Progress In Electromagnetics Research C, Vol. 172, 507–512, 2026

PIER C

(Received 10 July 2026, Accepted 13 August 2026, Scheduled 24 August 2026)

Chebyshev Polynomial-Based Adaptive Post-Distorter for Nonlinear Receiver Compensation

Chongchong Chen[1,2,*], Hongmin Lu[1,2,*], Fulin Wu[1,2], and Yangzhen Qin[1,2]

[1]*School of Electronic Engineering, Xidian University, Xi'an 710071, China*
[2]*China Academy of Space Technology, Xi'an 710100, China*

ABSTRACT: Receiver nonlinearity degrades signal quality in modern wireless systems. This paper proposes an adaptive post-distorter using Chebyshev polynomials as basis functions to improve numerical stability and convergence speed. Unlike conventional or distribution-matched bases, the Chebyshev basis ensures a well-conditioned input correlation matrix regardless of signal distribution. This property, together with the minimax approximation, enables the recursive least squares (RLS) algorithm to converge within a few pilot symbols — an order of magnitude faster than the power-series basis. Experiments on a real wideband receiver with OFDM and WCDMA signals show significant ACPR improvements over the power-series and Legendre bases. The proposed approach exhibits robust performance, fast convergence, and broad applicability.

1. INTRODUCTION

Modern wireless receivers employ wideband modulations with high peak-to-average power ratios (PAPR). Their front-end components — low-noise amplifiers (LNAs), mixers, and analog-to-digital converters (ADCs) — exhibit inherent nonlinearity, causing spectral regrowth and in-band distortion that severely degrade signal quality. Digital post-compensation using memory polynomials (MPs) is a common solution due to its structural simplicity [1–3].

The field of receiver nonlinear compensation has been developed by several approaches. Early approaches relied on Volterra-series-based equalizers [4], which provide a general framework for modeling nonlinear systems with memory. However, the number of coefficients grows exponentially with memory depth and nonlinear order, making real-time implementation impractical [5]. To reduce complexity, memory polynomial (MP) models were introduced [6, 7], retaining only diagonal terms. Yet the standard power-series basis is non-orthogonal, leading to numerical instability and slow convergence when adaptive algorithms are employed. More recently, orthogonal polynomial bases — such as Legendre, Hermite, and Laguerre — have been explored to improve conditioning [8–10]. These bases achieve low condition numbers only when the input distribution matches their respective weight functions (uniform for Legendre, Gaussian for Hermite, exponential for Laguerre), limiting their applicability in practical receivers where signal statistics are unknown or time-varying. Deep-learning-based approaches [11, 12] have also been proposed, but they require extensive offline training and significant computational resources, hindering real-time adaptation.

A key distinction exists between transmitter predistortion and receiver post-distortion. In transmitter predistortion, the original symbols are known, allowing direct error computation without pilots. In receiver post-distortion, however, transmitted symbols are unknown, making online adaptation fundamentally more challenging. Most existing MP compensators are offline fixed-coefficient models [4–9]. They are trained once on pre-collected input-output data and then deployed with static coefficients. They cannot track time-varying nonlinearities (temperature drift, aging) [13, 14] and require cumbersome data acquisition. Therefore, an online adaptive model that continuously updates its coefficients using the received signal is urgently needed.

To address these challenges, this paper proposes an RLS-based adaptive post-distorter. RLS convergence critically depends on the conditioning of the input correlation matrix. To ensure a well-conditioned matrix under arbitrary input distributions, we employ Chebyshev polynomials as basis functions. Chebyshev polynomials are orthogonal on $[-1, 1]$ and possess the minimax property [15], which guarantees uniform approximation error across the entire input range — beneficial for high-PAPR signals. Compared with the conventional power-series basis, the Chebyshev basis dramatically reduces the condition number of the input correlation matrix [17], thereby accelerating RLS convergence. Moreover, unlike Legendre, Hermite, or Laguerre bases that require exact input distribution matching, the Chebyshev basis maintains a well-conditioned correlation matrix under a wide variety of practical distributions [16]. This distribution-insensitive property makes the Chebyshev-enhanced RLS post-distorter universally applicable without requiring prior knowledge of signal statistics.

