Progress In Electromagnetics Research, PIER 8, 89–108, 1994

THE STUDY OF EFFECTIVE MEDIUM PARAMETERS FOR GRANULAR MEDIA

B. Jia, W. Lin, and S. Liu

1. Introduction

Granular media are random common media with strong fluctuation. They are composed of independent particles. The dielectric characteristic of the media is homogeneous within every particle, though it may be very different among them. The position and orientation of a particle in these media are completely stochastic. In nature, many materials can be divided into the category of granular media, such as soil, rock, grain, snow, forest, crops, etc. If the concept of the particle is extended to the molecule, nearly all materials belong in the category of granular media. Therefore, it is imperative to study the effective medium parameters of granular media for the benefit of industrial and agricultural production as well as national defense construction.

Effective medium parameters for granular media have been investigated by many researchers [1]. Although many formulas have been proposed, previous study is still limited to the same kind of particle, i.e., the media consists of either dielectric or magnetic particles. Recently, with the development of ferrite absorbed materials for stealth, more and more attention is being attracted to study the mixture of dielectric and magnetic particles [2]. We have presented a new formula to calculate the permittivity and permeability of a mixture of spherically shaped particles [3, 4]. In this paper, we will discuss the calculation method for a mixture with nonspherical particles. The influence of the particle shape and orientation distribution on the permittivity and permeability of the mixture are extensively discussed. As a result, we not only discover the optimum shape of the absorbent particles embedded in antiradar coating materials, but we also derive a formula for the effective permittivity and permeability for multi-shaped granular media. Experimental results show good agreement with this theory.

2. The General Theory of Effective Constitutive Tensor

The constitutive tensor is the basic physical quantity used to characterize a medium in electromagnetic field theory. The constitutive relation for general uniaxial media at random may have the form of

$$\left\|\overline{\overline{C}}\right\| = \begin{bmatrix} \overline{\overline{\varepsilon}}(\overline{x}) & 0 \\ 0 & \overline{\overline{\mu}}(\overline{x}) \end{bmatrix} \tag{1}$$

Here $\overline{\overline{\varepsilon}}(\overline{x})$ and $\overline{\overline{\mu}}(\overline{x})$ are the 3×3 tensors of permittivity and permeability, respectively. In general, both $\overline{\overline{\varepsilon}}(\overline{x})$ and $\overline{\overline{\mu}}(\overline{x})$ are stochastic functions of the space. Based on strong fluctuation theory, the effective constitutive tensor of a random medium is calculated by [5, 6]

$$\left\|\overline{\overline{C}}_{\text{eff}}\right\| = \left\|\overline{\overline{C}}_g\right\| + j\omega \int_{\Omega} \left\|\overline{\overline{R}}(\overline{r})\right\| \cdot d\overline{r} \tag{2}$$

where ω is the angle frequency of the electromagnetic wave, Ω is the region occupied by the medium, and $\left\|\overline{\overline{C}}_g\right\|$ is the quasistatic constitutive tensor of the random medium. It is the solution of

$$\langle \|\overline{\overline{\tau}}(\overline{x})\| \rangle = 0 \tag{3}$$

Here the angle bracket $\langle\ \rangle$ stands for ensemble average. $\left\|\overline{\overline{\tau}}(\overline{x})\right\|$ is an integral kernel of the scattering field in random media [4].

$$\left\|\overline{\overline{\tau}}(\overline{x})\right\| = \left[\left\|\overline{\overline{C}}(\overline{x})\right\| - \left\|\overline{\overline{C}}_g\right\|\right] \cdot \left\{\left\|\overline{\overline{E}}\right\| + j\omega \left\|\overline{\overline{S}}\right\| \cdot \left[\left\|\overline{\overline{C}}(\overline{x})\right| - \left\|\overline{\overline{C}}_g\right\|\right]\right\}^{-1} \tag{4}$$

where $\left\|\overline{\overline{E}}\right\|$ stands for a 6×6 unit tensor,

$$\left\|\overline{\overline{E}}\right\| = \begin{bmatrix} \overline{\overline{I}} & 0 \\ 0 & \overline{\overline{I}} \end{bmatrix} \tag{5}$$

$\left\|\overline{\overline{S}}\right\|$ stands for the singular part of the 6×6 Green's tensor,

$$\left\|\overline{\overline{S}}\right\| = \frac{1}{j\omega} \left\|\overline{\overline{C}}_g\right\|^{-1} \begin{bmatrix} \overline{\overline{S}}_e & 0 \\ 0 & \overline{\overline{S}}_m \end{bmatrix} \tag{6}$$

$\overline{\overline{S}}_e$ and $\overline{\overline{S}}_m$ are the singular terms corresponding to the electric and magnetic dyadic Green's functions, respectively. They are evaluated by [5]

$$\left\|\overline{\overline{S}}_e\right\| = \frac{-\overline{\overline{\varepsilon}}_g \cdot \overline{\overline{\mu}}_g}{4\pi \left(\det \left|\overline{\overline{\mu}}_g \cdot \overline{\overline{\varepsilon}}_g\right|\right)^{1/2}} \int_{\delta v} \nabla \left[\frac{1}{\overline{r} \cdot (\overline{\overline{\mu}}_g \cdot \overline{\overline{\varepsilon}}_g)^{-1} \cdot \overline{r}}\right]^{1/2} \widehat{n}\, ds \tag{7a}$$

$$\left\|\overline{\overline{S}}_m\right\| = \frac{-\overline{\overline{\mu}}_g \cdot \overline{\overline{\varepsilon}}_g}{4\pi \left(\det \left|\overline{\overline{\varepsilon}}_g \cdot \overline{\overline{\mu}}_g\right|\right)^{1/2}} \int_{\delta v} \nabla \left[\frac{1}{\overline{r} \cdot (\overline{\overline{\varepsilon}}_g \cdot \overline{\overline{\mu}}_g)^{-1} \cdot \overline{r}}\right]^{1/2} \widehat{n}\, ds \tag{7b}$$

where det means determinant and $\overline{r}$ is a vector from source point to field point.

