9

CALIBRATION OF POLARIMETRIC RADARS USING IN-SCENE REFLECTORS

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9.1   Introduction

Polarimetric radar backscatter data observed with satellite and airborne synthetic aperture radars (SAR) have demonstrated potential applications in geologic mapping and terrain cover classification [1–8]. Accurate calibration of such polarimetric radar systems is essential for polarimetric remote sensing of earth terrain. In this chapter, we will investigate the polarimetric calibration using three in-scene reflectors and provide a calibration algorithm which will be a useful tool in radar image interpretation.

The concept and formulation of polarimetric calibration using in-scene reflectors were developed by Barnes [9]. The transmitting and receiving ports of the polarimetric radar are modeled by two unknown polarization transfer matrices. The measured polarimetric scattering matrix is then expressed as the product of the transfer matrix of the receiving port, the scattering matrix of the illuminated target, the transfer matrix of the transmitting port, and a common factor. These two unknown polarization transfer matrices are determined by using the measured scattering matrices from targets with known polarization scattering parameters.

Two sets of simple targets, screened corner reflectors (SCRs) and top-hats with a trihedral, have been investigated [9]. The SCR approach uses three SCRs (wire grids in front of trihedral corner reflectors) oriented at 0°, 45°, and 90°, respectively. The top hats are dihedral-like reflectors oriented at 0° and 45°. The SCR approach solved for all unknowns exactly while the top-hat approach solved approximately for cross-talk and imbalance by assuming the cross-polarization coupling to be small. The top-hat approach was concluded to be superior to the SCR approach on the achievable polarimetric purity and broad angular pattern of targets, sensitivity on misalignment, and effect of noise.

Besides the passive corner reflectors, a set of Polarimetric Active Radar Calibrators (PARCs) is also used to calibrate the L-band and C-band airborne imaging radar images [10]. The advantage of the PARCs approach is that a very high signal-to-background-noise ratio (S/B) can be achieved. Initial results indicate that in some polarizations the S/B is as high as 60 dB, which will totally eliminate the effects of background noise. The tradeoff is that PARCs are more expensive, and the scattering characteristics may be more sensitive to the change of environmental temperature than the passive reflectors.

A polarimetric calibration algorithm using three reciprocal reflectors has been developed for the case where at least two of the scattering matrices can be simultaneously diagonalized [11]. The algorithm allows more degrees of freedom in the choice of calibration devices and makes it possible to analyze the sensitivities on noise and misalignment for each set of calibration targets.

All of the calibration procedures outlined above are based on the deployed point targets. When the point targets are not available, natural distributed targets (clutter) have good potential in polarimetric calibration. A technique to calibrate compressed polarimetric radar images using natural targets and trihedral corner reflectors was introduced by van Zyl [12]. The method is based on the theoretical results that the co- and cross-polarized components of the scattering matrix are uncorrelated for natural targets with azimuthal symmetry [13,14].

The objectives of this chapter are to generalize the polarimetric calibration algorithm developed in [11] for nonreciprocal calibrators, such as PARCs, and provide the performance analysis and comparison for several typical target sets. In section 9.2, a **Polarization-basis Transformation** technique (PT) is introduced to convert the scattering matrices into one of six sets of targets with simpler scattering matrices. The solution to the original problem then can be expressed in terms of the solution for simple scattering matrices. In section 9.3, the uniqueness of polarimetric calibration using three targets is addressed for all possible combinations of scattering parameters. Section 9.4 presents the polarimetric calibration solution for three sets of targets which correspond to the case where the scattering matrices of at least two of the calibration targets can be simultaneously diagonalized. Section 9.5 illustrates the effects of misalignment for six sets of calibration targets. In section 9.6, the effect of noise is illustrated for those six target sets and the PARCs [10]. Explicit polarimetric compensation formulas, which can be used to remove the effect of the radar distortion matrices from measured data, are given in section 9.7.

9.2 Polarimetric Calibration Using Three In-Scene Reflectors

The polarimetric calibration using three in-scene reflectors is formulated in this section. A set of coupled nonlinear equations is derived for the unknown system parameters including channel imbalance and

polarization cross-talk errors (isolation) for both transmitter and receiver.

The model of the polarimetric radar system defined in [9,11] is employed to formulate the problem. Assume that the measured polarization scattering matrices corresponding to three in-scene reflectors are

$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{bmatrix} = e^{i\phi_1} R S_1 T \tag{1a}$$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = e^{i\phi_2} R S_2 T \tag{1b}$$

$$Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = e^{i\phi_3} R S_3 T \tag{1c}$$

where S_1, S_2, and S_3 are the (undistorted) polarization scattering matrices of the calibration targets, and the matrices T and R account for the mismatch and cross-polarization coupling of the transmitting and receiving ports, respectively.

$$R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}, \quad T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \tag{2}$$

Note that the matrix R equals the transpose of the matrix B in [9]. The factor $\exp(i\phi_k)$ accounts for the phase delay and attenuation due to atmospheric effects and the path length between the radar and the target. This is treated as a nuisance parameter, and the objective of the polarimetric calibration is to solve for the normalized quantities of R and T matrices in terms of the normalized quantities of the X, Y, and Z matrices.

The normalized quantities, t_{12}, t_{21}, t_{22}, r_{12}, r_{21}, and r_{22} are defined by

$$t_{12} = T_{12}/T_{11} \tag{3a}$$

$$t_{21} = T_{21}/T_{11} \tag{3b}$$

$$t_{22} = T_{22}/T_{11} \tag{3c}$$

$$r_{12} = R_{12}/R_{11} \tag{3d}$$

$$r_{21} = R_{21}/R_{11} \tag{3e}$$

$$r_{22} = R_{22}/R_{11} \tag{3f}$$

r_{22} and t_{22} are the parameters describing channel imbalance, whereas r_{12}, r_{21}, t_{12}, and t_{21} are the parameters for polarization cross-talk. Note that (1) gives a set of coupled nonlinear equations for the system parameters.

In general, it is difficult to solve the unknowns directly from these nonlinear equations for three calibration targets with arbitrary scattering parameters. Another difficulty is in determining how many sets of solutions exist for a given set of targets. The latter issue involves whether it is sufficient to use only three targets. The **Polarization-basis Transformation** technique (PT) is presented to simplify the complexity of the problem and resolve these issues. The method will simultaneously diagonalize two matrices, or diagonalize one and transform the other one into a Jordan Canonical form.

The physical interpretation of the transformation scheme, PT, is that instead of working on the original polarization basis (e.g., linear basis) yielding complicated scattering matrices, we find a particular polarization basis so that the scattering matrices turn out to have a simpler form like (A)–(F) to be discussed below. Hence the transformation method is in effect performing a polarization basis transformation.

There are three positive features associated with this transformation scheme:

(i) It enables us to solve the calibration problem using only some simple targets. The polarimetric calibration using general targets can be obtained from these simple target cases by straightforward matrix inversion and multiplication.

(ii) The transformation method helps us to determine the number of solutions associated with any given set of calibration targets.

(iii) If there are multiple sets of solutions, the relationship between all the solutions can be clearly identified. Once we obtain a particular solution, the other solutions can be obtained by using the transformation relation.

The essence of PT is to find two nonsingular matrices A and B so that the scattering matrices of calibration targets can be transformed into one of the following cases:

$$(A) \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

$$(B) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

$$(C) \quad \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

$$(D) \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

$$(E) \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

$$(F) \quad \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$$

(A)–(C) correspond to the cases where S_1 and S_2 can be simultaneously diagonalized. (D)–(F) represent the cases where S_1 and S_2 cannot be simultaneously diagonalized. If it is possible to obtain the calibration solution using the targets with the above scattering matrices, then the solution for the general case can be obtained by an inverse transformation. The PT scheme will be described in detail in the remainder of this section.

a. S_1 and S_2 are Singular

Assuming that S_1 and S_2 are both singular, two nonsingular matrices A and B can be obtained such that S_1 and S_2 can be written into one of the following forms:

$$S_1 = A \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} B \tag{4a}$$

$$S_2 = A \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} B \tag{4b}$$

or

$$S_1 = A \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} B \tag{5a}$$

$$S_2 = A \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B \tag{5b}$$

or

$$S_1 = A \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} B \tag{6a}$$

$$S_2 = A \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} B \tag{6b}$$

Equation (4) corresponding to case (A) represents the situation where S_1 and S_2 can be simultaneously diagonalized. Equations (5) and (6) correspond to cases (E) and (F), respectively.

Because of the particularly simple equations (4)–(6), it is easy to solve for nonsingular matrices A and B in terms of the scattering parameters of S_1 and S_2 by explicitly multiplying out the right-hand-side of the equations and equating the results term-by-term with S_1 and S_2. After A and B are found, the scattering matrix of the third target is also transformed accordingly as follows

$$S_3 = A \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} B \tag{7}$$

or

$$\begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} = A^{-1} S_3 B^{-1} \tag{7'}$$

We will describe how PT helps to reduce the complexity of the problem for case (A) in detail. The same idea applies to all other cases. For case (A), we can pre and postmultiply (1) by A^{-1} and B^{-1}, respectively, and substitute (4) and (7) into the resulting equations. Thus, (1) becomes

$$X' = A^{-1} X B^{-1} = e^{i\phi_1} R' \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} T' \tag{8a}$$

$$Y' = A^{-1} Y B^{-1} = e^{i\phi_2} R' \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} T' \tag{8b}$$

$$Z' = A^{-1} Z B^{-1} = e^{i\phi_3} R' \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} T' \tag{8c}$$

where

$$R' = A^{-1} \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} A \tag{9a}$$

$$T' = B \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} B^{-1} \tag{9b}$$

Note that if the polarimetric calibration can be solved for the calibration targets with the scattering matrices given by cases (A)–(F), then it is straightforward to obtain the solution to the original problem by inverting (9a) and (9b).

It should also be noted that case (F) can be solved by taking advantage of similarity between case (E) and case (F). Taking the transpose of (1) results in

$$X^t = e^{i\phi_1} T^t S_1^t R^t \tag{10a}$$

$$Y^t = e^{i\phi_2} T^t S_2^t R^t \tag{10b}$$

$$Z^t = e^{i\phi_3} T^t S_3^t R^t \tag{10c}$$

where superscript t represents transpose. When S_1 and S_2 have the form of case (F), S_1^t and S_2^t become the form of case (E). If we know how to determine the solution for case (E), then it is straightforward to determine the solution for case (F).

b. S_1 and S_2 are Nonsingular

If both S_1 and S_2 are nonsingular, based on the same idea discussed above, we would like to transform the scattering matrices of these targets into the form of case (B) or (D). That is, to find the transformation matrices A and B such that

$$S_1 = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B \tag{11a}$$

$$S_2 = A \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} B \tag{11b}$$

if S_1 and S_2 can be diagonalized simultaneously (case B), otherwise

$$S_1 = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B \tag{12a}$$

$$S_2 = A \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} B \tag{12b}$$

which corresponds to case (D) with $a \neq 0$.

Inverting (11a) or (12a), we obtain

$$B = A^{-1} S_1. \tag{13}$$

If $S_2 S_1^{-1}$ has distinct eigenvalues, substituting (13) into (11b) gives

$$S_2 S_1^{-1} A = A \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{14}$$

This is a typical eigenvalue problem. a and b corresponding to eigenvalues and A can be obtained by solving for the eigenvectors of the corresponding eigenvalues.

If $S_2 S_1^{-1}$ has double-order eigenvalues $(a = b)$, we should perform the transformation (12b) yielding

$$S_2 S_1^{-1} A = A \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix} \tag{15}$$

This corresponds to transforming $S_2 S_1^{-1}$ into the Jordan canonical form.

Like the transformation method resulting in (8), the scattering matrix of the third target is also transformed according to (7). Hence, the polarimetric calibration using general targets can be transformed into the calibration problem using the simple targets: cases (B) or (D). The transformed and original R and T matrices are related to each other by (9).

c. S_1 is Singular and S_2 is Nonsingular

When S_1 is singular and S_2 is nonsingular, the transformation matrix A can be found so that

$$S_1 = A \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} B \tag{16a}$$

$$S_2 = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B \tag{16b}$$

when S_1 and S_2 can be simultaneously diagonalized, otherwise

$$S_1 = A \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} B \tag{17a}$$

$$S_2 = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B \tag{17b}$$

By inverting (16b) or (17b), matrix B is given as

$$B = A^{-1} S_2 \tag{18}$$

whereas A is obtained by solving the following eigenvalue problem:

$$S_1 S_2^{-1} A = A \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} \quad \text{for case (C),} \tag{19}$$

or

$$S_1 S_2^{-1} A = A \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{for case (D) with } a = 0. \tag{20}$$

Again (19) and (20) become the typical eigenvalue and eigenvector problem for matrix diagonalization and Jordan canonical form transformation, respectively. Subsequently, the third scattering matrix is transformed into the form of (7). Like the approach used in arriving at (8) and (9), once the normalized quantities of R' and T' are solved, the normalized quantities of R and T are obtained through (9).

In this section, we presented how the problem of polarimetric calibration using a general set of targets can be transformed into one of the simple target sets in (A)–(F). The transformation method enables us to work on the simple cases and also obtain the solution for more complicated cases. In general, there may be $3! (= 6)$ ways to arrange the order of the targets such that the transformed problem reduces to the cases shown in (A)–(F). It is always better to choose the order which makes the solution less sensitive to additive noise and misalignment of targets. This can be done by computer simulation of each possible order of calibration targets. Thus, the polarimetric radar calibration with three general targets can be completely solved using the method of PT if the polarimetric calibration using the simple targets (A)–(F) is solved.

9.3   Uniqueness

In this section, the uniqueness problems associated with each case are discussed. The method of PT is used to find out the number of solutions and the transformation relation between multiple solutions for each set of targets outlined in (A)–(F), section 9.2.

Suppose that the scattering matrices of three targets are S_1, S_2, and S_3. The method is to find all the possible A and B matrices such that

$$S_i = c_i A S_i B, \quad i = 1, 2, 3 \tag{21}$$

where c_i's are the appropriate scaling constant. which do not affect the solution of the normalized quantities (3). It is easy to see that the identity matrix is a trivial solution for both A and B in (21).

Substituting (21) into (1), we obtain

$$X = c_1 e^{i\phi_1} R A S_1 B T \tag{22a}$$

$$Y = c_2 e^{i\phi_2} RAS_2 BT \tag{22b}$$

$$Z = c_3 e^{i\phi_3} RAS_3 BT \tag{22c}$$

Comparing (1) and (22) indicates that if there are any solutions for A and B other than identity matrix, then the normalized quantities of

$$\begin{aligned} R' &= RA \\ T' &= BT \end{aligned} \tag{23}$$

are also solutions to the original problem. If a particular solution is solved from (1), (23) can be used to generate the other solutions.

