

A Novel Method for Calculating the Ground Loss Resistance of a Very Low Frequency T-Type Antenna Array

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ABSTRACT: This paper proposes a novel algorithm to calculate the ground-loss resistance of a very low-frequency (VLF) T-type antenna array with a ground screen. The near-ground region of the T-type antenna is divided into three zones: ground screen zone, ground screen outer-end zone, and ground screen outer lateral zone. Magnetic-field loss resistance and electric-field loss resistance generated by horizontal top wires and vertical down leads are calculated in all three zones to obtain the antenna's total ground loss resistance. Example calculations show that the results obtained by the proposed method are closer to the measured values than those calculated by the classical Wait-Pope method. The variation law of ground-loss resistance generated in different zones of the antenna site with earth conductivity is analyzed, and valuable conclusions are drawn.

1. INTRODUCTION

Very low frequency (VLF) communication (frequency: 3 ~ 30 kHz) is an important means for time service, navigation, and submarine communication. VLF transmitting antennas generally adopt vertical grounded antennas erected perpendicular to the ground, radiating vertically polarized waves with relatively low propagation attenuation. To improve the antenna's radiation performance, the size of the VLF transmitting antenna needs to be increased. For configurations such as T-type top-loaded antennas and umbrella antennas, the antenna structure is usually supported by iron towers with a height of 200 m to 400 m. Although the scale of VLF transmitting antennas is large, their geometric dimensions are far smaller than the wavelength (10 km to 100 km), leading to a fairly low radiation resistance, generally only several ohms. Therefore, the loss resistance of the antenna system must be minimized. The ground loss resistance constitutes the dominant component [1, 2, 10–13] of the total loss resistance, and its calculation is a key task in designing VLF transmitting antennas.

A typical VLF T-type antenna array consists of multiple vertical down-leads and horizontal top wires. Figure 1 shows the curtain structure of a T-type antenna element in a typical VLF T-type antenna array. Its horizontal top wires were suspended by three wire slings, while the down leads hung from the middle wire sling. The three wire slings were hung on six support masts via insulators, forming a T-type planar antenna. Beneath the antenna array lies a ground screen system. The main body of the ground screen was ground wires parallel to the horizontal top wires, acting as the primary source of losses in the ground screen system.

Ref. [7] presents the Wait-Pope analytical method for calculating the ground loss resistance of VLF transmitting antennas with large top loads. Refs. [8] and [9] calculate the ground loss

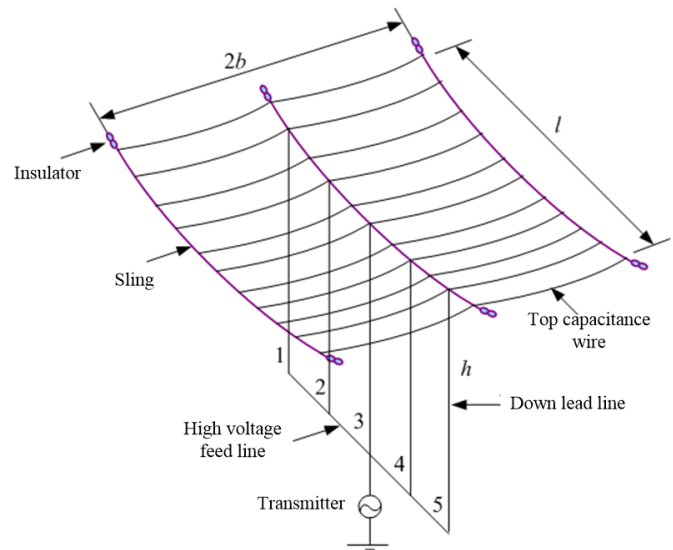


FIGURE 1. VLF T-type antenna curtain.

resistance of the T-type antenna array by using the moment-method-based NEC simulation software (with the Sommerfeld/Norton ground option) and the Sommerfeld integral-based algorithm combined with the Wait-Pope method, respectively. For T-type antennas equipped with top-loads, Ref. [7] neglects the contribution of horizontal top wires, which introduces considerable calculation errors for VLF T-type antennas with long top wires. This paper improves the Wait-Pope method by taking the magnetic-field loss contribution of top wires into account. The proposed approach is applicable to ground loss resistance calculation for T-type antenna with arbitrary-length top wires.