$$\overline{r} = \overline{x}' - \overline{x}$$

δv is the boundary of a volume whose shape is that of a limiting equicorrelation surface and $\widehat{n}$ is a unit vector to the surface element with area ds.

In the case of isotropic medium, $\left\|\overline{\overline{S}}_e\right\|$ and $\left\|\overline{\overline{S}}_m\right\|$ reduce to the simplest form and

$$\left\|\overline{\overline{S}}_e\right\| = \left\|\overline{\overline{S}}_m\right\| = \frac{-1}{4\pi}\int_{\delta v}\frac{\widehat{a}_r\,\widehat{n}}{r^2}\,ds \tag{8}$$

If we know the distribution function of $\|C(\overline{x})\|$ and the shape of the limiting equicorrelation surface, we will find the solution of $\left\|\overline{\overline{C}}_g\right\|$ by substituting (4) and (7) into (3).

In (2), $\|\overline{R}(\overline{x})\|$ is the correlative matrix, which stands for the correlative characteristic of the medium [6]

$$\left\|\overline{\overline{R}}(\overline{r})\right\| = \left\langle \|\overline{\overline{r}}(\overline{x})\| \cdot P.V.\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\| \cdot \|(\overline{\overline{r}}(\overline{x}')\| \right\rangle \tag{9}$$

where $P.V.\left\|\overline{\overline{G}}_g(\overline{x}\cdot\overline{x}')\right\|$ is the principal value of the matrix dyadic Green's function. $\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\|$ can be written in a general form [7]

$$\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\| = \begin{bmatrix} \overline{\overline{G}}_e(\overline{x},\overline{x}') & -\frac{\overline{\overline{\varepsilon}}_g^{-1}}{j\omega\varepsilon_0}\cdot\nabla\times\overline{\overline{G}}_m(\overline{x},\overline{x}') \\ -\frac{\overline{\overline{\mu}}_g^{-1}}{j\omega\mu_0}\cdot\nabla\times\overline{\overline{G}}_e(\overline{x},\overline{x}') & \overline{\overline{G}}_m(\overline{x},\overline{x}') \end{bmatrix} \tag{10}$$

Here $\overline{\overline{G}}_e(\overline{x},\overline{x}')$ and $\overline{\overline{G}}_m(\overline{x},\overline{x}')$ are the 3×3 dyadic Green's tensors of electric and magnetic types. $\overline{\overline{G}}_e(\overline{x},\overline{x}')$ and $\overline{\overline{G}}_m(\overline{x},\overline{x}')$ are the solutions of second order dyadic differential equations with some boundary condition.

$$\left[\left(\nabla\times\overline{\overline{\mu}}_g^{-1}\right)\cdot\left(\nabla\times\overline{\overline{I}} - k_0^2\cdot\overline{\overline{\varepsilon}}_g\right)\right]\cdot\overline{\overline{G}}_e(\overline{x},\overline{x}') = j\omega\mu_0\overline{\overline{I}}\delta(\overline{x}-\overline{x}') \tag{11a}$$

$$\left[\left(\nabla\times\overline{\overline{\varepsilon}}_g^{-1}\right)\cdot\left(\nabla\times\overline{\overline{I}} - k_0^2\cdot\overline{\overline{\mu}}_g\right)\right]\cdot\overline{\overline{G}}_m(\overline{x},\overline{x}') = j\omega\varepsilon_0\overline{\overline{I}}\delta(\overline{x}-\overline{x}') \tag{11b}$$

where $k_0^2 = \omega^2\mu_0\varepsilon_0$.

The evaluation method of (11) with some boundary condition has been studied extensively by many authors [7,8]. The solution of

$\overline{\overline{G}}_e(\overline{x},\overline{x}')$ and $\overline{\overline{G}}_m(\overline{x},\overline{x}')$ can be found in the literature and textbooks [7–10]. To compare with the other case, the solution of $\overline{\overline{G}}_e(\overline{x},\overline{x}')$ and $\overline{\overline{G}}_m(\overline{x},\overline{x}')$ are the simplest in isotropic media with infinite space. $\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\|$ can be written as [11]

$$\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\| = \begin{bmatrix} j\omega\mu_g\overline{\overline{G}}_0(\overline{x},\overline{x}') & -\nabla\times\overline{\overline{G}}_0(\overline{x},\overline{x}') \\ \nabla\times\overline{\overline{G}}_0(\overline{x},\overline{x}') & j\omega\varepsilon_g\overline{\overline{G}}_0(\overline{x},\overline{x}') \end{bmatrix} \tag{12}$$

where

$$\overline{\overline{G}}_0(\overline{x},\overline{x}') = \left(\overline{\overline{I}} + \frac{1}{k_0^2\varepsilon_g\mu_g}\nabla\nabla\right)\frac{\exp\left(jk_0\sqrt{\varepsilon_g\mu_g}|\overline{x}-\overline{x}'|\right)}{|\overline{x}-\overline{x}'|} \tag{13}$$

If the volume of the random media is large enough, the correlative matrix of the random media can be approximately evaluated by substituting (12) into (9).

