Progress In Electromagnetics Research, PIER 8, 161–171, 1994

AN AUTOMATED OPEN RESONATOR TECHNIQUE FOR MEASUREMENT OF EXTINCTION CROSS-SECTION OF SINGLE FALLING WATER DROPS OVER X-BAND

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1. Introduction

In the microwave and millimeter-wave frequency bands, interaction of a radio-wave with hydrometeors causes attenuation as well as scattering and depolarization of signals. In a detailed theoretical study of these effects, interaction of the radio-wave with a single particle of hydrometeor plays an important role [1]. However, very few attempts appear to have been made for the measurement of scattering properties of single falling water drops. This seems to be partly due to the inconvenience in handling water drops. With regards to single hailstones, detailed measurements have been reported in the literature [2, 3].

Extinction or the total cross-section of an individual water drop is a measure of its overall absorption and scattering characteristics. Total cross-section of water drops, calculated from the forward-scattering amplitudes of falling water drops measured at 60 GHz, was reported in [4]. More recently, measurements of total cross-sections of falling water drops were conducted at 100 GHz, using an open resonator [5]. Measurements of complex forward-scattering amplitudes with polarization effects taken into account have been described in [6]; in that study, the water drops were supported in a wind tunnel and measurements were carried out at 11 GHz. The open resonator method employed in [5] is direct in concept and, in essence, requires only the Q-factor measurement of an empty and 'loaded' resonator [7]; however, the results reported in [5] are at a single frequency, though water drops of different sizes are considered. Presented here is an open resonator system capable of yielding, simply and much less laboriously, extinction cross-section values for falling water drops of a given size over an entire microwave frequency band (X-band). A novel method has been adopted to estimate Q-factor of the resonator in the presence of a falling water drop.

Automation of open resonator measurements over X-band has been reported recently in [9] but the scattering objects considered so far have been stationary [3]. Further, in [9], Q-factors have been estimated in terms of the resonator output. Here, for a particular Q measurement, the full resonance curve is made use of and an absolute measurement of Q is carried out utilizing the peak and the 3 dB down frequencies.

2. Open Resonator Technique for Measurement of Extinction Cross-Section

Use of an open resonator for the measurement of extinction cross-section is well established. Proposed by Cullen [8], the technique has been well illustrated in, for example, [5, 7, 9]. Briefly, resonant frequencies for fundamental and higher order modes supported on a symmetric concave-concave open resonator are given by [10]:

$$f = (c/2d)[(q+1) + (1/\pi)(2p+l+1)\cos^{-1}(1-d/R)] \qquad (1)$$

where the symbols carry their usual meaning. The adjacent fundamental modes, $\mathrm{TEM}_{0,0,q}$, are separated by a frequency $c/2d$. The higher order modes, $\mathrm{TEM}_{p,l,q}$, are usually only weakly excited by a suitable choice of d/R and coupling aperture [7]. The extinction cross-section

(σ_{ext}) of an object placed axially in the open resonator is given by

$$\sigma_{\text{ext}} = \frac{\pi^2 w_0^2 d}{2\lambda}\left[\left(\frac{1}{Q_1} - \frac{1}{Q_0}\right) + \left(\frac{1}{Q_2} - \frac{1}{Q_0}\right)\right] \tag{2}$$

where w_0 is the beam waist radius, Q_0 is the quality factor of the empty resonator, Q_1 and Q_2 are the quality factors of the loaded resonator for axial positions of the scatterer separated by $\lambda/4$ at the frequency of interest. As is also pointed out in [11, 9], it is not necessary to physically shift the scatterer by $\lambda/4$. This makes it possible to gather complete data for σ_{ext} in a single setting of the open resonator - a feature necessary for automating the measurements. The method used here is slightly different from that described in [9] for estimating Q_2 in (2). If Q_0, Q_1, and Q_2 in (2) are required to correspond to the q^{th} dominant mode, Q_2 is taken to be the average of the loaded Q's for the $(q-1)$ and $(q+1)$th modes to calculate $\sigma_{\text{ext},q}$ for the q^{th} mode; i.e.,

$$\sigma_{\text{ext,q}} = \frac{\pi^2 w_0^2 d}{2\lambda}\left[\left(\frac{1}{Q'_q} - \frac{1}{Q_q}\right) + \left(\frac{2}{Q'_{q-1} + Q'_{q+1}} - \frac{1}{Q_{\ q}}\right)\right] \tag{3}$$

where the subscripts $q-1, q, q+1$ refer to the order of the dominant mode; unprimed Q-factors correspond to the empty resonator and primed Q-factors correspond to the loaded resonator.