Tables 9.1 through 9.5 list the solutions of A and B for cases (A)–(E) in section 9.2. In those tables, I denotes the identity matrix, and matrices A_1–A_6, B_1, D_1, and D_2 are defined as follows:

$$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}$$

$$A_2 = \begin{bmatrix} A_{22} & 0 \\ 0 & A_{11} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{11} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} A_{11} & (A_{22} - A_{11})c/d_2 \\ 0 & A_{22} \end{bmatrix}$$

$$A_5 = \begin{bmatrix} A_{11} & (A_{11} - A_{22})d_1/(c - e) \\ 0 & A_{22} \end{bmatrix}$$

$$A_6 = \begin{bmatrix} A_{11} & (A_{22} - A_{11})d_1/e \\ 0 & A_{22} \end{bmatrix}$$

$$B_1 = \begin{bmatrix} B_{11} & 0 \\ 0 & B_{22} \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix}$$

$$D_2 = \begin{bmatrix} 0 & d_1 \\ -d_2 & 0 \end{bmatrix}$$

No.	Parameters	Solution of A and B
1	$d_1 d_2 c \neq 0$	$A = B = I$
2	$d_1 d_2 e \neq 0$	$A = B = I$
3	$d_1 ce \neq 0$	$A = B = I$
4	$d_2 ce \neq 0$	$A = B = I$
5	$d_1 c \neq 0, d_2 = e = 0$	$A = A_1, B = I$
6	$d_1 e \neq 0, d_2 = c = 0$	$A = I, B = B_1$
7	$d_2 c \neq 0, d_1 = e = 0$	$A = I, B = B_1$
8	$d_2 e \neq 0, d_1 = c = 0$	$A = A_1, B = I$
9	$d_1 d_2 \neq 0, c = e = 0$	$A = B = A_1$
10	$ce \neq 0, d_1 = d_2 = 0$	$A = A_1, B = A_2$
11	$d_1 \neq 0, d_2 = c = e = 0$	$A = A_1, B = B_1$
12	$d_2 \neq 0, d_1 = c = e = 0$	$A = A_1, B = B_1$
13	$c \neq 0, d_1 = d_2 = e = 0$	$A = A_1, B = B_1$
14	$e \neq 0, d_1 = d_2 = c = 0$	$A = A_1, B = B_1$

Table 9.1 **All possible solutions of A and B matrices that may transform the scattering matrices of three calibration targets into themselves for case A with** $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$.

If the elements of A_i, B_1, and D_i are not assigned a definite number, then they are considered arbitrary as long as A_i, B_1, and D_i remain nonsingular. As can be seen from Tables 9.1–9.5, many cases do not provide a finite set of solutions for the polarimetric calibration using three targets. Note that case (F) is not shown separately because of the one to one correspondence between case (F) and case (E) (see the discussion given at the end of section 9.2.a).

9.4   Polarimetric Calibration Using Three Simple In-Scene Reflectors

Solutions of polarimetric calibration using three sets of simple cal-

ibrators are presented in this section. These three cases correspond to the situation that at least two of the scattering matrices can be simultaneously diagonalized. The solutions to the following cases are presented separately in the remainder of this section: (A) two dipoles and one general target, (B) one trihedral, one target with diagonal

No.	Parameters	Solution of A
1	$d_1 = d_2 = 0, \, c = \pm e$	$A_1, \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}$
2	$d_1 = d_2 = 0, \, c \neq \pm e$	A_1
3	$d_2 \neq 0, \, d_1 = c = e = 0$	A_1
4	$d_1 \neq 0, \, d_2 = c = e = 0$	A_1
5	$c \neq 0, \, d_1 \neq 0, \, a \neq -b$ or $c \neq \pm e$	I
6	$c \neq 0, \, d_2 \neq 0, \, a \neq -b$ or $c \neq \pm e$	I
7	$e \neq 0, \, d_1 \neq 0, \, a \neq -b$ or $c \neq \pm e$	I
8	$e \neq 0, \, d_2 \neq 0, \, a \neq -b$ or $c \neq \pm e$	I
9	$d_1 d_2 \neq 0, \, c = e \neq 0, \, a = -b$	I, D_1
10	$d_1 d_2 \neq 0, \, c = -e \neq 0, \, a = -b$	I, D_2
11	$d_1 d_2 \neq 0, \, c = e = 0, \, a \neq -b$	$I, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
12	$d_1 d_2 \neq 0, \, c = e = 0, \, a = -b$	$I, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, D_1, D_2$
13	$d_1 \neq 0, \, d_2 = 0, \, c = \pm e \neq 0, \, a = -b$	I
14	$d_2 \neq 0, \, d_1 = 0, \, c = \pm e \neq 0, \, a = -b$	I

Table 9.2 $B = A^{-1}$ and all possible solutions of A matrix that may transform the scattering matrices of three calibration targets into themselves for case B with $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, $\begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$.

No.	Parameters	Solution of A
1	$cd_1 \neq 0$	I
2	$cd_2 \neq 0$	I
3	$ed_1 \neq 0$	I
4	$ed_2 \neq 0$	I
5	$d_1 d_2 \neq 0, c = e = 0$	$I, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
6	$c \neq 0, d_1 = d_2 = 0$	A_1
7	$e \neq 0, d_1 = d_2 = 0$	A_1
8	$d_1 = 0$ or $d_2 = 0, c = e = 0$	A_1

Table 9.3 $B = A^{-1}$ and all possible solutions of A matrix that may transform the scattering matrices of three calibration targets into themselves for case C with $\begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}.$

No.	Parameters	Solution of A
1	$a \neq 0, d_2 = 0, c \neq e$	I
2	$a \neq 0, d_2 = 0, c = e$	A_3
3	$a \neq 0, d_2 \neq 0$	I
4	$a = 0, d_2 \neq 0, c \neq -e$	I
5	$a = 0, d_2 \neq 0, c = -e, ce = d_1 d_2$	A_4
6	$a = 0, d_2 \neq 0, c = -e, ce \neq d_1 d_2$	$I, \begin{bmatrix} 1 & -2c/d_2 \\ 0 & -1 \end{bmatrix}$
7	$a = 0, d_2 = 0, c \neq e$	A_5
8	$a = 0, d_2 = 0, c = e$	A_3

Table 9.4 $B = A^{-1}$ and all possible solutions of A matrix that may transform the scattering matrices of three calibration targets into themselves for case D with $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}, \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}.$

No.	Parameters	Solution of A and B
1	$d_2 \neq 0,\ d_1 = c = e = 0$	$A = A_1,\ B = B_1$
2	$d_1 \neq 0,\ d_2 = c = e = 0$	$A = A_1,\ B = B_1$
3	$c \neq 0,\ d_1 = d_2 = e = 0$	$A = A_1,\ B = B_1$
4	$e \neq 0,\ d_2 = d_1 = c = 0$	$A = A_1,\ B = B_1$
5	$d_1 c \neq 0,\ e = d_2 = 0$	$A = A_1,\ B = I$
6	$d_2 e \neq 0,\ c = d_1 = 0$	$A = A_1,\ B = I$
7	$d_2 c \neq 0,\ d_1 = e = 0$	$A = A_4,\ B = B_1$
8	$d_1 d_2 \neq 0,\ c = e = 0$	$A = B = A_1$
9	$ce \neq 0,\ d_2 = d_1 = 0$	$A = A_1,\ B = A_2$
10	$d_1 e \neq 0,\ d_2 = c = 0$	$A = A_6,\ B = B_1$
11	$e = 0,\ d_1 d_2 c \neq 0$	$A = A_4,\ B = A_1$
12	$d_2 = 0,\ cd_1 e \neq 0$	$A = A_6,\ B = A_2$
13	$d_1 = 0,\ d_2 e c \neq 0$	$A = B = I$
14	$c = 0,\ d_2 e d_1 \neq 0$	$A = B = I$
15	$d_2 e d_1 c \neq 0,\ ce \neq d_1 d_2$	$A = B = I$
16	$d_2 e d_1 c \neq 0,\ ce = d_1 d_2$	$A = A_6,\ B = I$

Table 9.5 **All possible solutions of A and B matrices that may transform the scattering matrices of three calibration targets into themselves for case E with** $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix}$.

scattering matrix, and one general reflector, and (C) one dipole, one trihedral, and one general reflector. These cases correspond to the cases (A)–(C) in section 9.2.

a. Two Dipoles and One General Target

The scattering matrices of three calibration targets are given as follows:

$$S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_3 = \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} \quad (24)$$

where the first two targets correspond to horizontal and vertical dipoles, respectively. The third target is a some other general reflector.

By substituting (24) into (1) and taking the ratio of the measurements, it can be easily shown that

$$t_{12} = \frac{X_{12}}{X_{11}} \tag{25a}$$

$$r_{21} = \frac{X_{21}}{X_{11}} \tag{25b}$$

$$r_{21}t_{12} = \frac{X_{22}}{X_{11}} \tag{25c}$$

$$\frac{t_{21}}{t_{22}} = \frac{Y_{21}}{Y_{22}} \tag{25d}$$

$$\frac{r_{12}}{r_{22}} = \frac{Y_{12}}{Y_{22}} \tag{25e}$$

$$\frac{r_{12}}{r_{22}}\frac{t_{21}}{t_{22}} = \frac{Y_{11}}{Y_{22}} \tag{25f}$$

and

$$\frac{Z_{12}}{Z_{11}} = \frac{t_{12}c + t_{22}d_1 + r_{12}t_{12}d_2 + r_{12}t_{22}e}{c + t_{21}d_1 + r_{12}d_2 + r_{12}t_{21}e} \tag{26a}$$

$$\frac{Z_{21}}{Z_{11}} = \frac{r_{21}c + r_{21}t_{21}d_1 + r_{22}d_2 + r_{22}t_{21}e}{c + t_{21}d_1 + r_{12}d_2 + r_{12}t_{21}e} \tag{26b}$$

$$\frac{Z_{22}}{Z_{11}} = \frac{r_{21}t_{12}c + r_{21}t_{22}d_1 + r_{22}t_{12}d_2 + r_{22}t_{22}e}{c + t_{21}d_1 + r_{12}d_2 + r_{12}t_{21}e} \tag{26c}$$

In (26), Z_{11} is used to normalize the measurements of the other polarization channels. This is justified if the magnitude of c is not small. In cases where c is small, we can also use Z_{22}, Z_{12}, or Z_{21} to normalize the other quantities when e or d_1 or d_2 is not small.

In the following, the solutions for the normalized quantities of the R and T matrices given in (3) are presented for the following subcases: (A.1) $c\,d_1d_2 \neq 0$, (A.2) $ed_1d_2 \neq 0$, (A.3) $d_1ce \neq 0$, (A.4) $d_2ce \neq 0$, and (A.5) $d_1d_2 = ce$ and $c\,d_1d_2e \neq 0$. Note that (A.1)–(A.4) correspond to the first four rows in Table 9.1. Case (A.5) can actually be covered by the cases (A.1)–(A.4). The reason the solution of this case is included is because of its particular simplicity. Also note that the cases (A.1) to (A.5) are not mutually exclusive to each other. For the parameters of S_3 in the common regime of validity, each formula can be applied.

a.1 H-dipole, V-dipole, and another reflector with $c\,d_1d_2 \neq 0$

For this case, the parameter e is arbitrary. By substituting (25) into (26), three linear equations are obtained with unknowns t_{22}, r_{22}, and $r_{22}t_{22}$. It is straightforward to solve for these unknowns. The solutions are given as

$$t_{22} = \frac{c}{d_1}\frac{Y_{22}}{X_{11}}\frac{\Delta_{1t}}{\Delta_1} \tag{27a}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}}t_{22} \tag{27b}$$

$$r_{22} = \frac{c}{d_2}\frac{Y_{22}}{X_{11}}\frac{\Delta_{1r}}{\Delta_1} \tag{27c}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}}r_{22} \tag{27d}$$

where Δ_1, Δ_{1t}, and Δ_{1r} are given by (A1) in appendix A.

If c is close to zero, it is impractical to apply the above solution to targets within a noisy background; if c is small, the determinant Δ_1 will also be small, and any additive noise will cause a large error in Δ_1.

a.2 H-dipole, V-dipole, and another reflector with $ed_1d_2 \neq 0$

When $e \neq 0$, we can use Z_{22} to normalize the other elements of the Z matrix and obtain three equations. Like the derivation of the previous case, three linear equations are obtained for the unknowns $1/t_{22}$, $1/r_{22}$, and $1/r_{22}t_{22}$, by substituting (25a), (25b), (25d), and (25e) into these equations. The solutions are given by,

$$t_{22} = \frac{-d_2}{e}\frac{Y_{22}}{X_{11}}\frac{\Delta_2}{\Delta_{2t}} \tag{28a}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}}t_{22} \tag{28b}$$

$$r_{22} = \frac{-d_1}{e}\frac{Y_{22}}{X_{11}}\frac{\Delta_2}{\Delta_{2r}} \tag{28c}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}}r_{22} \tag{28d}$$

where Δ_2, Δ_{2t}, and Δ_{2r} are given by (B1) in appendix B.

a.3 H-dipole, V-dipole, and another reflector with $d_1 ce \neq 0$

When $d_1 \neq 0$, we use Z_{12} to normalize the other elements of the Z matrix and obtain three equations. Substituting (25a), (25b), (25d), and (25e) into these equations gives three linear equations for the unknowns $1/t_{22}$, r_{22}, and r_{22}/t_{22}. The solutions are given by,

$$t_{22} = \frac{c}{d_1} \frac{Y_{22}}{X_{11}} \frac{\Delta_3}{\Delta_{3t}} \tag{29a}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}} t_{22} \tag{29b}$$

$$r_{22} = \frac{d_1}{e} \frac{Y_{22}}{X_{11}} \frac{\Delta_{3r}}{\Delta_3} \tag{29c}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}} r_{22} \tag{29d}$$

where Δ_3, Δ_{3t}, and Δ_{3r} are given by (C1) in appendix C.

a.4 H-dipole, V-dipole, and another reflector with $d_2 ce \neq 0$

When $d_2 \neq 0$, we use Z_{21} to normalize the other elements of the Z matrix and obtain three equations. By substituting (25a), (25b), (25d), and (25e) into these equations, three linear equations are obtained for the unknowns $1/r_{22}$, t_{22}, and t_{22}/r_{22}. The solution is given by,

$$t_{22} = \frac{d_2}{e} \frac{Y_{22}}{X_{11}} \frac{\Delta_{4t}}{\Delta_4} \tag{30a}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}} t_{22} \tag{30b}$$

$$r_{22} = \frac{c}{d_2} \frac{Y_{22}}{X_{11}} \frac{\Delta_4}{\Delta_{4r}} \tag{30c}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}} r_{22} \tag{30d}$$

where Δ_4, Δ_{4t}, and Δ_{4r} are given by (D1) in appendix D.

a.5 H-dipole, V-dipole, and inclined dipole with $d_1 d_2 = ce$ and $cd_1 d_2 e \neq 0$

The special case with $c = d_1 = d_2 = e = 1$ has been solved for the screened reflector approach in [9]. In this case the third target has a

singular matrix (i.e., $d_1 d_2 = ce$). When $d_1 d_2 = ce$, (26) will reduce to

$$\frac{Z_{12}}{Z_{11}} = \frac{t_{22}d_1 + t_{12}c}{t_{21}d_1 + c} \tag{31a}$$

$$\frac{Z_{21}}{Z_{11}} = \frac{r_{22}d_2 + r_{21}c}{r_{12}d_2 + c} \tag{31b}$$

$$Z_{22}Z_{11} = Z_{12}Z_{21} \tag{31c}$$

Then, the solution can be obtained from (25) and (31) and is given as

$$r_{22} = \frac{c}{d_2} \frac{Y_{22}}{X_{11}} \frac{X_{11}Z_{21} - X_{21}Z_{11}}{Y_{22}Z_{11} - Y_{12}Z_{21}} \tag{32a}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}} r_{22} \tag{32b}$$

$$t_{22} = \frac{c}{d_1} \frac{Y_{22}}{X_{11}} \frac{X_{11}Z_{12} - X_{12}Z_{11}}{Y_{22}Z_{11} - Y_{21}Z_{12}} \tag{32c}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}} t_{22} \tag{32d}$$

or

$$r_{22} = \frac{d_1}{e} \frac{Y_{22}}{X_{11}} \frac{X_{11}Z_{22} - X_{21}Z_{12}}{Y_{22}Z_{12} - Y_{12}Z_{22}} \tag{33a}$$

$$r_{12} = \frac{Y_{12}}{Y_{22}} r_{22} \tag{33b}$$

$$t_{22} = \frac{d_2}{e} \frac{Y_{22}}{X_{11}} \frac{X_{11}Z_{22} - X_{12}Z_{21}}{Y_{22}Z_{21} - Y_{21}Z_{22}} \tag{33c}$$

$$t_{21} = \frac{Y_{21}}{Y_{22}} t_{22} \tag{33d}$$

b. One Trihedral Reflector, One Target with Diagonal Scattering Matrix, and One General Target

For this case, the scattering matrices of the calibration targets are given as follows

$$S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_2 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}, \quad S_3 = \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} \tag{34}$$

where

$$ab \neq 0 \tag{35a}$$

$$a \neq b \tag{35b}$$

For this set of scattering matrices, the solutions are presented for the following subcases

(B.1) $c \neq e$, $d_1 \neq 0$ or $d_2 \neq 0$, and $ae \neq bc$,
(B.2) $a = -b$, $c = e \neq 0$, and $d_1 d_2 \neq 0$,
(B.3) $a \neq -b$, $c = e = 0$, and $d_1 d_2 \neq 0$,
(B.4) $a = -b$, $c = e = 0$, and $d_1 d_2 \neq 0$,
(B.5) $a \neq -b$, $c = e \neq 0$, and $d_1 \neq 0$ or $d_2 \neq 0$,
(B.6) $a = -b$, $c = e \neq 0$, $d_1 \neq 0$, and $d_2 = 0$,
(B.7) $a = -b$, $c = e \neq 0$, $d_2 \neq 0$, and $d_1 = 0$.