In the calculation of the correlative matrix, we must pay attention to some special points:

1. At point $|\overline{r}| = 0$, because $P.V.\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right\|$ equals the zero matrix, the correlative matrix must be zero.
2. In the region far from correlation, the medium properties of random media are not correlative with each other, so the ensemble average of (9) is evaluated separately.

$$\left\|\overline{\overline{R}}(\overline{r})\right\| = \langle\|\overline{\overline{\tau}}(\overline{x})\|\rangle \cdot P.V.\left\|\overline{\overline{G}}_g(\overline{x},\overline{x}')\right| \cdot \langle\|\overline{\overline{\tau}}(\overline{x}')\|\rangle \tag{14}$$

Substituting (3) into (14), the correlative matrix is also zero.

In the region of correlation, $\left\|\overline{\overline{R}}(\overline{r})\right\|$ is closely related to the correlative property of the random medium and the wavelength of incident electromagnetic waves. If the length of the correlation region is much smaller than the wavelength of the incident wave, $\left\|\overline{\overline{R}}(\overline{r})\right\|$ will be smaller than $\left\|\overline{\overline{C}}_g\right\|$. Therefore,

$$\|C_{\text{eff}}\| \approx \left\|\overline{\overline{C}}_g\right\| \tag{15}$$

Usually, (15) will be held with $1/\lambda \geq 10$, approximately. We call them long wavelength approximations.

3. The Effective Constitutive Tensor of Granular Media

The above discussion is a general theory for strong fluctuation random media. Now, we consider the specific case of granular media. Suppose the media is composed of N kinds of constituents with different shapes and medium properties. The constitutive tensor of the granular medium is [4, 5]

$$\left\|\overline{\overline{C}}(\overline{x})\right\| = \sum_{i=1}^{N} \left\|\overline{\overline{C}}_i\right\| \overline{U}_i(\overline{x}) \tag{16}$$

where $\left\|\overline{\overline{C}}_i\right\|$ is the constitutive tensor of the ith constituent in the medium. $\overline{U}_i(\overline{x})$ is a characteristic function the position of the ith constituent.

$$\overline{U}_i(\overline{x}) = \begin{cases} 1\ , & \text{within the } i\text{th constituent} \\ 0\ , & \text{otherwise} \end{cases} \tag{17}$$

Utilizing the property of orthogonality and normalization of $\overline{U}_i(\overline{x})$, $\|\overline{\overline{\tau}}(\overline{x})\|$ can be written in the form [12]

$$\|\overline{\overline{\tau}}(\overline{x})\| = \sum_{i=1}^{N} \|\overline{\overline{\tau}}_i\| \cdot \overline{U}_i(\overline{x}) \tag{18}$$

where

$$\|\overline{\overline{\tau}}_i\| = \left(\left\|\overline{\overline{C}}_i\right\| - \left\|\overline{\overline{C}}_g\right\|\right) \cdot \left[\|\overline{E}\| + j\omega \left\|\overline{\overline{S}}_i\right\| \cdot \left(\left\|\overline{\overline{C}}_i\right\| - \left\|\overline{\overline{C}}_g\right\|\right)\right]^{-1} \tag{19}$$

$\left\|\overline{\overline{S}}_i\right\|$ stands for the singular of the matrix dyadic Green's function in which the equicorrelation region corresponds to the particles of the ith constituent. If the orientation of the particles is stochastic, the effective property of the granular medium must be isotropic. $\left\|\overline{\overline{S}}_i\right\|$ is calculated by

$$\left\|\overline{\overline{S}}_i\right\| = \frac{1}{j\omega}\left\|\overline{\overline{C}}_g\right\|^{-1} \cdot \begin{bmatrix} \overline{\overline{L}}_i & 0 \\ 0 & \overline{\overline{L}}_i \end{bmatrix} \tag{20}$$

where $\overline{\overline{L}}_i$ is the polarization tensor of the i th constituent's particles.

$$\overline{\overline{L}}_i = -\frac{1}{4\pi}\int_{A_i} \frac{\widehat{a}_r\,\widehat{n}}{r^2}\,ds \tag{21}$$

Here the integral curvilinear surface A_i is the surface of the i th constituent's particle. Equation (21) shows that $\overline{\overline{L}}_i$ is only related to the particle shape.

Commonly, the particle surface of the i th constituent is quite irregular. If we calculate $\overline{\overline{L}}_i$ by the actual shape of the particle, the computative process will be very complex. Therefore, to calculate $\overline{\overline{L}}_i$, we usually choose an ellipsoid, which is similar to the shape of the i th constituent's particle, instead of the actual particle shape. When the axes of the ellipse coincide with the coordinate axes, the polarization tensor of the particle is a diagonal tensor. We call it the polarization tensor of the main coordinate.

$$\overline{\overline{L}}_{i0} = \begin{bmatrix} L_{ix} & 0 & 0 \\ 0 & L_{iy} & 0 \\ 0 & 0 & L_{iz} \end{bmatrix} \tag{22}$$

Here L_{ix}, L_{iy}, and L_{iz} are the polarization elements along x, y, and z of the particle. Table 1 is the main coordinate polarization tensor of some particle shapes, calculated by (21).

particle shape	polarization elements		
	L_{ix}	L_{iy}	L_{iz}
sphere	1/3	1/3	1/3
needle	1/2	1/2	0
dish	0	0	1

Table 1. The main coordinate polarization elements of some specific particle shapes.

