harmonic analysis of rotor radial displacement in Fig. 12, uncompensated displacement is mainly dominated by the fundamental harmonic component, indicating that rotor vibration is primarily caused by mass unbalance. Without compensation, the fundamental

TABLE 2. Main parameters of ORC-BPMSM.

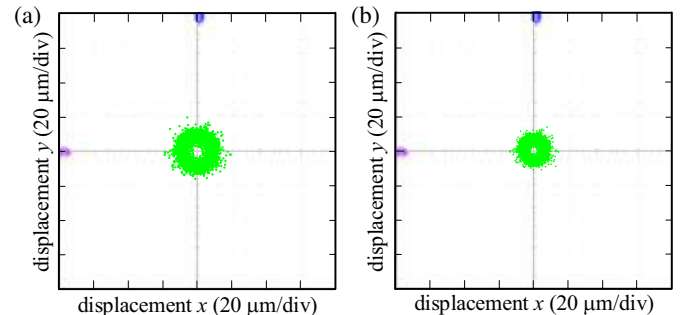
Parameters	Values	Parameters	Values
Rated power (kW)	2	Rotor outer diameter (mm)	200
Rated Voltage (V)	220	Rotor inner diameter (mm)	180
Rated speed (r/min)	5000	Stator outer diameter (mm)	160
Number of stator slots	36	Stator inner diameter (mm)	50
Polar arc coefficient	0.78	Air gap length (mm)	2
Number of pole pairs of torque windings	3	Number of pole pairs of suspension windings	2

**FIGURE 12.** Harmonic components of rotor radial displacement at 5000 r/min. (a) x -direction and (b) y -direction.

harmonic amplitudes in the x - and y -directions are $16.21 \mu\text{m}$ and $16.30 \mu\text{m}$, respectively, much higher than those of other harmonics. After compensation, they decrease to $0.35 \mu\text{m}$ and $0.32 \mu\text{m}$, respectively, demonstrating that the proposed method effectively suppresses synchronous vibration caused by rotor unbalance. Higher-order harmonics remain nearly unchanged, and the fundamental harmonic component no longer dominates the compensated displacement. The residual vibration is mainly attributed to higher-order harmonics and system non-idealities.

5.2. Different Control Methods Comparative Experiment

The ORC-BPMSM's rotor displacement trajectory under different compensation algorithms is shown in Fig. 13. For the comparative VSS-LMS and VSS-FxLMS methods, the step size is adjusted using a conventional sigmoid function. As shown in Figs. 13(a) and 13(b), the vibration signal amplitude is reduced to approximately 50% of the uncompensated case by the VSS-LMS algorithm at 5000 r/min and is further reduced to about 35% by the VSS-FxLMS algorithm. Compared with the results shown in Fig. 11, the proposed control method yields a smaller vibration amplitude at the rated speed condition. In addition, with the above two compensation methods, disturbance-induced discrete signals and displacement fluctuations are not significantly reduced. By contrast, the proposed method significantly reduces the displacement disturbances, indicating superior compensation performance.

**FIGURE 13.** Rotor displacement trajectories after compensation. (a) VSS-LMS and (b) VSS-FxLMS.

5.3. Acceleration Vibration Compensation Experiment

Figure 14 shows experimental waveforms of the actual speed and reference speed when the rotational speed increases from 3000 r/min to 5000 r/min. The ORC-BPMSM operates steadily at 3000 r/min during 0–0.5 s, and the corresponding operating frequency is 50 Hz. The motor then accelerates from 0.5 to 1.8 s until the speed reaches 5000 r/min. After a slight transient overshoot, the actual speed stabilizes at 5000 r/min, corresponding to an operating frequency of 83 Hz.

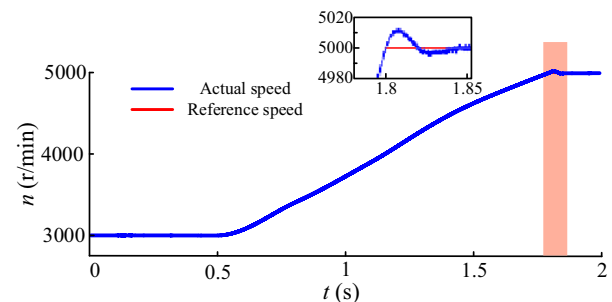
**FIGURE 14.** Waveform of reference speed and actual speed.

Figure 15 shows displacement waveforms in the x - and y -directions without compensation. It can be seen that at 3000 r/min, the vibration amplitudes of the displacement in the x - and y -directions are $37 \mu\text{m}$ and $36 \mu\text{m}$, respectively. After the acceleration is completed, the ORC-BPMSM operates steadily at 5000 r/min, where the vibration amplitudes in the x - and y -directions increase to $49 \mu\text{m}$ and $50 \mu\text{m}$, respectively. Fig. 16 shows the rotor displacement waveforms after introducing the OSPE-VSS-FxLMS algorithm. At 3000 r/min, the displacement's vibration amplitudes in the x - and y -directions are both $11 \mu\text{m}$, representing a reduction of about 69.4% compared