3. System Used for a Measurement

An open resonator routinely yields high Q-factors ($\approx 10^5$). This implies that, even in working with stationary objects, automation of data gathering and data processing is desirable. To carry out wideband measurements on moving objects, e.g. falling water drops, automation is indispensible.

A schematic block diagram of the measurement system is shown in Fig. 1. Various components of the system are described below.

Open Resonator:

Mechanical characteristics of the open resonator used are as follows:

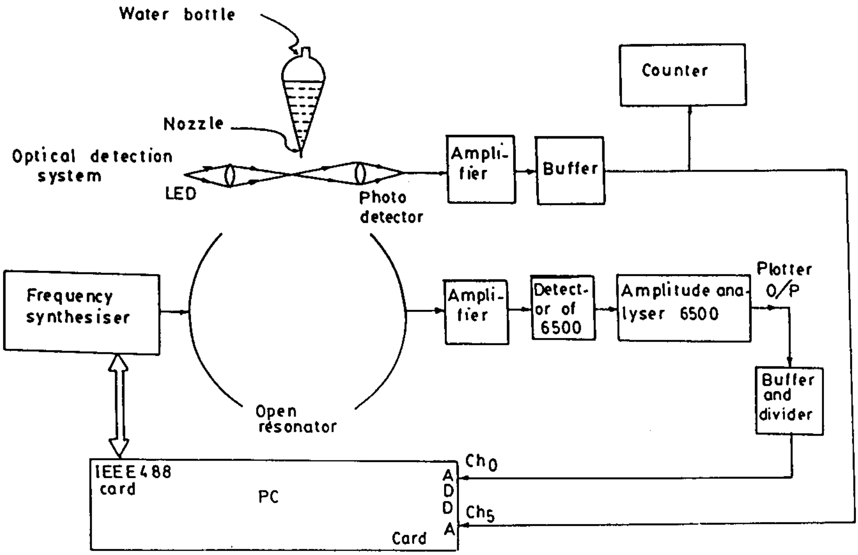

Figure 1. Schematic block diagram of the system configuration using a PC.

Radius of curvature of the reflectors, R	100 cm
Diameter of the reflectors, D	35 cm
Diameter of coupling holes	0.8 cm
Separation between the reflectors, d	54.37 cm

Source and Detector:

The resonator is excited by an HP frequency synthesizer controlled through an IEEE 488 bus.* The response of the resonator is obtained by measuring the output power using a Marconi amplitude analyser and its detector. The detector is preceded by a low-noise amplifier. The amplitude analyzer gives an 'on-line' output proportional to the y-axis display on the analyzer screen; this output is continuously sampled by the system controller.

* The synthesizer can operate from 2 to 26 GHz, with a switching time of 20 ms between different frequencies.

IEEE 488 System Controller:

An IBM compatible PC including an IEEE 488 card and an A/D and D/A card serves the purpose of controlling the instruments, acquiring data, and processing the acquired data.

Water Drops:

A bottle fitted with a nozzle of appropriate opening is used to make the water drops fall vertically through the resonator along its axis of symmetry. The rate at which water drops are produced is kept low enough to ensure that there is not more than one drop at any given time in the resonator volume. This necessarily slows down the measurements. Before any data for the loaded resonator is acquired, the presence of a water drop in the resonator is ensured through an optical detection system which is aligned such that a falling water drop would pass through the optical beam. The output of the optical detection system is also connected to a counter; this permits a simple estimation of the average size of the water drops by counting the total number of drops, weighing corresponding water collected, and assuming the drop shape to be spherical [5].