If the axes of the ellipse do not coincide with the coordinate axes and the spatial angles between them are θ_i, φ_i, and ψ_i, respectively, the polarization tensor can be derived from $\overline{\overline{L}}_{i0}$.

$$\overline{\overline{L}}_i = \alpha_i \cdot \overline{\overline{L}}_{i0} \cdot \alpha_i^T \tag{23}$$

Here α_i indicates the coordinate transform matrix between the two axes [13]. α_i^T is the transpose of α_i. From (23), we know $\overline{\overline{L}}_i$ is only related to the spatial orientation of the particle. However, the characteristic function $\overline{U}_i(x)$ is only a function of geometric position. Generally, both spatial orientation and geometric position of a particle are random in granular media and the stochastic processes of spatial orientation and geometric position are independent of each other. Therefore, the ensemble average of $\|\overline{\overline{\tau}}\|$ can be separated. Substituting (18) into (3), the ensemble average of $\|\overline{\overline{\tau}}(x)\|$ equals

$$\left\langle \|\overline{\overline{\tau}}(\overline{x})\| \right\rangle = \sum_{i=1}^{N} \left\langle \overline{U}_i(\overline{x}) \right\rangle \left\langle \|\overline{\overline{\tau}}_i\| \right\rangle \tag{24}$$

Here $\langle \overline{U}_i(\overline{x}) \rangle$ only contains the average of geometric position and $\langle \|\overline{\overline{\tau}}_i\| \rangle$ contains the average of spatial orientation.

When the position distribution of the i th constituent particles is an ergodic process, the ensemble average of $\overline{U}_i(\overline{x})$ can be calculated by [12]

$$\left\langle \overline{U}_i(\overline{x}) \right\rangle = \frac{1}{V} \int_V \overline{U}_i(\overline{x})\, d\overline{x} \tag{25}$$

where V is the volume of granular media. Substituting (17) into (25),

$$\langle \overline{U}_i(\overline{x}) \rangle = v_i \tag{26}$$

Here v_i indicates the total volume ratio of the i th constituent to the whole granular media.

If the orientation distribution of the particles is also an ergodic process, that is, if the orientation probability of a particle in every direction is a constant, $\langle \|\overline{\overline{T}}_i\| \rangle$ can be computed by

$$\langle \|\overline{\overline{T}}_i\| \rangle = \frac{1}{8\pi^2} \int_0^{\pi} \int_0^{2\pi} \int_0^{2\pi} \|\overline{\overline{T}}_i\| \sin\theta_i \, d\varphi_i \, d\psi_i \, d\theta_i \tag{27}$$

Under this condition, the effective property of granular media can not be anisotropic. $\left\|\overline{\overline{C}}_g\right\|$ may be written in the form

$$\left\|\overline{\overline{C}}_g\right\| = \begin{bmatrix} \varepsilon_g \overline{\overline{I}} & 0 \\ 0 & \mu_g \overline{\overline{I}} \end{bmatrix} \tag{28}$$

Substituting (28) into (27), we get

$$\langle \|\overline{\overline{T}}_i\| \rangle = \begin{bmatrix} \dfrac{\varepsilon_g}{3} \displaystyle\sum_{j=1}^{3} \dfrac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + L_{ij}(\varepsilon_i - \varepsilon_g)} \overline{\overline{I}} & 0 \\ 0 & \dfrac{\mu_g}{3} \displaystyle\sum_{j=1}^{3} \dfrac{\mu_i - \mu_g}{\mu_g + L_{ij}(\mu_i - \mu_g)} \overline{\overline{I}} \end{bmatrix} \tag{29}$$

Here the isotropic property of the i th constituent is supposed. $j = 1, 2, 3,$ indicate the x, y, z axes of the particle, respectively.

Substituting (24) into (3), we obtain the calculating formulas of ε_g and μ_g for granular media,

$$\sum_{i=1}^{N} v_i \sum_{j=1}^{3} \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + L_{ij}(\varepsilon_i - \varepsilon_g)} \tag{30a}$$

$$\sum_{i=1}^{N} v_i \sum_{j=1}^{3} \frac{\mu_i - \mu_g}{\mu_g + L_{ij}(\mu_i - \mu_g)} \tag{30b}$$

Equation (30) is a general formula to predict the quasistatic effective parameters for multiphase granular media. These formulas are different from Polder-Van Santern [14]. Here, we consider background material as particles and their shape is related to the orientation of the inclusion particles. When the orientation was the ergodic process, the particle shape of background material is a sphere. Substituting polarization factors of the background material into (30), the formula of effective medium parameters is given by

$$9v_b \frac{\varepsilon_b - \varepsilon_g}{2\varepsilon_g + \varepsilon_b} + \sum_{i=1}^{N-1} v_i \sum_{j=1}^{3} \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + L_j\left(\varepsilon_i - \varepsilon_g\right)} = 0 \tag{31a}$$

$$9v_b \frac{\mu_b - \mu_g}{2\mu_g + \mu_b} + \sum_{i=1}^{N-1} v_i \sum_{j=1}^{3} \frac{\mu_i - \mu_g}{\mu_g + L_j\left(\mu_i - \mu_g\right)} = 0 \tag{31b}$$

where subscript b indicates background material. In the case of a two-phase dielectric mixture, (31) is more accurate than the Polder-Van Santern formula.