The remainder of this paper is organized as follows. Section 2 presents the receiver model and the Chebyshev basis.

* Corresponding authors: Hongmin Lu (hmlu@mail.xidian.edu.cn); Chongchong Chen (ccchen@stu.xidian.edu.cn).

doi:10.2528/PIERC26071003

Section 3 describes the adaptive RLS post-distorter. Section 4 provides experimental results. Section 5 concludes the paper.

2. RECEIVER MODEL AND CHEBYSHEV BASIS

2.1. Receiver Nonlinear Model

The receiver front-end is modeled by a memory polynomial (MP) with complex baseband input $x(n)$ and output $y(n)$:

$$y(n) = \sum_{m=0}^{M} \sum_{k=1}^{K} a_{k,m} \phi_k(x(n-m)), \tag{1}$$

where M is the memory depth, K the nonlinear order, $\phi_k(\cdot)$ the kth basis function, and $a_{k,m}$ complex coefficients. For the conventional power-series basis, $\phi_k(x) = x|x|^{k-1}$. This basis suffers from high correlation among different orders, leading to an ill-conditioned Gram matrix that severely slows down adaptive algorithms.

The first-kind Chebyshev polynomial of degree k is defined on $[-1, 1]$ as $T_k(t) = \cos(k \arccos t)$. Important properties include [15]:

- **Three-term recurrence**: $T_0(t) = 1$, $T_1(t) = t$, $T_{k+1}(t) = 2tT_k(t) - T_{k-1}(t)$.
- **Orthogonality**: $\int_{-1}^{1} T_i(t)T_j(t)\frac{dt}{\sqrt{1-t^2}} = 0$ for $i \neq j$.
- **Minimax property**: Among all degree-k monic polynomials, $2^{1-k}T_k(t)$ minimizes maximum absolute value on $[-1, 1]$.

We evaluate the condition number κ of input correlation matrix $\mathbf{R} = \mathbb{E}[\phi(n)\phi^{\mathrm{H}}(n)]$ for different bases under uniform and Gaussian distributions with $M = 3$, $P = 5$. Fig. 1 shows that the power-series yields extremely high κ regardless of the distribution; Legendre, Hermite, and Laguerre yield low κ only when the input matches their weight functions (uniform, Gaussian, exponential). In contrast, the Chebyshev basis maintains low and steady κ for all tested distributions.

We further examine κ as a function of polynomial order P (with $M = 3$) and memory depth M (with $P = 3$) under a Gaussian input. Fig. 2(a) shows that the Chebyshev basis

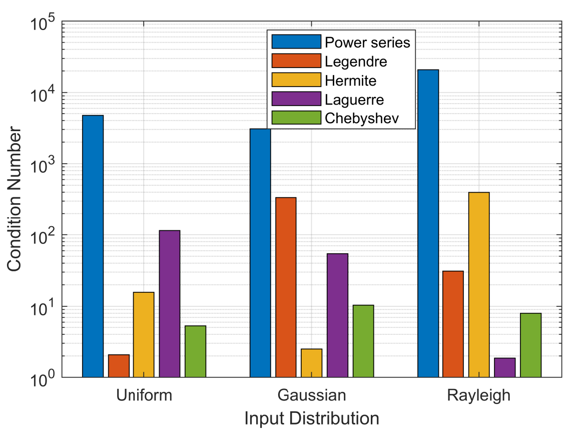

FIGURE 1. Condition number for different bases ($M = 3$, $P = 5$).

increases slowly and steadily, whereas the power-series basis grows exponentially. Fig. 2(b) shows that the Chebyshev basis remains well-conditioned for all memory depths. These results confirm the superior numerical properties of the Chebyshev basis.