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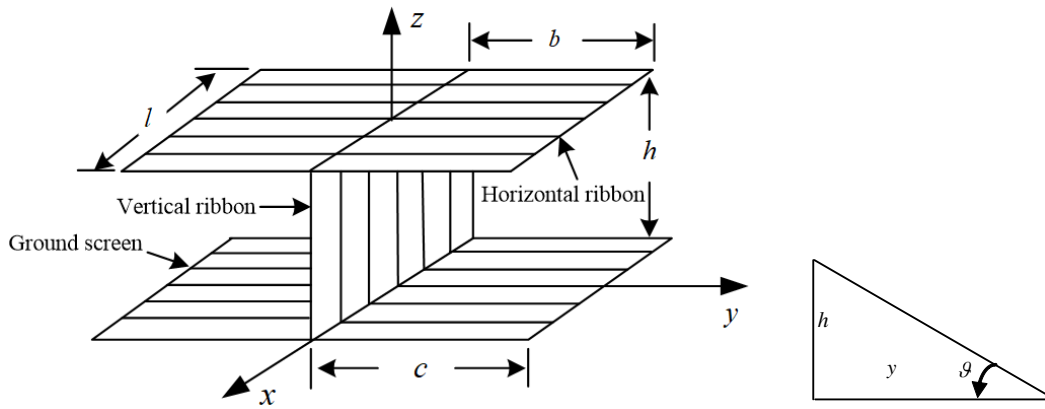


FIGURE 2. T-type planar antenna coordinate system.

2. CALCULATION PRINCIPLE

The ground loss power of an antenna system can be regarded as the sum of magnetic and electric field loss powers. Electric-field loss originates from the displacement current excited by the antenna flows from the antenna to the ground screen system. Its magnitude depends not only on the vertical electric field in the antenna's near region, but also on the antenna's top load and the electrical properties of the ground surface. Magnetic-field loss arises from currents induced by the tangential near-surface magnetic field flowing toward the antenna's vertical down leads. It corresponds to thermal loss generated by current propagating within the imperfectly conducting earth, excluding the copper loss inside ground screen conductors themselves. The calculation procedure for ground loss resistance is described as follows. First, the surface impedance and effective series resistance of the grounded system per unit area are determined. Next, the tangential magnetic field and vertical electric field on the ground surface within the antenna region are calculated. Finally, the magnetic-field loss power and electric-field loss power in the antenna's near zone are computed [3–6], yielding the magnetic-field loss resistance R_H and electric-field loss resistance R_E . The total ground loss resistance of the antenna system is given by $R_g = R_H + R_E$.

For VLF antennas with short electrical length, most losses concentrate in the antenna's near-field region where the surface tangential magnetic field $\mathbf{H}_t$ is in phase with the antenna current I_0 . The magnetic-field loss resistance R_H in the ground is expressed as

$$R_H = \frac{1}{I_0^2} \int_A R'_H |\mathbf{H}_t|^2 dA \quad (1)$$

where A denotes the entire near-region surrounding the antenna; I_0 is the current on the antenna wire; and R'_H is the equivalent surface resistance of the earth with a ground screen, which can be approximately calculated by the parallel combination of the earth surface impedance (without ground wires) and the grid impedance [7].

$$R'_H \approx \frac{2.1 \times 10^{-9} s^2 f^{3/2} \sigma_g^{1/2} [\log_{10}(s/\pi d_0)]^2}{1 + 1.06 \times 10^{-6} s^2 f \sigma_g [\log_{10}(s/\pi d_0)]^2 + 1.45 \times 10^{-3} s f^{1/2} \sigma_g^{1/2} \log_{10}(s/\pi d_0)} \quad (2)$$

In the formula, s is the spacing of ground wires; d_0 is the diameter of ground wires; f denotes the operating frequency; and σ_g stands for the earth conductivity.