The formulas of (31) indicate that the permittivity and permeability of the constituents only affect the quasistatic effective permittivity and permeability of the granular media, respectively. The cross interaction of the medium parameters of the particles merely appears in the term of the correlative matrix in (2). By substituting (18) into (9) and utilizing the orthogonality of the characteristic function $U_i(\overline{x})$, the correlative matrix is obtained.

$$\left\|\overline{\overline{R}}(\overline{r})\right\| = \sum_{i=1}^{N} v_i \left\langle \|\overline{\overline{\tau}}_i\| \right\rangle \cdot \text{P.V.} \left\|\overline{\overline{G}}_g(\overline{r})\right\| \cdot \left\langle \|\overline{\overline{\tau}}_i\| \right\rangle R_i(\overline{r}) \tag{32}$$

Here $R_i(\overline{r})$ is the correlative function of the ith constituent. $R_i(\overline{r})$ stands for the probability of source point $\overline{x}'$ and the field point $\overline{x}$ being in a particle of the ith constituent. For the medium in which the particle has ergodic orientation, the correlative region is a sphere whose radius is the particle length. If we know the distribution of the particle length for the i th constitutent, $R_i(\overline{r})$ can be determined by [5, 6]

$$R_i(\overline{r}) = \int_r^{\infty} P_i(l)\, dl \tag{33}$$

where $P_i(l)$ is the distribution function of the particle length of the i th constituent.

If all of the particle sizes for the i th constituent are the same, $P_i(l)$ is a Dirac delta function.

$$P_i(l) = \delta(l - l_0) \tag{34}$$

where l_0 is the particle length of the i th constituent. $R_i(\overline{r})$ equals

$$R_i(\overline{r}) = \begin{cases} 1\,, & r < l_0 \\ 0\,, & r > l_0 \end{cases} \tag{35}$$

If the particle length of the i th constituent is a Gaussian distribution,

$$P_i(l) = \frac{l}{2\sigma_i^2} \exp\left(-l/\sigma^2\right) \tag{36}$$

where σ_i stands for the correlation distance.

$$\sigma_i^2 = 2l_i^2/\pi$$

l_i is the average length of the i th constituent particles. Substituting (34) into (32)

$$R_i(\overline{r}) = \exp\left(-r^2/\sigma^2\right) \tag{37}$$

In practice, we usually use (37) to describe the correlative property of the constituent with different particle length.

After substituting (32) into (2), we can write the effective constitutive tensor of granular media in the following form,

$$\left\|\overline{\overline{C}}_{\text{eff}}\right\| = \left\|\overline{\overline{C}}_g\right\| + \sum_{i=1}^{N} v_i \cdot \langle\|\overline{\overline{r}}_i\|\rangle \cdot \left\|\overline{\overline{D}}_i\right\| \cdot \langle\|\overline{\overline{r}}_i\|\rangle \tag{38}$$

where $\left\|\overline{\overline{D}}_i\right\|$ represents the integral of dyadic Green's function and the correlative function.

$$\left\|\overline{\overline{D}}_i\right\| = j\omega \int_v P.V. \left\|\overline{\overline{G}}_g(\overline{x}, \overline{x}')\right\| R_i(\overline{r})\, d\overline{r} \tag{39}$$

Equation (38) shows that $\left\|\overline{\overline{C}}_{\text{eff}}\right\|$ consists of two terms. The first one is a quasistatic component, independent of the frequency of the incident wave. This first term is only affected by the composition ratio of granular media and particle shapes. The second term, however, is closely related to the frequency of the electromagnetic wave. It is not only affected by the composition ratio and particle shapes, but also by the particle length and the boundary condition of the granular media, because the derivation of $\left\|\overline{\overline{G}}_g(\overline{x}, \overline{x}')\right\|$ is based on the boundary condition. If the volume of the granular media is large enough, we can employ the dyadic Green's function of infinite medium to calculate $\left\|\overline{\overline{D}}_i\right\|$. By substituting (12) into (39) and under the Gauss correlation,

$$\left\|\overline{\overline{D}}_i\right\| = D_i \begin{bmatrix} \mu_g \overline{\overline{I}} & 0 \\ 0 & \varepsilon_g \overline{\overline{I}} \end{bmatrix} \tag{40}$$

where

$$D_i \approx 4\pi^2 \left[\frac{4}{3} \left(\frac{l_i}{\lambda_g} \right)^2 + j\pi^2 \left(\frac{l_i}{\lambda_g} \right)^3 \right] \tag{41}$$

Here λ_g is the wavelength of the electromagnetic wave in the medium whose permittivity and permeability are ε_g and μ_g, respectively. Substituting (29) and (40) into (38), the formulas of effective permittivity and permeability for granular media are given by

$$\varepsilon_{\text{eff}} = \varepsilon_g \left\{ 1 - \frac{1}{9} \sum_{i=1}^{N} v_i\, D_i \left[\sum_{j=1}^{3} \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + L_{ij}(\varepsilon_i - \varepsilon_g)} \right]^2 \right\} \tag{42a}$$

$$\mu_{\text{eff}} = \mu_g \left\{ 1 - \frac{1}{9} \sum_{i=1}^{N} v_i\, D_i \left[\sum_{j=1}^{3} \frac{\mu_i - \mu_g}{\mu_g + L_{ij}(\mu_i - \mu_g)} \right]^2 \right\} \tag{42b}$$

Because the real and imaginary parts of D_i are directly proportional to $(l_i/\lambda_g)^2$ and $(l_i/\lambda_g)^3$, respectively, the influence of the correlative term to the real and imaginary parts of the effective permittivity and permeability is also directly proportional to $(l_i/\lambda_g)^2$ and $(l_i/\lambda_g)^3$. When the average particle length is much smaller than the wavelength, the influence of the correlative term on the effective medium parameter of granular media can be neglected completely. This result is consistent with the conclusion of the general random media.