It will be shown in the following sections that the number of solutions is one for case (B.1), two for case (B.2), two for case (B.3), four for case (B.4), one for case (B.5), one for case (B.6), and one for case (B.7).

Note that the case with $c \neq e$, $d_1 \neq 0$ or $d_2 \neq 0$, and $ae = bc$ is not covered in (B.1)–(B.7). Under these circumstances, we should exchange the order of the first two targets and apply the method of PT [see (11)]. For example, we can select

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \tag{36}$$

such that

$$S_2 = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} B \tag{37a}$$

$$S_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} B \tag{37b}$$

$$S_3 = \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} = A \begin{bmatrix} c/a & d_1/b \\ d_2/a & e/b \end{bmatrix} B \tag{37c}$$

It is easy to see that the new transformed matrices will become one of the cases, (B.2), (B.5), (B.6) and (B.7) presented in this section. It should also be noted that other cases not presented in this section do not provide a finite number of solutions (see Table 9.2).

By substituting (34) into (1a) and (1b), we first obtain the following equalities

$$t_{12} = \frac{\lambda X_{12} Y_{11} + X_{11} Y_{12} + \lambda (X_{12} Y_{11} - X_{11} Y_{12}) r_{12} t_{21}}{(1 + \lambda) X_{11} Y_{11}} \tag{38a}$$

$$r_{12} t_{22} = \frac{X_{12} Y_{11} - X_{11} Y_{12} + (X_{12} Y_{11} + \lambda X_{11} Y_{12}) r_{12} t_{21}}{(1 + \lambda) X_{11} Y_{11}} \tag{38b}$$

$$r_{21} = \frac{\lambda X_{21}Y_{11} + X_{11}Y_{21} + \lambda(X_{21}Y_{11} - X_{11}Y_{21})r_{12}t_{21}}{(1+\lambda)X_{11}Y_{11}} \quad (38c)$$

$$r_{22}t_{21} = \frac{X_{21}Y_{11} - X_{11}Y_{21} + (X_{21}Y_{11} + \lambda X_{11}Y_{21})r_{12}t_{21}}{(1+\lambda)X_{11}Y_{11}} \quad (38d)$$

$$r_{21}t_{12} = \frac{\lambda X_{22}Y_{11} + X_{11}Y_{22} + \lambda(X_{22}Y_{11} - X_{11}Y_{22})r_{12}t_{21}}{(1+\lambda)X_{11}Y_{11}} \quad (38e)$$

$$r_{22}t_{22} = \frac{X_{22}Y_{11} - X_{11}Y_{22} + (X_{22}Y_{11} + \lambda X_{11}Y_{22})r_{12}t_{21}}{(1+\lambda)X_{11}Y_{11}} \quad (38f)$$

where

$$\lambda = -\frac{b}{a} \quad (39)$$

The quantities on the left-hand-side of the above equations are expressed in terms of the measurements of the first two targets and the unknown quantity $r_{12}t_{21}$. After $r_{12}t_{21}$ is solved using the measured Z matrix, all the normalized quantities (3) can be solved subsequently.

b.1 $c \neq e$, $d_1 \neq 0$ or $d_2 \neq 0$, and $ae \neq bc$

The solution for $r_{12}t_{21}$ shown below is valid only for $c \neq e$ and $ae \neq bc$. To solve for $r_{12}t_{21}$, firstly, two equations for r_{22} and t_{22} are obtained by substituting (46a) and (46b) into (26a) and (26b). Then r_{22} and t_{22} are solved which are given by (42a) and (43a). Thereafter using (38) to express (42a) and (43a) in terms of the unknown $r_{12}t_{21}$ and substituting the final expressions of r_{22}, t_{22}, and (38) into (26c) gives one equation for $r_{12}t_{21}$. After some algebraic manipulation, $r_{12}t_{21}$ can be solved and is given as

$$r_{12}t_{21} = \frac{\Delta_{rt}}{\Delta} \quad (40)$$

where Δ and Δ_{rt} are given by (E1) in appendix E. The products t_{12}, $r_{12}t_{22}$, r_{21}, $r_{22}t_{21}$, $r_{21}t_{12}$, and $r_{22}t_{22}$ can then be calculated from (38). Let

$$y'_{21} \equiv \frac{t_{21}}{t_{22}} = \frac{r_{22}t_{21}}{r_{22}t_{22}} \quad (41a)$$

$$y'_{12} \equiv \frac{r_{12}}{r_{22}} = \frac{r_{12}t_{22}}{r_{22}t_{22}} \quad (41b)$$

The following formulas are used to solve for r_{22} and t_{22}. When $c \neq 0$ and $d_1 \neq 0$

$$t_{22} = \frac{\Delta_{5t}}{\Delta_5} \qquad (42a)$$

$$r_{22} = \frac{r_{22}t_{22}}{t_{22}} \qquad (42b)$$

When $c \neq 0$ and $d_2 \neq 0$

$$r_{22} = \frac{\Delta_{5r}}{\Delta_5} \qquad (43a)$$

$$t_{22} = \frac{r_{22}t_{22}}{r_{22}} \qquad (43b)$$

When $e \neq 0$ and $d_1 \neq 0$,

$$t_{22} = \frac{\Delta_{6t}}{\Delta_6} \qquad (44a)$$

$$r_{22} = \frac{r_{22}t_{22}}{t_{22}} \qquad (44b)$$

When $e \neq 0$ and $d_2 \neq 0$,

$$r_{22} = \frac{\Delta_{6r}}{\Delta_6} \qquad (45a)$$

$$t_{22} = \frac{r_{22}t_{22}}{r_{22}} \qquad (45b)$$

where Δ_5, Δ_{5t}, Δ_{5r}, Δ_6, Δ_{6t}, and Δ_{6r} are given in appendix F. After t_{22} and r_{22} are determined, t_{21} and r_{12} can be solved from (41) and given as

$$t_{21} = y'_{21}t_{22} \qquad (46a)$$

$$r_{12} = y'_{12}r_{22} \qquad (46b)$$

b.2 $a = -b$, $c = e \neq 0$, and $d_1 d_2 \neq 0$

There are two sets of solutions for this case (No. 9, Table 9.2). Note that case No. 10 in Table 9.2 can be transformed into this case by exchanging the order of the first two targets and reapplying the

method of PT as discussed in section 9.2.b. From (38a), (38c), and (38e), the following quadratic equation for $r_{12}t_{21}$ can be obtained

$$a_1(r_{12}t_{21})^2 + 2a_2 r_{12}t_{21} + a_3 = 0 \tag{47}$$

where

$$a_1 = (X_{12}Y_{11} - X_{11}Y_{12})(X_{21}Y_{11} - X_{11}Y_{21})$$
$$a_2 = (X_{12}X_{21}Y_{11}^2 - X_{11}^2 Y_{12}Y_{21} - X_{11}X_{22}Y_{11}^2 + X_{11}^2 Y_{11}Y_{22})$$
$$a_3 = (X_{12}Y_{11} + X_{11}Y_{12})(X_{21}Y_{11} + X_{11}Y_{21})$$
$$\quad - 2X_{11}Y_{11}(X_{22}Y_{11} + X_{11}Y_{22})$$

After expressing a_1 and a_3 in terms of the elements of transfer matrices R and T and the scattering matrix elements a and b, it can be shown that

$$a_1 = a_3 \tag{48}$$

This means that these two roots of (47) are reciprocal to each other, i.e., if $r_{12}t_{21}$ is a solution, then $1/r_{12}t_{21}$ is the other solution. This obviously raises a problem in cases for which either r_{12} or t_{21} is zero.

In calculating the solutions for these cases, we find a root of (47) whose magnitude is smaller than one. For this root, we can employ (42)–(46) to solve for the corresponding t_{22}, r_{22}, t_{21}, and r_{12}. The second solution of (47) gives $r_{12}t_{21}$ with a magnitude greater than one and approaches infinity as r_{12} or t_{21} approaches zero. The relation between the solutions is discussed next, and an alternative way of obtaining the second solution is given.

The relation between two sets of solutions can be understood easily by exploring the symmetry property of this set of scattering matrices. Let the matrices A and B be

$$A = \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix} \tag{49}$$

By inserting the identity matrix between the scattering matrices and the transfer matrices in (1), it can be seen that

$$X = e^{i\phi_1} R A^{-1} A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} BB^{-1}T \tag{50a}$$

$$Y = e^{i\phi_2} R A^{-1} A \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} BB^{-1}T \tag{50b}$$

$$Z = e^{i\phi_3} R A^{-1} A \begin{bmatrix} c & d_1 \\ d_2 & c \end{bmatrix} BB^{-1}T \tag{50c}$$

Define

$$R' = RA^{-1} = \frac{1}{d_1 d_2} \begin{bmatrix} d_2 R_{12} & d_1 R_{11} \\ d_2 R_{22} & d_1 R_{21} \end{bmatrix} \qquad (51a)$$

$$T' = B^{-1}T = \frac{1}{d_1 d_2} \begin{bmatrix} d_1 T_{21} & d_1 T_{22} \\ d_2 T_{11} & d_2 T_{12} \end{bmatrix} \qquad (51b)$$

After straightforward multiplication, (50) becomes

$$X = e^{i\phi_1} R' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} T' \qquad (52a)$$

$$Y = e^{i\phi_2 + i\pi} R' \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} T' \qquad (52b)$$

$$Z = e^{i\phi_3} R' \begin{bmatrix} c & d_1 \\ d_2 & c \end{bmatrix} T' \qquad (52c)$$

Comparing (52) with (1) indicates that if the normalized quantities of R and T are the solutions of (1), then the normalized quantities of R' and T' are also the solutions of (1). These two sets of solutions are related by (51). Therefore, after the solution with smaller $r_{12}t_{21}$ is found, the other solution can be obtained by using the transformation relation given by (51).

b.3 $a \neq -b$, $c = e = 0$, and $d_1 d_2 \neq 0$

There are two sets of solutions for this case (No. 11, Table 9.2). By equating the product of (38a) and (38c) with (38e), a quadratic equation for $r_{12}t_{21}$ is given in (47). The other quadratic equation can be obtained by multiplying (38f) with $r_{12}t_{21}$ and equating the result with the product of (38b) and (38d). If $a \neq -b$ ($\lambda \neq 1$), these two second-order polynomial equations will be independent of each other. Thus, by canceling the second-order term from these two equations, $r_{12}t_{21}$ can be solved and is given as

$$r_{12}t_{21} = \frac{(1 - \lambda)(X_{12}Y_{11} - X_{11}Y_{12})(X_{21}Y_{11} - X_{11}Y_{21})}{\begin{bmatrix} \lambda(X_{12}Y_{11} - X_{11}Y_{12})(X_{21}Y_{11} - X_{11}Y_{21}) \\ -(X_{12}Y_{11} + \lambda X_{11}Y_{12})(X_{21}Y_{11} + \lambda X_{11}Y_{21}) \\ +(1 + \lambda)X_{11}Y_{11}(X_{22}Y_{11} + \lambda X_{11}Y_{22}) \end{bmatrix}} \qquad (53)$$

Then, t_{12}, $r_{12}t_{22}$, r_{21}, $r_{22}t_{21}$, $r_{21}t_{12}$, and $r_{22}t_{22}$ can be determined uniquely by (38).

There are two possible sets of solutions in this case for r_{12}, t_{21}, r_{22}, and t_{22},

$$r_{22} = \pm \sqrt{\frac{r_{22}t_{22}(Z_{21} - r_{21}y'_{21}Z_{12})}{Z_{12} - Z_{21}y'_{12}t_{12}}\frac{d_1}{d_2}} \tag{54a}$$

$$t_{22} = \frac{r_{22}t_{22}}{r_{22}} \tag{54b}$$

$$r_{12} = y'_{12}r_{22} \tag{54c}$$

$$t_{21} = y'_{21}t_{22} \tag{54d}$$

If the real parts of t_{22} or r_{22} can be required to be either greater or smaller than zero, then a unique solution can be chosen.

The relation between these two sets of solutions can also be understood by looking at the symmetry property of this set of targets. Let matrix A be

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{55}$$

By inserting the identity matrix between the scattering matrices and the transfer matrices in (1), it can be shown that

$$X = e^{i\phi_1} RA^{-1}A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} AA^{-1}T \tag{56a}$$

$$Y = e^{i\phi_2} RA^{-1}A \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} AA^{-1}T \tag{56b}$$

$$Z = e^{i\phi_3} RA^{-1}A \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix} AA^{-1}T \tag{56c}$$

After straightforward multiplication, the above equations become

$$X = e^{i\phi_1} R' \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} T' \tag{57a}$$

$$Y = e^{i\phi_2} R' \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} T' \tag{57b}$$

$$Z = e^{i\phi_3 + i\pi} R' \begin{bmatrix} 0 & d_1 \\ d_2 & 0 \end{bmatrix} T' \tag{57c}$$

where

$$R' = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} A^{-1} = \begin{bmatrix} R_{11} & -R_{12} \\ R_{21} & -R_{22} \end{bmatrix} \tag{58a}$$

$$T' = A^{-1} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ -T_{21} & -T_{22} \end{bmatrix} \tag{58b}$$

By comparing (57) with (1), it can be seen that if the normalized quantities of R and T are the solution of (1), then the normalized quantities of R' and T' are also a solution of (1). These two sets of solutions are related by (58).

b.4 $a = -b$, $c = e = 0$, and $d_1 d_2 \neq 0$

This case corresponds to case No. 12, Table 9.2. When $a = -b$ or $\lambda = 1$, there are two possible roots for $r_{12}t_{21}$ given by (47). For each root, there are two sets of solutions for r_{22}, t_{22}, r_{12}, and t_{21} which can be obtained by using (54). Therefore, there are four sets of solutions in this case. This case, $a = -b$ and $c = e = 0$, has both of the symmetry properties of cases B.2 and B.3. Therefore, these four sets of solutions can be related to each other by (51) or (58). This particular case ($d_1 = d_2$) has been solved approximately in [9] by assuming that the cross-polarization coupling is small, and the real parts of r_{22} and t_{22} are positive. With those assumptions, a particular set of solutions can be chosen.

b.5 $a \neq -b$, $c = e \neq 0$, and $d_1 \neq 0$ or $d_2 \neq 0$

This case can be solved by using (53) to determine $r_{12}t_{21}$ and then applying (38) and (42)–(46) for the unique solution of the normalized quantities defined in (3).

b.6 $a = -b$, $c = e \neq 0$, $d_1 \neq 0$, and $d_2 = 0$

This case corresponds to case No. 13 in Table 9.2. From the measurement of the first and second targets, the following equation is obtained

$$\frac{2t_{12}}{1 + t_{12}(t_{21}/t_{22})} = \frac{Y_{12}X_{22} - Y_{22}X_{12}}{Y_{11}X_{22} - Y_{21}X_{12}} \tag{59}$$

and from the measurement of the first and third targets, two equations are obtained

$$\frac{t_{21}}{t_{22}} = \frac{X_{22}Z_{11} - X_{12}Z_{21} + X_{21}Z_{12} - X_{11}Z_{22}}{2(X_{22}Z_{12} - X_{12}Z_{22})} \tag{60}$$

$$\frac{c}{d_1}\frac{1}{t_{22}}\left(1 - t_{12}\frac{t_{21}}{t_{22}}\right) = \frac{X_{22}Z_{11} - X_{12}Z_{21} - X_{21}Z_{12} + X_{11}Z_{22}}{2(X_{22}Z_{12} - X_{12}Z_{22})} \tag{61}$$

Again let

$$y'_{21} = \frac{t_{21}}{t_{22}} \tag{62}$$

Solving t_{12} from (59) gives us

$$t_{12} = \frac{Y_{12}X_{22} - Y_{22}X_{12}}{2(Y_{11}X_{22} - Y_{21}X_{12}) - y'_{21}(Y_{12}X_{22} - Y_{22}X_{12})} \tag{63}$$

Hence, we can solve t_{22} from (61) in terms of t_{12} and y'_{21}.

$$t_{22} = \frac{c}{d_1}(1 - t_{12}y'_{21})\frac{2(X_{22}Z_{12} - X_{12}Z_{22})}{X_{22}Z_{11} - X_{12}Z_{21} - X_{21}Z_{12} + X_{11}Z_{22}} \tag{64}$$

Therefore,

$$t_{21} = t_{22}y'_{21} \tag{65}$$

Once the normalized quantities of matrix T are all solved, the normalized quantities of matrix R can be obtained from any one of the following relations

$$R = e^{-i\phi_1}XT^{-1} \tag{66a}$$

$$R = e^{-i\phi_2}YT^{-1}\begin{bmatrix} 1/a & 0 \\ 0 & -1/a \end{bmatrix} \tag{66b}$$

$$R = e^{-i\phi_3}ZT^{-1}\begin{bmatrix} 1/c & -d_1/c^2 \\ 0 & 1/c \end{bmatrix} \tag{66c}$$

which are obtained by inverting (1).