Outside the ground screen region, s can be regarded as infinity. Then, the equivalent surface resistance of the earth is approximately

$$R'_H = R'_{H2} \approx 2.1 \times 10^{-3} f^{1/2} \sigma_g^{-1/2} \quad (3)$$

Assume that the vertical displacement current density J_d at an arbitrary position on the ground surface within the ground screen region is equal to the conduction current density J_z transmitted vertically underground. Given $J_d = \omega \epsilon E_z$, where E_z is the vertical component of the electric field, the electric field loss resistance can be written as:

$$R_E = \frac{1}{I_0^2} \int_A J_z^2 R'_E dA = \frac{1}{I_0^2} \int_A R'_E \cdot (\omega \epsilon)^2 E_z^2 dA \quad (4)$$

where R'_E is the effective series resistance per unit area [7].

$$R'_E = \frac{\bar{h} \sigma_g}{\sigma_g^2 + (2\pi f \epsilon_g)^2} \quad (5)$$

In the formula, $\bar{h}$ is the equivalent depth of ground screen wires, and ϵ and ϵ_g represent the dielectric constants in free space and the earth, respectively. Outside the ground screen zone, $\bar{h} = \delta/\sqrt{2}$. Inside the ground screen zone, if $h \gg s$ and $s \gg 2h' + d$, then $\bar{h}$ can be determined by:

$$\bar{h} = h' + 0.366s \left\{ \log_{10} \frac{s}{\pi d_0} + R'_0 \log_{10} \frac{s}{2\pi(2h' + d_0/2)} \right\} \quad (6)$$

For the VLF T-type antenna and ground screen system shown in Figure 1, let the height of the vertical antenna section (down-lead) above the ground be h , the length of the horizontal top wires be $2b$, the length of the antenna curtain be l , and the length of ground wires be $2c$. A Cartesian coordinate system was established with the intersection of the central down lead and ground surface as the origin. The xoy plane coincides with the ground surface, the z -axis runs along the down-lead, the y -axis is parallel to the top wires, and the x -axis is perpendicular to the horizontal top wires, as shown in Figure 2. All vertical

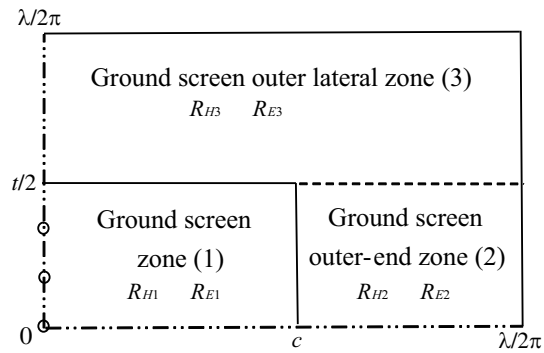


FIGURE 3. Schematic diagram of zone division on the ground near the antenna (1/4 Antenna).

down leads lie in the plane $y = 0$, the horizontal top wires is located on the plane $z = h$, and the ground surface corresponds to the plane $z = 0$. Since the calculation of the ground loss resistance relies on solving the near electromagnetic field on the ground surface close to the antenna, and VLF transmitting antennas have a complicated structure, it is difficult to accurately compute the electromagnetic field distribution in the region near the antenna. Therefore, both the down leads and top wires are approximated as a single planar surface [6, 7], and the ground surface was treated as a perfectly conducting plane to approximate the electromagnetic field distribution over the ground within the antenna near zone.

To facilitate the calculation of the magnetic field loss resistance, the near-ground region of the T-type antenna was divided into three zones (see Figure 3): ① the ground screen zone with ground screen wires laid, ② the ground screen outer-end zone on both ends of the T-type antenna top wires, ③ the ground screen outer lateral zone on both sides of the T-type antenna top wires. The outer boundary of the ground screen was set at a distance of $\lambda/2\pi$ from the antenna's central feed point. The total magnetic field loss resistance of the antenna consists of contributions from all three zones, expressed as:

$$R_g = R_{H1} + R_{H2} + R_{H3} + R_{E1} + R_{E2} + R_{E3} \quad (7)$$

where R_{H1} , R_{H2} , and R_{H3} denote the magnetic-field loss resistance generated jointly by the vertical down leads and horizontal top wires in the ground screen zone, ground screen outer-end zones, and ground screen outer lateral zones, respectively; R_{E1} , R_{E2} , and R_{E3} represent the electric-field loss resistances generated by the T-type antenna in all three zones above, respectively.