4. The Application of Effective Medium Theory in the Absorbing Materials of Coating Type

Coating type absorbing material is a kind of material used in stealth widely. In general, such material is a mixture of glutinous and absorbent particles. Therefore, we can apply the theory to estimate the permittivity and permeability of these materials. In most cases, the particle size in the material is only a few microns and much smaller than the wavelength. Therefore, we can directly utilize (31) to calculate the permittivity and permeability of absorbed materials. In doing so, we discover the optimum shape of absorbent particles in coating type material. The formulas of effective permittivity and permeability for ferrite absorbed material with multi-shapes are determined.

It is important to reduce the quantity of absorbing material in the practice of stealth. Therefore, the thickness of the absorbing coat must be as thin as possible. To reduce the thickness, a higher permittivity and permeability, as well as electric and magnetic loss, must be used. This can be obtained, however, by increasing the content of absorbent. Yet, the absorbing material must also satisfy the requirement of gravity, strength, and other physical parameters besides absorption. Therefore, there are some limits to the increase of absorbent. From (31), we see that the permittivity and permeability are affected by not only the content of the absorbent, but also by the particle shape of the absorbent. We can select the absorbent's particle shape to obtain the highest permittivity and permeability of the absorbing material of the same absorbent content.

If the absorbing material only contains one kind of absorbent with the same particle shape, (31) reduces to the form of

$$9v_b \frac{\varepsilon_b - \varepsilon_g}{2\varepsilon_g - \varepsilon_b} - v_i \sum_{j=1}^{3} \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g - L_j(\varepsilon_i - \varepsilon_g)} = 0 \tag{43a}$$

$$9v_b \frac{\mu_b - \mu_g}{2\mu_g - \mu_b} - v_i \sum_{j=1}^{3} \frac{\mu_i - \mu_g}{\mu_g - +L_j(\mu_i - \mu_g)} = 0 \tag{43b}$$

where the subscripts b and i stand for the glutinous substance and the absorbent, respectively. The polarization elements satisfy the relation

$$\sum_{j=1}^{3} L_j = 1 \tag{44}$$

Usually, a particle of the absorbent can be approximately regarded as a spinning ellipsoid. In this condition, there are two equal polarization elements among these elements. Substituting (44) into (43) and by evaluating the derivative of L_i in (43), we obtain extreme values of ε_g and μ_g. In the limited range of the polarization elements, the effective permittivity ε_g, and permeability μ_g will take the extreme values as the absorbent particles become shaped as spheres and disks. When the permittivity and permeability of the absorbent is higher than that of the glutinous substance, spherical particles of the absorbent will make ε_g and μ_g take the minimum values and disk-shaped absorbent will make ε_g and μ_g take the maximum values under the same absorbent content condition. Conversely, ε_s and μ_g will be at a maximum when the absorbent particle is spherical and at a minimum when the absorbent particle is disk-shaped. In practice, the permittivity and permeability are by far higher than those of the glutinous substance. Therefore, the disk is the optimum shape for the absorbent particle.

Figure 1 shows the effective permittivity and loss angle tangent curves for absorbing materials made of graphite powder, ethyne carbon black, and graphite helminth [16], respectively. The basic content of these absorbents is carbon. Observation with an electron microscope shows that the particles of graphite helminth are mainly disks. The ethyne carbon black particles are nearly needles, and most particles of graphite powder are spherical [16]. In Fig. 1, both permittivity and

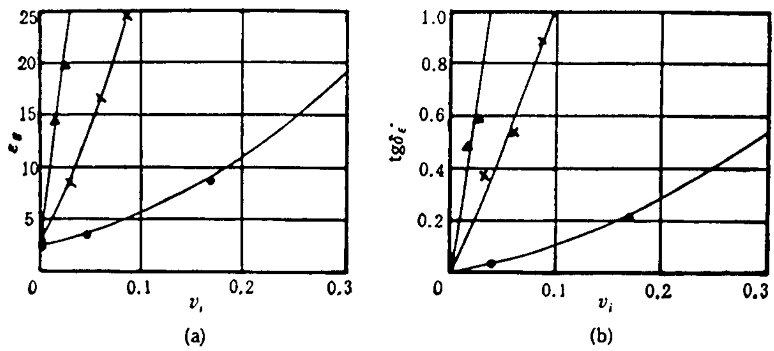

Figure 1. The permittivity and loss angle tangent curves of absorbed material. (• graphite powder; × ethyne carbon black; △ graphite helminth.)

loss angle tangent are highest when the particle shape of the absorbent have a disk form under the same v_i.

Although the conclusion of the optimum shape is obtained from medium absorbent, it can also be applied to metallic absorbent. Figure 2 shows the test result of absorbing material made of aluminum powder [16]. Here, the scale form powder is a typical disk, and granular form powder is close to spherical [16]. The test results of permittivity and loss angle tangent show the difference between the absorbing materials made up of scale form and granular form aluminum powder. Scale form powder also has the highest permittivity and loss angle tangent for the same absorbent content.