2.2. Complex Chebyshev Basis for Post-Distorter

To apply Chebyshev polynomials to complex signals, we first normalize the received signal magnitude:

$$\tilde{y}(n) = \frac{y(n)}{\alpha(n)}, \quad \alpha(n) = \rho\alpha(n-1) + (1-\rho)|y(n)|, \tag{2}$$

with $\rho = 0.98$ ensuring $|\tilde{y}(n)| \leq 1$. The complex Chebyshev basis functions are

$$\psi_k(\tilde{y}) = T_k(2|\tilde{y}| - 1) \cdot \frac{\tilde{y}}{|\tilde{y}| + \epsilon}, \tag{3}$$

where $\epsilon = 10^{-6}$ prevents division by zero. When $|\tilde{y}| = 0$, set $\psi_k(0) = 0$. The post-distorter output is

$$\hat{x}(n) = \sum_{m=0}^{M} \sum_{k=1}^{P} w_{m,k} \psi_k(\tilde{y}(n-m)), \tag{4}$$

with memory depth M, polynomial order P, and complex coefficients $w_{m,k}$. Define the regressor vector $\phi(n)$ of length $N = (M+1)P$; then $\hat{x}(n) = \mathbf{w}^{\mathrm{T}}\phi(n)$.

The low condition number of the Chebyshev basis, as demonstrated above, directly translates into fast and stable convergence of the adaptive RLS algorithm, which is the subject of the next section.

3. ADAPTIVE RLS POST-DISTORTER USING CHEBYSHEV BASIS

3.1. System Integration and Adaptive Procedure

Figure 3 illustrates the overall adaptive post-distortion scheme. The received signal $y(n)$ is first normalized and then processed by the Chebyshev basis to form the regressor vector $\phi(n)$. The RLS algorithm computes the error between the post-distorter output $\hat{x}(n)$ and the reference signal, and updates the coefficient vector $\mathbf{w}(n)$ accordingly. The compensated output $\hat{x}(n)$ is sent to the demodulator and, when in decision-directed mode, the hard decision is fed back as the reference.

The complete online adaptation procedure consists of three stages:

1. **Offline preparation:** Chebyshev basis functions are defined, and normalization parameters are computed from a short initial data segment.
2. **Pilot-assisted RLS training:** A short pilot sequence of $N_p = 30$ symbols is transmitted. For each pilot symbol, the RLS recursions are executed using the known pilot as the reference. The system monitors the normalized mean square error (NMSE) over a sliding window of 5 consecutive pilots. When the NMSE falls below -25 dB, the system is considered converged and switches to the next stage.

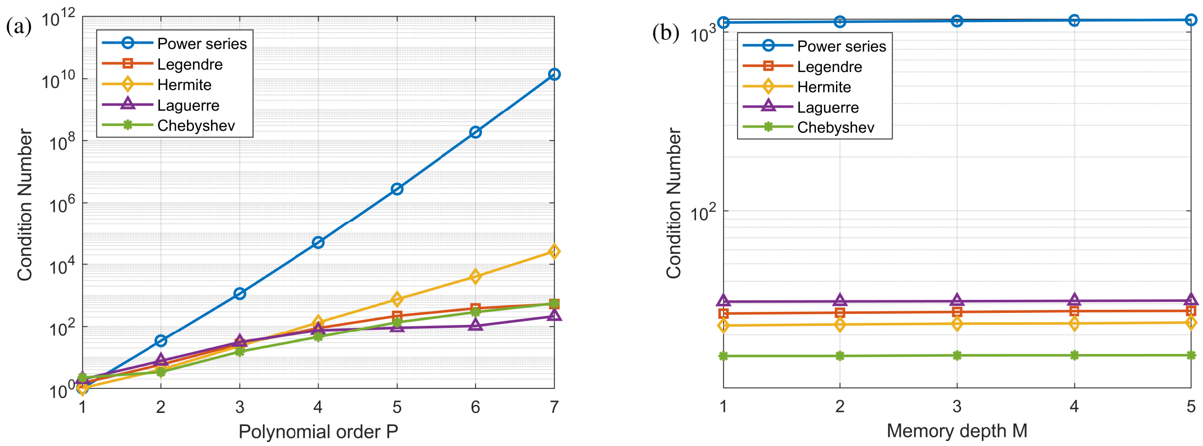

FIGURE 2. Condition number versus (a) polynomial order P and (b) memory depth M for different bases.

