

Microwave Measurements of Complex Permittivity by a Cylindrical Dielectric Resonator in a Closed Metal Cavity – A Further Analysis

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ABSTRACT: This paper describes various aspects of the analysis of a technique for measuring the dielectric constant and loss tangent using a cylindrical dielectric resonator in a closed metal cavity. The resonator's $TE_{01\delta}$ resonant mode was selected as the measurement target. The clear electric and magnetic field patterns inside the dielectric resonator were obtained from a theory derived by improving a very simple field model. Resonant frequencies under different dielectric constants were also studied. All possible sources of conducting dissipation were investigated to improve the accuracy of the loss tangent measurement. The influence of the energy storage factor on the calculation of the loss tangent measurement was also investigated. This cavity is also used for measuring complex permittivity over a sweeping frequency range, and the accuracy and limitations of its swept-frequency capability are discussed. The improved field model was compared with the primitive model through experimental measurements.

1. INTRODUCTION

A dielectric rod resonator placed inside a closed cylindrical cavity, as shown in Figure 1, has been widely used to measure complex permittivity ($\epsilon_r = \epsilon' + j\epsilon''$) at microwave frequencies, with the $TE_{01\delta}$ mode generally employed for such measurements. Various theoretical and experimental studies have been conducted [1–6]. Mathematical analyses of field distributions are usually relatively complicated. An accurate calculation of the resonant frequency was found in their research. Others have attempted to simplify the complex field expressions, but these simplified field equations usually suffer from high errors in depicting the electric and magnetic patterns [7, 8]. In recent years, different approaches have been studied. The traditional Hakki-Coleman dielectric resonance method was further investigated [9]. Gevorkyan et al. put the dielectric resonator inside a waveguide for measurements [10]. The perturbation technique, other than the dielectric resonance method, was also studied [11]. Various measurement techniques have been reviewed [12].

An improved theoretical modification to a simplified one, which can dramatically reduce measurement errors of the microwave complex permittivity, has been reported [13]. This improved theory was derived from the Itoh-Rudokas electromagnetic model, which was originally provided for a cylindrical dielectric between two parallel conducting shields [7, 8]. The Itoh-Rudokas model shows a high computational error in determining the resonant frequency; however, this model's electromagnetic field equations are very succinct compared to other existing models. This large error leads to significant uncertain-

ties in measuring microwave dielectric properties. To amend this concise but high-error field model, ref. [13] suggested an eigenvalue equation to improve field distributions for measuring the dielectric constant (ϵ'), and it has been proven to provide accurate results for samples with various dielectric characteristics and parameters. The loss tangents ($\tan \delta = \epsilon'' / \epsilon'$) of different dielectric samples were also measured, and an acceptable low error was obtained. To provide a more complete understanding of this modified model, this article provides some significant aspects that have not yet been completely investigated.

For accurate measurements of dielectric properties at microwave frequencies, conductor loss must be computed correctly to determine the loss tangent, which requires careful investigation. Because conductor losses exist on both the side-wall and the upper and lower parallel shields, it is also necessary to clarify the relative quantity of conducting dissipation from these two sources. Second, another important factor in computing the dielectric losses in this resonator system is the relative amount of energy stored in the dielectric compared to that stored in the surrounding regions of the sample. Therefore, an investigation of this relative energy factor is necessary. Third, resonant techniques using a dielectric resonator, in this study, typically measure only a single frequency point but with significantly higher accuracy than transmission techniques [14–17], which measure the transmission or reflection signal to a slab-shaped sample and possess frequencyswept capability. In this research, by adjusting the height between the two parallel conducting shields, the variation in the resonant frequency allows the measurement of the complex permittivity within a