$b.7$ $a = -b$, $c = e \neq 0$, $d_1 = 0$, and $d_2 \neq 0$

This case is similar to case 6 (No. 14 in Table 9.2). As a matter of fact, after we take the transpose of the measured polarization matrices,

$$X^t = e^{i\phi_1}T^t\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}R^t \tag{67a}$$

$$Y^t = e^{i\phi_2}T^t\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}R^t \tag{67b}$$

$$Z^t = e^{i\phi_3}T^t\begin{bmatrix} c & d_2 \\ 0 & c \end{bmatrix}R^t \tag{67c}$$

we can use the following mapping

$$X \to X^t \tag{68a}$$

$$Y \to Y^t \tag{68b}$$

$$Z \to Z^t \tag{68c}$$

$$R \to T^t \tag{68d}$$

$$T \to R^t \tag{68e}$$

to obtain the solution of this case from those of case B.6.

c. One Dipole, One Trihedral Reflector, and One General Reflector

The scattering matrices of the calibration targets are given as follows

$$S_1 = \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad S_3 = \begin{bmatrix} c & d_1 \\ d_2 & e \end{bmatrix} \tag{69}$$

By substituting (69) into (1a) and (1b), the following equalities can be obtained

$$t_{12} = \frac{X_{12}}{X_{11}} \tag{70a}$$

$$r_{21} = \frac{X_{21}}{X_{11}} \tag{70b}$$

$$y'_{21} \equiv \frac{t_{21}}{t_{22}} = \frac{Y_{21}X_{11} - Y_{11}X_{21}}{Y_{22}X_{11} - Y_{12}X_{21}} \tag{70c}$$

$$y'_{12} \equiv \frac{r_{12}}{r_{22}} = \frac{Y_{12}X_{11} - Y_{11}X_{12}}{Y_{22}X_{11} - Y_{21}X_{12}} \tag{70d}$$

and

$$r_{22}t_{22} = \frac{Y_{22}X_{11}^2 - Y_{11}X_{21}X_{12}}{X_{11}^2(Y_{11} - y'_{12}y'_{21}Y_{21})} \tag{71}$$

Hence, we can obtain

$$r_{22}t_{21} = r_{22}\,t_{22}\,y'_{21} \tag{72a}$$

$$r_{12}t_{21} = r_{22}\,t_{22}\,y'_{12} \tag{72b}$$

$$r_{12}t_{21} = r_{22}\,t_{22}\,y'_{21}\,y'_{12} \tag{72c}$$

by multiplying (71) with (70c) or (70d). Thus, all of the quantities appearing in (38) are obtained.

Discussed below are five subcases (C.1) $c \neq 0$ and $d_1 \neq 0$ (No. 1, Table 9.3), (C.2) $c \neq 0$ and $d_2 \neq 0$ (No. 2, Table 9.3), (C.3) $e \neq 0$ and $d_1 \neq 0$ (No. 3, Table 9.3), (C.4) $e \neq 0$ and $d_2 \neq 0$ (No. 4, Table 9.3), and (C.5) $c = e = 0$ (No. 5, Table 9.3).

For cases (C.1)–(C.4), we can use (42)–(45), respectively, to solve for r_{22} and t_{22}, and then apply (46) for r_{12} and t_{21}. Note that t_{12} and r_{21} are given in (70). Thus, all the normalized quantities are uniquely determined.

For case (C.5), when both c and e are zero, there are two sets of solutions (see No. 5, Table 9.3). Note that this set of targets has exactly the same kind of symmetry as case B.3. Equation (58) gives the relation of these two solutions. The solutions are solved in the following manner. Taking the ratio of Z_{12} and Z_{21} and making use of (41), we obtain

$$
r_{22} = \frac{X_{11}Z_{21} - X_{21}Z_{12}y'_{21}}{X_{11}Z_{12} - X_{12}Z_{21}y'_{12}} \frac{d_1}{d_2} t_{22}
\tag{73}
$$

where y'_{12} and y'_{21} denote r_{12}/r_{22} and t_{21}/t_{22}, respectively. Then, from the ratio of Y_{22} and Y_{11}, we obtain the solution of t_{22} as

$$
t_{22} = \pm \sqrt{\frac{(X_{11}Z_{12} - X_{12}Z_{21}y'_{12})\,(Y_{22}X_{11}^2 - Y_{11}X_{12}X_{21})}{(X_{11}Z_{21} - X_{21}Z_{12}y'_{21})\,(Y_{11}X_{11}^2 - Y_{22}X_{11}^2 y'_{12}y'_{21})} \frac{d_2}{d_1}}
\tag{74}
$$

The rest of the quantities can be obtained from (70).

9.5   Misalignment

Effects of misalignment between calibration targets and the polarimetric radar are considered in this section. When the calibration reflectors are not aligned correctly relative to the radar system, the misalignment will manifest its effects in the estimated normalized quantities of R and T matrices. The misalignment may be due to the misplaced roll or yaw angles during the deployment of the calibration targets or the uncertain pitch and roll angles of the platform which carries the radar system.

The effect of misalignment in the pointing direction (parallel to the line-of-sight) is determined by the angular scattering pattern of the target. Calibration targets with broad angular beam width are desired to reduce this error. The above misalignment will not be discussed
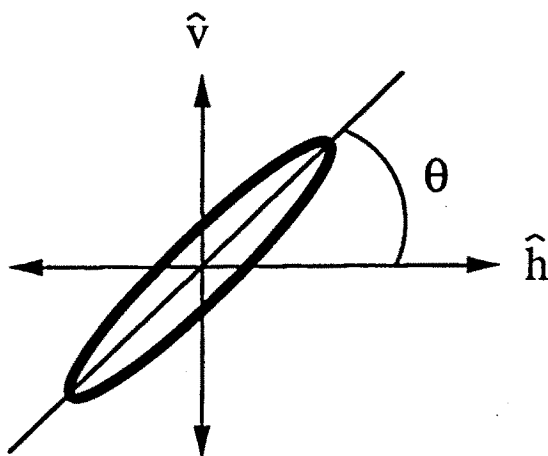
Figure 9.1 Configuration of rolling a target by θ.

further. In the remainder of this section, we will focus on the effects of misalignment in roll angles.

Let S be the scattering matrix of a calibration target. Thus, if the target is rolled by an angle θ on the plane perpendicular the line-of-sight (Fig. 9.1) the scattering matrix S_θ of the rolled target becomes

$$S_\theta = ASA^{-1} \tag{75}$$

where

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \tag{76}$$

is the coordinate rotation matrix.

Note that a linear combination of the following two matrices is invariant under arbitrary rolling angle (roll-invariant)

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The former corresponds to a sphere or trihedral reflector. The latter represents a non-reciprocal target which always rotates the polarization of the electric field by 90° and could be simulated by a polarimetric active calibrator or manufactured by use of the Faraday rotation phenomena exhibited in some passive gyrotropic materials. It can be

shown that there are only two linear independent roll-invariant targets. All other roll-invariant targets must be a linear combination of these two roll-invariant targets.

a. Uniform Misalignment

To illustrate the effects of misalignment, let us consider a simple case that the first two targets are roll-invariant and the third target has a general scattering matrix S_3. Hence, the rolling angles of the first two targets need not be concerned. If the third target is misaligned by a roll angle θ, it is equivalent to say that all the targets are off by the same roll angle. Thus the corresponding measurements become

$$X = e^{i\phi_1} RAS_1 A^{-1}T \tag{77a}$$

$$Y = e^{i\phi_2} RAS_2 A^{-1}T \tag{77b}$$

$$Z = e^{i\phi_3} RAS_3 A^{-1}T \tag{77c}$$

Assume that there is no knowledge of the roll angle θ. The estimated R and T matrices will be

$$R' = RA, \quad T' = A^{-1}T \tag{78}$$

for the given measurements X, Y, and Z. Expressing (78) explicitly gives

$$R' = RA = \begin{bmatrix} R_{11}\cos\theta + R_{12}\sin\theta & -R_{11}\sin\theta + R_{12}\cos\theta \\ R_{21}\cos\theta + R_{22}\sin\theta & -R_{21}\sin\theta + R_{22}\cos\theta \end{bmatrix} \tag{79a}$$

$$T' = A^{-1}T = \begin{bmatrix} T_{11}\cos\theta + T_{21}\sin\theta & T_{12}\cos\theta + T_{22}\sin\theta \\ -T_{11}\sin\theta + T_{21}\cos\theta & -T_{12}\sin\theta + T_{22}\cos\theta \end{bmatrix} \tag{79b}$$

If the cross-polarization coupling of the polarimetric radar is relatively small and the misaligned roll angle θ is no more than a few degrees, the normalized quantities of estimated matrices R' and T' are

$$\frac{R'_{12}}{R'_{11}} \simeq \frac{R_{12}}{R_{11}} - \tan\theta \tag{80a}$$

$$\frac{R'_{21}}{R'_{11}} \simeq \frac{R_{21}}{R_{11}} + \frac{R_{22}}{R_{11}}\tan\theta \tag{80b}$$

$$\frac{R'_{22}}{R'_{11}} \simeq \frac{R_{22}}{R_{11}} - \left(\frac{R_{22}R_{12}}{R_{11}^2} + \frac{R_{21}}{R_{11}}\right)\tan\theta \qquad (80c)$$

$$\frac{T'_{12}}{T'_{11}} \simeq \frac{T_{12}}{T_{11}} + \frac{T_{22}}{T_{11}}\tan\theta \qquad (80d)$$

$$\frac{T'_{21}}{T'_{11}} \simeq \frac{T_{21}}{T_{11}} - \tan\theta \qquad (80e)$$

$$\frac{T'_{22}}{T'_{11}} \simeq \frac{T_{22}}{T_{11}} - \left(\frac{T_{22}T_{21}}{T_{11}^2} + \frac{T_{12}}{T_{11}}\right)\tan\theta \qquad (80f)$$

Note that the terms describing the channel imbalance, (80c) and (80f), are not sensitive to the misalignment when the cross-polarization coupling of the radar is small. However, the errors in the estimated cross-polarization couplings (80a), (80b), (80d), and (80e) are of the order of $\tan\theta$. For a perfect polarimetric system without cross-polarization coupling and mismatch ($R = T =$ Identity Matrix), the effect of misalignment appears purely as a pseudo cross-polarization coupling $\tan\theta$. If the error in the estimated cross-talk is to be smaller than -30 dB, the roll angle of the target must be within $\pm 1.8°$ accuracy.

The above conclusion also holds for arbitrary sets of calibration targets uniformly misplaced by the same roll angle (uniform misalignment). Nevertheless, if the previous system parameters for the radar are available, we can always check to see whether the newly estimated system parameters are related to the previous parameters by the simple coordinate transformation (79). If that is the case, we can remove the errors by an inverse rotation transformation.

b. Relative Misalignment

Next to be considered is the case that each calibration reflector may be misaligned by a different roll angle (nonuniform misalignment). This nonuniform misalignment problem can be thought of as a uniform misalignment problem plus a relative misalignment problem where the first calibrator is aligned correctly, and all the other calibrators are misaligned relative to the first one. As discussed before, a uniform misalignment simply introduces a rotation-transformed error and could be corrected by an inverse rotation. Hence, we may restrict the discussion to the case that the first target is in alignment with the radar and the other calibration targets may be misplaced by some other angles.

The effect of relative misalignment will be analyzed in the following manner. The measurements from these misaligned targets are simu-

lated, and the normalized quantities of R and T matrices are estimated by assuming all the targets are aligned. Comparing the estimated solution for t'_{mn} and r'_{mn} with the true system parameters, t_{mn} and r_{mn}, gives the square error defined as

$$E_\theta = \sum_{m,n=1}^{2} \left| t'_{mn} - t_{mn} \right|^2 + \sum_{m,n=1}^{2} \left| r'_{mn} - r_{mn} \right|^2 \qquad (81)$$

In general, if we substitute the estimated quantities t'_{mn} and r'_{mn} back to calculate the corresponding measurements X', Y', and Z' for perfectly aligned targets

$$X' = R'S_1T' \qquad (82a)$$
$$Y' = R'S_2T' \qquad (82b)$$
$$Z' = R'S_3T' \qquad (82c)$$

The calculated normalized quantities of X', Y', and Z' may not be consistent with the true normalized quantities of X, Y, and Z. It is, therefore, necessary to check whether the solutions are consistent with the measurements or not. Let us define D(M) as the distance between the true and calculated normalized measurements

$$D(M) = \sum_{m,n=1}^{2} \left| x'_{mn} - x_{mn} \right|^2 + \left| y'_{mn} - y_{mn} \right|^2 + \left| z'_{mn} - z_{mn} \right|^2 \quad (83)$$

where

$$x_{mn} = \frac{X_{mn}}{\sqrt{\sum_{m,n} |X_{mn}|^2} \exp(i\theta_{ij})} \qquad (84a)$$

$$x'_{mn} = \frac{X'_{mn}}{\sqrt{\sum_{m,n} |X'_{mn}|^2} \exp(i\theta'_{ij})} \qquad (84b)$$

and θ_{ij} is the phase angle of X_{ij} which has the largest magnitude among the elements of X. Note that θ'_{ij}, the phase angle of X'_{ij} (not necessarily the largest element of X'), is used for the phase reference of X'. Likewise, $Y(Z)$ and $Y'(Z')$ are normalized in the same manner.

Under the situation that the radar is perfect ($R = T = I$), Figs. 9.2–9.7 illustrate E_θ and $D(M)$ for the following sets of calibration

targets (i)–(vi) where the first target is assumed to be in perfect align-
ment with the radar (relative misalignment). θ_2 and θ_3 represent, re-
spectively, the actual roll angle of the second and third targets.