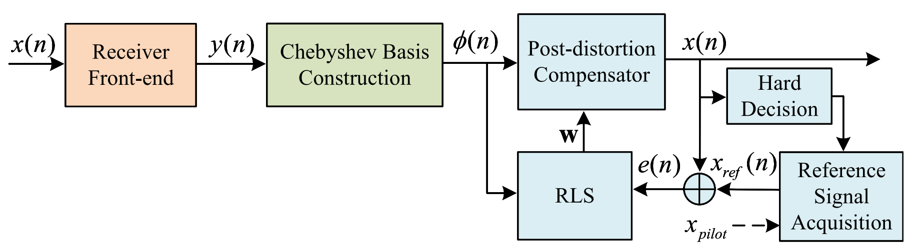

FIGURE 3. Block diagram of the proposed Chebyshev-based adaptive RLS post-distorter.

3. **Decision-directed tracking:** After convergence, the system switches to decision-directed mode. The RLS updates continue using hard decisions as the reference. To prevent error propagation, an error gate is applied: if the instantaneous error magnitude exceeds a threshold $\theta = 0.2$, the coefficient update is skipped for that symbol. This mechanism enables the compensator to slowly follow time-varying nonlinearities while avoiding divergence due to incorrect decisions.

3.2. RLS Algorithm

We employ the recursive least squares (RLS) algorithm [18] to update the post-distorter's coefficient vector $\mathbf{w}(n)$. RLS minimizes the exponentially weighted sum of squared errors, leading to faster convergence than gradient-based methods, especially when the input correlation matrix is well conditioned.

Define the error signal as

$$e(n) = x_{\mathrm{ref}}(n) - \mathbf{w}^{\mathrm{T}}(n-1)\phi(n), \tag{5}$$

where $x_{\mathrm{ref}}(n)$ is a reference signal, and $\phi(n)$ is the regressor vector built from the Chebyshev basis (see Section 2). The cost function minimized by RLS is

$$J(\mathbf{w}) = \sum_{i=1}^{n} \lambda^{n-i} |e(i)|^2, \tag{6}$$

with forgetting factor $0 < \lambda \leq 1$.

The standard RLS recursions are as follows. Initialize $\mathbf{w}(0) = \mathbf{0}$, $\mathbf{P}(0) = \delta^{-1}\mathbf{I}$ (with a small positive δ). For each time step $n = 1, 2, \ldots$:

$$\mathbf{g}(n) = \frac{\mathbf{P}(n-1)\phi(n)}{\lambda + \phi^{\mathrm{T}}(n)\mathbf{P}(n-1)\phi(n)}, \tag{7}$$

$$\mathbf{P}(n) = \lambda^{-1}\left(\mathbf{P}(n-1) - \mathbf{g}(n)\phi^{\mathrm{T}}(n)\mathbf{P}(n-1)\right), \tag{8}$$

$$\mathbf{w}(n) = \mathbf{w}(n-1) + \mathbf{g}(n)e^{*}(n). \tag{9}$$

The per-iteration complexity is $O(N^2)$, where $N = (M+1)P$ is the number of coefficients. For typical values (e.g., $M = 3$, $P = 5$), this is modest and easily handled by modern digital signal processors or field programmable gate arrays (FPGAs).