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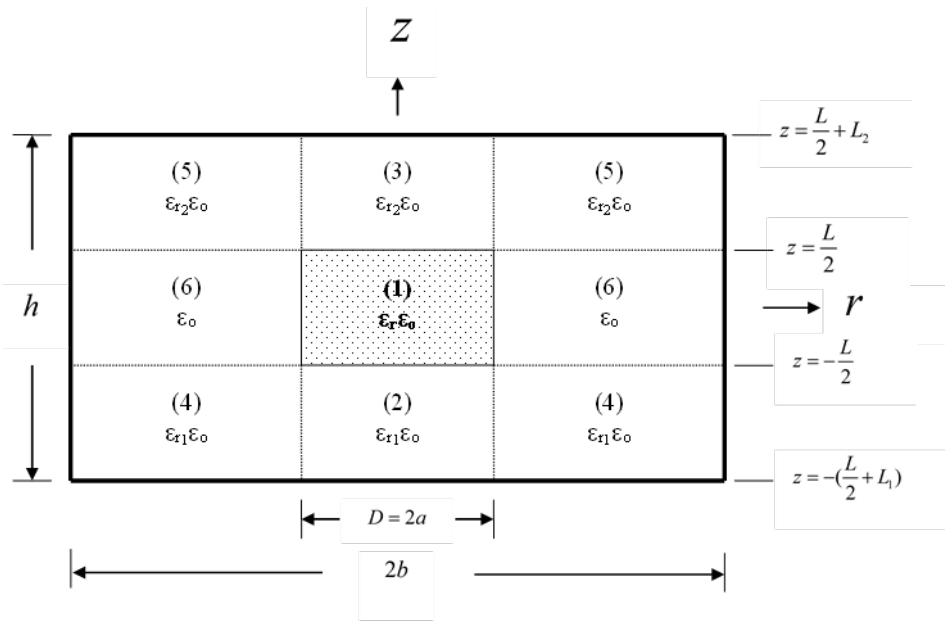


FIGURE 1. Side view of a cylindrical dielectric sample located inside a closed conducting shield (Adopted from [19], reproduced courtesy of The Electromagnetics Academy).

specific sweeping frequency band. The reliability of this resonant technique's swept-frequency capability should be investigated. Finally, a juxtaposition of the measurement results of the improved theory model and the original Itoh-Rudokas theory can help assess the progress and prove the practicability of this modified model in dielectric property calculations. A comprehensive investigation of these four key issues using the improved field model is presented here to enhance the understanding of the effectiveness and limitations of this modified field model.

A preliminary study on the computation of conducting losses was reported in [18]. In that study, the relative contributions of conducting losses from the sidewall and two parallel plates were evaluated. The variation in the conductor loss with different sample dimensions and varying distances between the sample and cavity walls is also described. The results of that study are referenced here for the microwave measurements of dielectric properties. Preliminary research on the energy factor was also presented in another study [19]. The relationship between the energy storage factor and the dielectric properties and dimensions of the resonators is discussed and adopted for measurements in this study. To examine the swept-frequency capability of this technique, complex permittivity measurements of the two samples at various frequencies were conducted by adjusting the distance between conducting plates. This swept-frequency bandwidth is also compared with a newly proposed cylindrical dielectric resonator antenna design by Khan et al. [20], which can support clearly separated resonant modes in two frequency bands and provide a different approach to frequency-swept measurements. Experimental dielectric properties were then compared with those obtained using the widely adopted Hakki-Coleman method, which is known to provide reliable results, particularly for the real part of the complex permittivity, thereby confirming the measurement accuracy of the

proposed model. Measurement results demonstrate that accurate dielectric properties at microwave frequencies can be obtained over a finite frequency range. Furthermore, results from the unmodified model are computed and juxtaposed with those from the modified model. This modified field model has proven to provide much more reliable experimental results for the real part of dielectric properties than the unmodified theoretical expression.

2. THEORETICAL ANALYSIS

2.1. Modified Field Model

The specimen was fabricated into a disk- or rod-shaped dielectric with relative complex permittivity $\varepsilon_r = \varepsilon' + j\varepsilon''$. It has a thickness L and diameter D and is placed inside a cylindrical conducting cavity. This metal cavity has an inner height h and diameter $2b$, as shown in Figure 1. Inside the cavity, it was divided into six spaces, with the sample in the center for analysis. The magnetic-field expressions of the improved model for the $TE_{01\delta}$ mode in each region are presented in [13]. The electric field equations indicated from 1 to 6 describe the fundamental patterns in each region as follows:

$$E_{\phi 1} = E_o J_1(k_{c1}r) \cos(\beta_1 z - \phi_1) \quad (1a)$$

$$E_{\phi 2} = E_o \frac{\cos\left(\beta_1 \frac{L}{2} + \phi_1\right)}{\sinh(\alpha_1 L_1)} J_1(k_{c1}r) \sinh \alpha_1 \left(z + L_1 + \frac{L}{2}\right) \quad (1b)$$

$$E_{\phi 3} = E_o \frac{\cos\left(\beta_1 \frac{L}{2} + \phi_1\right)}{\sinh(\alpha_2 L_2)} J_1(k_{c1}r) \sinh \alpha_2 \left(L_2 + \frac{L}{2} - z\right) \quad (1c)$$