(i) Trihedral and two nonreciprocal reflectors

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 3.2 & -1 \\ 1 & -1 \end{bmatrix}$$

(ii) Horizontal (0°), Vertical (90°) and 45° dipoles

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

(iii) Horizontal dipole, Vertical dipole, and 22.5° dihedral

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(iv) Trihedral, 0° dihedral, and 22.5° dihedral

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

(v) Trihedral, 0° dihedral, and 45° dihedral

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

(vi) Horizontal dipole, trihedral, and 22.5° dihedral

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

Figure 9.2 corresponds to target set (i). Because trihedral and the
second reflector are roll-invariant, this case corresponds to the case of
uniform misalignment. Hence, the total square error E_θ is independent
of θ_2 and equal to $4 \tan^2 \theta_3$ [see (80a,b,d,e)]. Also D(M) is zero for all
angles. The performance is the optimum in all cases.

Figure 9.3 for target set (ii) shows larger E_θ than Fig. 9.2. Note
that $D(M)$ shown in Fig. 9.3(b) is also zero everywhere. This is because
the measurement from each dipole gives two independent equations [see

$4 \tan^2 2° = 0.00488$

(a)

(b)
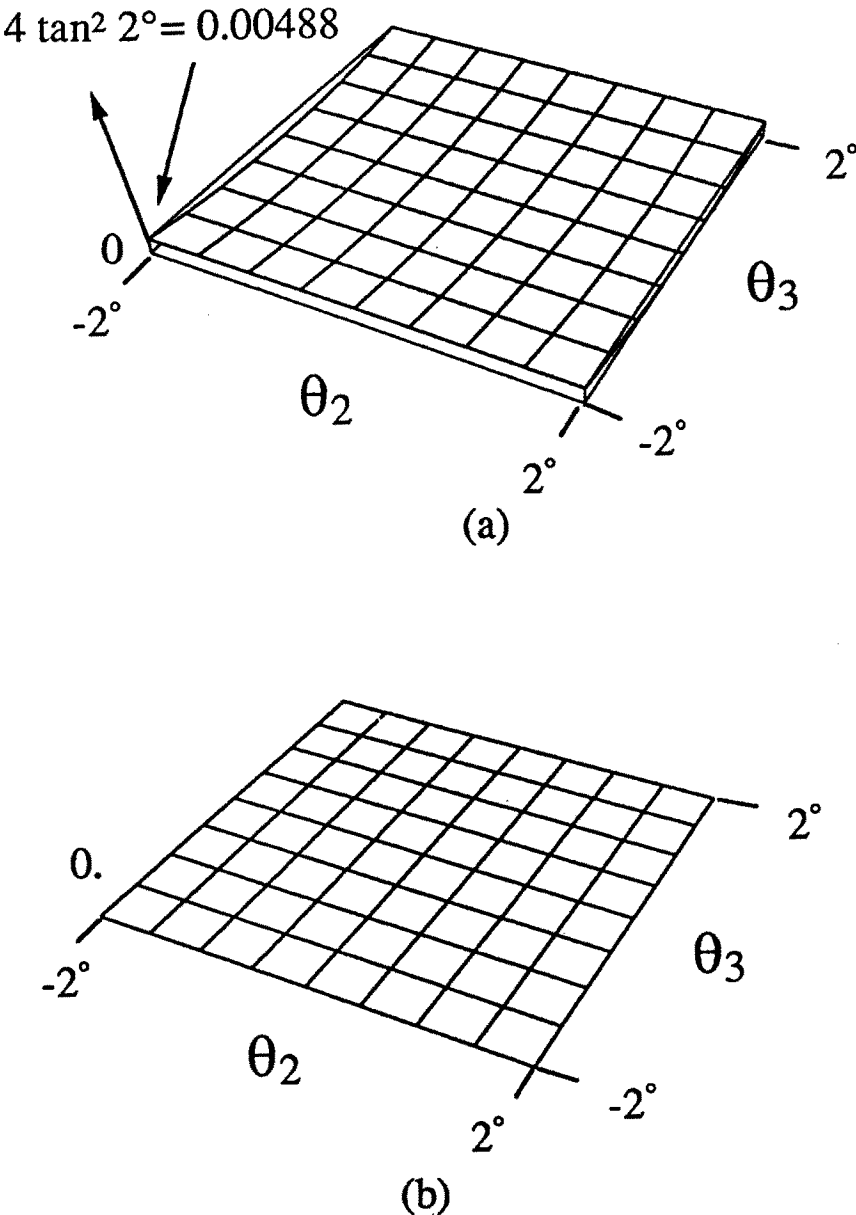
Figure 9.2 Effect of misalignment for target set (i). (a) E_θ and (b) $M(D)$.

(a)

(b)
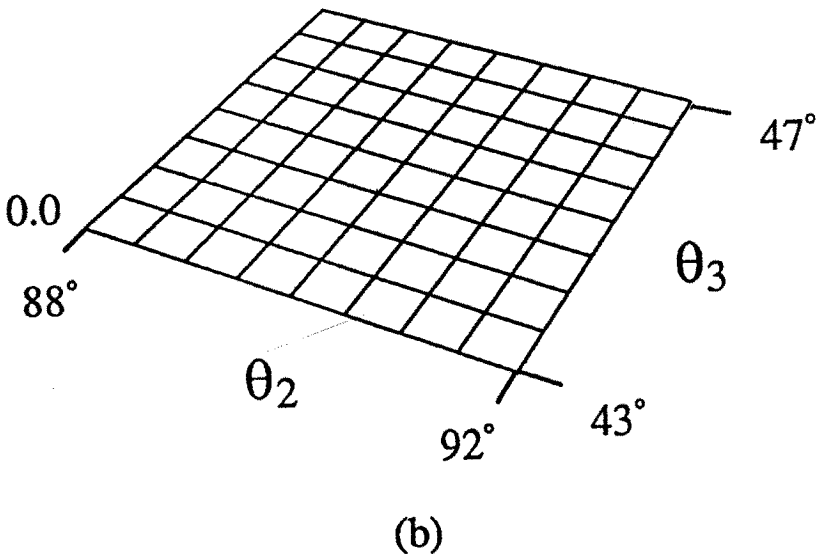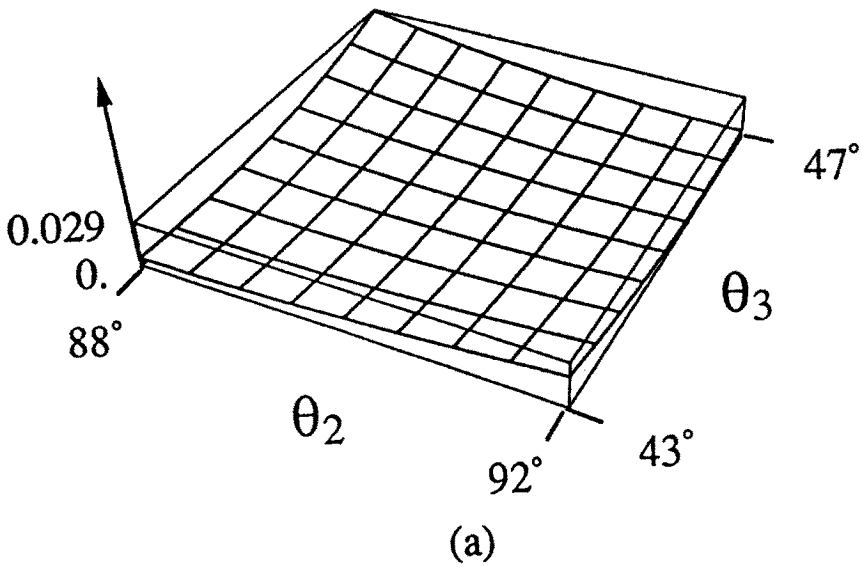
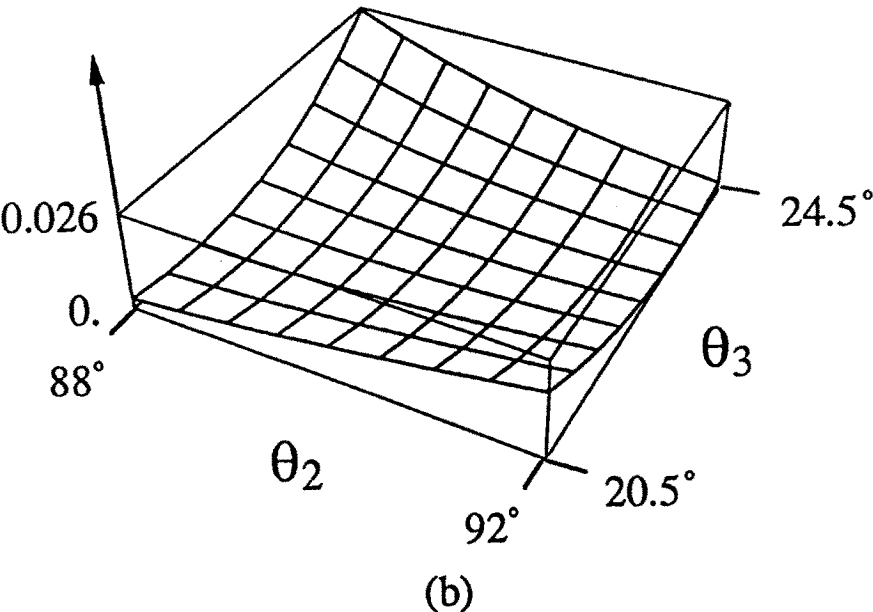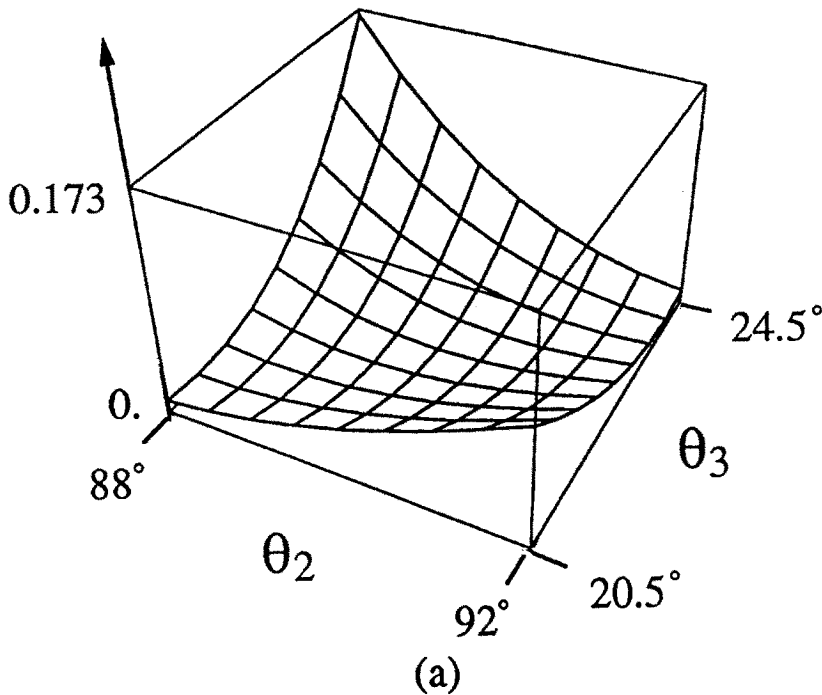Figure 9.3 Effect of misalignment for target set (ii). (a) E_θ and (b) $M(D)$.

(a)

(b)

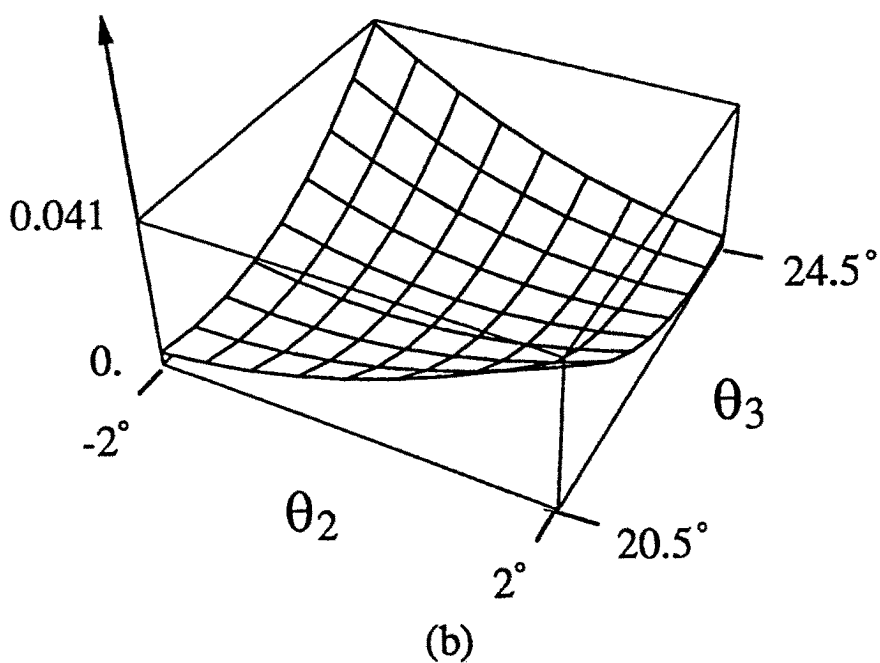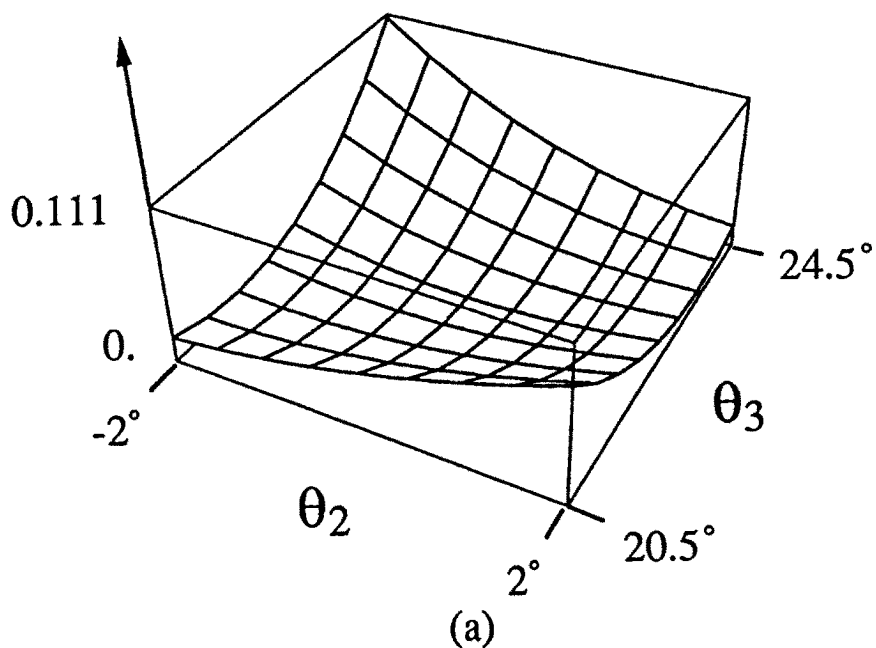Figure 9.4 Effect of misalignment for target set (iii). (a) E_θ and (b) $M(D)$.