3.3. Reference Signal Acquisition

The choice of reference signal $x_{\mathrm{ref}}(n)$ determines the operating mode:

- **Pilot-assisted mode (initial convergence):** At system start-up or after a significant change in the nonlinearity, a short pilot sequence of known symbols is transmitted. The receiver uses these known symbols as $x_{\mathrm{ref}}(n)$. Because the Chebyshev basis provides a well-conditioned correlation matrix, the RLS algorithm converges very quickly, requiring only a few tens of pilot symbols.
- **Decision-directed mode (steady-state tracking):** After initial convergence, the system switches to decision-directed mode. The reference signal is then obtained by making hard decisions on the post-distorter output:

$$x_{\mathrm{ref}}(n) = \mathrm{dec}\left(\hat{x}(n)\right), \tag{10}$$

where $\mathrm{dec}(\cdot)$ maps the soft estimate to the nearest constellation point. The algorithm continues with a slightly larger

TABLE 1. Average number of pilot symbols to reach -25 dB NMSE and steady-state NMSE.

Method	Mean pilots	Steady-state NMSE (dB)
Power-series RLS	320	-25
Legendre RLS	156	-30
Volterra RLS	350	-39
Chebyshev RLS (proposed)	26	-37.5

TABLE 2. Complexity comparison.

Method	Coeff.	Complexity	Rel. time	Tot. cost
Power-series	$(M+1)P$	$O((M+1)^2P^2)$	1.0	320
Legendre	$(M+1)P$	$O((M+1)^2P^2)$	1.1	172
Volterra	$\sum_{i=0}^{P}(M+1)^i$	$O((M+1)^{2P})$	8.5	2975
Chebyshev	$(M+1)P$	$O((M+1)^2P^2)$	1.2	31

forgetting factor to track slow variations (e.g., temperature drift, component aging) without requiring additional pilots.

The convergence rate of RLS is directly determined by the eigenvalue spread of the input correlation matrix $\mathbf{R} = \mathbb{E}[\phi(n)\phi^{\mathrm{H}}(n)]$. While RLS converges in at most N steps for a white input, an ill-conditioned $\mathbf{R}$ can severely slow down the convergence. Thanks to the low condition number ensured by the Chebyshev basis, the RLS algorithm using Chebyshev polynomials is expected to converge an order of magnitude faster than the power-series counterpart, regardless of input signal statistics. Moreover, the distribution-insensitive property of the Chebyshev basis guarantees that fast convergence is preserved even when the input distribution changes. Thus, the proposed Chebyshev-RLS post-distorter is universally applicable without requiring any prior knowledge of signal statistics.

4. EXPERIMENTAL RESULTS

To validate these points, we evaluate the performance using two high-PAPR realistic signals: an OFDM signal (64 subcarriers, 16-QAM modulation, cyclic prefix length 16) and a 3-carrier WCDMA signal. The input-output data used in this paper are obtained from an actual wideband direct-conversion receiver [19].

Figure 4 shows the normalized mean-square error (NMSE) convergence during pilot training (mean over 100 runs). The power-series RLS converges very slowly, requiring hundreds of pilot symbols to reach -25 dB NMSE. Legendre RLS converges faster (about 150 pilots) but still suffers from distribution mismatch. Volterra RLS converges even slower than power-series, taking about 350 pilots to reach -25 dB NMSE, although its steady-state NMSE reaches -39 dB, slightly better than Chebyshev. In contrast, the proposed Chebyshev RLS converges within only 20–30 pilots — an order of magnitude faster — achieving -37.5 dB NMSE. This speedup results from the low condition number ensured by the Chebyshev basis, as demonstrated in Section 2. Table 1 lists the average number of pilots required for each method.

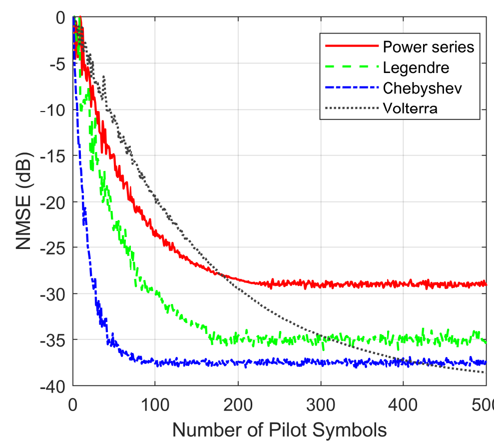

FIGURE 4. NMSE convergence during pilot training.