$$E_{\phi 4} = E_o \frac{J_1(k_{c1}a) \cos\left(\beta_1 \frac{L}{2} + \phi_2\right)}{K_1(k_{co}a) \sinh(\alpha_1 L_1)}$$

$$[K_1(k_{co}r) - K_1(k_{co}b)] \sinh \alpha_1 \left(z + L_1 + \frac{L}{2} \right) \quad (1d)$$

$$E_{\phi 5} = E_o \frac{J_1(k_{ci}a) \cos \left(\beta_1 \frac{L}{2} + \phi_2 \right)}{K_1(k_{co}a) \sinh(\alpha_2 L_2)}$$

$$[K_1(k_{co}r) - K_1(k_{co}b)] \sinh \alpha_2 \left(L_2 + \frac{L}{2} - z \right) \quad (1e)$$

$$E_{\phi 6} = E_o \frac{J_1(k_{ci}a)}{K_1(k_{co}a)} [K_1(k_{co}r) - K_1(k_{co}b)] \cos(\beta_1 z - \phi_1) \quad (1f)$$

with [13],

$$\frac{J_0(k_{ci}a)}{J_1(k_{ci}a)} = -\frac{k_{co}a}{k_{ci}a} \frac{1}{K_1(k_{co}a)} \left[K_0(k_{co}a) + \frac{1}{k_{co}a} K_1(k_{co}b) \right] \quad (2a)$$

$$k_{co}^2 = \left(\frac{\pi}{h} \right)^2 - k_o^2 \quad (2b)$$

$$k_{ci}^2 = k_o^2 \varepsilon - \beta_1^2 \quad (2c)$$

$$k_o = \omega \sqrt{\mu_o \varepsilon_o} \quad (2d)$$

$$\phi_1 + \phi_2 = 0 \quad (2e)$$

$$\phi_{1,2} = \tan^{-1} \left(\frac{\alpha_{1,2}}{\beta_1} \coth \alpha_{1,2} L_{1,2} \right) - \beta_1 \frac{L}{2} \quad (2f)$$

$$\alpha_{1,2}^2 = k_{ci}^2 - k_o^2 \varepsilon_{r1,2} \quad (2g)$$

where E_o is the amplitude of the electric field; ϕ_1 and ϕ_2 are parameters to be resolved; k_{ci} and k_{co} are wavenumbers in the r axis; β_1 is the phase factor in the z axis; α_1 and α_2 are attenuation factors in the z axis in spaces 2 and 3, respectively; and J and K are Bessel and modified Bessel functions, respectively. Eq. (2a) is the eigenvalue equation acquired from the boundary condition between the specimen and surrounding air at $r = a$, as referenced in [13].

For the $TE_{01\delta}$ mode, $H_\phi = E_r = E_z = 0$, and by adopting the equations in (1), magnetic-field distributions in the z and radial directions can be obtained from Eqs. (3a) and (3b), respectively:

$$H_z = \frac{j}{\omega \mu_o} \frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) \quad (3a)$$

$$H_r = -\frac{j}{\omega \mu_o} \frac{\partial}{\partial z} E_\phi \quad (3b)$$

2.2. Calculations of Dielectric Properties

The real part of the complex permittivity, namely the dielectric constant, of the measured sample was determined using eigenvalue equations of the $TE_{01\delta}$ mode, as given in Eq. (2). The calculation requires parameters such as the specimen thickness and diameter, the cavity internal dimensions, and the resonant frequency. Figure 2 shows the variation in resonant frequency

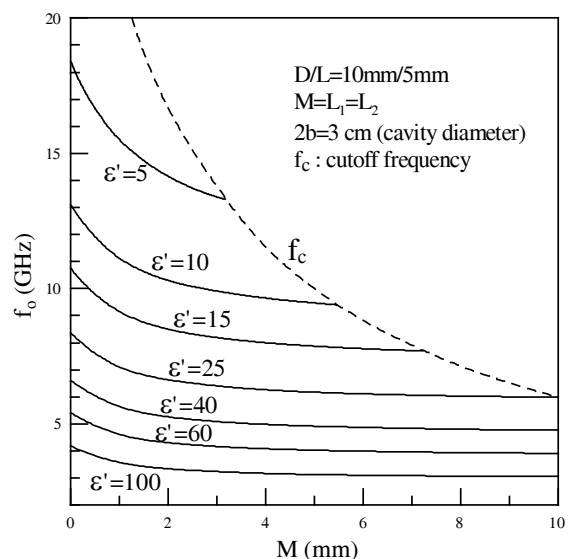


FIGURE 2. Variation of the resonant frequency with the gap height between the sample and the two parallel conducting shields under different dielectric constant values ε' .