(a)

(b)

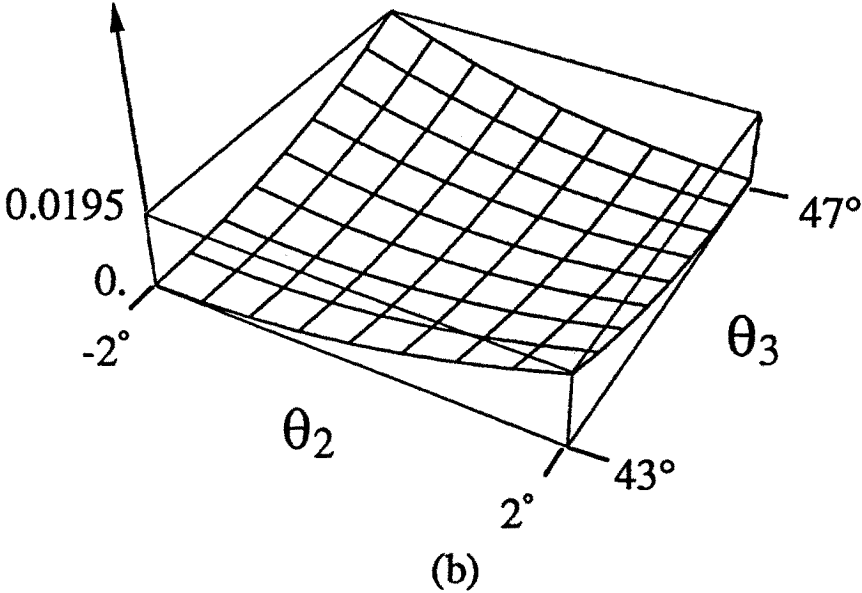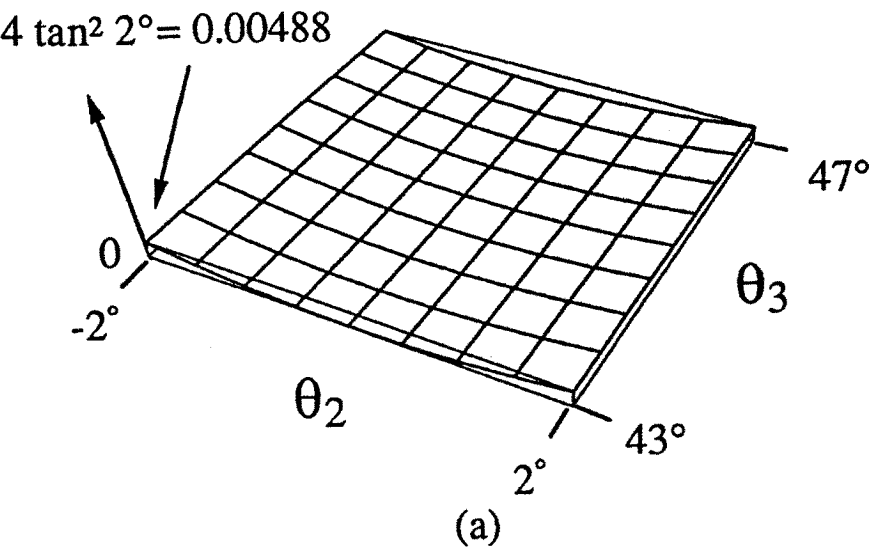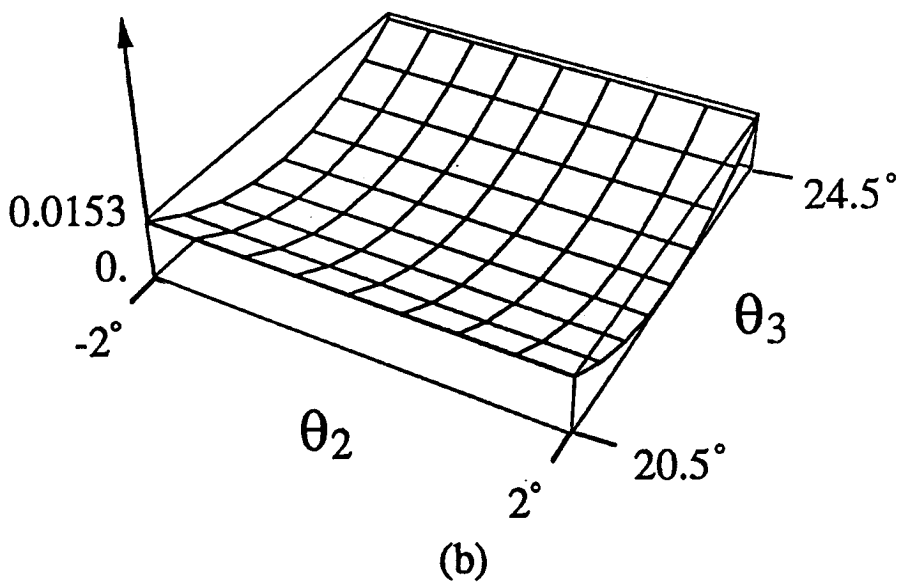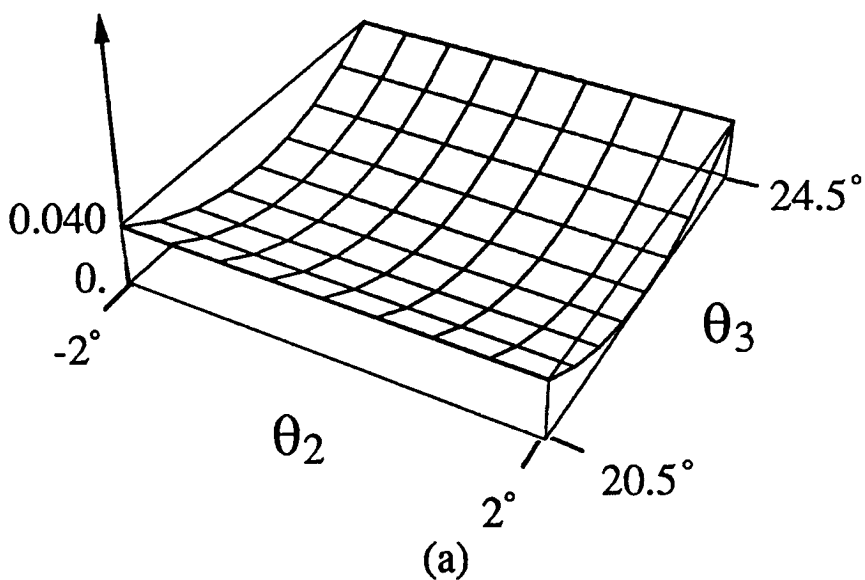Figure 9.5 Effect of misalignment for target set (iv). (a) E_θ and (b) $M(D)$.

(a)

(b)

Figure 9.6 Effect of misalignment for target set (v). (a) E_θ and (b) $M(D)$.

(a)

(b)

Figure 9.7 Effect of misalignment for target set (vi). (a) E_θ and (b) $M(D)$.

(31)]. Hence, we have only six equations to determine six unknowns (3a–f). If a set of solutions is found, then it will satisfy all equations even though some targets are misaligned. Thus, $D(M)$ is zero and does not provide us with any information regarding the misalignment of the targets.

Figure 9.4 shows the effect of misalignment for target set (iii). Note that E_θ is the largest among all six cases. Nevertheless, $D(M)$, which is zero only at the center of the figure ($\theta_2 = \theta_3 = 0$), carry the information about the alignment of the targets. This is useful in self-calibrating the roll angles of calibration targets. We can use $D(M)$ to estimate the roll angles of targets two and three relative to the first target even when the first target is not aligned with the radar. After θ_2 and θ_3 are corrected with respect to the first one, the residual error will become that of uniform alignment.

E_θ for target set (iv) shown in Fig. 9.5 is smaller than that in Fig. 9.4. $D(M)$ is not zero when the second or the third targets are misaligned. Note that this case corresponding to case B.2, section 9.4, has two sets of solutions. We select the solution that makes the cross-talk smaller than the one for the results shown in Fig. 9.5.

Figure 9.6 illustrates E_θ and $D(M)$ for target set (v), corresponding to case B.4, section 9.4. One of the four solutions which is closest to the correct solution is selected. Note that E_θ is equal to $4\tan^2\theta_2$ as compared with $4\tan^2\theta_3$ in Fig. 9.2. It is also interesting to see that E_θ is not a function of θ_3 whereas $D(M)$ is still a function of θ_3. This is due to the fact that Z_{11} and Z_{22} are not used to calculate the solution [see (54)]. As a matter of fact, if the radar does have some cross-polarization coupling, E_θ will show the dependence on θ_3.

The results of target case (vi) are illustrated in Fig. 9.7. Because the second target is roll-invariant, neither E_θ nor $D(M)$ is a function of θ_2. E_θ is larger than those of target sets (i) and (v) and smaller than all the others.

As a summary of the investigation of these six target sets, target sets (i) and (v) provide better performance than the others if $D(M)$ is not used to self-calibrate the relative roll angles. Regarding the disadvantage of these two sets of targets, using set (i) requires a nonreciprocal reflector for one of the roll-invariant targets whereas using set (v) requires extra information in selecting the correct one from four possible solutions. If $D(M)$ is used for self-calibration, E_θ for all the other cases can be corrected to that of the optimum case (i), except

case (ii) using three dipoles.

9.6   Effect of Noise

In general, the measurements cannot be free from noise which will affect the estimated system parameters. In this section the effect of noise will be analyzed for those sets of calibration targets discussed in the previous section. Let us consider the case that the noises are additive to the returns from the targets and are independent complex Gaussian with equal variance for each polarization channel.

When the measurements are contaminated by additive noise,

$$X = e^{i\phi}RST + N \tag{85}$$

where N is a two by two matrix with each element accounting for the noise received by the corresponding polarization channel.

As a result of noise, the estimated normalized quantities of R and T matrix will contain some errors,

$$R' = R'_{11} \begin{bmatrix} 1 & r_{12} + \epsilon_{r12} \\ r_{21} + \epsilon_{r21} & r_{22} + \epsilon_{r22} \end{bmatrix} \tag{86a}$$

$$T' = T'_{11} \begin{bmatrix} 1 & t_{12} + \epsilon_{t12} \\ t_{21} + \epsilon_{t21} & t_{22} + \epsilon_{t22} \end{bmatrix} \tag{86b}$$

where ϵ_{r22} and ϵ_{t22} are the estimated errors in channel imbalance, and ϵ_{r12}, ϵ_{r21}, ϵ_{t12}, and ϵ_{t21} are errors for the estimated cross-talk.

The following mean square errors (MSE) will be evaluated by the Monte Carlo simulation for each set of calibration targets.

$$MSE_{r,tij} = \langle |\epsilon_{r,tij}|^2 \rangle \tag{87}$$

Under the assumption that the radar is perfect $(R = T = I)$, Figs. 9.8–9.14 illustrate the MSEs for six sets of calibration targets described in section 9.5 and one additional set of targets composed of three PARCs with the following scattering parameters [10]

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$$

Note that the MSEs are approximately linearly proportional to the power of noise in all cases. It should be noticed that there are actually

four solutions for case (v) and two solutions for case (iv). In plotting
Figs. 9.11 and 12, we chose the solution which is closest to the correct
solution. This requires extra information about the radar system.

Figure 9.8 illustrates the results for case (i) which is the most
robust to the misalignment as discussed in the previous section. The
diagonal elements, c and e, of the third target have been varied so that
smaller MSEs are achieved. c and e turn out to be 3.2 and -1, re-
spectively, for minimum MSE. Comparing Figs. 9.8 and 9.12 indicates
that the performance of target set (v) is slightly better than set (i).
However, unlike case (v), set (i) provides a unique solution. Also the
first two targets are roll-invariant, and only the third target needs a
special treatment in alignment. The disadvantage for target set (i) is
that the second and third targets are not reciprocal reflectors which
may be difficult to make.

Figures 9.9 and 9.10 show that the MSEs for cross-talk are equal
to the power of noise. However, the MSEs for channel imbalance are 6
dB and 9 dB higher than the power of noise for case (ii) and case (iii),
respectively. Comparing Fig. 9.9 and Fig. 9.11 shows that case (iv) is
3 dB better in cross-talk than case (ii), and about the same in channel
imbalance.

Figure 9.13 shows the MSEs for case (vi). The results indicate that
the targets used in case (vi) are not good for polarimetric calibration
purposes.

Figure 9.14 illustrates the approach of the three PARCs [10]. In
evaluating the MSEs for this set of targets, they are transformed into
(24) in section 9.4.a, according to (4) with the transformation matrices
A and B given by

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{88a}$$

$$B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \tag{88b}$$

indicating that the polarization bases of these two polarization chan-
nels in the transmitting port are switched, whereas those of receiving
port remain unchanged. The performance is shown to be almost iden-
tical to the three-dipole case by comparing Fig. 9.9 with 9.14.

Overall, target set (v) gives the best performance. This is due to
the fact that Z_{11} and Z_{22}, supposed to be pure noises in case (v), are not
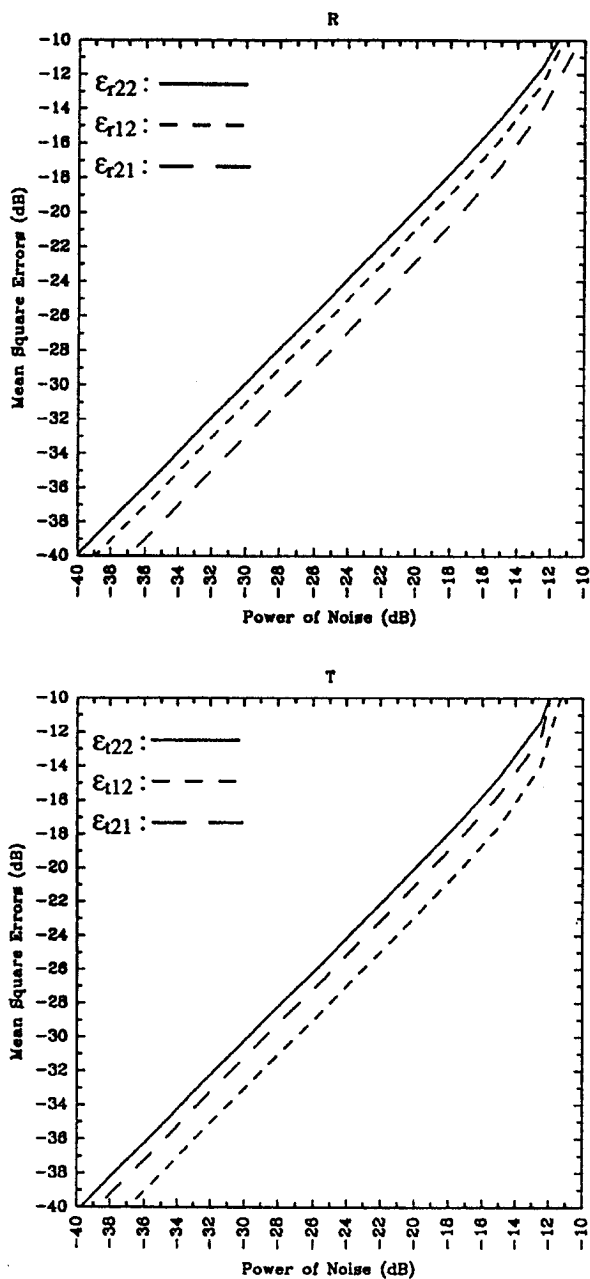used to evaluate the solutions. In contrast, using Z_{11} and Z_{22} degrades