4. Measurement Algorithm

For the determination of the total cross-section of a falling water drop, it is required to measure the Q-factor of the open resonator when it is empty and when the falling drop is near the center of the resonator. This task is divided into two main parts – data acquisition and data processing. In the first part, the empty and loaded resonator frequency responses are obtained around a resonant frequency and are stored. In the second part, the data so acquired is processed to find the resonant frequencies and Q-factors. This process is repeated at different resonant frequencies covering the frequency range of interest, which in this case is X-band. Finally, the total cross-section is calculated at different frequencies, using (3). The two main parts of the algorithm are described below in more detail.

(i) *Data acquisition:*

To obtain the frequency response of the empty and loaded resonator, 200 readings are taken at a frequency interval of 6 kHz, around an estimated resonant frequency. A typical plot of the resonator output at a particular frequency as a water drop falls through the resonator is shown in Fig. 2. At its lowest value, the output is relatively flat for some time, corresponding to the passage of the water drop through the maximum field strength of the Gaussian beam profile. A reading corresponding to this minimum value gives the response of the system when the water drop is at the resonator axis of symmetry. This procedure gives one of the 200 readings taken for obtaining the frequency response of the loaded system. Typical data acquisition time for these 200 readings is 3–4 minutes.

Readings for the empty system do not involve such a complicated procudure and are taken in a straightforward manner. A frequency interval of 6 kHz between adjacent readings represents a tradeoff between the time taken and the accuracy realized. Typical overall empty and loaded system frequency responses so obtained are displayed in Fig. 3.

(ii) *Data processing*:

As is evident from Fig. 3, the frequency response of the loaded system is noisy, perhaps because of minor size-variations from drop-to-drop and mechanical vibrations. For the purpose of estimating the resonant frequency and Q-factor, a smooth curve is 'fitted' to the noisy one, minimizing the mean squared error. An 8th order curve fitted to a noisy one is included in Fig. 3. Curve fitting can be restricted to the region of interest, i.e., peak and the 3dB down points, to improve the fit of the curve. The frequency response for the empty system is not so noisy and is used directly to extract the information of interest.

5. Results

The algorithm described in the previous section has been used for determining the total cross-section of falling water drops for vertical polarization. The mean radii of the drops are 0.308 cm, 0.256 cm, 0.196 cm, and 0.165 cm. The corresponding measured normalized total cross-sections of the drops $\sigma_{\text{ext}}/2\pi a^2$, are plotted in Fig. 4. Table I also lists the same quantity, together with frequencies and Q-factors, for a drop-size with mean radius $a = 0.256$ cm.

The fall velocity of the water drops in the present case is about

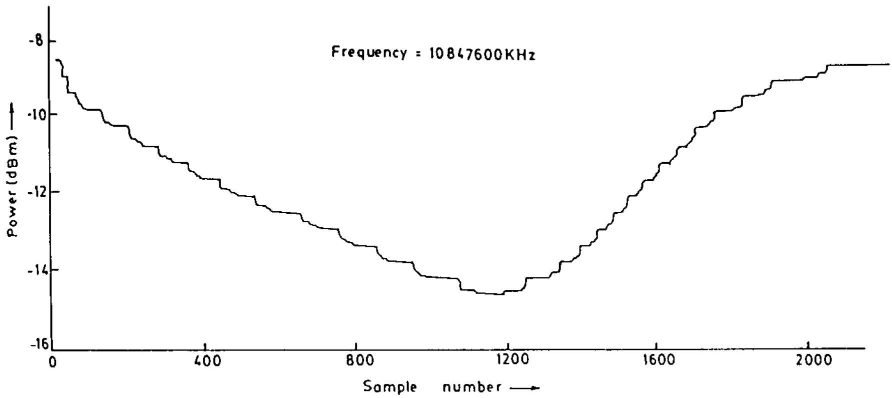

Figure 2. Resonator output at a particular frequency as a water drop falls through the system.