as a function of the distance between the sample and the two parallel conducting shields for different dielectric constant values ε' . A simple computation program was developed for the dielectric constant calculations. The Newton-Raphson iterative method was employed to solve Eq. (2). The procedure starts by computing the wavenumber k_{co} from the measured frequency and cavity height using Eqs. (2b) and (2d). An initial value of k_{ci} is then proposed, and the iteration procedure is implemented in Eq. (2a) to obtain a converged solution for k_{ci} . Once the converged value of k_{ci} is acquired, the dielectric constant ε' is calculated from Eqs. (2e)–(2g) using the thickness-matching approach with a similar iterative procedure [21]. For $f_o > f_c$ ($h > \lambda_o/2$, where f_c is the cutoff frequency, and λ_o is the wavelength at the resonant frequency in free space), from Eq. (2b), the height h exceeds the cutoff condition, and k_{co} becomes imaginary. The Hankel function replaces the modified Bessel function, and a convergent solution of Eq. (2a) is not reached.

The quality factor Q_u of the resonant $TE_{01\delta}$ mode was measured to compute the loss tangent. Assuming that the height between the two parallel shields is not over the cutoff ($h < \lambda_o/2$), the loss tangent is obtained by:

$$\tan \delta = A \left(\frac{1}{Q_u} - \frac{1}{Q_c} \right) \quad (4a)$$

$$Q_u = \frac{f_o}{\Delta f_{3dB}} \frac{1}{|1 - S_{21}|} \quad (4b)$$

where Q_u and Q_c are the calculated unloaded quality factor in (4b) and the quality factor as the result of the conducting loss, respectively; A is the energy ratio of that stored in the system to that stored only in the dielectric sample; f_o , Δf_{3dB} , and S_{21} are the measured resonant frequency, half-power (3 dB) bandwidth, and insertion S -parameter, respectively. The next two sections present further studies on Q_c and A .

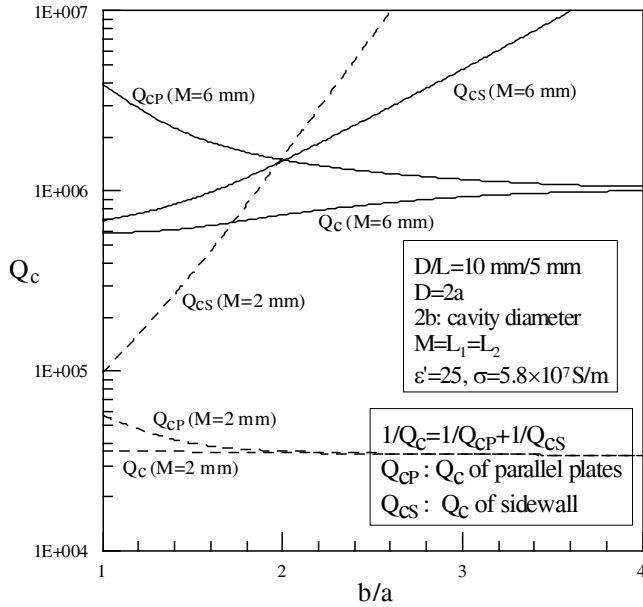


FIGURE 3. Relationship of Q_c with the air gap dimensions.

2.3. Calculations of Q_c

In Eq. (4a), the accurate calculation of Q_c , from Eq. (4), is a key factor in the loss-tangent computation. The Q factor due to the conducting loss can be obtained as follows:

$$Q_c = \frac{2\omega W_e}{P_c} \quad (5a)$$

$$P_c = \sum_{i=2}^5 \frac{1}{2} R_s \iint_S |J_{s,i}|^2 dS \quad (5b)$$

$$W_e = W_d + W_a \quad (5c)$$

$$W_d = \frac{1}{4} \varepsilon' \varepsilon_o \int_V |E_{\phi 1}|^2 dV \quad (5d)$$