Figure 9.8 Effect of noise for target set (i). (a) MSE for r_{22}, r_{12}, and r_{21} (b) MSE for t_{22}, t_{12}, and t_{21}.
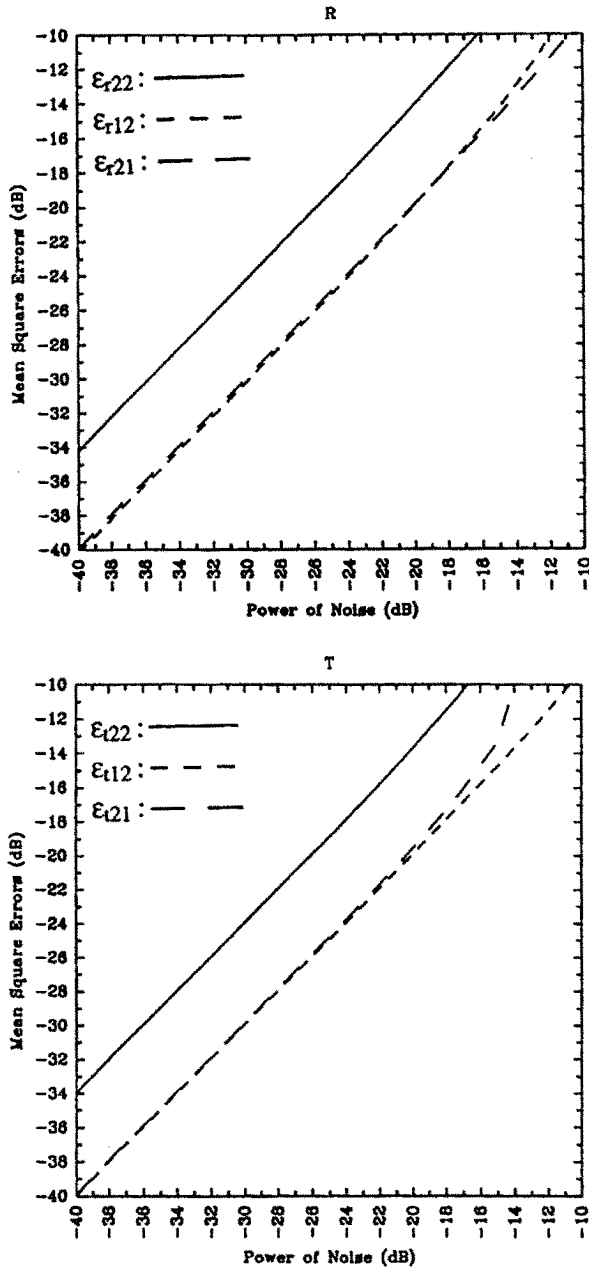
Figure 9.9 Effect of noise for target set (ii). (a) MSE for r_{22}, r_{12}, and r_{21} (b) MSE for t_{22}, t_{12}, and t_{21}.
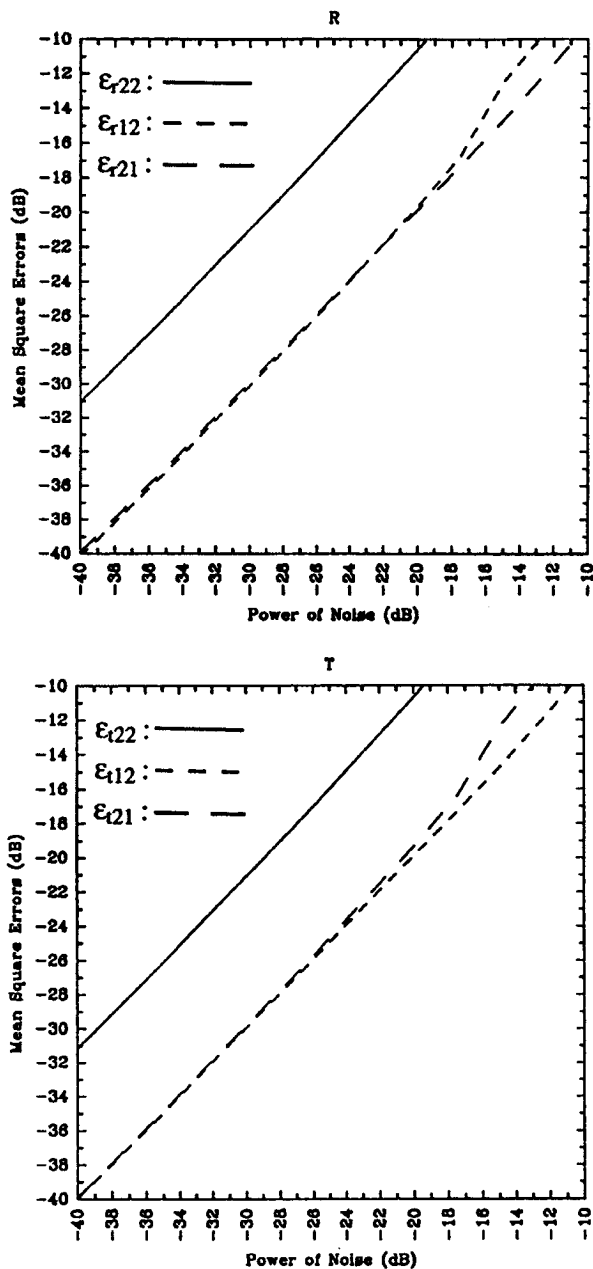
Figure 9.10 Effect of noise for target set (iii). (a) MSE for r_{22}, r_{12}, and r_{21} (b) MSE for t_{22}, t_{12}, and t_{21}.
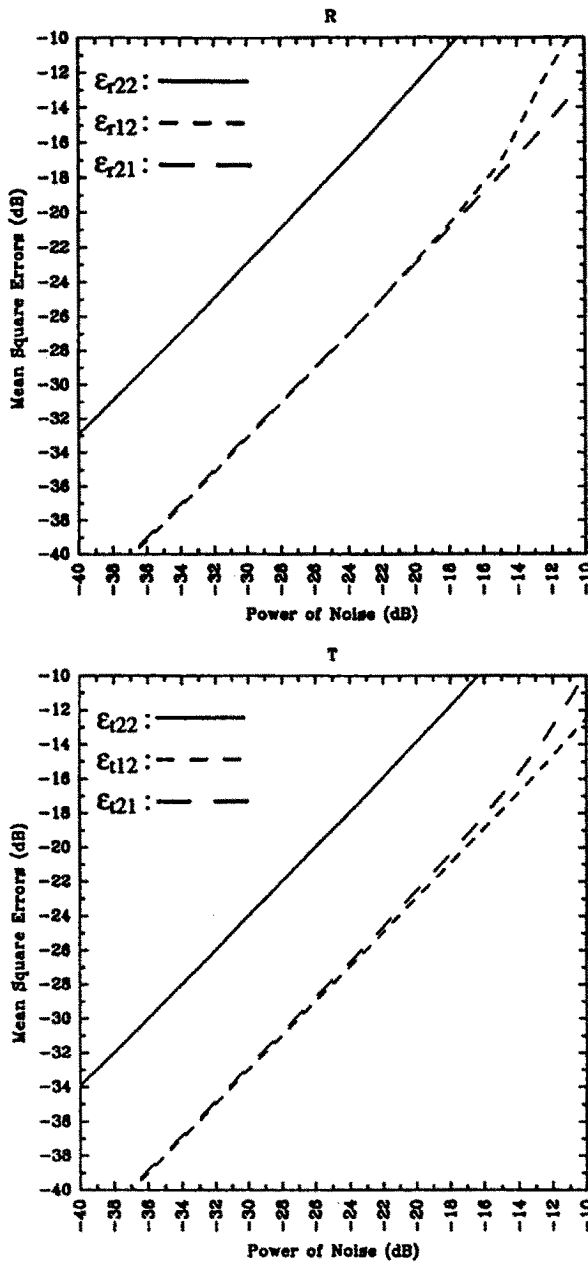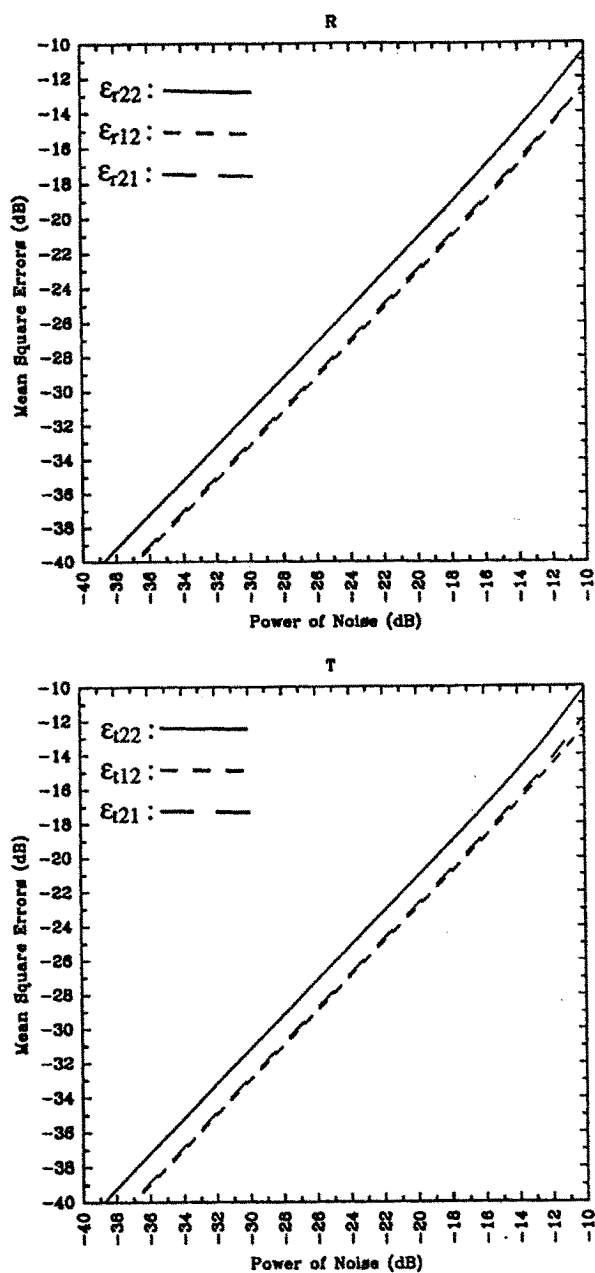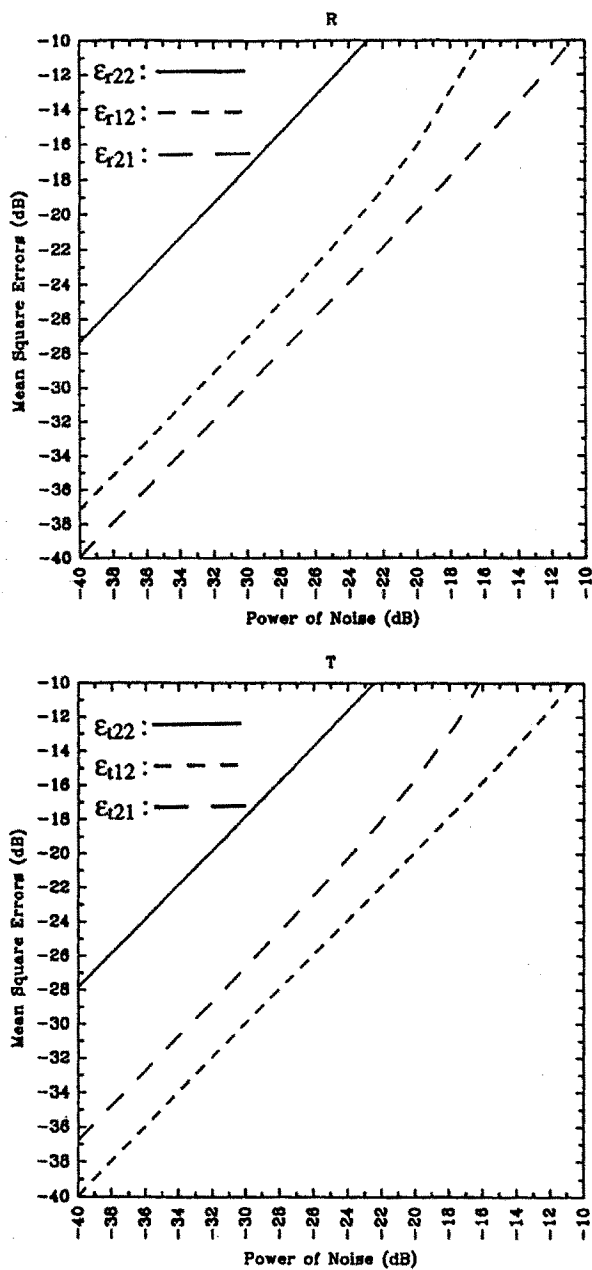
Figure 9.11 Effect of noise for target set (iv). (a) MSE **for** r_{22}, r_{12}, **and** r_{21} **(b)** MSE **for** t_{22}, t_{12}, **and** t_{21}.

Figure 9.12 Effect of noise for target set (v). (a) MSE for r_{22}, r_{12}, and r_{21} (b) MSE for t_{22}, t_{12}, and t_{21}.
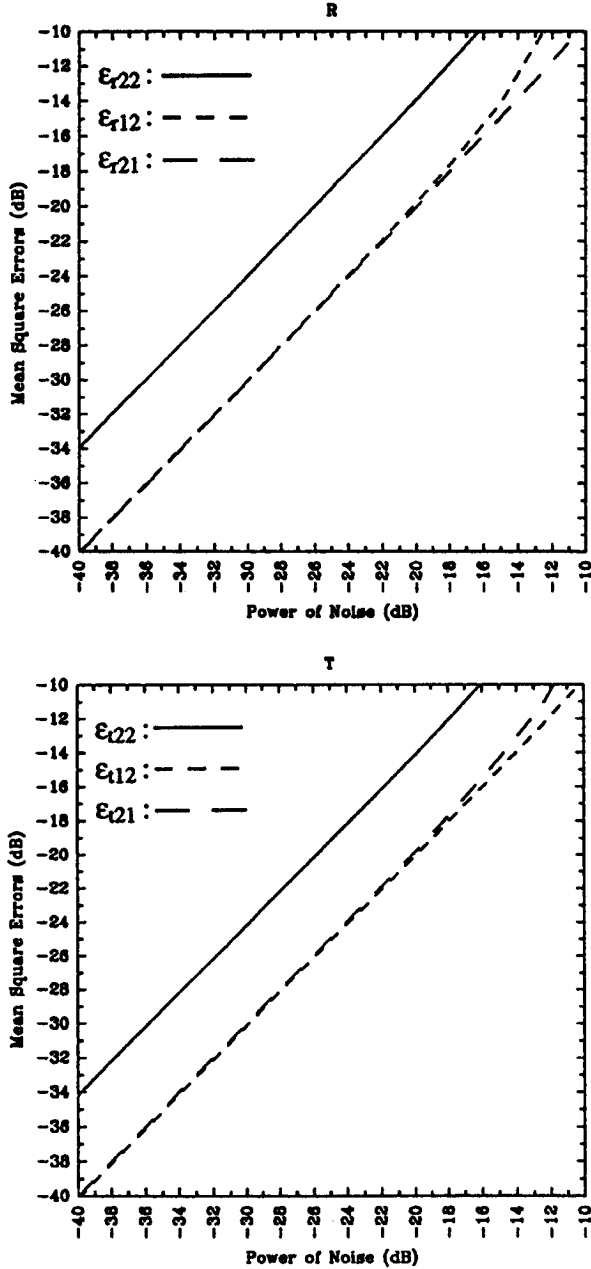
Figure 9.13 Effect of noise for target set (vi). (a) MSE **for** r_{22}, r_{12}, **and** r_{21} **(b)** MSE **for** t_{22}, t_{12}, **and** t_{21}.

Figure 9.14 Effect of noise for three PARCs [10]. (a) MSE for r_{22}, r_{12}, and r_{21} (b) MSE for t_{22}, t_{12}, and t_{21}.

the MSE for channel imbalance as shown in Fig. 9.11 for target set (iv) which offers similar MSEs for cross-talk (3 dB lower than the noise) as compared with Fig. 9.12.

9.7   Polarimetric Compensation

In this section, the explicit polarimetric compensation formula is given. Once radar returns from the three calibration targets have been measured, the polarimetric radar can be calibrated so that only the absolute phase is undetermined. Assuming the measured polarization matrix $[V_{ij}]$ of a target with scattering matrix S is given as

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} = \exp(i\phi) \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} S_{hh} & S_{hv} \\ S_{vh} & S_{vv} \end{bmatrix} \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (89)$$

it can be shown that the calibrated scattering matrix elements are given by

$$\begin{bmatrix} S_{hh} \\ S_{hv} \\ S_{vh} \\ S_{vv} \end{bmatrix} = \frac{\exp(-i\phi)}{R_{11}T_{11}(r_{22} - r_{12}r_{21})(t_{22} - t_{12}t_{21})}$$

$$\times \begin{bmatrix} r_{22}t_{22} & -r_{22}t_{21} & -r_{12}t_{22} & r_{12}t_{21} \\ -r_{22}t_{12} & r_{22} & r_{12}t_{12} & -r_{12} \\ -r_{21}t_{22} & r_{21}t_{21} & t_{22} & -t_{21} \\ r_{21}t_{12} & -r_{21} & -t_{12} & 1 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \\ V_{21} \\ V_{22} \end{bmatrix} \quad (90)$$

where r_{12}, r_{21}, r_{22}, t_{12}, t_{21}, and t_{22} have been obtained, in terms of the measurements of the calibration targets, in sections 9.2 and 9.3.