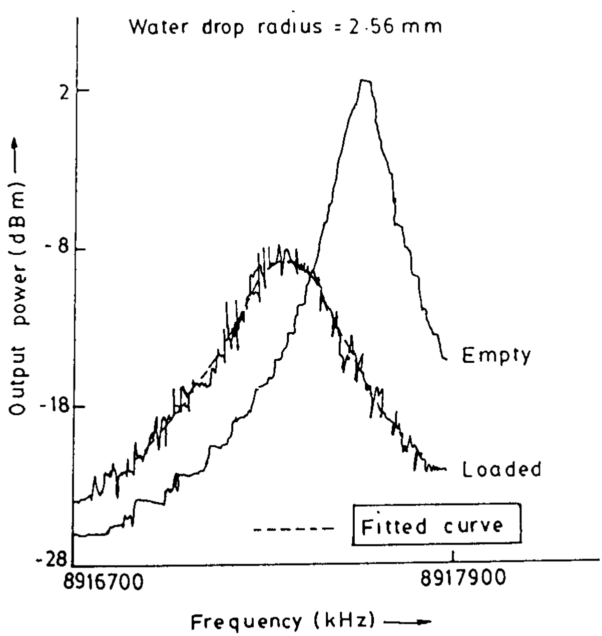

Figure 3. Empty and loaded system frequency response. Also shown is an 8th order curve fitted to the loaded system frequency response.