$$W_a = \sum_{i=2}^6 \frac{1}{4} \varepsilon_i \varepsilon_o \int_V |E_{\phi i}|^2 dV, \quad \varepsilon_i = 1, \varepsilon_{r1}, \text{ or } \varepsilon_{r2} \quad (5e)$$

where P_c represents the power dissipated in the conductor; W_e , W_d , and W_a indicate the total energy stored, storage energy in the sample, and storage energy in spaces surrounding the sample, respectively; and R_s and J_s denote the surface resistance and current density of the metal cavity, respectively. Figures 3 and 4 show the schematic descriptions of the conductor losses. Similar figures with different parameters are also shown in [18]. Both the parallel shields and sidewall contribute to the conductor loss, as shown in Figure 3. From the curves, it can be observed that the conducting loss due to the two parallel shields is significantly higher than that caused by the sidewall when $b/a > 3$. Consequently, the sidewall's power dissipation can be neglected in this study. Figure 4 illustrates the relationship between Q_c and the sample diameter to thickness ratio for different air gap distances.

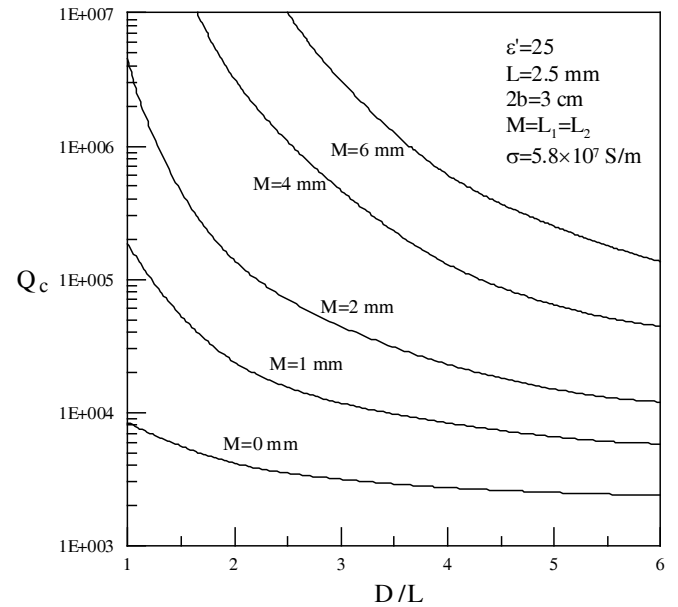


FIGURE 4. Variations of Q_c with the sample diameter/thickness ratios under different air gap distances.

In addition to adopting Eq. (5) to compute Q_c , if the sidewall's power dissipation can be overlooked, Q_c can also be calculated using a perturbation method [21, 22]. A detailed study of this perturbation technique and further discussions on the Q_c topic are presented in [18].

Figure 5 shows the relationship among $Q_d(1/\tan \delta)$, Q_c , and Q_u in Eq. (4). For materials with low dielectric loss, the inaccuracy of Q_c requires careful handling. Another observation worth noting in Figure 4 is that the conducting loss increases with increasing the D/L value because of the sample's larger cross-section.

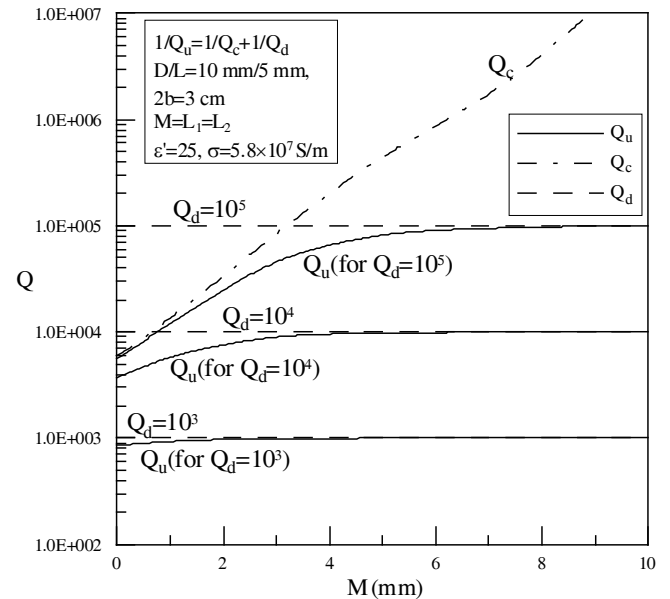


FIGURE 5. Relationship of Q_u , Q_c , and $Q_d(1/\tan \delta)$.