The calibration of absolute radar cross section can be carried out by using (1) to solve for the absolute magnitude of the transfer matrices

$$|R_{11}T_{11}| = \frac{|X_{11}|}{|S_{1hh} + (t_{21} + r_{12})S_{1hv} + r_{12}t_{21}S_{1vv}|} \quad (91a)$$

$$= \frac{|X_{22}|}{|r_{21}t_{12}S_{1hh} + (r_{21}t_{22} + r_{22}t_{12})S_{1hv} + r_{22}t_{22}S_{1vv}|} \quad (91b)$$

$$= \frac{|X_{12}|}{|t_{12}S_{1hh} + (t_{22} + r_{12}t_{12})S_{1hv} + r_{12}t_{22}S_{1vv}|} \quad (91c)$$

$$= \frac{|X_{21}|}{|r_{21}S_{1hh} + (r_{21}t_{21} + r_{22})S_{1hv} + r_{22}t_{21}S_{1vv}|} \quad (91d)$$

Equations (91a) and (91b) are useful when the calibration target has large like-like polarized returns HH and VV. Equations (91c) and (91d) are useful when the calibration target can significantly depolarize the radar signal.

9.8   Summary

Polarimetric calibration algorithms have been found for a general choice of three calibration targets. The algorithm presented is not limited to any particular polarization basis although the examples are given on the linear polarization basis. A method of **Polarization-basis Transformation** technique (PT) is employed to convert the scattering matrices into one of six canonical cases. The uniqueness problem for those six canonical cases is studied in detail. If there are multiple solutions for a given target choice, the relation between these solutions is clearly identified. The solution is presented for three canonical cases which correspond to the general cases that at least two of the scattering matrices of three calibration targets can be simultaneously diagonalized.

Based on the analysis on the effects of noise and misalignment for some typical calibration targets, target sets (i) and (v) show superior performance. However, the implementation of the second and third reflectors in target set (i) needs to be further investigated. Also it should be noted that target set (v) corresponding to case B.4 gives four solutions. Two of these solutions can be rejected if we know that the cross-polarization couplings are not greater than unity. The other two solutions give the same magnitude for the channel imbalance but with a plus or minus sign difference. Extra information is needed to select the correct solution for target set (v) [9]. The evaluation of PARCs [10] shown to be equivalent to the three-dipole approach indicates that PARC having a dipole-like scattering matrix can be used to replace the more complicated design used [10]. The analysis of misalignment also indicates that a possible self-calibration procedure could be used to remove the effects of misalignment based on the distance measure $D(M)$.

Acknowledgments

Appendix A

Δ_1, Δ_{1t}, and Δ_{1r} are given as

$$\Delta_1 = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{1t} = \begin{vmatrix} q_1 & p_{12} & p_{13} \\ q_2 & p_{22} & p_{23} \\ q_3 & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{1r} = \begin{vmatrix} p_{11} & q_1 & p_{13} \\ p_{21} & q_2 & p_{23} \\ p_{31} & q_3 & p_{33} \end{vmatrix} \tag{A1}$$

where

$$p_{11} = X_{11}(Y_{22}Z_{11} - Y_{21}Z_{12})$$
$$p_{12} = Y_{12}(X_{12}Z_{11} - X_{11}Z_{12})$$
$$p_{13} = Y_{12}(Y_{22}Z_{11} - Y_{21}Z_{12})$$
$$p_{21} = Y_{21}(X_{21}Z_{11} - X_{11}Z_{21})$$
$$p_{22} = X_{11}(Y_{22}Z_{11} - Y_{12}Z_{21})$$
$$p_{23} = Y_{21}(Y_{22}Z_{11} - Y_{12}Z_{21})$$
$$p_{31} = X_{21}Y_{22}Z_{11} - X_{11}Y_{21}Z_{22}$$
$$p_{32} = X_{12}Y_{22}Z_{11} - X_{11}Y_{12}Z_{22}$$
$$p_{33} = Y_{22}^2 Z_{11} - Y_{12}Y_{21}Z_{22}$$

and

$$q_1 = X_{11}(X_{11}Z_{12} - X_{12}Z_{11})$$
$$q_2 = X_{11}(X_{11}Z_{21} - X_{21}Z_{11})$$
$$q_3 = X_{11}^2 Z_{22} - X_{21}X_{12}Z_{11}$$

Appendix B

Δ_2, Δ_{2t}, and Δ_{2r} are given as

$$\Delta_2 = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{2t} = \begin{vmatrix} p_{11} & q_1 & p_{13} \\ p_{21} & q_2 & p_{23} \\ p_{31} & q_3 & p_{33} \end{vmatrix}$$

$$\Delta_{2r} = \begin{vmatrix} q_1 & p_{12} & p_{13} \\ q_2 & p_{22} & p_{23} \\ q_3 & p_{32} & p_{33} \end{vmatrix} \qquad \text{(B1)}$$

where

$$p_{11} = Y_{22}(X_{21}Z_{12} - X_{11}Z_{22})$$
$$p_{12} = X_{12}(Y_{22}Z_{12} - Y_{12}Z_{22})$$
$$p_{13} = X_{12}(X_{21}Z_{12} - X_{12}Z_{22})$$
$$p_{21} = X_{21}(Y_{22}Z_{21} - Y_{21}Z_{22})$$
$$p_{22} = Y_{22}(X_{12}Z_{21} - X_{11}Z_{22})$$
$$p_{23} = X_{21}(X_{12}Z_{21} - X_{11}Z_{22})$$
$$p_{31} = X_{21}Y_{22}Z_{11} - X_{11}Y_{21}Z_{22}$$
$$p_{32} = X_{12}Y_{22}Z_{11} - X_{11}Y_{12}Z_{22}$$
$$p_{33} = X_{12}X_{21}Z_{11} - X_{11}^2 Z_{22}$$

and

$$q_1 = Y_{22}(Y_{22}Z_{12} - Y_{12}Z_{22})$$
$$q_2 = Y_{22}(Y_{22}Z_{21} - Y_{21}Z_{22})$$
$$q_3 = Y_{22}^2 Z_{11} - Y_{21}Y_{12}Z_{22}$$

Appendix C

Δ_3, Δ_{3t}, and Δ_{3r} are given as

$$\Delta_3 = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{3t} = \begin{vmatrix} q_1 & p_{12} & p_{13} \\ q_2 & p_{22} & p_{23} \\ q_3 & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{3r} = \begin{vmatrix} p_{11} & p_{12} & q_1 \\ p_{21} & p_{22} & q_2 \\ p_{31} & p_{32} & q_3 \end{vmatrix} \tag{C1}$$

where

$$p_{11} = X_{11}(X_{11}Z_{12} - X_{12}Z_{11})$$
$$p_{12} = Y_{12}(X_{11}Z_{12} - X_{12}Z_{11})$$
$$p_{13} = Y_{12}(Y_{21}Z_{12} - Y_{22}Z_{11})$$
$$p_{21} = X_{11}(X_{21}Z_{12} - X_{12}Z_{21})$$
$$p_{22} = X_{11}Y_{22}Z_{12} - X_{12}Y_{12}Z_{21}$$
$$p_{23} = Y_{22}(Y_{21}Z_{12} - Y_{12}Z_{21})$$
$$p_{31} = X_{12}(X_{21}Z_{12} - X_{11}Z_{22})$$
$$p_{32} = X_{12}(Y_{22}Z_{12} - Y_{12}Z_{22})$$
$$p_{33} = Y_{22}(Y_{22}Z_{12} - Y_{12}Z_{22})$$

and

$$q_1 = X_{11}(Y_{22}Z_{11} - Y_{21}Z_{12})$$
$$q_2 = X_{11}Y_{22}Z_{21} - X_{21}Y_{21}Z_{12}$$
$$q_3 = Y_{22}(X_{11}Z_{22} - X_{21}Z_{12})$$

Appendix D

Δ_4, Δ_{4t}, and Δ_{4r} are given as

$$\Delta_4 = \begin{vmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{4t} = \begin{vmatrix} q_1 & p_{12} & p_{13} \\ q_2 & p_{22} & p_{23} \\ q_3 & p_{32} & p_{33} \end{vmatrix}$$

$$\Delta_{4r} = \begin{vmatrix} p_{11} & p_{12} & q_1 \\ p_{21} & p_{22} & q_2 \\ p_{31} & p_{32} & q_3 \end{vmatrix} \tag{D1}$$

where

$$p_{11} = X_{11}(X_{11}Z_{21} - X_{21}Z_{11})$$
$$p_{12} = Y_{21}(X_{11}Z_{21} - X_{21}Z_{11})$$
$$p_{13} = Y_{21}(Y_{12}Z_{21} - Y_{22}Z_{11})$$
$$p_{21} = X_{11}(X_{12}Z_{21} - X_{21}Z_{12})$$
$$p_{22} = X_{11}Y_{22}Z_{21} - X_{21}Y_{21}Z_{12}$$
$$p_{23} = Y_{22}(Y_{12}Z_{21} - Y_{21}Z_{12})$$
$$p_{31} = X_{21}(X_{12}Z_{21} - X_{11}Z_{22})$$
$$p_{32} = X_{21}(Y_{22}Z_{21} - Y_{21}Z_{22})$$
$$p_{33} = Y_{22}(Y_{22}Z_{21} - Y_{21}Z_{22})$$

and

$$q_1 = X_{11}(Y_{22}Z_{11} - Y_{12}Z_{21})$$
$$q_2 = X_{11}Y_{22}Z_{12} - X_{12}Y_{12}Z_{21}$$
$$q_3 = Y_{22}(X_{11}Z_{22} - X_{12}Z_{21})$$

Appendix E

Δ and Δ_{rt} are give as

$$\Delta = p_1 r_1 + p_2 r_2 + p_3 r_3$$
$$\Delta_{rt} = -(p_1 q_1 + p_2 q_2 + p_3 q_3)$$

$$(E1)$$

where

$$p_1 = Z_{11}(X_{22}Y_{11} - X_{11}Y_{22}) - Z_{21}(X_{12}Y_{11} - X_{11}Y_{12})$$
$$- Z_{12}(X_{21}Y_{11} - X_{11}Y_{21})$$
$$p_2 = Z_{11}(X_{12}Y_{22} - X_{22}Y_{12}) + Z_{12}(X_{21}Y_{12} - X_{12}Y_{21})$$
$$+ Z_{22}(X_{12}Y_{11} - X_{11}Y_{12})$$
$$p_3 = Z_{11}(X_{21}Y_{22} - X_{22}Y_{21}) + Z_{21}(X_{12}Y_{21} - X_{21}Y_{12})$$
$$+ Z_{22}(X_{21}Y_{11} - X_{11}Y_{21})$$
$$q_1 = (1 + \lambda)cX_{11}Y_{11}Z_{22} - cZ_{11}(\lambda X_{22}Y_{11} + X_{11}Y_{22})$$
$$- eZ_{11}(X_{22}Y_{11} - X_{11}Y_{22})$$
$$q_2 = (1 + \lambda)cX_{11}Y_{11}Z_{21} - cZ_{11}(\lambda X_{21}Y_{11} + X_{11}Y_{21})$$
$$- eZ_{11}(X_{21}Y_{11} - X_{11}Y_{21})$$
$$q_3 = (1 + \lambda)cX_{11}Y_{11}Z_{12} - cZ_{11}(\lambda X_{12}Y_{11} + X_{11}Y_{12})$$
$$- eZ_{11}(X_{12}Y_{11} - X_{11}Y_{12})$$
$$r_1 = (1 + \lambda)eX_{11}Y_{11}Z_{22} - c\lambda Z_{11}(X_{22}Y_{11} - X_{11}Y_{22})$$
$$- eZ_{11}(X_{22}Y_{11} + \lambda X_{11}Y_{22})$$
$$r_2 = (1 + \lambda)eX_{11}Y_{11}Z_{21} - c\lambda Z_{11}(X_{21}Y_{11} - X_{11}Y_{21})$$
$$- eZ_{11}(X_{21}Y_{11} + \lambda X_{11}Y_{21})$$
$$r_3 = (1 + \lambda)eX_{11}Y_{11}Z_{12} - c\lambda Z_{11}(X_{12}Y_{11} - X_{11}Y_{12})$$
$$- eZ_{11}(X_{12}Y_{11} + \lambda X_{11}Y_{12})$$

Appendix F

Δ_5, Δ_{5t}, Δ_{5r} Δ_6, Δ_{6t}, and Δ_{6r} are given as

$$
\begin{aligned}
\Delta_5 =& (Z_{11} - Z_{12}y'_{21})(Z_{11} - Z_{21}y'_{12}) \\
& - y'_{12}(Z_{11}t_{12} - Z_{12})y'_{21}(Z_{11}r_{21} - Z_{21}) \\
\Delta_{5t} =& (Z_{11} - Z_{21}y'_{12})[\frac{c}{d_1}(Z_{12} - Z_{11}t_{12}) \\
& + \frac{e}{d_1}(r_{12}t_{21}Z_{12} - r_{12}t_{22}Z_{11}] \\
& - y'_{12}(t_{12}Z_{11} - Z_{12})[\frac{c}{d_1}(Z_{21} - r_{21}Z_{11}) \\
& + \frac{e}{d_1}(r_{12}t_{21}Z_{21} - r_{22}t_{21}Z_{11}] \\
\Delta_{5r} =& (Z_{11} - Z_{12}y'_{21})[\frac{c}{d_2}(Z_{21} - Z_{11}r_{21}) \\
& + \frac{e}{d_2}(r_{12}t_{21}Z_{21} - r_{22}t_{21}Z_{11})] \\
& - y'_{21}(r_{21}Z_{11} - Z_{21})[\frac{c}{d_2}(Z_{12} - t_{12}Z_{11}) \\
& + \frac{e}{d_2}(r_{12}t_{21}Z_{12} - r_{12}t_{22}Z_{11})]
\end{aligned}
\tag{F1}
$$

$$
\begin{aligned}
\Delta_6 =& (Z_{21} - Z_{12}y'_{21}r_{21})(Z_{12} - Z_{22}y'_{12})t_{12} \\
& - (Z_{22} - Z_{12}r_{21})(Z_{12} - Z_{21}y'_{12}t_{12}) \\
\Delta_{6t} =& (Z_{12} - Z_{21}t_{12}y'_{12})[\frac{c}{d_1}(Z_{22}t_{12} - Z_{12}r_{21}t_{12}) \\
& + \frac{e}{d_1}(Z_{22}r_{12}t_{22} - Z_{12}r_{22}t_{22})] \\
& - t_{12}(Z_{12} - Z_{22}y'_{12})[\frac{c}{d_1}(Z_{21}t_{12} - Z_{12}r_{21}) \\
& + \frac{e}{d_1}(r_{12}t_{22}Z_{21} - r_{22}t_{21}Z_{12})] \\
\Delta_{6r} =& (Z_{21} - Z_{12}r_{21}y'_{21})[\frac{c}{d_2}(Z_{22}t_{12} - Z_{12}r_{21}t_{12}) \\
& + \frac{e}{d_2}(Z_{22}r_{12}t_{22} - Z_{12}r_{22}t_{22})] \\
& - (Z_{22} - Z_{12}r_{21})[\frac{c}{d_2}(Z_{21}t_{12} - Z_{12}r_{21}) \\
& + \frac{e}{d_2}(r_{12}t_{22}Z_{21} - r_{22}t_{21}Z_{12})]
\end{aligned}
\tag{F2}
$$

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