3. MAGNETIC FIELD LOSS RESISTANCE OF T-TYPE ANTENNA

3.1. Surface Magnetic Field in Ground Screen Zone and Ground Screen Outer-End Zone

Because $b \ll \lambda$, the current distribution on the antenna top wires can be approximated as a linear distribution. For a T-type antenna curtain that consists of multiple horizontal top wires, every single top wire is treated as a thin strip with nearly constant current, and the corresponding strips together form a complete conductor body. Assuming that the current of the T-type

antenna top wires is uniformly distributed along the x -direction, the magnetic field on the ground surface induced by the T-type antenna can be calculated via surface integration. When l is relatively large and the surface magnetic field is assumed to be uniform along the x -direction, the magnetic field intensity at an arbitrary point on the ground surface from the vertical down-lead can be approximated as

$$H_{x1}^h(y) = \frac{I_0}{\pi b l} \left[\frac{h}{2} \ln \frac{h^2 + (y+b)^2}{h^2 + (y-b)^2} + (2b) \arctan \frac{y}{h} + (y-b) \arctan \frac{y-b}{h} - (y+b) \arctan \frac{y+b}{h} \right] \quad (8)$$

The magnetic field generated by the down lead in the ground screen zone and ground screen outer-end zones can be expressed as

$$H_{x1}^v(y) = \frac{I_0}{\pi l} \arctan \left(\frac{h}{y} \right) \quad (9)$$

3.2. Surface Magnetic Field in Ground Screen Outer Lateral Zone

The surface magnetic field in the ground screen outer lateral zone is generated by the horizontal top wires and vertical down leads. Excluding the ground screen zone on both sides of the horizontal top wire, the magnetic field can be considered as that generated by a single horizontal wire carrying a uniform current I_0 in the middle of the horizontal top wires. The tangential magnetic field generated by the horizontal top wires in the ground screen outer lateral zones can be expressed as follows:

$$H_{x3}^h(x, y) = \frac{I_0 h}{2\pi b (h^2 + x^2)} \left[\frac{2yb}{\sqrt{h^2 + x^2 + y^2}} + \sqrt{h^2 + x^2 + (y-b)^2} - \sqrt{h^2 + x^2 + (y+b)^2} \right] \quad (10)$$

In the ground screen outer lateral zone at both ends of the down-lead plane, the magnetic field can be approximated as the field excited by a single vertical straight wire (a vertical antenna) carrying a uniform current I_0 . According to the approximate treatment method of Wait-Pope [7], the magnetic-field loss produced by a single vertical antenna in the ground screen outer lateral zone on both ends of the down-lead plane can be approximated by the loss from a uniform magnetic field within a finite strip of width $2c$ instead of the loss from a magnetic field that decays with increasing y . Provided that $2c$ is sufficiently large relative to the antenna height h and coordinate x , the approximated magnetic-field loss resistance in this zone will not be underestimated. For approximation, within the width $2c$ zone, the magnetic field induced by vertical down leads is identical for all points at equal distance from the ground screen edge. Outside the $2c$ width in the outer ground screen region, the magnetic field from vertical down leads is neglected. The magnetic field is then approximated as

$$H_{x3}^v(x, y) \approx \begin{cases} \frac{I_0 h}{2\pi x^2} & (-c \leq y \leq c) \\ 0 & (-c > y > c) \end{cases} \quad (11)$$

3.3. Magnetic Field Loss Resistance in Ground Screen Zone

Because the magnetic field inside the ground screen zone beneath the antenna curtain is assumed to be uniformly distributed along the x -direction, the ground loss resistance within the ground screen zone can be calculated using a ground screen zone of length l . We do not consider the edge effect of the antenna wire. The magnetic-field loss resistance jointly generated by both horizontal top wires and down leads within the ground screen zone is

$$R_{H1} = 2lR'_H \int_0^c \frac{|H_{x1}^v(y) + H_{x1}^h(y)|^2}{I_0^2} dy = \frac{2}{\pi^2 b^2 l} R'_H F_1(c) \quad (12)$$

where R'_H is calculated using Eq. (2), and $F_1(\tau)$ is defined as

$$F_1(\tau) = \int_0^\tau \left[\frac{h}{2} \ln \frac{h^2 + (y+b)^2}{h^2 + (y-b)^2} + (2b) \arctan \frac{y}{h} + (y-b) \arctan \frac{y-b}{h} - (y+b) \arctan \frac{y+b}{h} + b \left(\arctan \frac{h}{y} \right)^2 \right] dy \quad (13)$$