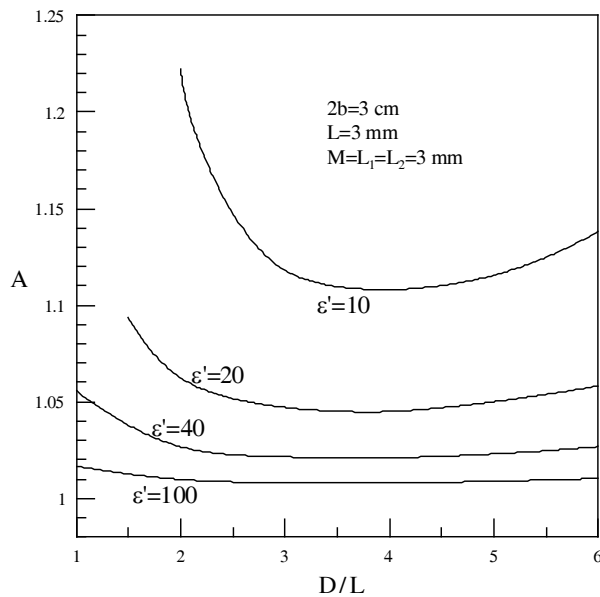


FIGURE 6. Variations of A factor with the diameter/thickness ratios of samples (Redrawn from [19], reproduced courtesy of The Electromagnetics Academy).

2.4. Calculations of Factor A

Based on the introduction of Eq. (4), the traditional calculation of A is from the ratio of stored energies in different regions:

$$A = 1 + \frac{W_a}{W_d} \quad (6)$$

This traditional calculation method by Eq. (6) was adopted in our experiments. However, another perturbation theory can also be applied to compute factor A . A detailed comparison of the traditional and perturbation methods can be found in the other publication [19]. Figure 6 compares the A factors calculated for various diameter/thickness ratios of the samples by Eq. (6) [19]. As shown in the figure, the stored energy increases as ϵ' increases. The A approaches 1.0 if the specimen is a high ϵ' material. Therefore, for the loss-tangent measurement of a high-dielectric-constant material using Eq. (4), the stored energy in the sample's surroundings can be neglected directly, without significant error [13, 18]. From Figure 6, simply letting $A = 1$ permits control of the error within 10% if the ϵ' value is larger than 20. Furthermore, Figure 6 shows that the energy factor increases as the D/L value decreases, which implies that W_a is mainly dominated by the side region (region 6) for a smaller sample diameter. It should also be noted that A increases with increasing L_1 or L_2 . Therefore, the A factor is close to 1.0 if L_1 or L_2 is close to zero.

3. EXPERIMENTS AND DISCUSSIONS

Two commercial ceramic materials (Token Co.) with different dielectric constant values were selected to study the effectiveness of adopting this closed dielectric resonant cavity for dielectric property measurements using the improved electromagnetic model in various dielectric constants and frequency ranges. The frequency variation was realized by tuning the

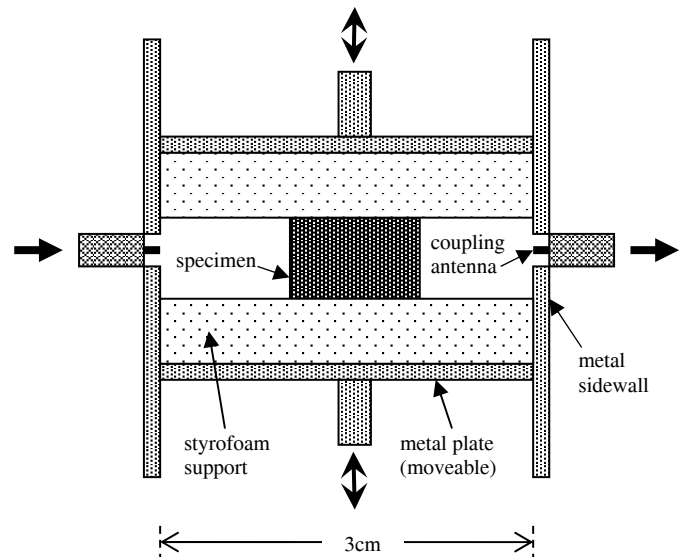


FIGURE 7. Measurement configuration of the closed cylindrical cavity dielectric resonator.