Table 2 compares the number of coefficients, RLS update complexity, single-iteration relative runtime, and the total computational cost for each method, where M and P denote memory depth and nonlinear order, respectively. The total computational cost is defined as the product of the single-iteration relative runtime and the number of iterations required to reach convergence (from Table 1). The Chebyshev basis requires slightly more computation per iteration due to the polynomial recurrence and normalization step. However, this overhead is negligible compared to the overall convergence time, as Chebyshev requires far fewer iterations to converge. Consequently, the total computational cost of the proposed method is roughly one order of magnitude lower than that of the power-series baseline.

After convergence, we compute the power spectral density (PSD) and adjacent channel power ratio (ACPR). Fig. 5 shows spectra for uncompensated, power-series RLS, Legendre RLS, and the proposed Chebyshev RLS for two different input signals, 16-QAM OFDM and 3-carrier WCDMA. Table 3 lists ACPR values. The uncompensated signal exhibits strong spectral regrowth. Power-series RLS reduces out-of-band emissions only moderately. Legendre RLS provides a clear improvement, but still leaves visible sidelobes. The proposed Chebyshev RLS achieves the most effective suppression with

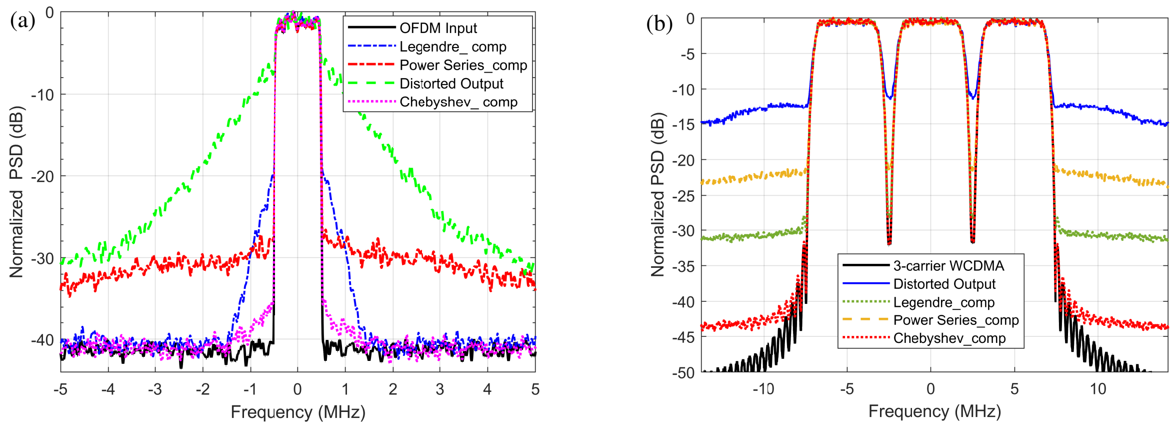

FIGURE 5. Power spectral density after compensation with different input signals. (a) OFDM and (b) 3-carrier WCDMA.

TABLE 3. ACPR obtained with different polynomial bases.

Signal	Power series	Legendre	Chebyshev
OFDM	-23.6	-31.5	-37.5
3-carrier WCDMA	-22.8	-34.4	-39.3

almost no discernible regrowth, demonstrating the best steady-state compensation quality regardless of signal type.

To evaluate robustness, we add artificial additive Gaussian white noise (AWGN) to the received signal. Table 4 reports the ACPR for both OFDM (16-QAM, 10 MHz) and 3-carrier WCDMA (QPSK, 5 MHz) signals at different signal-to-noise ratio (SNR) levels (15, 25, 35 dB). For each signal and SNR condition, the three columns correspond to Power series (PS), Legendre (Leg), and Chebyshev (Cheb), respectively. The proposed Chebyshev method consistently achieves the best ACPR across all tested conditions. The advantage of the Chebyshev method is particularly pronounced at higher SNR levels, where numerical conditioning becomes the dominant factor. These results confirm that the proposed method maintains its superiority across multiple modulation formats and SNR conditions, without requiring prior knowledge of the input signal statistics.