3.4. Magnetic Field Loss Resistance in Ground Screen Outer-End Zone

The analysis method is identical to that for the ground screen zone. The magnetic field loss resistance generated by the top wire within the ground screen outer-end zone is

$$R_{H2} = 2l R'_{H2} \int_c^{\lambda/2\pi} \frac{|H_{x1}^v(y) + H_{x1}^h(y)|^2}{I_0^2} dy = 2.1 \times 10^{-3} f^{1/2} \sigma_g^{-1/2} \frac{2}{\pi^2 b^2 l} F_2(c) \quad (14)$$

where R'_{H2} is calculated using Eq. (3), and $F_2(\tau)$ is defined as.

$$F_2(\tau) = \int_\tau^{\lambda/2\pi} \left[\frac{h}{2} \ln \frac{h^2 + (y+b)^2}{h^2 + (y-b)^2} + (2b) \arctan \frac{y}{h} + (y-b) \arctan \frac{y-b}{h} - (y+b) \arctan \frac{y+b}{h} + b \left(\arctan \frac{h}{y} \right)^2 \right] dy \quad (15)$$

3.5. Magnetic Field Loss in Ground Screen Outer Lateral Zone

The magnetic-field loss resistance jointly generated by both horizontal top wires and down-leads in the ground screen outer lateral zone is expressed as

$$R_{H3} = 4R'_{H2} \int_{t/2}^{\lambda/2\pi} \int_0^{\lambda/2\pi} \frac{|H_{x3}^v(x, y) + H_{x3}^h(x, y)|^2}{I_0^2} dy dx$$

$$= \frac{h^2 c R'_{H2}}{3\pi^2} \left[\left(\frac{2}{t} \right)^3 - \left(\frac{2\pi}{\lambda} \right)^3 \right] + \frac{R'_{H2}}{2\pi^2 b^2 h} \left[\frac{\pi}{2} + \frac{2\pi h}{\lambda} - \arctan \frac{t}{2h} - \frac{ht/2}{h^2 + (t/2)^2} \right] F_3(\lambda/2\pi) + \frac{2R'_{H2}}{\pi^2 b h} \left[\frac{2h}{t} - \frac{2\pi h}{\lambda} + \arctan \frac{t}{2h} - \arctan \frac{\lambda}{2\pi h} \right] F_3(c) \quad (16)$$

where

$$F_3(\tau) = b \left[2\sqrt{h^2 + (t/2)^2 + \tau^2} - 2\sqrt{h^2 + (t/2)^2} + \sqrt{h^2 + (t/2)^2 + b^2} \right] + \frac{\tau - b}{2} \sqrt{h^2 + (t/2)^2 + (\tau - b)^2} - \frac{c + b}{2} \sqrt{h^2 + (t/2)^2 + (\tau + b)^2} + \frac{h^2 + (t/2)^2}{2} \ln \frac{[\tau - b + \sqrt{h^2 + (t/2)^2 + (\tau - b)^2}][b + \sqrt{h^2 + (t/2)^2 + b^2}]}{[\tau + b + \sqrt{h^2 + (t/2)^2 + (\tau + b)^2}][-b + \sqrt{h^2 + (t/2)^2 + b^2}]} \quad (17)$$

In the T-type antenna array composed of multiple elements, each antenna element is independently tuned. We mainly focus on the elements within the array. When using Eq. (7) to calculate the ground loss resistance of a single T-type antenna element, we need to distinguish whether the element is located at the array center or the array edge. For the central elements, the ground loss resistance R_{H3} on both outer lateral sides of the ground screen zone of this element can be neglected. For edge elements, half of the ground loss resistance R_{H3} on both outer lateral sides of the element's ground screen zone shall be incorporated into the total loss resistance.