height between the two parallel metal plates, as shown in Figure 7. The conducting cavity was fabricated from copper with a high conductivity of 5.8×10^7 S/m, and its inner diameter was 3 cm. To reduce the influence of the surface roughness on our experimental accuracy, the sidewall surface and the two parallel shields were all carefully polished. The cavity's surface resistance was derived based on the copper's electrical conductivity to facilitate conductor-loss calculations. Two antennas made of thin semi-rigid coaxial cables were designed to couple the transmission signal (S_{21}) in and out for the experiments. At the end of each antenna, a small metal loop was designed and horizontally oriented to excite the desired $TE_{01\delta}$ mode. The two antennas were retreated to near the sidewall surface to minimize their interference with the electromagnetic field distributions inside the cavity. A rod-shaped sample was placed between two styrofoam supports inside the cavity. The styrofoam has a dielectric constant ($\epsilon' = 1.03$) near 1.0, like air, which has the benefit that the effect of specimen support in the measurements is minimized. Theoretically, dielectric dissipation is proportional to the dielectric constant and the square of the amplitude of the electric field. Due to the low dielectric constant of the styrofoam and the decay of electromagnetic fields outside the measured dielectric samples [13], the loss from the styrofoam support was neglected in our measurement. To compute the samples' dielectric properties using Eqs. (2) and (4), the resonant frequency f_o , half-power bandwidth Δf_{3dB} , and transmission coefficient S_{21} at f_o for the $TE_{01\delta}$ mode were recorded with a network analyzer via the swept frequency method. To further verify the experimental results, the real and imaginary parts of these two samples were investigated again using the popular Hakki-Coleman post-resonance technique, in which the disk-shaped specimen is placed between and directly contacts two parallel metal plates [24–26].

Table 1 lists measured dielectric properties of the two materials. The calculations of the real part of permittivity using the original Itoh-Rudokas theory (see the Appendix) are juxtaposed with the experimental data obtained using this improved

TABLE 1. Measurements of dielectric constant and loss tangent of two materials by the improved dielectric resonator method in a closed cylindrical copper cavity by adjusting the distances of the sample and parallel metal plates (M). Dielectric properties obtained by the Hakki-Coleman method are also presented for comparison.

(a) Sample # 1:

M (mm)	f_o (GHz)	Modified model		Unmodified model	A factor
		$\tan \delta$	ϵ'	ϵ'	
4	5.37	4.42×10^{-4}	35.57	39.08	1.025
3	5.44	4.84×10^{-4}	36.13	38.54	1.020
2	5.55	4.63×10^{-4}	37.21	38.41	1.015
1	6.03	7.19×10^{-4}	36.12	36.44	1.012
0	7.03	8.47×10^{-4}	36.24	36.41	1.004
Hakki-Coleman technique	7.01	9.07×10^{-4}	36.45		
$M = L_1 = L_2, D = 9.7 \text{ mm}, L = 5.0 \text{ mm}$					

(b) Sample # 2:

M (mm)	f_o (GHz)	Modified model		Unmodified model	A factor
		$\tan \delta$	ϵ'	ϵ'	
4	6.92	1.47×10^{-4}	20.87	23.20	1.047
3	7.06	1.43×10^{-4}	20.94	22.56	1.037
2	7.22	1.58×10^{-4}	21.55	22.41	1.028
1	7.68	2.02×10^{-4}	21.92	22.21	1.020
0	9.16	3.09×10^{-4}	21.17	21.15	1.007
Hakki-Coleman technique	9.20	3.67×10^{-4}	21.01		
$M = L_1 = L_2, D = 9.8 \text{ mm}, L = 5.0 \text{ mm}$					

TABLE 2. Comparison of swept-frequency bandwidth.

	ϵ'	Frequency band (GHz)	Bandwidth percentage	Modes
Sample #1	36	5.4–7.0	26.7%	TE _{01δ}
Sample #2	21	6.9–9.2	27.8%	
Ref. [20]	9.9	8–16	66.7%	TE _{0nδ}
		24–30	22.2%	HE _{y11}