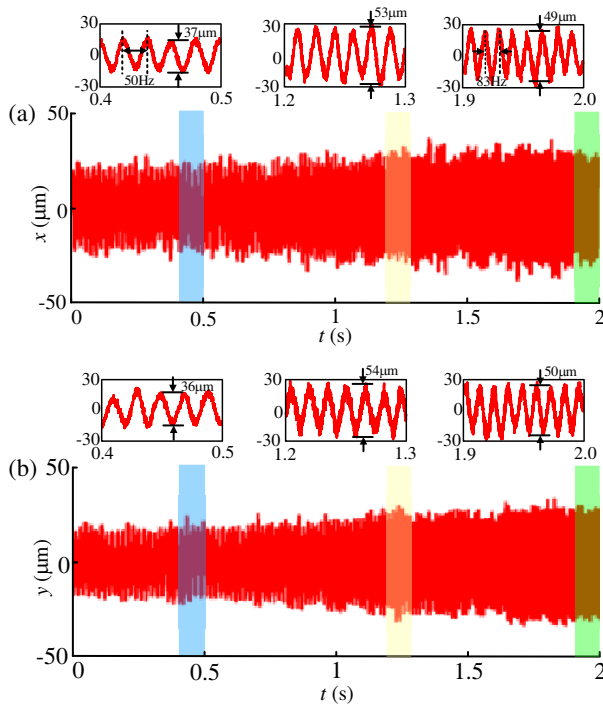


FIGURE 15. Displacement waveform of rotor acceleration experiment without vibration compensation. (a) x -direction and (b) y -direction.

with the uncompensated condition. When the speed increases to 5000 r/min, the average displacement amplitude is approximately 13 μm , and the vibration amplitude remains essentially stable throughout the acceleration process.

The experimental data indicate that the proposed method yields superior performance over a range of operating speeds and remains effective in compensating for rotor vibration during speed variations.

6. CONCLUSION

To suppress rotor unbalance vibration in the ORC-BPMSM, a variable step-size FxLMS feedforward compensation method with optimized secondary path estimation is proposed. The proposed method employs a tanh-exp function and NLMS-based secondary-path estimation optimization to compensate for the vibration components in the displacement signal. The simulated and experimental results lead to the following conclusions:

1. Based on the established mathematical model of the ORC-BPMSM's suspension force, the mechanism of rotor unbalance vibration is analyzed, and characteristics of the rotor vibration signal are further analyzed based on rotor motion equations.
2. A tanh-exp-based step-size optimization method and an NLMS-based online secondary-path estimation method are proposed to address the fixed step-size and secondary-path mismatch problems of the conventional FxLMS algorithm, thereby improving convergence speed and steady-state accuracy and avoiding update-gradient deviation caused by secondary-path estimation error.

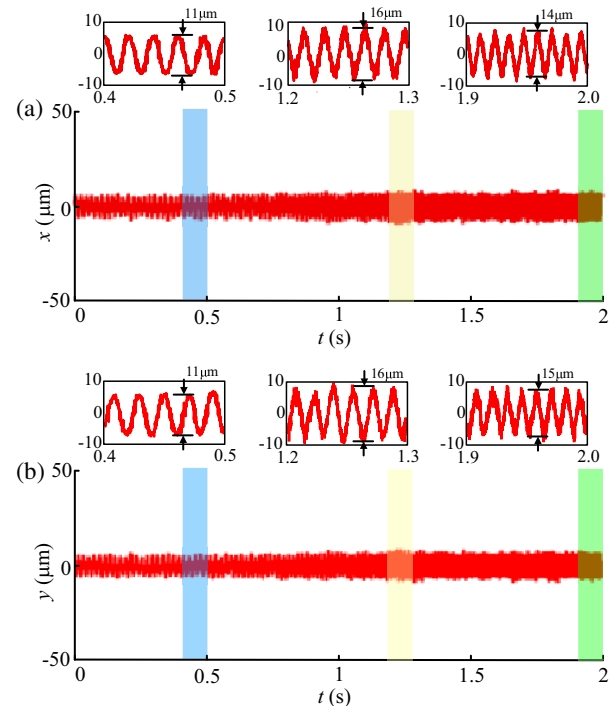


FIGURE 16. Displacement waveform of rotor acceleration experiment with the OSPE-VSS-FxLMS algorithm. (a) x -direction and (b) y -direction.

3. Experimental results show that the proposed control method provides effective compensation for rotor unbalance vibration over a range of operating speeds. At 3000 r/min, the vibration amplitudes in the x - and y -directions are both reduced to 11 μm , corresponding to a reduction of about 69.4% compared with the uncompensated condition. At 5000 r/min, the displacement amplitudes in the x - and y -directions are reduced from 49 μm and 50 μm to 14 μm and 15 μm , respectively. FFT results further show that 1X components in the x - and y -directions decrease from 16.21 μm and 16.30 μm to 0.35 μm and 0.32 μm , respectively, indicating that the synchronous vibration caused by rotor mass unbalance is effectively suppressed.
4. It should be noted that the proposed method mainly targets the 1X synchronous vibration component caused by rotor mass unbalance. High-order harmonic components caused by inverter nonlinearity, current distortion, electromagnetic force ripple, and sensor noise are not directly compensated. In addition, the compensation performance depends on the accuracy of secondary-path estimation and the selection of algorithm parameters. Future research will focus on multi-harmonic vibration compensation and parameter adaptation techniques under a wider range of operating conditions, including wider speed ranges and variable load conditions.

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