To evaluate tracking capability under time-varying nonlinearities, we simulate a temperature-drift scenario, where receiver nonlinearity coefficients are abruptly increased by 10% at symbol index 1000. Fig. 6 compares the NMSE performance of a fixed model (trained offline with no subsequent adaptation) and the proposed decision-directed RLS adaptation. The fixed model exhibits a permanent degradation from -35 dB to -25 dB after the drift occurs. In contrast, the proposed method temporarily degrades to -25.6 dB but recovers to approximately -33 dB within 200 symbols, demonstrating its ability to effectively track temperature-induced nonlinear variations. This confirms the practical advantage of the proposed online adaptive strategy for real-world receivers operating under changing environmental conditions.

TABLE 4. ACPR (dBc) at different SNR levels.

Signal	SNR	PS	Leg	Cheb
OFDM	15 dB	-18.2	-24.1	-29.3
	25 dB	-21.5	-28.7	-34.1
	35 dB	-23.6	-31.5	-37.5
3-carrier WCDMA	15 dB	-19.5	-26.6	-30.8
	25 dB	-20.8	-28.7	-35.5
	35 dB	-24.8	-33.9	-38.1

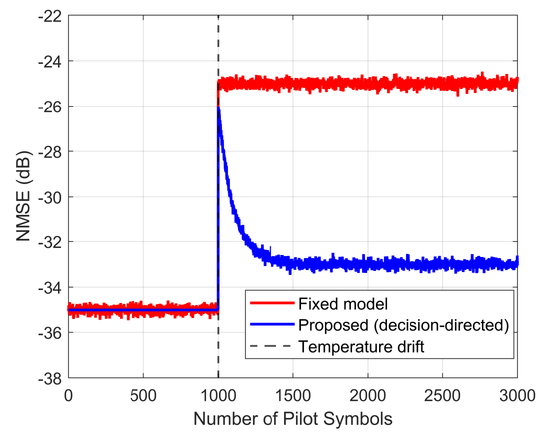

FIGURE 6. NMSE tracking performance under simulated temperature drift.

These results confirm that the Chebyshev-based post-distorter converges fast, provides superior steady-state linearization, and maintains these advantages across different input signal distributions without requiring prior knowledge of signal statistics. This distribution-robust property makes it highly attractive for practical receivers and a universally applicable solution for online nonlinear compensation.

5. CONCLUSION

This paper presents a Chebyshev polynomial-based adaptive RLS post-distorter for nonlinear receiver compensation. The Chebyshev basis maintains a well-conditioned input correlation matrix regardless of signal distribution, unlike power-series or distribution-matched bases that require perfect distribution matching. Experiments on OFDM and WCDMA signals demonstrate fast convergence, excellent ACPR improvement, and robust performance under varying input distributions. The proposed method is a practical, fast-converging, universally applicable solution for online nonlinear compensation in modern wireless receivers.

REFERENCES

[1] Singerl, P. and G. Kubin, "Chebyshev approximation of baseband Volterra series for wideband RF power amplifiers," in *2005 IEEE International Symposium on Circuits and Systems*, 2655–2658, Kobe, Japan, 2005.

[2] Morgan, D. R., Z. Ma, J. Kim, M. G. Zierdt, and J. Pastalan, "A generalized memory polynomial model for digital predistortion of RF power amplifiers," *IEEE Transactions on Signal Processing*, Vol. 54, No. 10, 3852–3860, Oct. 2006.

[3] Ding, L., G. T. Zhou, D. R. Morgan, Z. Ma, J. S. Kenney, J. Kim, and C. R. Giardina, "A robust digital baseband predistorter constructed using memory polynomials," *IEEE Transactions on Communications*, Vol. 52, No. 1, 159–165, Jan. 2004.