4. ELECTRIC FIELD LOSS RESISTANCE OF T-TYPE ANTENNA

For the T-type antenna, electric-field loss was considered in two zones: inside the ground screen zone and outside the ground screen zone. Within the ground screen zone, the horizontal wires of each wire structure were approximated as parallel plates; the ground surface electric field was considered as a uniform field, while the influence of the vertical down-leads can be neglected. Outside the ground screen zone, only the contribution of the antenna's vertical down leads was considered, while the influence of horizontal wires can be neglected [7].

If $\sigma_g \gg \omega \epsilon_g$, based on the electric-field analysis method for parallel-plate capacitors, the approximate upper-bound formula

for the electric-field loss resistance inside the ground screen zone is given by

$$R_{E1} \approx 1.778\bar{h}/[2b l \sigma_g + (2l + 4b)h\sigma_g] \quad (18)$$

When the antenna array length l is relatively large, the electric-field loss resistance in the ground screen outer-end zone generated by the vertical down-leads of the T-type antenna can be calculated by

$$R_{E2} = 70.4h^2 f^{-\frac{1}{2}} \sigma_g^{-\frac{3}{2}} (l + 2h) \left(\frac{1}{3l^2 c^3} - \frac{4}{l^4 c} + \frac{4\pi}{l^5} - \frac{8}{l^5} \arctan \frac{2c}{l} \right) \quad (19)$$

Using the Wait-Pope treatment method [7], the electric-field loss resistance in the ground screen outer lateral zone generated by vertical down leads of the T-type antenna can be approximated as

$$R_{E3} \approx 7.04h^2 f^{-\frac{1}{2}} \sigma_g^{-\frac{3}{2}} c \left(\frac{l}{2} + h \right)^{-5} \quad (20)$$

5. NUMERICAL EXAMPLES

Calculations were carried out for a typical VLF T-type antenna array. The array consisted of five T-type antenna elements with an inter-element spacing of 500 m. Each antenna element is composed of 11 top wires and 5 vertical down leads. The length of each top wire was 1500 m, with a wire spacing of 45 m, and the height of the vertical down leads was 200 m. The length of ground wires beneath the top wires was 1800 m, with a wire spacing of 10 m. The earth conductivity was 0.025 S/m; the diameter of ground copper wires was 3 mm; and the copper conductivity was 5.6×10^7 S/m.

Ground loss resistances of the edge element and inner element of the practical antenna array were calculated using the algorithm proposed in this paper. The results were compared with the measured ground loss resistance of the inner element and the calculation results from the Wait-Pope algorithm [7], as shown in Figure 4. Because the antenna loss resistance is a component

of the antenna input resistance, it cannot be measured directly. In practice, only the antenna input impedance and antenna system efficiency can be measured, and the measured loss resistance is obtained as the difference between the measured input resistance and radiation resistance. The antenna radiation efficiency must be acquired via field-strength measurement. This measurement procedure is complicated and strongly affected by the actual site environment, leading to inevitable measurement errors. The slight rise in the measured ground loss resistance near 30 kHz in the figure should be attributed to measurement errors. During measuring input resistance and radiation field strength for the measured inner element, all other antenna elements were open-circuited to ground.

It can be observed from Figure 4 that ground loss resistances calculated using both the algorithm proposed in this study and the Wait-Pope algorithm are all lower than measured values. The measured loss resistance includes not only the ground loss resistance but also other loss components of the antenna system, such as tuning coil loss, antenna wire loss, dielectric loss, and so on. Therefore, it is reasonable that the calculated ground loss resistance is less than the measured total loss resistance. The ground loss resistance calculated using the Wait-Pope algorithm was obviously underestimated. The reason is that the top wire length of the practical antenna is far greater than the antenna height, and the Wait-Pope method neglects the contribution of the top-wire current to the magnetic loss resistance, which inevitably makes the calculated results smaller than the actual values. Hence, the improved algorithm proposed in this study achieved higher calculation accuracy. The results also indicate that the ground loss resistance of edge elements is larger than that of central elements, caused by the extra ground loss resistance from the ground screen outer lateral zone of edge elements. The ground loss resistance calculated by the method in this paper is basically comparable with the calculation results in the literature [8, 9], while the proposed method is simpler, faster, and convenient for analyzing the contribution of each zone of the antenna site to the total ground loss resistance.