field model (Eq. (2)) in Table 1 to understand the improvement by this proposed model. The variations of ϵ' are about 2.3% and 2.5% for samples #1 and #2, respectively. The samples were attached to the two conducting plates in the Hakki-Coleman post-resonance method, which is similar to the condition of $L_1 = L_2 = M = 0$. Therefore, very close measurement frequencies were observed between the post-resonance method and the case of $M = 0$. For low M values, both the ϵ' data from the modified and unmodified models agree well with the measurements of the Hakki-Coleman method. However, by increasing the height of the two parallel conducting shields, the dielectric constant ϵ' from the improved method has a much better agreement with the data obtained using the widely adopted Hakki-Coleman measurement method than the unmodified method, which proves the distinct gain in the experimental accuracy of the improved theory over the unimproved one. The good consistency in the dielectric constant values between the improved method of the closed cavity dielectric resonator and the Hakki-Coleman technique also demonstrates the experimental reliability of the measurements of the dielectric constant ϵ' . Regarding the results of the loss tangent at $M = 0$,

where the measurement frequency is very close to the frequency of the Hakki-Coleman technique, a reasonable agreement at the same frequency between the two methods was also obtained. Table 1 also lists the calculated energy factor A . It can be found that simply letting $A = 1$ permits control of the error to less than 2.6% for sample #1 ($\epsilon' \sim 36$) and 4.8% for sample #2 ($\epsilon' \sim 21$). This result confirms our previous analysis, and the error is lower for the higher dielectric constant sample.

To observe the swept-frequency capability, measurement frequencies were found to span approximately 1.7 and 2.2 GHz, acquired by adjusting the height of the two parallel metal plates shielding the two ceramic materials. The frequency range increases with decreasing dielectric constant. Figure 2 shows this phenomenon, where the slopes of the curves increase while the dielectric constants decrease. Table 2 shows the comparison of the swept-frequency bandwidth with the newly published cylindrical dielectric resonator antenna design by Khan et al. The swept-frequency bandwidths are smaller than those by [20] but still provide acceptable swept-frequency capability. For the dielectric property dispersion with frequencies, the loss tangent values tend to increase with increasing frequency, and the

variation in ε' with frequency is relatively small. This phenomenon is understandable because the ε'' value, based on the analysis of the classic oscillator model, increases with increasing frequency in the microwave region with the multiplication of frequency and quality factor close to a constant ($f \times Q = f / \tan \delta \approx \text{constant}$), whereas the variation in ε' with frequency is not always obvious [27]. However, in our experimental results, although loss tangents have the trend of increasing with frequency, they do not strictly follow the rule of $f / \tan \delta \approx \text{constant}$.

4. CONCLUSIONS

This article presents an integrated investigation of an improved dielectric resonator technique in a cylindrical closed conducting cavity for dielectric property measurements at microwave frequencies. The various contributions of conductor losses were examined to improve the accuracy and understanding of the loss-tangent measurements. The relative quantity of conducting losses due to the sidewall and two parallel shields was studied. The conducting dissipation from the sidewall is much lower than that from parallel plates and can be ignored for some sample dimensions of the experimental configuration. In the study of the energy factor A , the energy factor increased as the diameter/thickness of the samples decreased. It was also found that the A factor approaches 1.0 for high ε' samples or low gap height M values. A swept-frequency capability in the range of 1.7–2.2 GHz for dielectric constants between 21 and 36 was achieved by adjusting the height of the two parallel metal shields in our experimental setup. Moreover, an analysis of the real part of the complex permittivity confirmed, via the modified electromagnetic eigenvalue equation, a significant improvement in experimental reliability compared with the original Itoh-Rudokas theory. Finally, this dielectric resonator method with a closed cylindrical conducting cavity based on a modified and uncomplicated field theory introduced in this research, represents simple and accurate measurements for both the real and imaginary parts of permittivity by comparing with the results from other well-known measurement methods.

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APPENDIX A. THE ITOH-RUDOKAS MODEL

The Itoh-Rudokas electromagnetic theory is derived from modifications to Cohn's Model for the $TE_{01\delta}$ mode [21]. Cohn's model uses the perfect magnetic conductor assumption on the interface of the dielectric and air, $r = a$, and Eqs. (2a) and (2b) are expressed as,

$$k_{ci} = \frac{2.4048}{a} \quad (A1)$$

$$k_{co}^2 = \beta_I^2 - k_o^2 \quad (A2)$$

In addition to adopting the assumption of a perfect magnetic conductor, Itoh-Rudokas proposed a more physical condition for arranging the continuous condition at the boundary between regions 1 and 6 to acquire the following eigenvalue equation, replacing Eqs. (A1) and (A2) [22]:

$$\frac{J_0(k_{ci}a)}{J_1(k_{ci}a)} = -\frac{k_{co}a K_0(k_{co}a)}{k_{ci}a K_1(k_{co}a)} \quad (A3)$$

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