[4] Marttila, J., M. Allén, M. Kosunen, K. Stadius, J. Ryynänen, and M. Valkama, "Reference receiver enhanced digital linearization of wideband direct-conversion receivers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 65, No. 2, 607–620, Feb. 2017.

[5] Grimm, M., M. Allén, J. Marttila, M. Valkama, and R. Thomä, "Joint mitigation of nonlinear RF and baseband distortions in wideband direct-conversion receivers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 62, No. 1, 166–182, Jan. 2014.

[6] Rebeiz, E., A. S. H. Ghadam, M. Valkama, and D. Cabric, "Spectrum sensing under RF non-linearities: Performance analysis and DSP-enhanced receivers," *IEEE Transactions on Signal Processing*, Vol. 63, No. 8, 1950–1964, Apr. 2015.

[7] Chen, C., H. Lu, F. Wu, X. Liu, and Y. Qin, "Behavioral modeling of direct-conversion receivers for nonlinear distortion mitigation," *IEEE Access*, Vol. 12, 15 390–15 398, 2024.

[8] Huang, X., A. T. Le, H. Zhang, J. A. Zhang, and Y. J. Guo, "Orthogonal expanded memory polynomial model for circular complex Gaussian processes," *IEEE Transactions on Signal Processing*, Vol. 72, 3287–3302, Jul. 2024.

[9] Lee, H., J. Kim, G. Choi, I. P. Roberts, J. Choi, and N. Lee, "Nonlinear self-interference cancellation with adaptive orthonormal polynomials for full-duplex wireless systems," *IEEE Transactions on Wireless Communications*, Vol. 24, No. 7, 5796–5810, Jul. 2025.

[10] Huang, L., Z. Dai, and J. Yao, "Modified ito generalization of the Hermite polynomials for the linearization of radio over fiber links with increased numerical stability," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 73, No. 3, 1752–1760, Mar. 2025.

[11] Jin, Q., X. Li, X. Li, and F. Liu, "A deep learning-based receiver for LEO satellite communications with nonlinear distortion," *IEEE Wireless Communications Letters*, Vol. 15, 16–20, Sep. 2025.

[12] Jin, Q., X. Li, R. Guo, and F. Liu, "DRNet: Deep learning-based dual-branch receiver for OFDM with nonlinear distortions," *IEEE Communications Letters*, Vol. 30, 357–361, Dec. 2025.

[13] Islam, S. S. and A. F. M. Anwar, "Temperature-dependent nonlinearities in GaN/AlGaN HEMTs," *IEEE Trans. Electron Devices*, Vol. 49, No. 5, 710–717, May 2002.

[14] Boumaiza, S. and F. M. Ghannouchi, "Thermal memory effects modeling and compensation in RF power amplifiers and predistortion linearizers," *IEEE Transactions on Microwave Theory and Techniques*, Vol. 51, No. 12, 2427–2433, Dec. 2003.

[15] Mason, J. C. and D. C. Handscomb, *Chebyshev Polynomials*, CRC Press, 2002.

[16] Xu, G., H. Yu, C. Hua, and T. Liu, "Chebyshev polynomial-LSTM model for 5G millimeter-wave power amplifier linearization," *IEEE Microwave and Wireless Components Letters*, Vol. 32, No. 6, 611–614, Jun. 2022.

[17] Chen, C., H. Lu, F. Wu, and Y. Qin, "Conditioning analysis of orthogonal polynomial models for receiver nonlinear behavioral model," *Electronics*, Vol. 15, No. 9, 1892, 2026.

[18] Haykin, S., *Adaptive Filter Theory*, 5th ed., Prentice Hall, 2014.

[19] Chen, C., H. Lu, Y. Zhang, and G. Zhang, "Behavioral modeling for nonlinear effects of receiver front-ends based on block-oriented structure," *Progress In Electromagnetics Research C*, Vol. 115, 205–217, Sep. 2021.