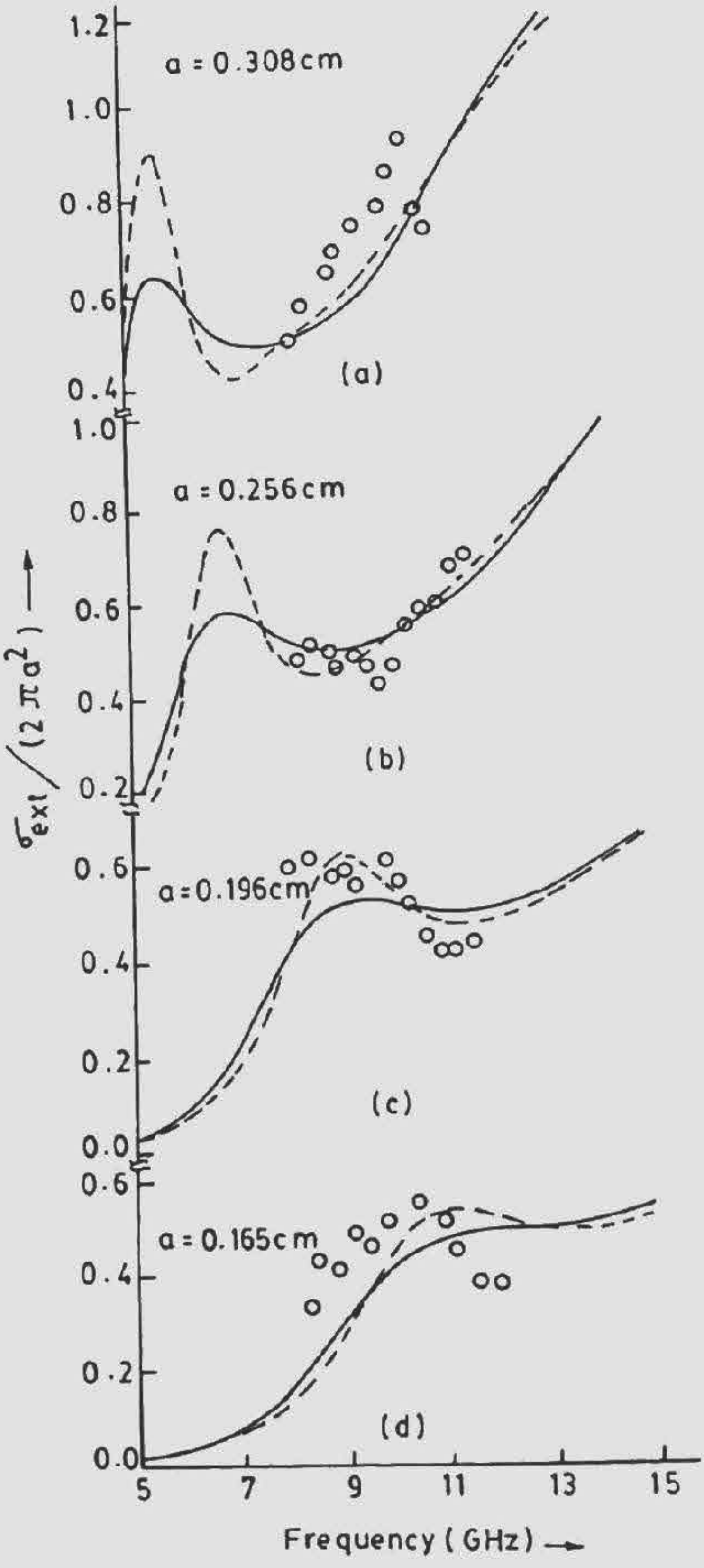

Figure 4. Normalized total cross-section of falling water drops as a function of frequency. (a) mean radius a = 0.308 cm, temperature = 31°C; (b) a = 0.256 cm, temperature = 30°C; (c) a = 0.196 cm, temperature = 29.5°C; (d) a = 0.165 cm, temperature = 28°C. – – – – theoretical, for a fixed drop radius. ____ theoretical, for drop radius varying in a Gaussian manner with a standard deviation of 0.1, oooo measured.

Resonant Frequencies, Q-factors, and normalized total Cross Section for a Water Drop of Radious a = 0.256 cm.

EMPTY RESONATOR Resonant Frequency (KHz)	EMPTY RESONATOR Q-Factor Q_0	LOADED RESONATOR Resonant Frequency (KHz)	LOADED RESONATOR Q-Factor Q	Normalised Total Cross-section $\sigma_{ext}/(2\pi a^2)$
8090428	42137.7	8090362	18727.7	-
8366172	34008.8	8366094	19366.0	0.49
8641876	60013.0	8641822	20575.8	0.52
8917642	67557.9	8917401	23971.5	0.50
9193262	35632.8	9193250	19643.7	0.47
9468998	65756.9	9468680	22544.5	0.49
9744506	77337.4	9744602	26195.2	0.47
10020190	46389.8	10019812	22266.3	0.44
10295962	53624.8	10296058	23833.5	0.47
10571678	73414.4	10571558	22024.1	0.56
10847412	66959.3	10847232	20313.2	0.60
11123118	56177.4	11122794	22334.4	0.60
11398830	67850.2	11398764	17922.6	0.69
11674562	62766.5	11674052	17372.1	0.71
11950252	60354.8	11950354	22130.3	-

Table I.

$3ms^{-1}$, which is quite less than the terminal velocity. Therefore one expects little deformation of the drop-shape from spherical [5]. Thus, for comparison with theory, the Mie solution for a spherical shape is used; the computer program given in [12] for homogeneous spheres is utilized for this purpose. The small random drop-to-drop size variation which may take place during the experiment is accounted for in the calculation by assuming that the drop radius varies in a Gaussian manner around the mean radius with a standard deviation of 0.1 times the mean radius. Further, the complex refractive index of water at X-band, required for theoretical estimates of the total cross-section, is calculated using the empirical model proposed in [13]; the effect of ambient temperature on the total cross-section is automatically incorporated in the estimate since the refractive index is calculated at the appropriate temperature. The agreement between the theoretical estimate and the measured values of the extinction cross-section is good for moderate drop sizes, and otherwise is quite satisfactory.

6. Conclusion

A microwave open resonator system controlled by a personal computer has been configured and used for measuring the total scattering cross-section of single falling water drops over X-band (8–12 GHz). The system has worked satisfactorily and the measured results compare well with the theoretical estimates. The technique described makes it possible to estimate over a wide band, the extinction cross-section of water drops in flight.

Acknowledgment

The work reported here was undertaken as part of a grants-in-aid project entitled "Microwave open Resonators" and funded by Defence Research & Development Organization. Also gratefully acknowledged are the kind encouragement received from Professor Rajendra K. Arora, helpful discussions with Dr. R. K. Patney, assistance in theoretical calculations from Capt. Rajendra Singh, and excellent technical support for measurements from Mr. Sunil Khanna. Finally, the authors express their deep appreciation of the pertinent and constructive comments of the reviewers.

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