Ferrite absorbing material is an important material which has been widely used recently. Due to the limits of manufacturing technology, it is difficult to make a ferrite absorbent which has uniformly shaped particles. Figure 3 is a microscope photograph of the ferrite absorbent powder. From the picture, we will find that the particle shape is quite irregular. If we use (31) directly to calculate the effective permittivity and permeability of the ferrite absorbing material, the ferrite absorbent must be classified into many types, dependent on the particle shape and medium property. Even though there is only one sort of ferrite absorbent in absorbing material, the constituent of the media must be of many types, according to their shape. In order to use (31), we must determine the content of every kind. This is a very difficult task. Up until now, there has been no method for us to do this work.

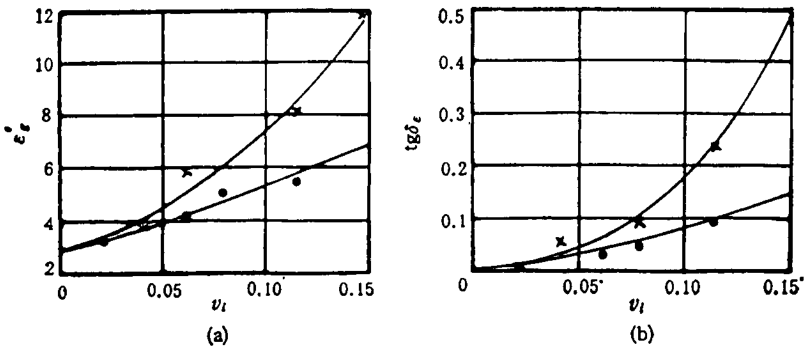

Figure 2. The permittivity and loss angle tangent curves of the absorbed material made up of aluminum powder. (× scale form powder; • granular form powder.

Therefore, we must look for new ways to solve the problem. Here, we use the mean value theorem to deal with the calculation of ferrite absorbing material of multi-shape absorbent.

Suppose there are various particle shapes of the inclusion particle with the same permittivity or permeability. The sum for i can be divided into two parts. The first sum is carried out over particles of the same permittivity or permeability with different shapes. The second sum is over the medium property (permittivity or permeability) of the particles. If the particle shape is so varied that the variety of the shape is considered as that of continuous change, the first sum is replaced by an integral. Equation (31) becomes

$$3v_b \frac{\varepsilon_b - \varepsilon_g}{2\varepsilon_g + \varepsilon_b} + \sum_{i=1}^{N_1} v_i \int_0 \frac{P_i(L)(\varepsilon_i - \varepsilon_g)}{\varepsilon_g + L(\varepsilon_i - \varepsilon_g)} \, dL = 0 \tag{45a}$$

$$3v_b \frac{\mu_b - \mu_g}{2\mu_g + \mu_b} + \sum_{j=1}^{N_2} v_i \int_0 \frac{Q_i(L)(\mu_i - \mu_g)}{\mu_g + L(\mu_i - \mu_g)} \, dL = 0 \tag{45b}$$

Here N_1 and N_2 stand for the number of permittivity and permeability, respectively. $P_i(L)$ and $Q_i(L)$ are the distribution functions

Figure 3. **The microscope photograph of ferrite absorbent** ($\times 3000$).

of the particle shape whose permittivity and permeability are ε_i and μ_i, respectively, and

$$\int_0^1 P_i(L)\, dL = 1 \tag{46a}$$

$$\int_0^1 Q_i(L)\, dL = 1 \tag{46b}$$

According to the mean value theorem, (45) can be written in the form

$$3v_b \frac{\varepsilon_b = \varepsilon_g}{2\varepsilon_g + \varepsilon + b} + \sum_{i=1}^{N_1} v_i \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + \overline{L}_{i_1} (\varepsilon_i - \varepsilon_g)} = 0 \tag{47a}$$

$$3v_b \frac{\mu_b - \mu_g}{2\mu_g + \mu_b} + \sum_{i=1}^{N_2} v_i \frac{\mu_i - \mu_g}{\mu_g + \overline{L}_{i_2} (\mu_i - \mu_g)} = 0 \tag{47b}$$

where the integral relation of $P_i(L)$ and $Q_i(L)$ in (46) is used. $\overline{L}_{i_1}$ and $\overline{L}_{i_2}$ are the middle values of the polarization elements for permit-

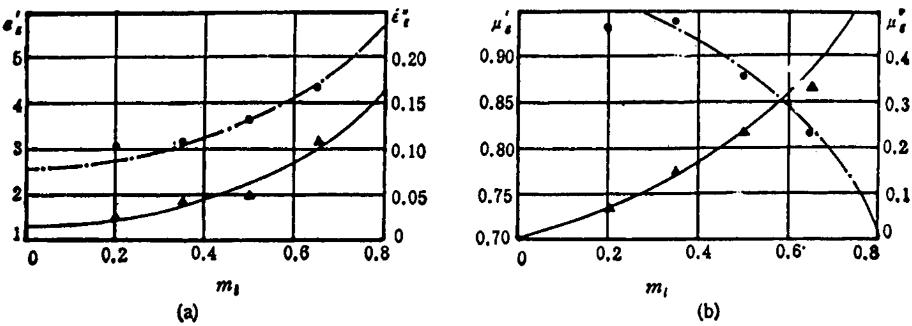

Figure 4. The permittivity and permeability of the ferrite absorbed material A18-1. —·—· : the calculation value of ε_g' and μ_g', ——— : the calculation value of ε_g'' and μ_g'', ● ● ● : the test value of ε_g' and μ_g', △ △ △ : the test value of ε_g'' and μ_g''.

tivity and permeability, respectively. $0 \leq \overline{L}_{i1} \leq 1$; $0 \leq \overline{L}_{i2} \leq 1$. In practice, they can be determined by the measurement of the effective permittivity or permeability with a different content of the absorbent.