Under conditions that the burial depth of the ground wire was 0.3 m, the spacing between ground wires was 10 m, and the operating frequency was 20 kHz. Figure 5 shows calculated ground loss resistance varying with earth conductivity.

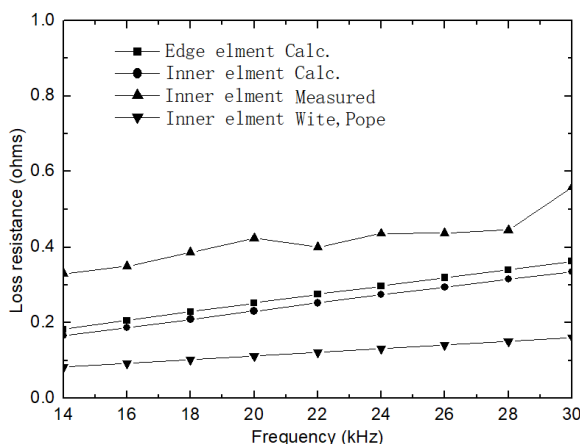


FIGURE 4. Variation of ground loss resistance versus frequency. ($h' = 0.3$ m, $s = 10$ m, $h = 200$ m, $\sigma_g = 2.5 \times 10^{-2}$ S/m).

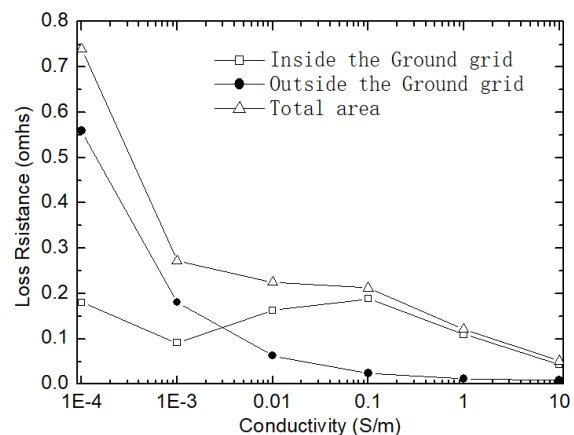


FIGURE 5. Calculated ground loss resistance under different earth conductivities. ($f = 20$ kHz, $h' = 0.3$ m, $s = 10$ m, $h = 200$ m).

As earth conductivity decreases, the loss resistance outside the ground screen increases significantly, and when $\sigma_g < 3 \times 10^{-3}$ S/m, the loss resistance outside the ground screen becomes the main component of the total ground loss resistance. Because the loss resistance outside the ground screen is one of the major losses of VLF transmitting antennas, the earth conductivity around the antenna is a key factor determining the ground loss resistance. The internal loss resistance of the ground screen originates from the parallel combination of the earth surface impedance (without ground wires) and grid impedance, which is related to parameters such as earth conductivity and ground-wire spacing. Accordingly, the internal loss resistance of the ground screen does not decrease monotonically with earth conductivity. Only when the earth surface impedance (without ground wires) is much smaller than the grid impedance will the internal loss resistance of the ground screen decrease monotonically as earth conductivity increases. For a multi-layer earth structure, the apparent conductivity of the multi-layer earth should be calculated first, which is then used to replace the homogeneous-earth conductivity σ_g adopted in this paper.

6. CONCLUSIONS

This paper proposes a novel method to calculate the ground-loss resistance of VLF T-type antennas. The calculated ground-loss resistance of the example antenna matches well with the measured data. Compared with the Wait-Pope method, the calculated results obtained by the proposed algorithm are closer to measured values, which verifies the algorithm's effectiveness. In contrast to numerical calculation methods for ground-loss resistance in [8] and [9], the presented algorithm is simple and efficient, and facilitates analyzing the contribution of each zone in the antenna site to the total ground loss resistance. The calculated results show that the ground loss resistance of edge antenna elements is larger than that of central elements in the array. This phenomenon arises from the extra ground loss outside the ground screen on the side of edge elements. Because the ground-loss resistance outside the ground screen constitutes one of the primary losses of VLF transmitting antennas, sites for VLF transmitting antennas should be in regions with high earth conductivity.

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