Now we consider a simple case, where there is only one kind of absorbing material. Suppose the particle shape of the ferrite absorbent is quite irregular. Because the particle shape of the glutinous substance is spherical, (47) becomes

$$3v_b \frac{\varepsilon_b - \varepsilon_g}{2\varepsilon_g + \varepsilon_b} + v_i \frac{\varepsilon_i - \varepsilon_g}{\varepsilon_g + \overline{L}_{i_1}(\varepsilon_i - \varepsilon_g)} = 0 \tag{48a}$$

$$3v_b \frac{\mu_b - \mu_g}{2\mu_g + \mu_b} + v_i \frac{\mu_i - \mu_g}{\mu_g + \overline{L}_{i_2}(\mu_i - \mu_g)} = 0 \tag{48b}$$

where the subscripts b and i stand for the glue and the absorbent, respectively. The experimental results show that the middle values of the polarization elements in (48) are equal, $\overline{L}_{i_1} = \overline{L}_{i_2} = 0.08$. Figure 4 shows the effective permittivity and permeability of the ferrite absorbed material, with the curves calculated by (48), and the points are the test values. Figure 4 shows very good agreement between the test and the calculation. In Fig. 4, the abscissa indicates the weight ratio of the absorbent. The relation between weight and volume ratio is

$$m_i = v_i \rho_i / \rho \tag{49}$$

where ρ_i and ρ are the gravity of the absorbent and the mixture, respectively.

5. Conclusions

In this paper, the calculation method of effective permittivity and permeability for granular media has been studied with the strong fluctuation theory. The calculation for the absorbing material is based on the long wavelength approximation. If the particle size is long enough to compare with the wavelength, the influence of the correlative term must be considered. In practice of ferrite absorbing material, this phenomenon only occurs at optical frequency. Therefore, the formulas can be used at wide microwave frequency range.

Acknowledgments

The author wishes to thank Zhou Lishan, Xiong Kemin, and his colleagues in the 621 Institute of Aviation and Space Industry Department for their help in making the absorbed material. This work was supported by the National Natural Science Foundation of China.

References

[1] Van Beek, L. K. H., "Dielectric behaviour of heterogeneous systems," *Prog. Diele.*, Heywood, London, No. 7, 69–114, 1969.
[2] Varadan, V., "Design of ferrite-impregnated plastics (PVC) as microwave absorbers," *Proc. IEEE,* Vol. MTT-34, No. 2, 251–258, 1986.
[3] Baofu, J., "The electromagnetic wave scattering by strong fluctuations random media," *ISRAMT 1989,* Beijing, 157–160, 1989.
[4] Baofu, J., "Effective dielectric constant and permeability of strong fluctuation in random media," *Chinese Journal of Radio Science,* Vol. 5, No. 2, 28–34, June 1990.

[5] Baofu, J., *Strong Fluctuation Theory and Applied in Absorbed Material for Stealth,* Univ. Electronic Science and Technology of China, Doctoral Thesis, 1989.

[6] Baofu, J., "Strong fluctuation theory and development," *Journal of UEST of China,* Vol. 20, No. 5, 473–480, Oct. 1990.

[7] Weiglhofer, W. S., "Dyadic Green's functions for general uniaxial media," *IEE Proc.,* Vol. 137, Pt. H, No. 1, 5–10, Feb. 1990.

[8] Weiglhofer, W., "Reduction of dyadic Green's functions to scalar Hertz potentials for gyrotropic media," *Radio Science,* Vol. 22, No. 2, 209–215, 1987.

[9] Chew, W. C., *Waves and Fields in Inhomogeneous Media,* New York, VNR, Chapter 7, 1990.

[10] Tai, C. T., *Dyadic Green's Function in Electromagnetic Theory,* Scranton, PA, Intext Educational Publishers, 1972.

[11] Chen, K. M., "Interaction of electromagnetic field with biological bodies," in *Research Topics in Electromagnetic Wave Theory,* J. A. Kong, ed., 291–346, 1981.

[12] Tsang, L., and J. A. Kong, "Scattering of electromagnetic waves from random media with strong permittivity fluctuation," *Radio Science,* Vol. 16, No. 3, 303–320, 1981.

[13] Wang, L. X., *The Handbook of Mathematics,* People's Education Publishing House, Beijing, 330, 1979.

[14] Polder, D., and J. H. Van Santern, "Effective permeability of mixtures of solids," *Physica,* Vol. 12, 257–271, 1946.

[15] Jin, H., and W. Lin, "Mixture granular media electromagnetic wave theory," *Journal of China Institute of Communication,* Vol. 11, No. 3, 1–10, May 1990.

[16] Wang, J. H., *The Influence of Absorbed Material Performance by Electromagnetic Parameter of the Substance,* Dept. Aviation and Space Industry, 703 Insti., Master's Thesis, 1988.